Radar and Remote Sensing

Formulas

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Contents

1 Constants

Constant which appear in the equations and useful quantities to help in computations.

$c \simeq 3 \times 10^8\,\mathrm{m/s}$[1] $\tau_c = 6.67\,\mathrm{ns/m}$[2] $k_B = 1.38 \times 10^{-23}\,\mathrm{m^2 kg/s^2 K}$[3]

Notes

1. Speed of light
2. Time needed for light to travel for 1 m considering the full roundtrip. This is helpful for quick computations like "what's the range of an object if the time of flight is 120 ns?"
3. Boltzmann's constant

2 Generic Radar

2.1 Geometry

$$A_e = \frac{G\lambda^2}{4\pi} = \rho A \quad (4)$$

$$\sigma = \frac{P_R \frac{4\pi}{\Omega}}{\frac{P_c}{A_{\text{target}}}} = G_{\text{target}} A_{\text{target}} \quad (6)$$

$$\sigma_{\text{plate}} = \frac{4\pi A^2}{\lambda^2} \quad (8)$$

$$G = 4\pi \frac{A_e}{\lambda^2} \quad (5)$$

$$\sigma_{\text{sphere}} = \pi r^2 \quad (7)$$

$$\sigma_{\text{corner}} = \frac{4\pi A_{\text{eff}}^2}{\lambda^2} \quad (9)$$

2.2 Power

$$P_D = \frac{P_t}{4\pi R^2} G \quad (10)$$

$$P_r = P_t \frac{G_0^2 \lambda^2 \sigma}{(4\pi)^3 h^4} \quad (12)$$

$$\mathrm{SNR}_s = \frac{P_{\text{avg}} G \lambda^2 \sigma T_{\text{scan}}}{(4\pi)^2 R^4\ kT_e\ F\ L\ \Omega} \quad (14)$$

$$P_r = P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (11)$$

$$\mathrm{SNR} = \frac{P_t G^2 \lambda^2 \sigma n_p}{(4\pi)^3 R^4\ k_B T_e B\ F\ L} \quad (13)$$

Notes

4. Effective area of the antenna
5. Gain
6. Radar Cross Section
7. RCS of a sphere; r is the radius
8. RCS of a plate
9. RCS of a corner reflector
10. Transmitted power density over the surface of a sphere with radius R
11. Radar equation
12. Radar equation for $cosec^2$ pattern
13. Signal to Noise Ratio in the radar equation. n_p is the number of transmitted pulses
14. Signal to Noise Ratio in a surveillance Radar

3 Pulsed Radar

3.1 Characteristics

$$f_r = \frac{1}{T} \quad (15)$$

$$\tau = \frac{1}{B} \quad (16)$$

3.2 Range

$$R = \frac{c}{2t} \quad 17$$
$$R_{\text{max}} = \frac{c}{2T} \quad 18$$
$$R_{\text{min}} = \delta R = \frac{c}{2\tau} \quad 19$$
$$\theta_B = R\frac{\lambda}{d} \quad 20$$
$$M = \frac{R_{\text{max}} - R_{\text{min}}}{\delta R} \quad 21$$

3.3 Power

$$d_t = \frac{\tau}{T} \quad 22$$
$$P_{\text{avg}} = P_t \cdot d_t \quad 23$$

Notes

15. Pulse Repetition Interval (PRF) versus Pulse Repetition Interval (PRI)
16. Bandwidth
17. Range
18. Max. unambiguous range
19. Min. range or range resolution
20. Azimuth resolution; d is the length of the antenna
21. Number of space bins
22. Duty Cycle
23. Average power

4 Doppler

$$f_d = \frac{2v_{\text{target}}}{\lambda} \cos\theta_e \cos\theta_a \quad 24$$
$$\delta f_d = \frac{1}{NT} \quad 25$$
$$f_r \geq 2f_{d,\text{max}} \quad 26$$
$$\gamma = 1 + \frac{2v_{\text{target}}}{c} \quad 27$$

Notes

24. Doppler frequency. Normally the cosines are not needed
25. Resolution of the doppler frequency
26. PRF must follow the Nyquist rule to sample the Doppler frequency
27. Time dilation

5 Linear FM Radar

5.1 Characteristics

$$B\tau = B\Delta t \quad 28$$

5.2 Range

$$f_{bd} = \frac{2R}{c}\frac{B}{\Delta t} + f_d$$
$$f_{bu} = \frac{2R}{c}\frac{B}{\Delta t} - f_d$$
$$\frac{f_{bd} - f_{bu}}{2} = f_d$$
$$\frac{f_{bd} + f_{bu}}{2} = \frac{2R}{c}\frac{B}{\Delta t}$$
$$\delta R = \frac{c}{2B} \quad 29$$
$$R_{\text{max}} = \frac{\Delta t\ c}{4\Delta f} f_s \quad 30$$

Notes

28. Time-Bandwidth product. Δt is the duration of the chirp.
29. Range resolution
30. Max. range

6 Scattering Matrix

$$\vec{b} = S\vec{a}$$ [31]

6.1 Useful matrices

$$S_{\text{c, cw}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$ [32]

$$S_{\text{c, ccw}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ [33]

$$S_{\text{ins}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ [34]

$$S_{dc} = \begin{pmatrix} 0 & \sqrt{1-k^2} & ke^{j\theta} & 0 \\ \sqrt{1-k^2} & 0 & 0 & ke^{j\theta} \\ ke^{j\theta} & 0 & 0 & \sqrt{1-k^2} \\ 0 & ke^{j\theta} & \sqrt{1-k^2} & 0 \end{pmatrix}$$ [35]

6.2 Other quantities

$$C = 10\log_{10}\frac{P_1}{P_3} = 20\log_{10}\frac{1}{S_{3,1}} = -20\log_{10}|k|$$ [36]

Notes

31. Definition of the scattering matrix
32. Clockwise circulator
33. Counter-clockwise circulator
34. Insulator
35. Directional coupler
36. Coupling factor for a directional coupler