Radar and Remote Sensing

Formulas

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Contents

1 Constants

Constant which appear in the equations and useful quantities to help in computations.

$$c \simeq 3 \times 10^8\,\mathrm{m/s}^{1} \qquad \tau_c = 6.67\,\mathrm{ns/m}^{2} \qquad k_B = 1.38 \times 10^{-23}\,\mathrm{m^2 kg/s^2 K}\ ^{3}$$

Notes

1. Speed of light
2. Time needed for light to travel for 1 m considering the full roundtrip. This is helpful for quick computations like "what's the range of an object if the time of flight is 120 ns?"
3. Boltzmann's constant

2 Generic Radar

2.1 Geometry

$$A_e = \frac{G\lambda^2}{4\pi} = \rho A \ ^{4}$$

$$G = 4\pi \frac{A_e}{\lambda^2} \ ^{5}$$

$$\sigma = \frac{P_R \frac{4\pi}{\Omega}}{\frac{P_c}{A_{\text{target}}}} = G_{\text{target}} A_{\text{target}} \ ^{6}$$

$$\sigma_{\text{sphere}} = \pi r^2 \ ^{7}$$

$$\sigma_{\text{plate}} = \frac{4\pi A^2}{\lambda^2} \ ^{8}$$

$$\sigma_{\text{corner}} = \frac{4\pi A_{\text{eff}}^2}{\lambda^2} \ ^{9}$$

2.2 Power

$$P_D = \frac{P_t}{4\pi R^2} G \ ^{10} \qquad P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \ ^{11} \qquad \text{SNR} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k_B T_e B F L R^4} \ ^{12}$$

Notes

4. Effective area of the antenna
5. Gain
6. Radar Cross Section
7. RCS of a sphere; r is the radius
8. RCS of a plate
9. RCS of a corner reflector
10. Transmitted power density over the surface of a sphere with radius R
11. Radar equation
12. Signal to Noise Ratio in the radar equation

3 Pulsed Radar

3.1 Characteristics

$$f_r = \frac{1}{T} \ ^{13} \qquad \tau = \frac{1}{B} \ ^{14}$$

3.2 Range

$$R = \frac{c}{2t} \ ^{15}$$

$$R_{\max} = \frac{c}{2T} \ ^{16}$$

$$R_{\min} = \delta R = \frac{c}{2\tau} \ ^{17} \qquad \theta_B = R\frac{\lambda}{d} \ ^{18} \qquad M = \frac{R_{\max} - R_{\min}}{\delta R} \ ^{19}$$

3.3 Power

$$d_t = \frac{\tau}{T} \text{ [20]} \qquad P_{\text{avg}} = P_t \cdot d_t \text{ [21]}$$

Notes

13. Pulse Repetition Interval (PRF) versus Pulse Repetition Interval (PRI)
14. Bandwidth
15. Range
16. Max. unambiguous range
17. Min. range or range resolution
18. Azimuth resolution; d is the length of the antenna
19. Number of space bins
20. Duty Cycle
21. Average power

4 Doppler

$$f_d = \frac{2v_{\text{target}}}{c} \cos\theta_e \cos\theta_a \text{ [22]} \qquad \delta f_d = \frac{\text{PRF}}{N} \text{ [23]} \qquad \gamma = 1 + \frac{2v_{\text{target}}}{c} \text{ [24]}$$

Notes

22. Doppler frequency. Normally the cosines are not needed.
23. Resolution of the doppler frequency
24. Time dilation

5 Linear FM Radar

5.1 Characteristics

$$B\tau = B\Delta t \text{ [25]}$$

5.2 Range

$$f_{bd} = \frac{2R}{c}\frac{B}{\Delta t} + f_d \qquad \frac{f_{bd} + f_{bu}}{2} = \frac{2R}{c}\frac{B}{\Delta t}$$

$$f_{bu} = \frac{2R}{c}\frac{B}{\Delta t} - f_d \qquad R_{\max} = \frac{\Delta t}{4\Delta f}\frac{c}{}f_s \text{ [27]}$$

$$\frac{f_{bd} - f_{bu}}{2} = f_d \qquad \delta R = \frac{c}{2B} \text{ [26]}$$

Notes

25. Time-Bandwidth product. Δt is the duration of the chirp.
26. Range resolution
27. Max. range

6 Scattering Matrix

$$\vec{b} = S\vec{a} \text{ [28]}$$

6.1 Useful matrices

$$S_{\text{c, cw}} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$ [29]

$$S_{\text{c, ccw}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$ [30]

$$S_{\text{ins}} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ [31]

$$S_{dc} = \begin{pmatrix} 0 & \sqrt{1-k^2} & ke^{j\theta} & 0 \\ \sqrt{1-k^2} & 0 & 0 & ke^{j\theta} \\ ke^{j\theta} & 0 & 0 & \sqrt{1-k^2} \\ 0 & ke^{j\theta} & \sqrt{1-k^2} & 0 \end{pmatrix}$$ [32]

6.2 Other quantities

$$C = 10 \log_{10} \frac{P_1}{P_3} = 20 \log_{10} \frac{1}{S_{3,1}} = -20 \log_{10} |k|$$ [33]

Notes

28. Definition of the scattering matrix
29. Clockwise circulator
30. Counter-clockwise circulator
31. Insulator
32. Directional coupler
33. Coupling factor for a directional coupler