

Radar and Remote Sensing

Formulas

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1 Constants

Constant which appear in the equations and useful quantities to help in computations.

$$c \simeq 3 \times 10^8 \text{ m/s}^1 \quad \tau_c = 6.67 \text{ ns/m}^2 \quad k_B = 1.38 \times 10^{-23} \text{ m}^2\text{kg/s}^2\text{K}^3$$

Notes

1. Speed of light
2. Time needed for light to travel for 1 m considering the full roundtrip. This is helpful for quick computations like "what's the range of an object if the time of flight is 120 ns?"
3. Boltzmann's constant

2 Generic Radar

2.1 Geometry

$$\begin{aligned} A_e &= \frac{G\lambda^2}{4\pi} = \rho A^4 \\ G &= 4\pi \frac{A_e}{\lambda^2}^5 \\ \sigma &= \frac{P_R \frac{4\pi}{\Omega}}{\frac{P_c}{A_{\text{target}}}} = G_{\text{target}} A_{\text{target}}^6 \\ \sigma_{\text{plate}} &= \frac{4\pi A^2}{\lambda^2}^8 \\ \sigma_{\text{sphere}} &= \pi r^2^7 \\ \sigma_{\text{corner}} &= \frac{4\pi A_{\text{eff}}^2}{\lambda^2}^9 \end{aligned}$$

2.2 Power

$$\begin{aligned} P_D &= \frac{P_t}{4\pi R^2} G^{10} \\ P_r &= P_t \frac{G_0^2 \lambda^2 \sigma}{(4\pi)^3 h^4}^{12} \\ P_r &= P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}^{11} \\ \text{SNR}_s &= \frac{P_{\text{avg}} G \lambda^2 \sigma T_{\text{scan}}}{(4\pi)^2 R^4 k T_e F L \Omega}^{14} \\ \text{SNR} &= \frac{P_t G^2 \lambda^2 \sigma n_p}{(4\pi)^3 R^4 k_B T_e B F L}^{13} \end{aligned}$$

Notes

4. Effective area of the antenna
5. Gain
6. Radar Cross Section
7. RCS of a sphere; r is the radius
8. RCS of a plate
9. RCS of a corner reflector
10. Transmitted power density over the surface of a sphere with radius R
11. Radar equation
12. Radar equation for cosec^2 pattern
13. Signal to Noise Ratio in the radar equation. n_p is the number of transmitted pulses
14. Signal to Noise Ratio in a surveillance Radar

3 Pulsed Radar

3.1 Characteristics

$$f_r = \frac{1}{T}^{15} \quad \tau = \frac{1}{B}^{16}$$

3.2 Range

$$R = \frac{c}{2t} \quad R_{\min} = \delta R = \frac{c}{2\tau} \quad \theta_B = R \frac{\lambda}{d} \quad M = \frac{R_{\max} - R_{\min}}{\delta R}$$

$$R_{\max} = \frac{c}{2T}$$

3.3 Power

$$d_t = \frac{\tau}{T} \quad P_{\text{avg}} = P_t \cdot d_t$$

Notes

15. Pulse Repetition Interval (PRF) versus Pulse Repetition Interval (PRI)
16. Bandwidth
17. Range
18. Max. unambiguous range
19. Min. range or range resolution
20. Azimuth resolution; d is the length of the antenna
21. Number of space bins
22. Duty Cycle
23. Average power

4 Doppler

$$f_d = \frac{2v_{\text{target}}}{\lambda} \cos \theta_e \cos \theta_a \quad \delta f_d = \frac{1}{NT} \quad \gamma = 1 + \frac{2v_{\text{target}}}{c}$$

$$f_r \geq 2f_{d,\max}$$

Notes

24. Doppler frequency. Normally the cosines are not needed
25. Resolution of the doppler frequency
26. PRF must follow the Nyquist rule to sample the Doppler frequency
27. Time dilation

5 Linear FM Radar

5.1 Characteristics

$$B\tau = B\Delta t$$

5.2 Range

$$f_{bd} = \frac{2R}{c} \frac{B}{\Delta t} + f_d \quad \frac{f_{bd} + f_{bu}}{2} = \frac{2R}{c} \frac{B}{\Delta t} \quad \delta v = \frac{\lambda_0}{2\Delta t}$$

$$f_{bu} = \frac{2R}{c} \frac{B}{\Delta t} - f_d$$

$$\frac{f_{bd} - f_{bu}}{2} = f_d \quad \delta R = \frac{c}{2B} \quad R_{\max} = \frac{c}{2} \frac{\Delta t}{2\Delta f}$$

Notes

28. Time-Bandwidth product. Δt is the duration of the chirp.
29. Range resolution
30. Speed resolution
31. Max. range

6 Scattering Matrix

$$\vec{b} = S\vec{a} \quad 32$$

6.1 Useful matrices

$$S_{c, \text{ cw}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad 33$$

$$S_{c, \text{ ccw}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad 34$$

$$S_{\text{ins}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad 35$$

$$S_{dc} = \begin{pmatrix} 0 & \sqrt{1-k^2} & ke^{j\theta} & 0 \\ \sqrt{1-k^2} & 0 & 0 & ke^{j\theta} \\ ke^{j\theta} & 0 & 0 & \sqrt{1-k^2} \\ 0 & ke^{j\theta} & \sqrt{1-k^2} & 0 \end{pmatrix} \quad 36$$

6.2 Other quantities

$$C = 10 \log_{10} \frac{P_1}{P_3} = 20 \log_{10} \frac{1}{S_{3,1}} = -20 \log_{10} |k| \quad 37$$

Notes

- 32. Definition of the scattering matrix
- 33. Clockwise circulator
- 34. Counter-clockwise circulator
- 35. Insulator
- 36. Directional coupler
- 37. Coupling factor for a directional coupler

7 Microstrps

7.1 Characteristics

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/W}} \quad 38 \quad \lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{\text{eff}}}} \quad 39$$

Notes

- 38. Effective dielectric constant
- 39. Guided wavelength