faiii_ha6_julia

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```
In [15]: using PyPlot
```

###
Homework Assignment 6
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Lets define first the 0-spline as the characteristic function in $[0, 1]$

$$B_0(x) = \chi_{[0,1]}(x)$$

```
In [111]: function B0(x)
              if 0<=x && x<=1
                  1.0
              else
                  0.0
              end
          end

Out[111]: B0 (generic function with 1 method)
```

The next N-splines are defined as the convolution $B_N(x) = (B_0 * B_{N-1})(x)$, the analytical computation of this is quite costly, the first spline will be:

$$B_1(x) = B_0 * B_0(x) = \int_{\mathbb{R}} \chi_{[0,1]}(y)\chi_{[0,1]}(x-y)dy = \int_{[0,1]} \chi_{[0,1]}(x-y)dy = |[0,1] \cap (x - [0,1])| \quad (1)$$

$B_1(x)$ has as support $[0, 2]$ (in general B_N has support $[0, N+1]$ (as it was proven in the third exercise), and is defined as:

$$B_1(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 < x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

```
In [112]: function B1(x)
              if 0<=x && x<=1
                  float(x)
              elseif 1<x && x<=2
                  float(2-x)
```

```
        else
            0.0
        end
    end
```

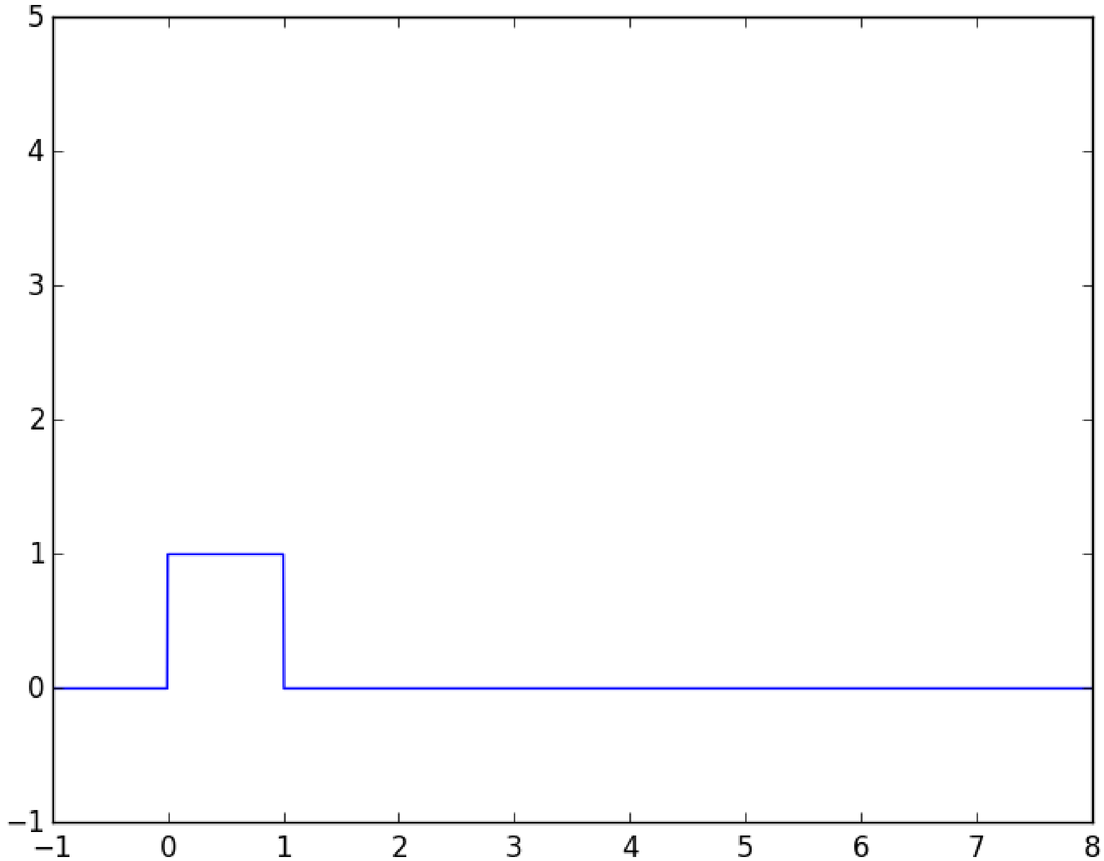
```
Out[112]: B1 (generic function with 1 method)
```

Lets plot this two first splines

```
In [173]: B_0=[B0(x) for x in -1:0.005:8]
          B_1=[B1(x) for x in -1:0.005:8]
          x=[x for x in -1:0.005:8];
```

0.0.1 B_0

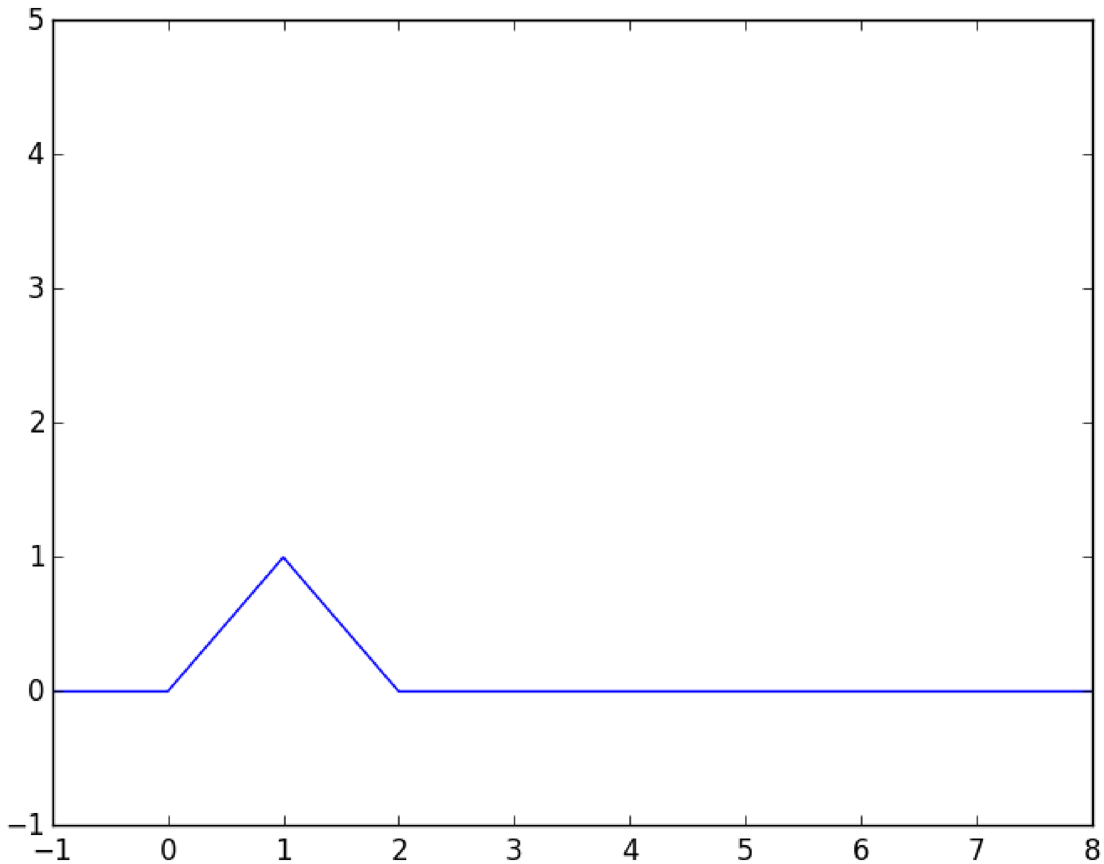
```
In [174]: plot(x,B_0)
          ax=axes()
          ax[:set_ylim]([-1,5])
```




```
Out[174]: (-1,5)
```

0.0.2 B_1

```
In [175]: plot(x,B_1)
          ax=axes()
          ax[:set_ylim]([-1,5])
```



```
Out[175]: (-1,5)
```

As the other N-splines might be too costly to compute we can use the properties of the fourier transform acting on convolutions, i.e.

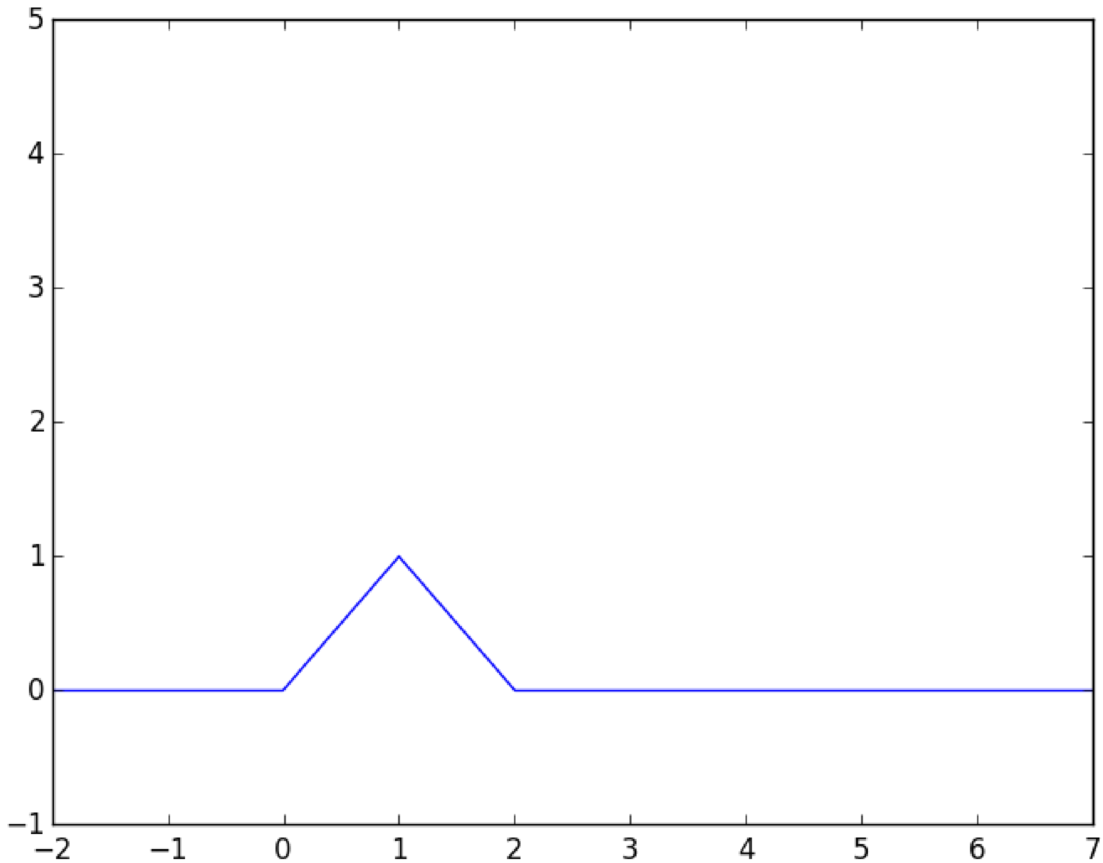
$$B_N(x) = \mathcal{F}^{-1}(\mathcal{F}(B_0 * B_{N-1}))(x) = \mathcal{F}^{-1}(\mathcal{F}(B_0)\mathcal{F}(B_{N-1}))(x)$$

As Benchmark we will use the already known $B_1(x)$

0.0.3 B_1 with Fourier convolution

```
In [176]: B_1test=abs(ifft(fft(B_0).*fft(B_0)));
```

```
In [177]: plot(x-1,B_1test/maximum(B_1test))
          ax=axes()
          ax[:set_ylim]([-1,5])
```



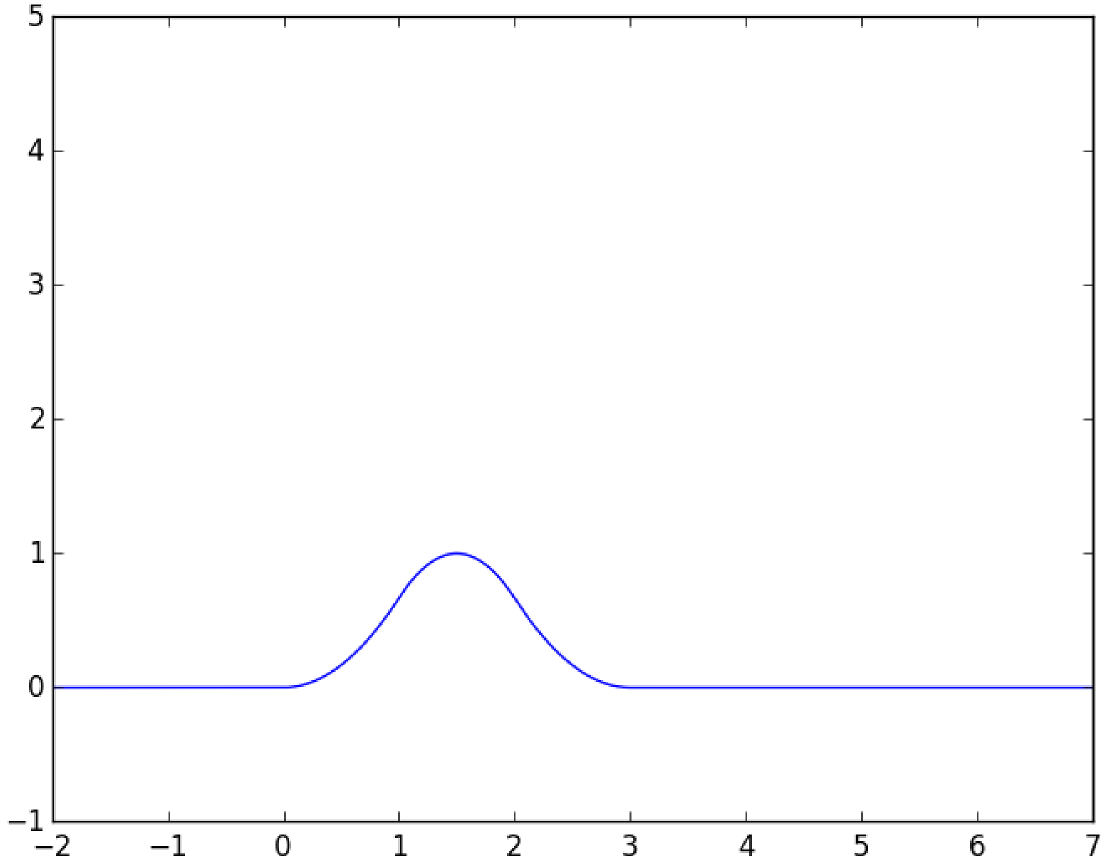

```
Out[177]: (-1,5)
```

It is working!, so we will apply to calculate the next 4 N-splines

0.0.4 B_2 with Fourier convolution

```
In [178]: B_2=abs(ifft(fft(B_0).*fft(B_1)));
```

```
In [185]: plot(x-1,B_2/maximum(B_2))
          ax=axes()
          ax[:set_ylim]([-1,5])
```

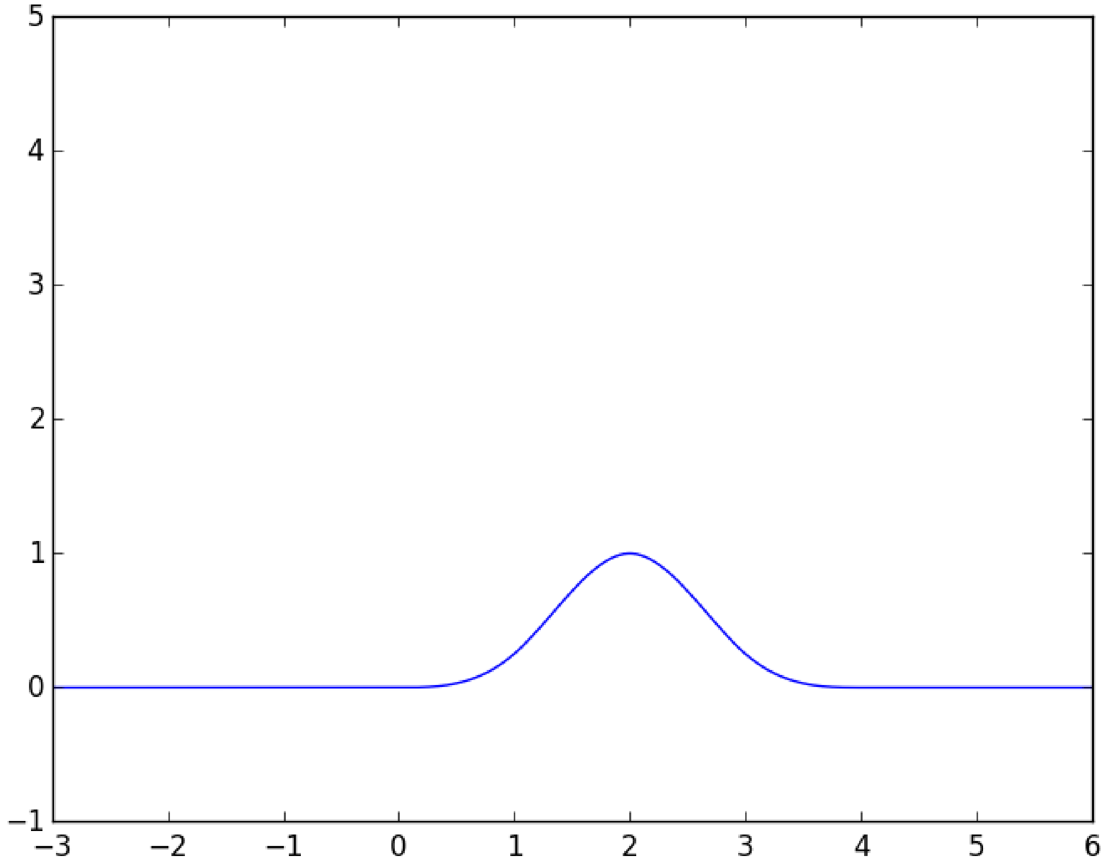



```
Out[185]: (-1,5)
```

0.0.5 B_3 with Fourier convolution

```
In [192]: B_3=abs(ifft(fft(B_0).*fft(B_2)));
```

```
In [193]: plot(x-2,B_3/maximum(B_3))
          ax=axes()
          ax[:set_ylim]([-1,5])
```

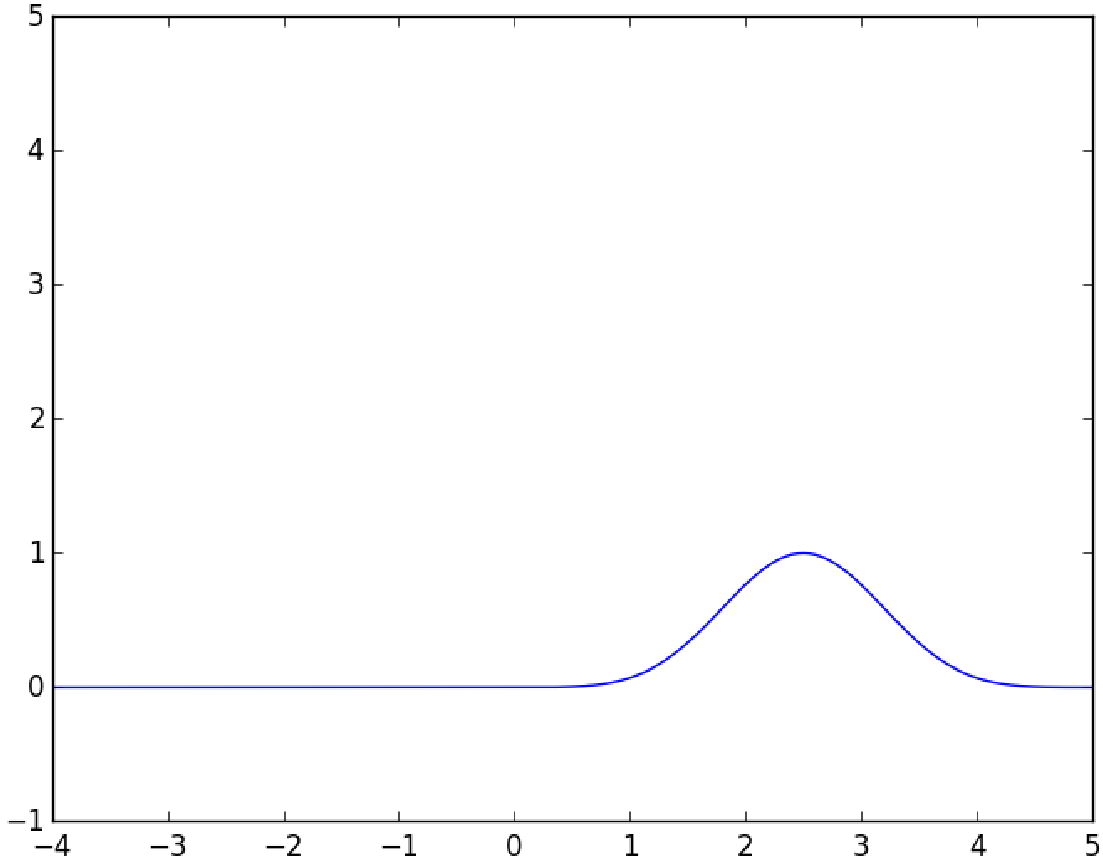



```
Out[193]: (-1,5)
```

0.0.6 B_4 with Fourier convolution

```
In [188]: B_4=abs(ifft(fft(B_0).*fft(B_3)));
```

```
In [191]: plot(x-3,B_4/maximum(B_4))
          ax=axes()
          ax[:set_ylim]([-1,5])
```




```
Out[191]: (-1,5)
```

Finally we can answered the question responding to the questions in the exercise with this observations

- What can you say about the support of B_N for increasing N? It is visible in the plots that the support of B_N is [0,N+1].
- What can you say about the regularity of B_N for increasing N? It is also visible that the regularity increases with N, since B_0 is piecewise continuous, B_1 is continuous but not differentiable, and B_2,B_3,B_4 are differentiable.