

**Lecture 2: August 19**

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## **2.1 Administrative details**

### **2.1.1 Consultation hours**

1. Prof. Kuldeep: Tuesdays 3-4pm
2. Dr. Ler: Fridays 10-11am

### **2.1.2 Miscellaneous**

1. Submissions due on Week 3:
  - (a) Lecture notes scribing
  - (b) Assignment 1
2. Projects:
  - Project 1 description and grouping will be released next week.
  - There will be a peer review component to ensure fairness. However, teams should have the following to assist in the peer review process:
    - (a) Open communication channel (partners should respond to each other within 24 hours).
    - (b) List of tasks to be done by each person.

## **2.2 Recap of Week 1**

In week 1, we have covered:

1. Reflex agents
  - Passive.
  - Next state depends only on percept.
2. Model-based reflex agents
  - Passive.
  - Next state depends on the percept, and the model of the world.
3. Goal-based reflex agents
  - Active.

- Next state depends on the current state, the percept, and the model of the world.
  - Typically only have a single goal.
4. Utility-based agents
- Might have multiple conflicting goals, and the agent might not be able to achieve all of its goals.
  - Agent may assign a value (i.e. weight) to each goal, and attempt to balance them.

## 2.3 More on... agents

**Definition 2.1** (Autonomous agents).

*Autonomous agents are agents which can update its agent program based on its experiences.*

The goal of this module is to design *rational autonomous agents*.

### 2.3.1 How do we model a problem?

Abstraction: we can abstract away the unnecessary details in the problem, and attempt to capture the basic properties of the problem. Then, we can model these properties as a graph.

Let's assume that we have a goal-based agent, in a deterministic and fully-observable task environment, e.g. a mopbot. What will be some useful abstractions in this case?

1. State:  $\langle L, A, B \rangle$ 
  - $L$  represents the location of the mopbot.
  - $A$  and  $B$  represent the status of locations  $A$  and  $B$  respectively, i.e. whether they are clean or dirty.
2. Actions:  $\{\text{left, right, clean, idle}\}$ 
  - The list of actions that can be performed by the mopbot.
3. Transition model:  $g : \text{State} \times \text{Action} \rightarrow \text{State}$ .
  - Difference between a transition model and an agent function:
    - Agent function: given the current state and percept, what should I do?
    - Transition model: given the current state, what will happen if I perform a certain action?
4. Performance measure (a.k.a. cost function):  $h : \text{State} \times \text{Action} \rightarrow \mathbb{R}$ .
5. Goal state:  $\langle *, C, C \rangle$ , where  $C$  indicates that the location is clean.
  - The goal state may not necessarily only consist a single state.
  - In the case of the mopbot, the goal state can be both  $\langle A, C, C \rangle$  and  $\langle B, C, C \rangle$ .
6. Start state

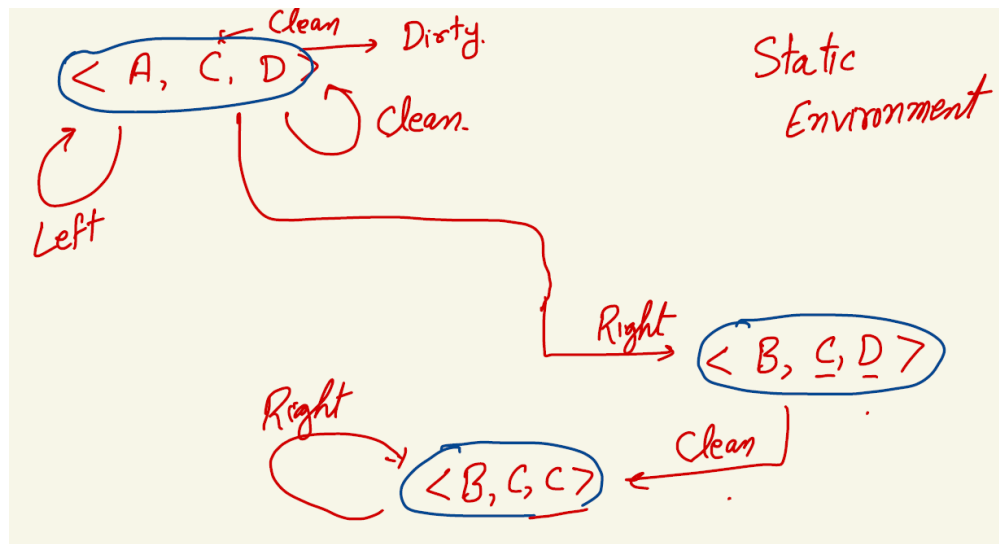


Figure 2.1: Transition model of the mopbot (incomplete)

We can illustrate the transition model using a graph, as shown above. However, the graph representing the transition model can potentially be infinitely large, even for seemingly small problems.<sup>1</sup>

After modelling our problem as a graph, we can now begin exploring the techniques used in graph search.

**Remark:** In fact, mathematics can be seen as a form of goal-based learning.

- When proving mathematical statements, we often start from an initial statement, and try to reach our final statement (complete our proof).

Initial statement  $\rightarrow \rightarrow \dots \rightarrow \rightarrow$  Final statement

<sup>1</sup>Examples include the  $(3n + 1)$  problem, and the Four Fours problem

## 2.4 Uninformed search

Let us assume that we have obtained the following graph: How can we find a path from  $s_0$  to  $s_2$ ?

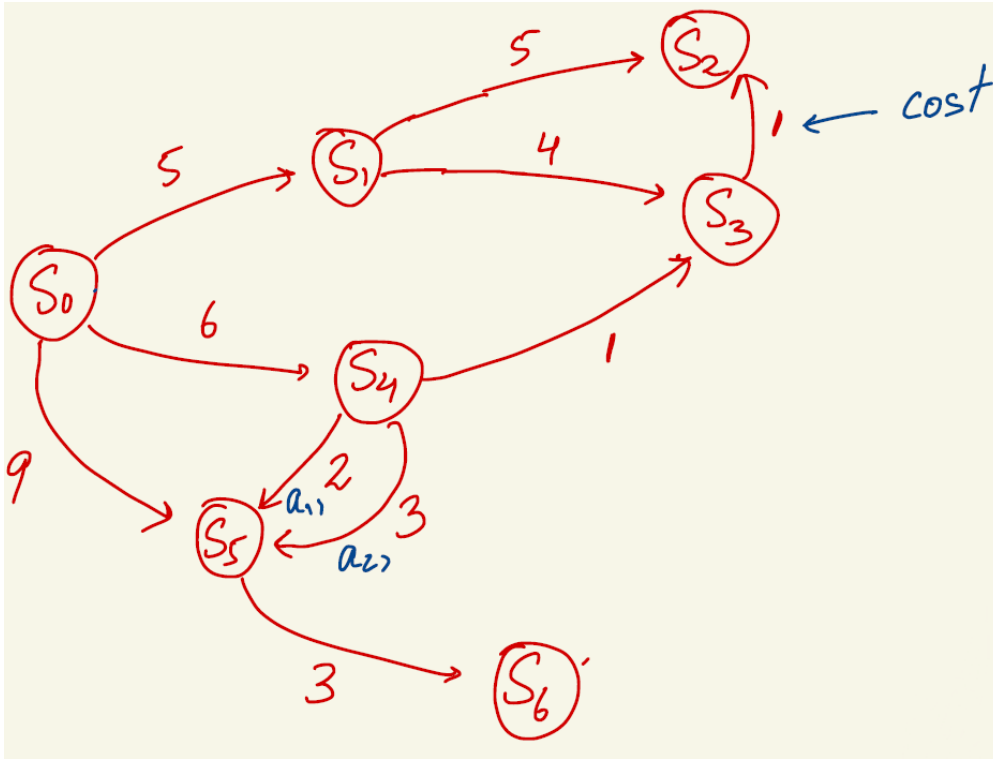


Figure 2.2: A graph with start state  $s_0$  and end state  $s_2$

### 1. Tree search:

- (a) At our current state, we review our list of performable actions.
- (b) If there is at least one performable action, we will execute the action.
- (c) If we hit a dead end, we will go back and try out another action which hasn't been executed.
- Limitation: this algorithm makes many unnecessary repeated moves.
  - E.g. after exploring  $s_6$  through  $s_0 \rightarrow s_5 \rightarrow s_6$ , we will hit a dead end and start exploring another route from  $s_0$ . Let's say we take the route  $s_0 \rightarrow s_4 \rightarrow s_5 \rightarrow \dots$ . Then, we will continue traversing  $s_5$  for a second time, even though we have already traversed it before.

### 2. Graph search:

- The algorithm is similar to that of tree search, but we will now memoize the nodes that we have traverse.
  - What do we memoize?
    - (a) State
    - (b) Parent

- (c) Action (that can be carried out at the node)
- (d) Path cost

- Limitation: we will require additional memory to store our memo table (i.e. space-time tradeoff).

### 3. Breadth-first search

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#### Algorithm 1 BFS

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1: procedure BFS( $u$ )
2:   if GOALTEST( $u$ ) then
3:     return path( $u$ )
4:   end if
5:    $F \leftarrow \text{queue}(u)$ 
6:    $E \leftarrow \{u\}$ 
7:   while  $F$  is not empty do
8:      $u \leftarrow F.\text{pop}()$ 
9:     for all children  $v$  in  $u$  do
10:      if GOALTEST( $v$ ) then
11:        return path( $v$ )
12:      else
13:        if  $v$  not in  $E$  then
14:           $E.\text{add}(v)$ 
15:           $F.\text{push}(v)$ 
16:        end if
17:      end if
18:    end for
19:  end while
20:  return FAIL
21: end procedure

```

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- Limitation: not optimal.

### 4. Dijkstra.

#### 2.4.1 Algorithmic analysis

How do we determine whether an algorithm is good?

1. Completeness: the algorithm will find a path to the goal, if the goal is reachable from the starting state.
2. Optimality: if the algorithm finds a path, the path will be of minimum cost.
3. Time complexity.
4. Space complexity.

**Exercise:** Is BFS complete?

**Proof:**

Assume that there is a path from the start state  $s_0$  to the goal state  $s_g$ , i.e.  $\pi : s_0, s_{i_1}, \dots, s_{i_k}, s_g$ , of length  $k + 1$ .

Base case: when  $k + 1 = 0$ , it means that  $s_0 = s_g$ .

IH: we assume that the algorithm is able to reach nodes at distance  $\leq k$ .

When we have reached  $s_{i_k}$ , we should have either:

- already explored  $s_g$ , i.e.  $s_g$  is along the path from  $s_0$  to  $s_{i_k}$ , or
- $s_g$  will be explored next.

Thus, BFS is complete. □

**Exercise:** Is BFS optimal?

**Proof:**

No. We will prove this by contradiction.

Referring to Figure 2.2, BFS will return  $s_0 \rightarrow s_1 \rightarrow s_2$ , instead of  $s_0 \rightarrow s_4 \rightarrow s_3 \rightarrow s_2$ , which is the actual optimal path. □

**Exercise:** What are the time and space complexities of BFS?

Taking  $b$  to be the branching factor of our graph, and  $d$  to be the depth (i.e. the minimum path length from the start to our goal),

- Time complexity:  $b + b^2 + \dots + b^d = O(b^{d+1})$
- Space complexity:  $O(b^{d+1})$

So BFS is not optimal. How can we do better?

1. We can replace the queue used by our frontier  $F$  in Algorithm 1 with a priority queue.
  - The nodes will be ordered according to their cost from the starting node.
  - When node  $u$  is popped from  $F$ , it will be the minimum cost node that is unexplored.
2. We should not mark the child nodes as “explored” too early in Algorithm 1.
  - By marking them as “explored” early on, we will not update/relax their weights after going through them once more from another path.