11.06 Леонтьева Анна Викторовна

12 вариант

① $\int \frac{2x\,dx}{(22-19x^2)^{11}} =$ Пусть $t = 22-19x^2$

$x\,dx = -\frac{1}{38}dt$

$= 2\int -\frac{1}{38t^{11}}\,dt = -\frac{1}{19}\cdot\left(-\frac{1}{10t^{10}}\right) = \frac{1}{190\,t^{10}}$ [обр. замена] $= \frac{1}{190(22-19x^2)^{10}} + C$

② $\int -1{,}5\cdot(-3{,}4\cdot x^5 + 2{,}2\cdot x^2)\cdot \ln(x^2)\cdot dx =$

$= \int \frac{-3\left(\frac{11x^2}{5} - \frac{17x^5}{5}\right)\ln(x^2)\,dx}{2} = \int -3\left(\frac{11x^2}{5} - \frac{17x^5}{5}\right)\ln x\,dx =$

$= \frac{3}{5}\int(17x^5 - 11x^2)\ln x\,dx$

$\int(17x^5 - 11x^2)\ln x\,dx =$

$= \begin{bmatrix} f = \ln x & g' = 17x^5 - 11x^2 \\ f' = 1/x & g = \frac{17x^6}{6} - \frac{11x^3}{3} \end{bmatrix} = \left(\frac{17x^6}{6} - \frac{11x^3}{3}\right)\ln x - \int \frac{\frac{17x^6}{6} - \frac{11x^3}{3}}{x}\,dx =$

$= \left(\frac{17x^6}{6} - \frac{11x^3}{3}\right)\ln x - \int\left(\frac{17x^5}{6} - \frac{11x^2}{3}\right)dx =$

$= \left[\int\left(\frac{17x^5}{6} - \frac{11x^2}{3}\right) = \frac{17}{6}\int x^5dx - \frac{11}{3}\int x^2dx = \frac{17x^6}{36} - \frac{11x^3}{9}\right] =$

$= \left(\frac{17x^6}{6} - \frac{11x^3}{3}\right)\ln x - \frac{17x^6}{36} + \frac{11x^3}{9}$

$\frac{3}{5}\int(17x^5 - 11x^2)\ln x\,dx = \frac{3\left(\frac{17x^6}{6} - \frac{11x^3}{3}\right)\ln x}{5} - \frac{17x^6}{60} + \frac{11x^3}{15} + C$

③ $\int \frac{x-\frac{17}{5}}{x^3-5x^2+4x-20} = \frac{1}{5}\int \frac{5x-17}{x^3-5x^2+4x-20}dx =$

$= \frac{1}{5}\int \frac{5x-17}{x^2(x-5)+4(x-5)}dx = \frac{1}{5}\int \frac{5x-17}{(x-5)(x^2+4)}dx =$

$= \int \frac{5x-17}{(x-5)(x^2+4)} = \left[\frac{A}{x-5} + \frac{Bx+C}{x^2+4} = \frac{5x-17}{(x-5)(x^2+4)};\ x=5 \Rightarrow 29A=8 \Rightarrow A=\frac{8}{29} \right.$
$\left. A(x^2+4)+(Bx+C)(x-5)=5x-17;\ Ax^2+Bx^2=0;\ B=-A=-\frac{8}{29} \right] =$

$= \left[4A-5C=-17 \quad C=\frac{4A+17}{5} = \frac{4\cdot\frac{8}{29}+17}{5} = \frac{32/29+17}{5} = \frac{525}{29\cdot5} = \frac{105}{29} \right] =$

$= \int \left(\frac{8}{29(x-5)}\right) - \frac{8x-105}{29(x^2+4)}\Big) dx = \frac{8}{29}\int \frac{1}{x-5}dx - \frac{1}{29}\int \frac{8x-105}{x^2+4}dx =$

$= \frac{8}{29}\ln|x-5| - \frac{1}{29}\int \frac{8x-105}{x^2+4}dx$

$\int \frac{8x-105}{x^2+4}dx = \int \frac{8x}{x^2+4}dx - \int \frac{105}{x^2+4}dx = 8\int \frac{x}{x^2+4}dx - 105\int \frac{1}{x^2+4}dx =$

$= \Big[$ 1) $\int \frac{x}{x^2+4}dx = \quad t=x^2+4;\ \frac{dt}{dx}=2x \Rightarrow dx=\frac{1}{2x}dt = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\ln|t|$

Обр замена: $\frac{1}{2}\ln(x^2+4)$

2) $\int \frac{1}{x^2+4}dx;\ t=\frac{x}{2} \Rightarrow \frac{dt}{dx}=\frac{1}{2} \Rightarrow dx=2dt$

$\int \frac{2}{4t^2+4} = \frac{1}{2}\int \frac{1}{t^2+1} = \frac{1}{2}\operatorname{arctg} t = $ [обр.зам.] $= \frac{1}{2}\operatorname{arctg}\frac{x}{2} \Big] =$

$= \frac{8}{29}\int \frac{1}{x-5}dx - \frac{1}{29}\int \frac{8x-105}{x^2+4}dx = \frac{8\ln(x-5)}{29} - \frac{4\ln(x^2+4)}{29} + \frac{105\operatorname{arctg}\frac{x}{2}}{58}$

$\frac{1}{5}\int \frac{5x-17}{x^3-5x^2+4x-20}dx = \frac{8\ln(x-5)}{145} - \frac{4\ln(x^2+4)}{145} + \frac{21\operatorname{arctg} x/2}{58} + C$

11.06.22 12 вар

Леонтьева А.В.

④ $\int \frac{2\,dx}{12+\sqrt{13x+43}} = 2\int \frac{dx}{\sqrt{13x+43}+12} = \left[\begin{matrix} t = 13x+43 \\ x = \frac{t-43}{13} \end{matrix} \quad dx = \frac{1}{13}dt \right] =$

$= 2\int \frac{dt}{13(\sqrt{t}+12)} = \left[\begin{matrix} u = \sqrt{t}+12 \\ 2du = \frac{1}{\sqrt{t}}dt \end{matrix} \right] = \frac{2}{13}\int \frac{2u-24}{u}\,du =$

$= \frac{2}{13}\int \left(2 - \frac{24}{u}\right)du = \frac{2}{13}\left(2\int du - 24\int \frac{1}{u}du\right) = \frac{4}{13}u - \frac{48\ln|u|}{13} =$

$= [\text{обр. замен.}] = \frac{4\sqrt{t}}{13} - \frac{48\ln|\sqrt{t}+12|}{13} + \frac{48}{13} = [\text{обр. замена}] =$

$= -\frac{48\ln\sqrt{13x+43}+12}{13} + \frac{4\sqrt{13x+43}}{13} + \frac{48}{13} + C$

⑤ $\int \frac{2}{12\cos^2 x + 17\sin^2 x}\,dx = \left[tg\,x = \frac{\sin x}{\cos x} \right] =$

$= 2\int \frac{dx}{\cos^2 x(17tg^2 x + 12)} = \left[\begin{matrix} t = tg\,x \\ dt = 1/\cos^2 x\,dx \end{matrix} \right] =$

$= 2\int \frac{1}{17t^2+12}dt = \left[u = \frac{\sqrt{17}t}{2\sqrt{3}};\ t = \frac{2\sqrt{3}u}{\sqrt{17}} \right] =$

$= 2\int \frac{2\sqrt{3}}{\sqrt{17}(12u^2+12)}\,du = \left[dt = 2\sqrt{3}/\sqrt{17}\,du \right] = 2\int \frac{1\,du}{2\sqrt{3}\sqrt{17}(u^2+1)} =$

$= \frac{1}{\sqrt{3}\sqrt{17}}\,arctg\,u = \frac{arctg\,u}{\sqrt{3}\sqrt{17}} = [\text{обр. зам.}] = \frac{arctg\left(\frac{\sqrt{17}t}{2\sqrt{3}}\right)}{\sqrt{3}\sqrt{17}} =$

$= [\text{обр. зам.}] = \frac{arctg\left(\frac{\sqrt{17}\,tg\,x}{2\sqrt{3}}\right)}{\sqrt{3}\sqrt{17}} + C = \frac{arctg\,\frac{\sqrt{17}}{2\sqrt{3}}\,tg\,x}{\sqrt{51}} + C$

⑦ $\int_{-5}^{9} \frac{dx}{\sqrt{32-4x-x^2}} \textcircled{=} \int \frac{dx}{\sqrt{32-4x-x^2}} = \int \frac{dx}{\sqrt{-(x+2)^2+36}} =$

$= \left[t = \frac{x+2}{6} \Rightarrow \frac{dt}{dx} = \frac{1}{6};\ dx = 6dt \right] = \int \frac{6}{\sqrt{36-36t^2}}dt = \int \frac{1}{\sqrt{1-t^2}}dt = \arcsin t =$

$= [\text{обр. замен}] = \arcsin\left(\frac{x+2}{6}\right) + C$

$\textcircled{=} \arcsin \frac{x+2}{6}\Big|_{-5}^{9} = \arcsin \frac{11}{6} + \frac{\pi}{6}$

11.06.22 12 вар

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2.1

$y' = -2y$ $\quad *y \neq 0$

$\frac{dy}{dx} = -2y$; $dy = -2y\,dx$; $\frac{1}{y}dy = -2dx$ * ;

$\int \frac{1}{y}dy = \int -2dx$ $\quad \left[\int \frac{1}{y}dy = \ln(|y|) + C_1 \text{ ; } \int -2dx = -2x + C_2 \right.$ $\left. C_1 \in \mathbb{R} \text{ ; } C_2 \in \mathbb{R} \right]$

$\ln(|y|) = -2x + C \quad (C \in \mathbb{R})$

$\ln(|y|) = -2x + C \to [\ln(x) = b \Rightarrow x = e^b] \Rightarrow |y| = e^{-2x+C}$

$|y| = e^{-2x} \cdot e^C$; $|y| = Ce^{-2x}$

$\cancel{|y| = e^{-2x} \cdot C}$

При $y = 0$

$\frac{d}{dx}(0) = -2 \cdot 0$

$0 = 0 \Rightarrow y = 0$ - решение

I сл: $y = Ce^{-2x}$ $\quad$ II сл. $y = -Ce^{-2x}$

Общ. реш: $y = Ce^{-2x} \quad (C \in \mathbb{R})$

$y(0) = 13$

$13 = Ce^{-2x}$; $13 = Ce^0$; $13 = C \cdot 1 \Rightarrow C = 13$

~~[illegible]~~ $\quad$ ч/р: $y = 13e^{-2x}$

11.06.22 12 вар

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