Radar and Remote Sensing

Formulas

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Contents

1 Constants

Constant which appear in the equations and useful quantities to help in computations.

$$c \simeq 3 \times 10^8 \, \text{m/s}[1] \qquad \tau_c = 6.67 \, \text{ns/m}[2] \qquad k_B = 1.38 \times 10^{-23} \, \text{m}^2\text{kg/s}^2\text{K} \,[3]$$

Notes

1. Speed of light
2. Time needed for light to travel for 1 m considering the full roundtrip. This is helpful for quick computations like "what's the range of an object if the time of flight is 120 ns?"
3. Boltzmann's constant

2 Generic Radar

2.1 Geometry

$$A_e = \frac{G\lambda^2}{4\pi} = \rho A \,[4] \qquad \sigma = \frac{P_R \frac{4\pi}{\Omega}}{\frac{P_c}{A_{\text{target}}}} = G_{\text{target}} A_{\text{target}} \,[6] \qquad \sigma_{\text{plate}} = \frac{4\pi A^2}{\lambda^2} \,[8]$$

$$G = 4\pi \frac{A_e}{\lambda^2} \,[5] \qquad \sigma_{\text{sphere}} = \pi r^2 \,[7] \qquad \sigma_{\text{corner}} = \frac{4\pi A_{\text{eff}}^2}{\lambda^2} \,[9]$$

2.2 Power

$$P_D = \frac{P_t}{4\pi R^2} G \,[10] \qquad P_r = P_t \frac{G_0^2 \lambda^2 \sigma}{(4\pi)^3 h^4} \,[12] \qquad \text{SNR}_s = \frac{P_{\text{avg}} G \lambda^2 \sigma T_{\text{scan}}}{(4\pi)^2 R^4 \; kT_e \; F \; L \; \Omega} \,[14]$$

$$P_r = P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \,[11] \qquad \text{SNR} = \frac{P_t G^2 \lambda^2 \sigma n_p}{(4\pi)^3 R^4 \; k_B T_e B \; F \; L} \,[13]$$

Notes

4. Effective area of the antenna
5. Gain
6. Radar Cross Section
7. RCS of a sphere; r is the radius
8. RCS of a plate
9. RCS of a corner reflector
10. Transmitted power density over the surface of a sphere with radius R
11. Radar equation
12. Radar equation for $cosec^2$ pattern
13. Signal to Noise Ratio in the radar equation. n_p is the number of transmitted pulses
14. Signal to Noise Ratio in a surveillance Radar

3 Pulsed Radar

3.1 Characteristics

$$f_r = \frac{1}{T} \,[15] \qquad\qquad\qquad \tau = \frac{1}{B} \,[16]$$

3.2 Range

$$R = \frac{c}{2t} \quad [17] \qquad\qquad R_{\min} = \delta R = \frac{c}{2\tau} \quad [19] \qquad\qquad \theta_B = R\frac{\lambda}{d} \quad [20] \qquad\qquad M = \frac{R_{\max} - R_{\min}}{\delta R} \quad [21]$$

$$R_{\max} = \frac{c}{2T} \quad [18]$$

3.3 Power

$$d_t = \frac{\tau}{T} \quad [22] \qquad\qquad\qquad\qquad\qquad\qquad P_{\mathrm{avg}} = P_t \cdot d_t \quad [23]$$

Notes

15. Pulse Repetition Interval (PRF) versus Pulse Repetition Interval (PRI)

16. Bandwidth

17. Range

18. Max. unambiguous range

19. Min. range or range resolution

20. Azimuth resolution; d is the length of the antenna

21. Number of space bins

22. Duty Cycle

23. Average power

4 Doppler

$$f_d = \frac{2v_{\mathrm{target}}}{\lambda} \cos\theta_e \cos\theta_a \quad [24] \qquad\qquad \delta f_d = \frac{1}{NT} \quad [25] \qquad\qquad \gamma = 1 + \frac{2v_{\mathrm{target}}}{c} \quad [27]$$

$$f_r \geq 2f_{d,\max} \quad [26]$$

Notes

24. Doppler frequency. Normally the cosines are not needed

25. Resolution of the doppler frequency

26. PRF must follow the Nyquist rule to sample the Doppler frequency

27. Time dilation

5 Linear FM Radar

5.1 Characteristics

$$B\tau = B\Delta t \quad [28]$$

5.2 Range

$$f_{bd} = \frac{2R}{c}\frac{B}{\Delta t} + f_d \qquad\qquad \frac{f_{bd} + f_{bu}}{2} = \frac{2R}{c}\frac{B}{\Delta t}$$

$$f_{bu} = \frac{2R}{c}\frac{B}{\Delta t} - f_d \qquad\qquad\qquad\qquad\qquad\qquad R_{\max} = \frac{c\,\Delta t}{2}\frac{f_s}{2\Delta f} \quad [30]$$

$$\frac{f_{bd} - f_{bu}}{2} = f_d \qquad\qquad\qquad \delta R = \frac{c}{2B} \quad [29]$$

Notes

28. Time-Bandwidth product. Δt is the duration of the chirp.

29. Range resolution

30. Max. range

6 Scattering Matrix

$$\vec{b} = S\vec{a} \ ^{31}$$

6.1 Useful matrices

$$S_{\text{c, cw}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \ ^{32}$$

$$S_{\text{ins}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \ ^{34}$$

$$S_{\text{c, ccw}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \ ^{33}$$

$$S_{dc} = \begin{pmatrix} 0 & \sqrt{1-k^2} & ke^{j\theta} & 0 \\ \sqrt{1-k^2} & 0 & 0 & ke^{j\theta} \\ ke^{j\theta} & 0 & 0 & \sqrt{1-k^2} \\ 0 & ke^{j\theta} & \sqrt{1-k^2} & 0 \end{pmatrix} \ ^{35}$$

6.2 Other quantities

$$C = 10\log_{10}\frac{P_1}{P_3} = 20\log_{10}\frac{1}{S_{3,1}} = -20\log_{10}|k| \ ^{36}$$

Notes

31. Definition of the scattering matrix
32. Clockwise circulator
33. Counter-clockwise circulator
34. Insulator
35. Directional coupler
36. Coupling factor for a directional coupler

7 Microstrps

7.1 Characteristics

$$\epsilon_{\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12h/W}} \ ^{37} \qquad \lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{\text{eff}}}} \ ^{38}$$

Notes

37. Effective dielectric constant
38. Guided wavelength