

Master Thesis: Fast Sparse Light Field Reconstruction with Shearlet-Based Inpainting

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TU Berlin, BMS

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- ▶ The thesis presents all the steps in the reconstruction pipeline with theory, algorithms and code.

Involved concepts

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- ▶ Compressed sensing techniques (ℓ^1 optimization algorithms for inverse problems).

Light Field Theory

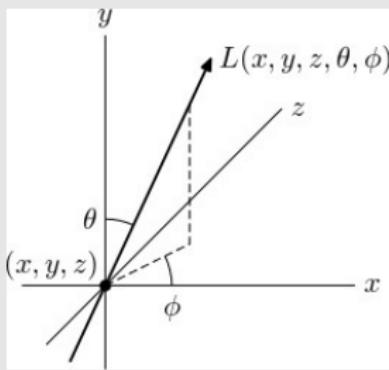
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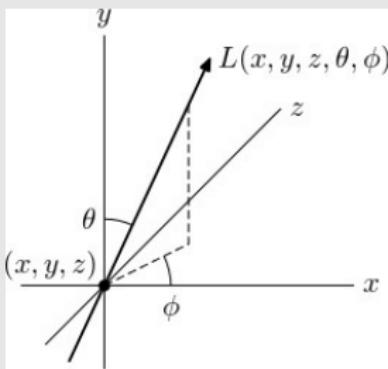
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- ▶ The plenoptic function can be simplified to a 4D function L_4 , called 4D Light Field or simply Light Field, which quantifies the intensity of static and monochromatic light rays propagating in half space.

4D Light Field Representation

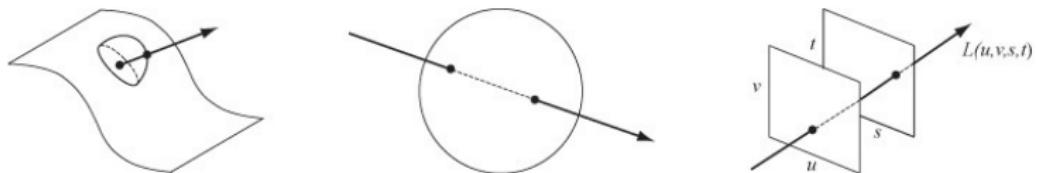


Figure: Three different representation of 4F LF. Left: $L_4(u, v, \phi, \theta)$. Center: $L_4(\phi_1, \theta_1, \phi_2, \theta_2)$. Right: $L_4(u, v, s, t)$.

4D Light Field Representation

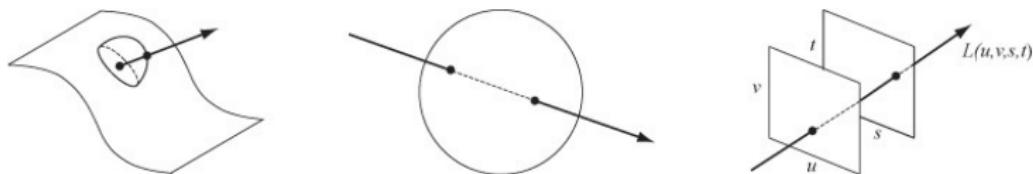


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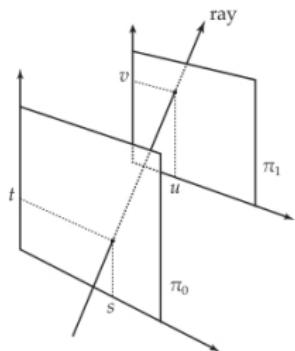


Figure: Used representation: "Two plane parametrization".

An article about computational result is advertising, not scholarship. The actual scholarship is the full software environment, code and data, that produced the result.

Buckheit and Donoho (1995)

Stereo Vision and Epipolar Geometry

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Stereo Vision and Epipolar Geometry

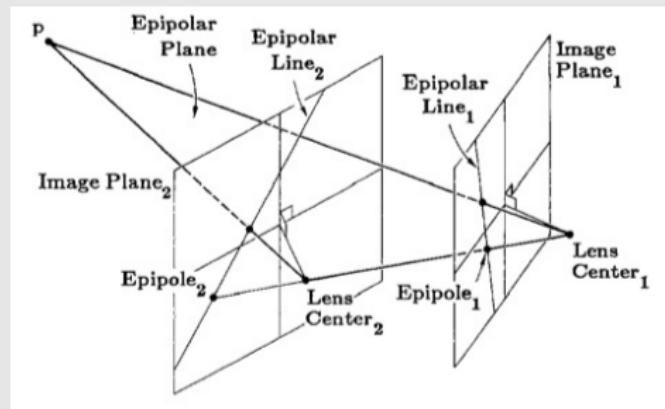
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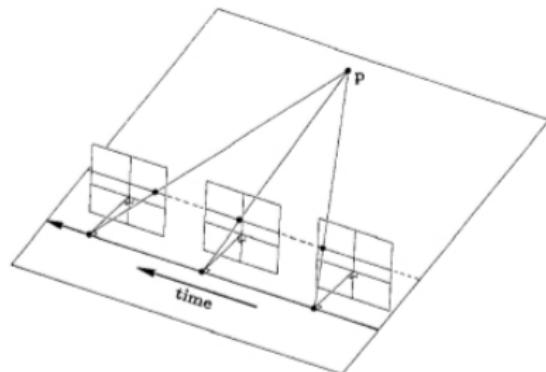
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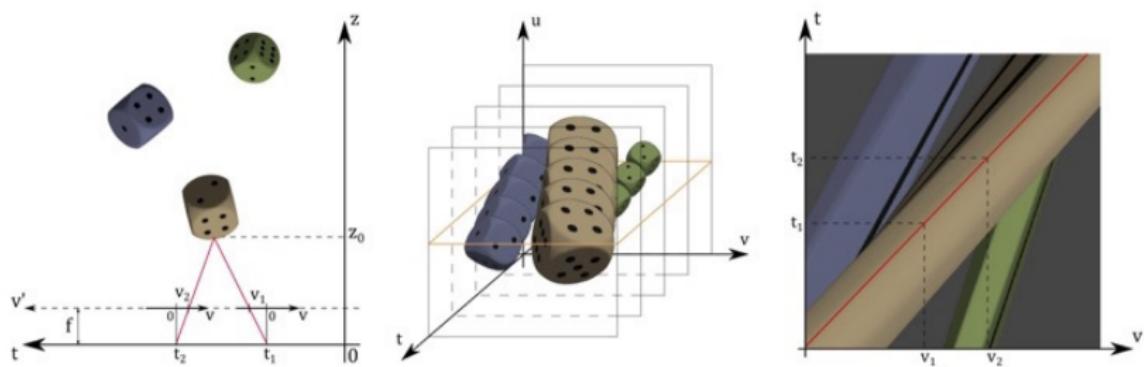
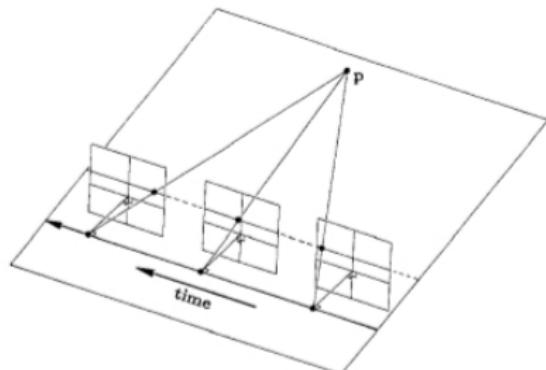
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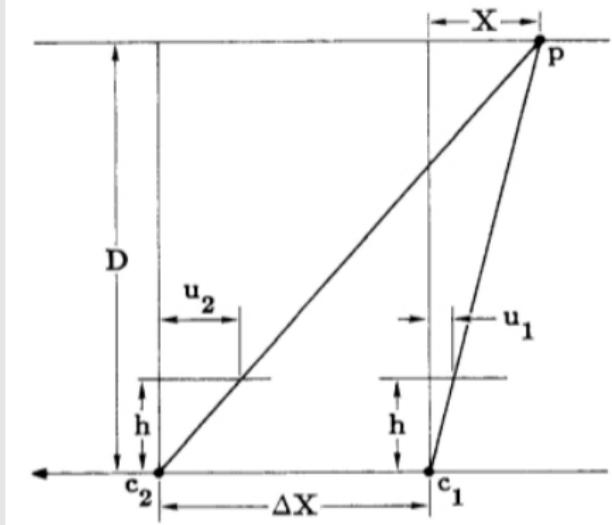
Epipolar Plane Images (EPIs) on Straight Line Trajectories



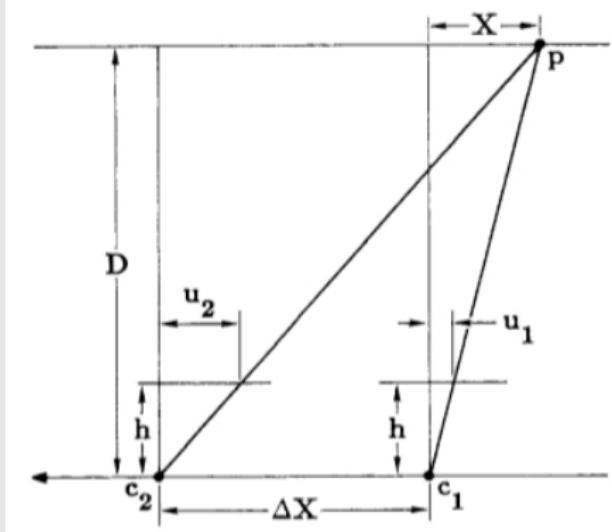
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Depth map Estimation with EPIs

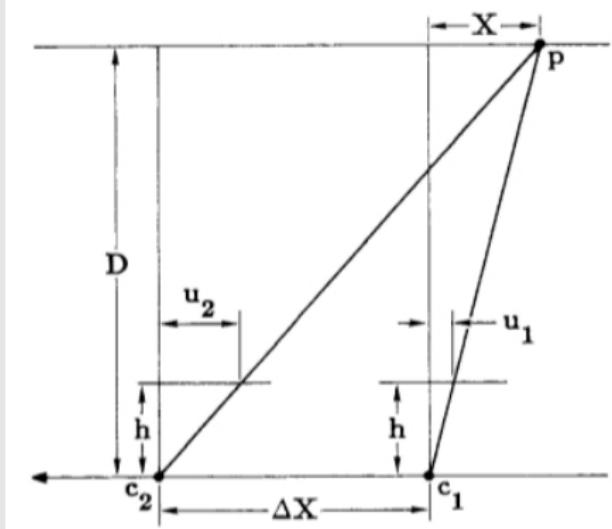


Depth map Estimation with EPIs



► **Point-depth formula:** $D = h \frac{\Delta X}{\Delta u} = h \frac{\Delta X}{u_1 - u_2}$.

Depth map Estimation with EPIs



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- ▶ **Sampling rate (Nyquist criterion):** $\Delta X \leq \frac{D_{min}}{h} \Delta u$.

Physical Acquisition Setup

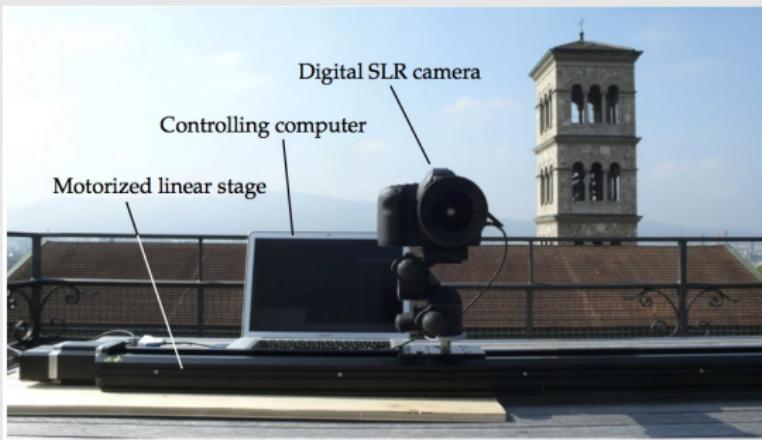
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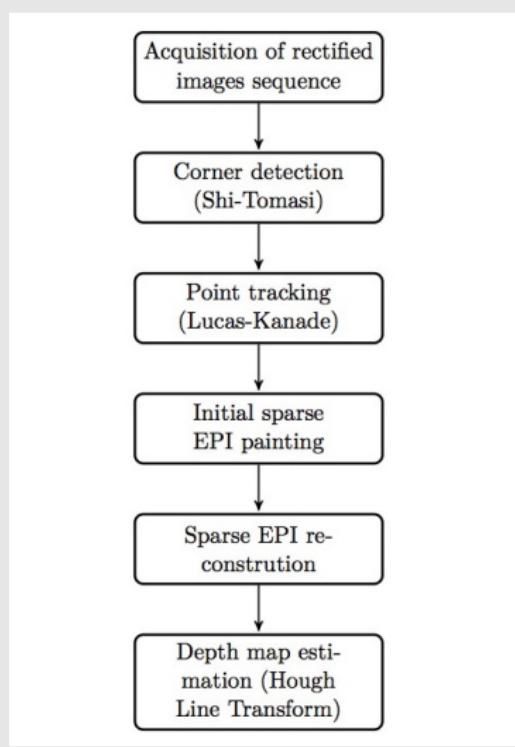
Used Data Set: Church



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Followed Pipeline



Point Tracking Results



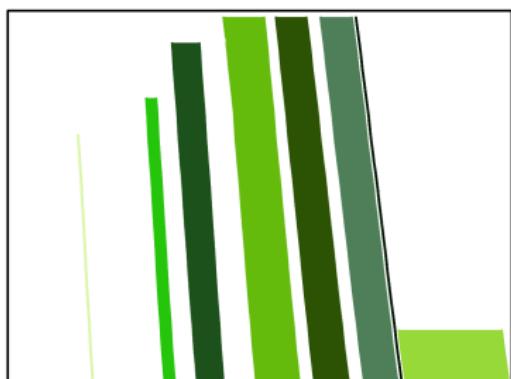
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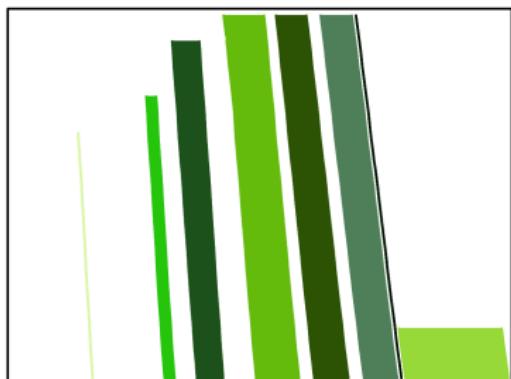
Particular EPI Example



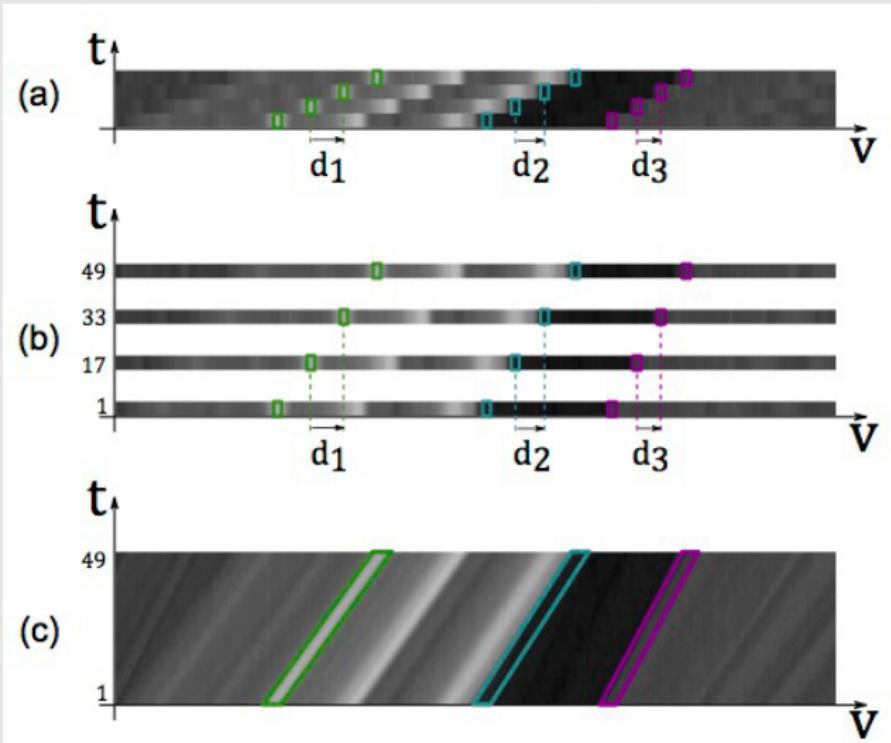
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Reconstruction Method with Inpainting



Important Tool: Frame Theory

Frames

Let I be a set of countable indices. A sequence $(\phi_i)_{i \in I}$ in a Hilbert space \mathcal{H} is called a **frame** of H , if there exist constants $0 < A \leq B < \infty$ such that

$$A\|x\|^2 \leq \sum_{i \in I} |\langle x, \phi_i \rangle|^2 \leq B\|x\|^2, \forall x \in \mathcal{H}$$

A and B are called lower and upper frame bound. Moreover, if A and B can be chosen to be equal, we call it (A -)tight frame. If $A = B = 1$ is possible, $(\phi_i)_{i \in I}$ forms a Parseval frame. A frame $(\phi_i)_{i \in I}$ span \mathcal{H} .

Analysis, synthesis and frame operator

$T : \mathcal{H} \rightarrow \ell_2(I)$ given by $f \mapsto (\langle f, \phi_i \rangle)_{i \in I}$ is called **the analysis operator**, and $T^* : \ell_2(I) \rightarrow \mathcal{H}$, given by $(c_i)_{i \in I} \mapsto \sum_{i \in I} c_i \phi_i$ is called **the synthesis operator**.

$S = T^* T : \mathcal{H} \rightarrow \mathcal{H}$, given by $f \mapsto \sum_{i \in I} \langle f, \phi_i \rangle \phi_i$ is called **the frame operator** and is an invertible operator.

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Reconstruction and Decomposition Formula

If $(\phi_i)_{i \in I} \subseteq \mathcal{H}$ be a frame for \mathcal{H} , and S its frame operator, then

$$f = \sum_{i \in I} \langle f, \phi_i \rangle S^{-1} \phi_i, \quad \forall f \in \mathcal{H} \text{ (Reconstruction)}$$

$$f = \sum_{i \in I} \langle f, S^{-1} \phi_i \rangle \phi_i, \quad \forall f \in \mathcal{H} \text{ (Decomposition)}$$

Abstract Inpainting Framework

- ▶ Let \mathcal{H} a separable Hilbert space and $x^0 \in \mathcal{H} = \mathcal{H}_K \oplus \mathcal{H}_M = P_K \mathcal{H} \oplus P_M \mathcal{H}$. Then, given a corrupt signal $P_K x^0$, we want to recover the missing part $P_M x^0$.

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- ▶ In image inpanting $\mathcal{H} = L^2(\mathbb{R}^2)$, the missing space $H_M = L^2(\mathcal{M})$ for some measurable set $\mathcal{M} \subset \mathbb{R}^2$. Given $x_k \in \mathcal{H}_K$ we want to find $x^0 \in \mathcal{H}$ such that $x_K = P_K x^0$ (underdetermined problem).

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- ▶ We will assume that x^0 can be efficiently represented by some Parseval frame $\Phi = (\phi_i)_{i \in I}$ for \mathcal{H} , the inpainting will be translated as asking for the solution of the ℓ^0 -minimization problem

$$\min_{c \in \ell^2(I)} \|c\|_{\ell^0(I)} \quad \text{subject to} \quad P_K x^0 = P_K T_\Phi^* c = \sum_{i \in I} c_i P_K \phi_i$$

Analysis approach

Algorithm 1: Inpainting via ℓ^1 -minimization

Input : Corrupted signal $P_Kx^0 \in \mathcal{H}_K$, Parseval frame $\Phi = (\phi_i)_{i \in I}$ for \mathcal{H}

Compute:

$$x^* = \underset{x \in \mathcal{H}}{\operatorname{argmin}} \|T_\Phi x\|_{\ell^1(I)} \quad \text{subject to} \quad P_K x^* = P_K x^0 = x_K$$

Output : recovered signal $x^* \in \mathcal{H}$

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Algorithm 2: Inpainting via ℓ^1 -minimization

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Best N-term approx. error (Donoho, 2001)

Let $\{\psi_\lambda\}_{\lambda \in \Lambda} \subseteq L^2(\mathbb{R}^2)$ a frame. The optimal best N -term approximation error for any $f \in \mathcal{E}^2(\mathbb{R}^2)$ (cartoon-like functions space) is

$$\sigma_N(f, \{\psi_\lambda\}_{\lambda \in \Lambda}) = O(N^{-1})$$

In the case of 2D-wavelets

$$\sigma_N(f, \{\psi_{j,m}\}_{j,m}) \sim N^{-1/2}$$

Discrete Shearlet System

For $j \in \mathbb{Z}$, let

$$A_j := \begin{pmatrix} 2^j & 0 \\ 0 & 2^{j/2} \end{pmatrix}$$

be the **parabolic scaling matrix**, and for $k \in \mathbb{Z}$, let

$$S_k := \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$

be the **shearing matrix**. Given $\psi \in L^2(\mathbb{R}^2)$, the **shearlet system** associated with ψ is defined as

$$\mathcal{SH}(\psi) := \{2^{3j/4}\psi(S_k A_j x - m) : j \in \mathbb{Z}, k \in \mathbb{Z}, m \in \mathbb{Z}^2\}$$

Shearlets

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Discrete Shearlet Transform

For $f \in L^2(\mathbb{R}^2)$ the associated **discrete shearlet transform** is defined by

$$f \mapsto \mathcal{SH}_\psi f(j, k, m) = \langle f, \psi_{j,k,m} \rangle, (j, k, m) \in \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^2$$

Classical Shearlets

Let $\psi \in L^2(\mathbb{R}^2)$ defined by $\hat{\psi}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_1)\hat{\psi}_2(\xi_2/\xi_1)$, where $\psi_1, \psi_2 \in L^2(\mathbb{R})$ satisfy the following properties:

- ▶ $\sum_{j \in \mathbb{Z}} |\hat{\psi}_1(2^{-j}\xi)|^2 = 1$ for a.e. $\xi \in \mathbb{R}$ ("wavelet-like"),
 $\text{supp}(\hat{\psi}_1) \subseteq [-1/2, -1/16] \cup [1/16, 1/2]$ and $\hat{\psi}_1 \in C^\infty(\mathbb{R})$.
- ▶ $\sum_{k=-1,0,1} |\hat{\psi}_2(\xi + k)|^2 = 1$ for a.e. $\xi \in [-1, 1]$ ("bump-like"),
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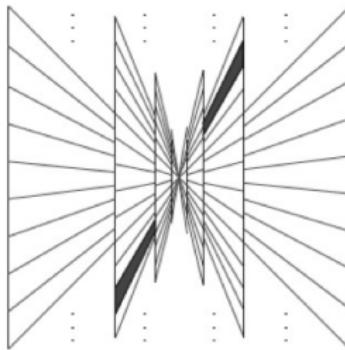
Then, we call ψ a **classical shearlet**. Moreover, $S\mathcal{H}(\psi)$ will form a Parseval frame for $L^2(\mathbb{R}^2)$.

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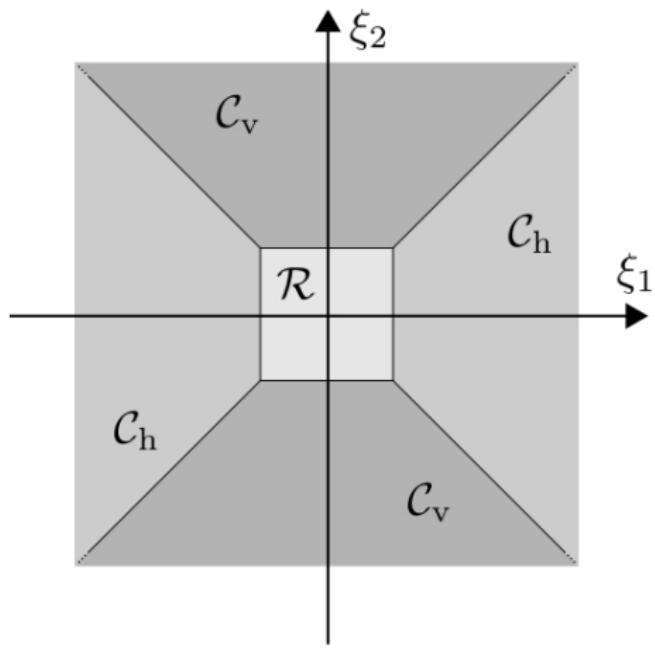
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Cone Adapted Shearlets

For $\psi, \tilde{\psi}, \phi \in L^2(\mathbb{R}^2)$ and $c \in \mathbb{Z}^2$ the **cone adapted shearlet system** is defined by

$$\mathcal{SH}(\phi, \psi, \tilde{\psi}, c) := P_{\mathcal{R}}\Phi(\phi, c_1) \cup P_{\mathcal{C}_h}\Psi(\psi, c) \cup P_{\mathcal{C}_v}\tilde{\Psi}(\tilde{\psi}, c)$$



Universal Shearlets and 0-Shearlets

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- ▶ With A_{j,α_j} we can define the **Universal Shearlet System** (Kutyniok, Genzel, 2014), a generalization of the cone-adapted shearlet system for different level of anisotropy.

Schwartz Functions Space

$$\mathbb{S}(\mathbb{R}^d) := \{\phi \in C^\infty(\mathbb{R}^d) | \forall K, N \in \mathbb{N}_0 : \sup_{x \in \mathbb{R}^d} (1 + |x|^2)^{-N/2} \sum_{|\alpha| \leq K} |D^\alpha \phi(x)| < \infty\}$$

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Meyer and Corona Scaling Functions

Let $\phi \in \mathbb{S}(\mathbb{R})$ with $0 \leq \hat{\phi} \leq 1$, $\hat{\phi}(u) = 1$ for $u \in [-1/16, 1/16]$ and $\text{supp}(\hat{\phi}) \subset [-1/8, 1/8]$; then ϕ is usually called **Meyer scaling function**. One can define then the **corona scaling function** for $j \in \mathbb{N}_0$ for $\xi = (\xi_1, \xi_2) \in \mathbb{R}^2$ by

$$\hat{\Phi}(\xi) := \hat{\phi}(\xi_1)\hat{\phi}(\xi_2)$$

$$W(\xi) := \sqrt{\hat{\Phi}^2(2^{-2}\xi) - \hat{\Phi}^2(\xi)}$$

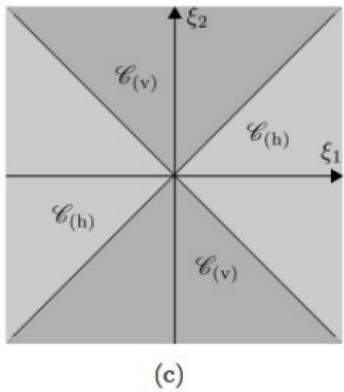
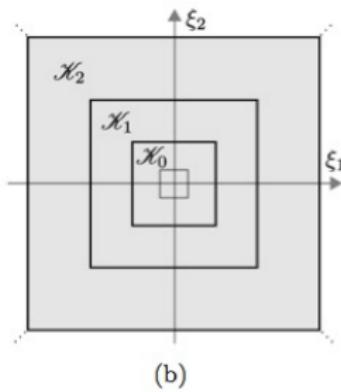
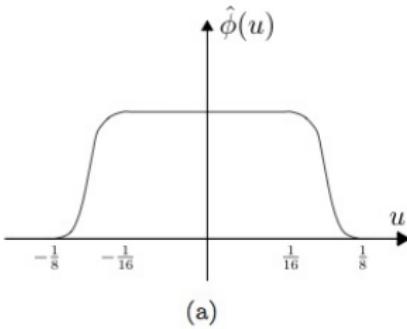
$$W_j(\xi) := W(2^{-2j}\xi)$$

The functions W_j are compactly supported in corona-shaped scaling levels $\mathcal{K}_j := [2^{-2j-1}, 2^{2j-1}]^2 \setminus (-2^{2j-4}, 2^{2j-4})^2$.

Bump-like Function

A **bump-like** function is defined as $v \in C^\infty(\mathbb{R})$ such that $\text{supp}(v) \subset [-1, 1]$ and

$$|v(u-1)|^2 + |v(u)|^2 + |v(u+1)|^2 = 1 \quad \text{for } u \in [-1, 1] \quad , \text{and}$$
$$v(0) = 1 \quad \text{and} \quad v^{(n)}(0) = 0 \quad \text{for } n \geq 1$$



Scaling and Shearing Matrices

Let $j \in \mathbb{Z}$, $k \in \mathbb{N}$ and $\alpha_j \in (-\infty, 2)$ for all $j \in \mathbb{Z}$, then

$$A_{j,\alpha_j,(h)} := \begin{pmatrix} 2^j & 0 \\ 0 & 2^{\alpha_j j/2} \end{pmatrix}, \quad S_{k,(h)} := \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$$

$$A_{j,\alpha_j,(v)} := \begin{pmatrix} 2^{\alpha_j j/2} & 0 \\ 0 & 2^j \end{pmatrix}, \quad S_{k,(v)} := \begin{pmatrix} 1 & 0 \\ 1 & k \end{pmatrix}$$

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Adapted Cone Functions

Let $\iota \in \{h, v\}$ and $\xi = (\xi_1, \xi_2) \in \mathbb{R}^2$, then

$$V_{(h)}(\xi) := v(\xi_2/\xi_1), \quad V_{(v)}(\xi) := v(\xi_1/\xi_2), \quad \xi \in \mathbb{R}^2.$$

Ingredients for the Universal Shearlet System

Let $\Phi, W, V_{(h)}, V_{(v)} \in L^2(\mathbb{R}^2)$ be defined as before.

1. **Coarse scaling functions:** For $k \in \mathbb{Z}^2$, we set

$$\psi_{-1,k} := \Phi(x - k), \quad x \in \mathbb{R}^2.$$

2. **Interior shearlets:** Let $\alpha_j \in (-\infty, 2)$, $j \in \mathbb{N}_0$, $k \in \mathbb{Z}$ with $|k| < 2^{(2-\alpha)j/2}$, $m \in \mathbb{Z}^2$ and $\iota \in \{h, v\}$. The shearlets will be given by

$$\begin{aligned}\hat{\psi}_{j,k,m}^{\alpha_j,(\iota)}(\xi) := & 2^{-(\alpha_j+2)j/4} W(2^{-j}\xi) V_{(\iota)}(\xi^\top A_{-j,\alpha_j,(\iota)} S_{-k,(\iota)}) \\ & e^{-2\pi i \xi^\top A_{-j,\alpha_j,(\iota)} S_{-k,(\iota)}}, \quad \xi \in \mathbb{R}^2\end{aligned}$$

3. **Boundary shearlets:** For $\alpha_j \in (-\infty, 2)$, $j \geq 1$, $k = \pm \lceil 2^{(2-\alpha_j)j/2} \rceil$ and $k \in \mathbb{Z}^2$, we define

$$\hat{\psi}_{j,k,m}^{\alpha_j} := \begin{cases} 2^{-(\alpha_j+2)j/4-1/4} & W(2^{-j}\xi)V_{(h)}(\xi^\top A_{-j,\alpha_j,(h)}S_{-k,(h)}) \\ & e^{-\pi i \xi^\top A_{-j,\alpha_j,(h)} S_{-k,(h)} m}, \quad \xi \in \mathcal{C}_{(h)}, \\ 2^{-(\alpha_j+2)j/4-1/4} & W(2^j\xi)V_{(v)}(\xi^\top A_{-j,\alpha_j,(v)}S_{-k,(v)}) \\ & e^{-\pi i \xi^\top A_{-j,\alpha_j,(v)} S_{-k,(h)} m}, \quad \xi \in \mathcal{C}_{(v)} \end{cases}$$

and in the case $j = 0$, $k = \pm 1$, we define

$$\hat{\psi}_{0,k,m}^{\alpha_j} := \begin{cases} W(\xi)V_{(h)}(\xi^\top S_{-k,(h)})e^{-2\pi i \xi^\top m}, & \xi \in \mathcal{C}_{(h)}, \\ W(\xi)V_{(v)}(\xi^\top S_{-k,(v)})e^{-2\pi i \xi^\top m}, & \xi \in \mathcal{C}_{(v)}. \end{cases}$$

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Scaling Sequence

A sequence $(\alpha_j)_{j \in \mathbb{N}_0} \subset \mathbb{R}$ is called a scaling sequence if

$$\alpha_j \in A_j := \{2n/j | n \in \mathbb{Z}, n \leq j-1\} = \{\dots, -4/j, -2/j, 0, 2/j, \dots, 2-2/j\}$$

Universal Shearlet System

Let $(\alpha_j)_{j \in \mathbb{N}_0}$ be a scaling sequence, ϕ a Meyer scaling function and v a bump-like function, then **the universal shearlet system** is given by

$\mathcal{SH}(\phi, v, (\alpha_j)_j) := \mathcal{SH}_{\text{Low}}(\phi) \cup \mathcal{SH}_{\text{Int}}(\phi, v, (\alpha_j)_j) \cup \mathcal{SH}_{\text{Bound}}(\phi, v, (\alpha_j)_j)$,
where

$$\mathcal{SH}_{\text{Low}}(\phi) := \{\psi_{-1,m} | m \in \mathbb{Z}^2\}$$

$$\mathcal{SH}_{\text{Int}}(\phi, v, (\alpha_j)_j) := \{\psi_{j,k,m}^{\alpha_j, (\iota)} | j \geq 0, |k| < 2^{(2-\alpha_j)j/2}, m \in \mathbb{Z}^2, \iota \in \{h, v\}\},$$

$$\mathcal{SH}_{\text{Bound}}(\phi, v, (\alpha_j)_j) := \{\psi_{j,k,m}^{\alpha_j} | j \geq 0, |k| = \pm 2^{(2-\alpha_j)j/2}, m \in \mathbb{Z}^2\}.$$

Parseval Frame Property

If $(\alpha_j)_j$ is a scaling sequence then $\mathcal{SH}(\phi, v, (\alpha_j)_j)$ is a Parseval frame for $L^2(\mathbb{R}^2)$.

0-Shearlets System

The 0-Shearlets System are obtained by selecting the scaling sequence $(\alpha_j)_{j \in \mathbb{Z}} = (-2/j)_{j \in \mathbb{Z}}$ on the Universal Shearlets System; they are sensible to linear singularities and have an associated scaling matrix given by

$$A_j = \begin{pmatrix} 2^j & 0 \\ 0 & 2^{-1} \end{pmatrix}$$

We used the 0-Shearlets System as the selected sparsifying system for the sparse EPIs inpainting.

Shearlet-based inpainting with iterative hard thresholding

Algorithm

Input : Sparse EPI y , sampling matrix M , $\delta_{init}, \delta_{min}$, iterations

Compute: $x_0 := 0;$

$$\delta_0 := \delta_{init};$$

$$\lambda := (\delta_{min})^{1/(iterations-1)};$$

$$\Gamma_0 := supp(T(x_0));$$

$$\beta_0 := T_{\Gamma_0}(y - Mx_0);$$

$$\alpha_0 = \frac{||\beta_0||_2^2}{||MT^*(\beta_0)||_2^2};$$

for $n := 0$ **to** $(iterations-1)$ **do**

$$x_{n+1} = T^*(Thr_{\delta_n}(T(x_n + \alpha_n(y - Mx_n))));$$

$$\Gamma_{n+1} := supp(T(x_{n+1}));$$

$$\beta_{n+1} := T_{\Gamma_{n+1}}(y - Mx_{n+1});$$

$$\alpha_{n+1} := \frac{||\beta_{n+1}||_2^2}{||MT^*(\beta_{n+1})||};$$

$$\delta_{n+1} := \lambda \delta_n;$$

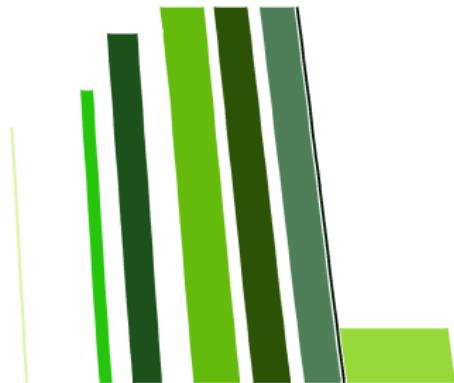
end

Output : Inpainted EPI $x_{iterations}$

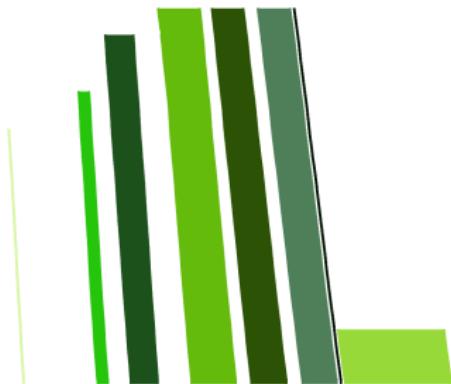
Results on EPIs inpainting



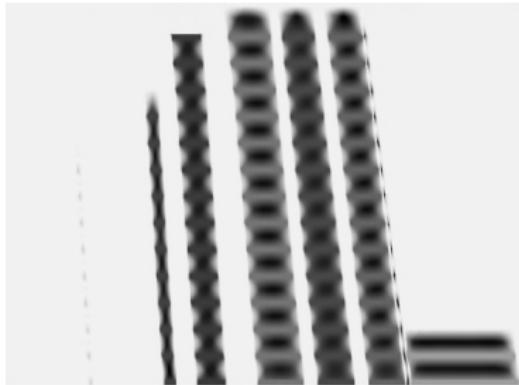
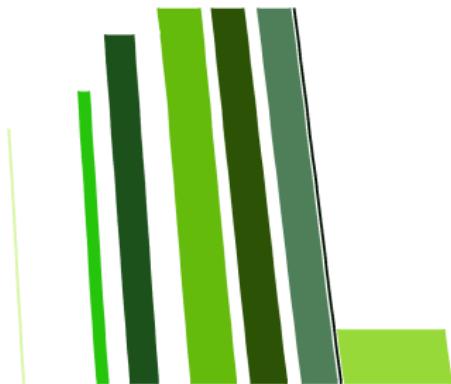
Results on EPIs inpainting



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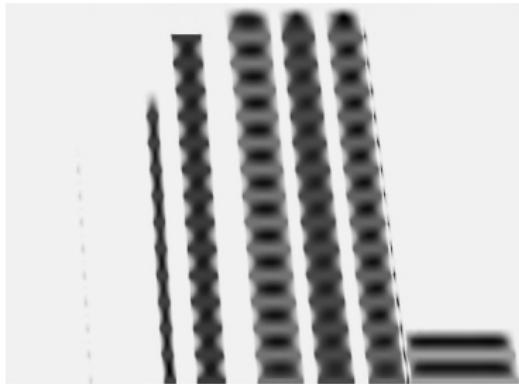
Results on EPIs inpainting



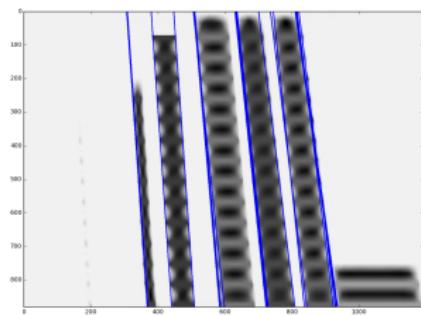
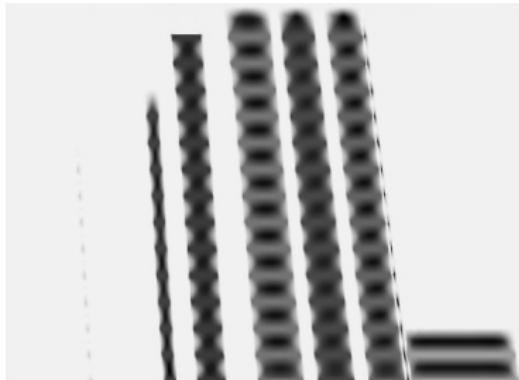
Results on line detection and depth map estimation



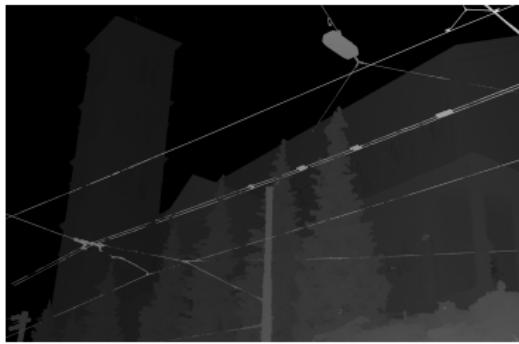
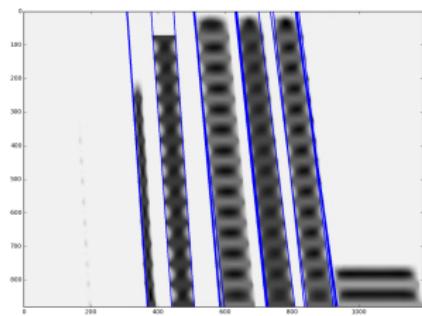
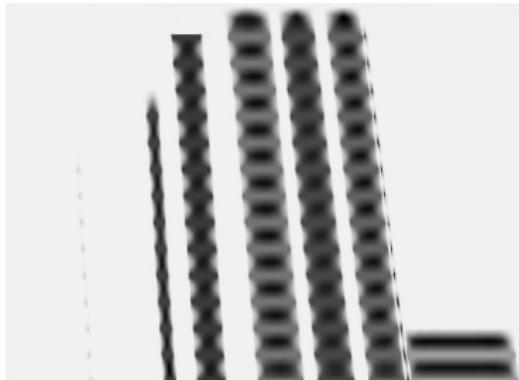
Results on line detection and depth map estimation



Results on line detection and depth map estimation



Results on line detection and depth map estimation



Conclusions and Outlook

Thanks!

Questions?

