EE 25088 Convex Optimization II

Problem Set I

Due Data: Farvardin 16, 1401

1) A bipartite graph $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ is a graph whose vertices can be divided into two disjoint sets $\mathcal{U}$ and $\mathcal{V}$ such that each edge $(u_i, v_j) \in \mathcal{E}$ connects a vertex $u_i \in \mathcal{U}$ and one $v_j \in \mathcal{V}$. If each edge in graph G has an associated weight w_{ij}, the graph $\mathcal{G}$ is called a weighted bipartite graph. In a bipartite graph $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$, a matching M of graph $\mathcal{G}$ is a subset of $\mathcal{E}$ such that no two edges in M share a common vertex. If the graph $\mathcal{G}$ is a weighted bipartite graph, the minimum weighted bipartite matching (MWBM) is a matching whose sum of the weights of the edges is minimum.
   a) Formulate the MWBM problem as an integer programming.
   b) Formulate the *relaxation* of the above problem as a linear programming (LP).

2) Let $P = \{x \in \mathbb{R}^n : Ax \leq b, A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m\}$ be a bounded, non-empty set. Formulate a linear program that computes the radius of the largest ball that can be inscribed into P. Feel free to define as many variables as necessary.

3) Assume that you are a production manager. There are n production tasks to be executed, and task j requires p_j working hours to be completed. You have m employees that can each work on a subset of the tasks. Denote by S_i the set of jobs that employee i can work on. A work allocation plan has to ensure that all tasks are completed. You want to create an allocation which is also fair, i.e., the maximum number of working hours assigned to an employee is to be minimized. Formulate this problem as a linear programming problem.

4) A set of n jobs must be carried out on a single machine that can only do one job at a time. Each job j takes p_j hours to complete. Given job weights w_j for $j = 1, \cdots, n$, we seek to find in what order should the jobs be carried out in order to minimize the weighted sum of their completion times. Formulate this problem as a mixed integer linear programming (MINLP) problem. Feel free to define any as many variables as necessary.

5) A factory produces two different products. To create one unit of product 1, it needs one unit of raw material A and one unit of raw material B. To create one unit of product 2, it needs one unit of raw material B and two units of raw material C. Raw material B needs preprocessing before it can be used, which takes one minute per unit. At most 20 hours of time is available per day for the preprocessing. Raw materials of capacity at most 1200 can be delivered to the factory per day. One unit of raw material A, B and C has size 4, 3 and 2, respectively. At most 130 units of the first and 100 units of the second product can be sold per day. The first product sells for 6 dollars per unit and the second one for 9 dollars per unit. Formulate the problem of maximizing turnover as a linear program in TWO variables

6) Consider the problem

$$\begin{aligned} \text{minimize} \quad & 2x + 3|y - 10| \\ \text{subject to} \quad & |x + 2| + y \leq 5 \end{aligned}$$

and reformulate it as a LP problem

7) Consider the following network flow shown in Fig. 1. The values assigned to each edge of graph corresponds to the maximum flow allowed to be transferred on that edge. Write the problem of finding maximum flow from s to t as a linear program. What is the optimum value of the objective function?

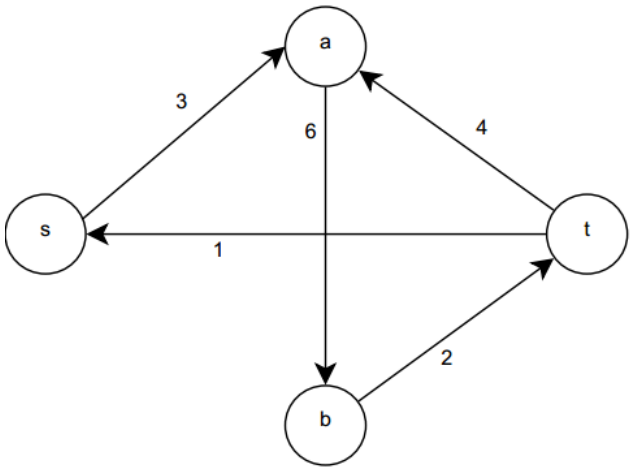

Fig. 1: Flow graph of Problem 6 .

8) Consider the following IP problem.

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} i x_i \\ \text{subject to} \quad & \sum_{i=1}^{n} x_i = K \\ & x_i \in \{0, 1, 2\}, i = 1, \cdots, n \end{aligned}$$

a) Solve the problem using Matlab and determine its optimal value as well as the time Matlab takes to find the solution for $(n, K) = (200, 15)$ and $(n, K) = (2000, 150)$.

b) Relax the integer variables and solve the corresponding LP problem. Compare the optimal values and the time with that of previous part.

Include the source code as well as the results when you submit the assignment.