## Introduction

Allen’s Algebra is a set of qualitative relations to deal with entities that have projection on time. By the virtue of a finite set of *jointly exhaustive and pairwise disjoint relationships*, called the base relations, Allen’s algebra facilitates Qualitative Spatial and Temporal reasoning (QSTR). Practical applications of this calculi can be found planning and scheduling, natural language processing, robotics, supply-chain, health care, and business workflows. In most of these applications, temporal data such as relations between different tasks and events are modeled with these relationships. However, tasks, process, and events have relations to many other concepts apart from their relationship with time. These concepts and relations need a rich ontology providing a more expressive model. Basic Formal Ontology (BFO) is one of such ontology. In the following sections, an alignment of Allen’s algebra to BFO is provided.

## BFO Temporal Region

D1: **temporal region**: A **temporal region** (TRegion) is an occurrent over which processes can unfold [100-BFO]

D2: **zero-dimensional temporal region**: A **zero‐dimensional temporal region** (TRegion0) is a temporal region that is a whole consisting of one or more separated temporal instants as parts.

D3: **one-dimensional temporal region**: A **one-dimensional temporal** (TRegion1) is a temporal region is a whole that has a temporal interval and zero or more temporal intervals and temporal instants as parts.

D4: **temporal interval**: a **temporal interval** (TInterval) is a **one‐dimensional temporal region** that is continuous, thus without gaps or breaks.

D5: **temporal instant**: a **temporal instant** (TInstant) is a **zero‐dimensional temporal region** that has no proper temporal part.

D6: **first instance of** (*firstInstanceOf*)

Def: temporal instant i first instant of temporal region t =Def. i precedes all temporal parts of t other than i [268-BFO].

D7: **last instance of** (*firstInstanceOf*)

Def: temporal instant i last instant of temporal region t ́ =Def. all temporal parts of t other than i precedes i [268-BFO].

D6: **precedes** (*precedes*)

Def: If o, o' are occurrents and t is the temporal extent of o and t' is the temporal extent of o' then o precedes o’ means:

either last instant of o is before first instant of o’ or

last instant of o = first instant of o’ and neither o nor o’ are temporal instants [270-BFO]

Note: Domain and range of *precedes* relationship are Occurrent.

Issues:

1. *precedes* is defined using *firstInstanceOf* and *lastInstanceOf* relationship which are themselves defined using *precedes* relationship.
2. Definition of *precedes* relationship uses *before* relationship which is not defined.

However, one important observation is that for one interval preceding another interval, the last temporal instant of the first interval is placed either before the first temporal instant of the second interval or they are the same temporal instant. In both cases in the following figure, interval *i* *precedes* *i’*.

t

t'

t'

t

(a)

(b)

## Allen’s Interval Algebra

Allen's interval algebra defines 13 relations, which represent all the possible relations between two intervals: 1) precedes/before, 2) meets, 3) overlaps, 4) finished by, 5) contains, 6) starts, 7) equals, 8) started by, 9) during, 10) finishes, 11) overlapped by, 12) met by, and 13) preceded by/after.

*Precedes* and *preceded* by relationships are also labeled as *before* and *after* in some literature. We will prefer before and after to avoid confusing them with bfo:precedes and bfo:preceded by.

Six pairs of the relations are inverses. For example, the inverse of "a *precedes* b" is "b *preceded* by a"; whenever the first relation is true, its inverse is true also. The thirteenth, "equals", is its own inverse. We will therefore use the

There are two ways to model the seven Allen relationships (inverses are not considered as they will be implied). The first model is based on point calculus and uses the start and end time of the intervals. The second model is point-free, that all other relationships are defined using *meets* relationship. We will present both models below.

**Point-based model:**

Three relationship before, after and equal

**Point-free model:**