Convex Optimization II

Lecture 0: Introduction and Motivation

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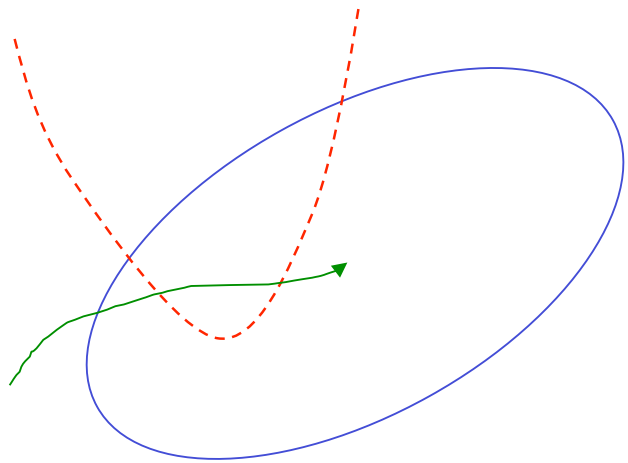
Mathematical Optimization

Optimizing an objective function subject to a set of constraint

$$\begin{aligned} &\underset{\mathbf{x}}{\text{minimize}}\ f_0(\mathbf{x}) \\ &\text{subject to } \mathbf{x} \in \mathcal{X}. \end{aligned}$$



- $\mathbf{x} = (x_1, \ldots, x_n)$: Optimization variables
- $f_0 : \mathbb{R}^n \to \mathbb{R}$: Objective function
- $\mathcal{X}$: Constraints set

Optimal solution $\boldsymbol{x}^*$ has smallest value of f_0 among all vectors that satisfy the constraints.

Solving Optimization Problems

General optimization problem

- very difficult to solve
- methods involve some compromise, e.g., very long computation time, or not always finding the solution

Exceptions: certain problem classes can be solved efficiently and reliably

- Linear problems
- Least-square problems
- Convex problems

APPLICATIONS

- Computer science
- Machine learning & Data science
- Data fitting
- Computer networks
- Communication systems and signal processing
- Control theory
- VLSI design (routing, placement)
- Power distribution and market
- Portfolio optimization

Course Goals

We learn

- How to formulate optimization problems.
- How to recognize the problem type and perform transformations.
- How to characterize the optimal solution.
- How to use optimization theory in our research areas and design optimal/suboptimal algorithms and protocols.

How do we find the solution? **We don't**.

- There are a variety of algorithms that solve some types of optimization problems.
- There are powerful tools (e.g., Matlab, CVX, ...) that have implemented the algorithms.

What This Course Is About

- Insight: Understanding the optimization theory and how we can formulate optimization problems.
- Techniques: Learning several useful techniques and tools to analyze and design efficient algorithms.
- Case Studies: Optimal design in different research topics.

What This Course Is NOT About

- Not a math course on optimization theory or stochastic optimization. We learn optimization and modeling techniques only to be able to use them as a tool (not many rigorous proofs).
- Not an electrical engineering course for a certain field. Several selected topics of different areas will be covered.

Why Take This Course

- Learn the tools and mentality of optimization (surprisingly useful for other study you may engage in later on)
- Learn classic and recent results on optimization covering a wide range of applications
- Train the ability to conduct research in academia or industry