Radar and Remote Sensing

Formulas

Alessandro Scisca

Contents

1 Constants 2

2 Generic Radar 3
2.1 Geometry . . . 3
2.2 Power . . . 3

3 Pulsed Radar 4
3.1 Characteristics . . . 4
3.2 Range . . . 4
3.3 Power . . . 4

4 Doppler 5

5 Linear FM Radar 6
5.1 Characteristics . . . 6
5.2 Range . . . 6

6 Scattering Matrix 7
6.1 Useful matrices . . . 7
6.2 Other quantities . . . 7

7 Microstrps 8
7.1 Characteristics . . . 8

1 Constants

Constant which appear in the equations and useful quantities to help in computations.

$c \simeq 3 \times 10^8\,\mathrm{m/s}$[1] $\qquad \tau_c = 6.67\,\mathrm{ns/m}$[2] $\qquad k_B = 1.38 \times 10^{-23}\,\mathrm{m^2kg/s^2K}$ [3]

Notes

1. Speed of light
2. Time needed for light to travel for 1 m considering the full roundtrip. This is helpful for quick computations like "what's the range of an object if the time of flight is 120 ns?"
3. Boltzmann's constant

2 Generic Radar

2.1 Geometry

$$A_e = \frac{G\lambda^2}{4\pi} = \rho A \quad 4$$

$$\sigma = \frac{P_R \frac{4\pi}{\Omega}}{\frac{P_c}{A_{\text{target}}}} = G_{\text{target}} A_{\text{target}} \quad 6$$

$$\sigma_{\text{plate}} = \frac{4\pi A^2}{\lambda^2} \quad 8$$

$$G = 4\pi \frac{A_e}{\lambda^2} \quad 5$$

$$\sigma_{\text{sphere}} = \pi r^2 \quad 7$$

$$\sigma_{\text{corner}} = \frac{4\pi A_{\text{eff}}^2}{\lambda^2} \quad 9$$

2.2 Power

$$P_D = \frac{P_t}{4\pi R^2} G \quad 10$$

$$P_r = P_t \frac{G_0^2 \lambda^2 \sigma}{(4\pi)^3 h^4} \quad 12$$

$$\text{SNR}_s = \frac{P_{\text{avg}} G \lambda^2 \sigma T_{\text{scan}}}{(4\pi)^2 R^4 \ k T_e \ F \ L \ \Omega} \quad 14$$

$$P_r = P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad 11$$

$$\text{SNR} = \frac{P_t G^2 \lambda^2 \sigma n_p}{(4\pi)^3 R^4 \ k_B T_e B \ F \ L} \quad 13$$

Notes

4. Effective area of the antenna
5. Gain
6. Radar Cross Section
7. RCS of a sphere; r is the radius
8. RCS of a plate
9. RCS of a corner reflector
10. Transmitted power density over the surface of a sphere with radius R
11. Radar equation
12. Radar equation for *cosec*^{2} pattern
13. Signal to Noise Ratio in the radar equation. n_p is the number of transmitted pulses
14. Signal to Noise Ratio in a surveillance Radar

3 Pulsed Radar

3.1 Characteristics

$$f_r = \frac{1}{T}$$ [15]

$$\tau = \frac{1}{B}$$ [16]

3.2 Range

$$R = \frac{c}{2t}$$ [17]

$$R_{\max} = \frac{c}{2T}$$ [18]

$$R_{\min} = \delta R = \frac{c}{2\tau}$$ [19]

$$\theta_B = R\frac{\lambda}{d}$$ [20]

$$M = \frac{R_{\max} - R_{\min}}{\delta R}$$ [21]

3.3 Power

$$d_t = \frac{\tau}{T}$$ [22]

$$P_{\text{avg}} = P_t \cdot d_t$$ [23]

Notes

15. Pulse Repetition Interval (PRF) versus Pulse Repetition Interval (PRI)
16. Bandwidth
17. Range
18. Max. unambiguous range
19. Min. range or range resolution
20. Azimuth resolution; d is the length of the antenna
21. Number of space bins
22. Duty Cycle
23. Average power

4 Doppler

$$f_d = \frac{2v_{\text{target}}}{\lambda} \cos\theta_e \cos\theta_a$$ [24]

$$\delta f_d = \frac{1}{NT}$$ [25]

$$f_r \geq 2f_{d,\text{max}}$$ [26]

$$\gamma = 1 + \frac{2v_{\text{target}}}{c}$$ [27]

Notes

24. Doppler frequency. Normally the cosines are not needed
25. Resolution of the doppler frequency
26. PRF must follow the Nyquist rule to sample the Doppler frequency
27. Time dilation

5 Linear FM Radar

5.1 Characteristics

$$B\tau = B\Delta t$$ [28]

5.2 Range

$$f_{bd} = \frac{2R}{c}\frac{B}{\Delta t} + f_d$$

$$f_{bu} = \frac{2R}{c}\frac{B}{\Delta t} - f_d$$

$$\frac{f_{bd} - f_{bu}}{2} = f_d$$

$$\frac{f_{bd} + f_{bu}}{2} = \frac{2R}{c}\frac{B}{\Delta t}$$

$$\delta R = \frac{c}{2B}$$ [29]

$$R_{\max} = \frac{c\,\Delta t}{2}\frac{f_s}{2\Delta f}$$ [30]

Notes

28. Time-Bandwidth product. Δt is the duration of the chirp.
29. Range resolution
30. Max. range

6 Scattering Matrix

$$\vec{b} = S\vec{a}$$ [31]

6.1 Useful matrices

$$S_{\text{c, cw}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$ [32]

$$S_{\text{c, ccw}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ [33]

$$S_{\text{ins}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ [34]

$$S_{dc} = \begin{pmatrix} 0 & \sqrt{1-k^2} & ke^{j\theta} & 0 \\ \sqrt{1-k^2} & 0 & 0 & ke^{j\theta} \\ ke^{j\theta} & 0 & 0 & \sqrt{1-k^2} \\ 0 & ke^{j\theta} & \sqrt{1-k^2} & 0 \end{pmatrix}$$ [35]

6.2 Other quantities

$$C = 10\log_{10}\frac{P_1}{P_3} = 20\log_{10}\frac{1}{S_{3,1}} = -20\log_{10}|k|$$ [36]

Notes

31. Definition of the scattering matrix
32. Clockwise circulator
33. Counter-clockwise circulator
34. Insulator
35. Directional coupler
36. Coupling factor for a directional coupler

7 Microstrps

7.1 Characteristics

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12h/W}}$$ [37]

$$\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{\text{eff}}}}$$ [38]

Notes

37. Effective dielectric constant
38. Guided wavelength