Homework 2

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Prove that if the reliability formula is exponential, then the failure of the system is memoryless.

$$R(t) = e^{-\lambda t} \implies P(T > t) = P(T > t + s \mid T > s)$$

Proof:
To show that the system is memoryless, we need to prove the following expression:

$$P(T > t) = P(T > t + s \mid T > s)$$

First of all we expand the following expression:

$$P(T > t + s \mid T > s) = \frac{P(T > t+s \cap T > s)}{P(T > s)}$$

We know $T > t + s$ then $T > s$. Thus we can say $P(T > t + s \cap T > s) = P(T > t + s)$.

Therefore:

$$P(T > t + s \mid T > s) = \frac{P(T > t+s)}{P(T > s)}$$

On the one hand, we have the reliability formula, which is the assumption of the problem.

$$R(t) = e^{-\lambda t} \implies P(T > t) = e^{-\lambda t}$$

Thus:

$$P(T > s) = e^{-\lambda s} \quad \text{and} \quad P(T > t + s) = e^{-\lambda(t+s)} = e^{-\lambda t} \cdot e^{-\lambda s}$$

Now we put the expression $P(T > t)$ and $P(T > t + s \cap T > s) = P(T > t + s)$ in reliability formula:

$$P(T > t + s \mid T > s) = \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t}$$

$$P(T > t) = e^{-\lambda t}$$

Thus we prove:

$$P(T > t) = P(T > t + s \mid T > s)$$