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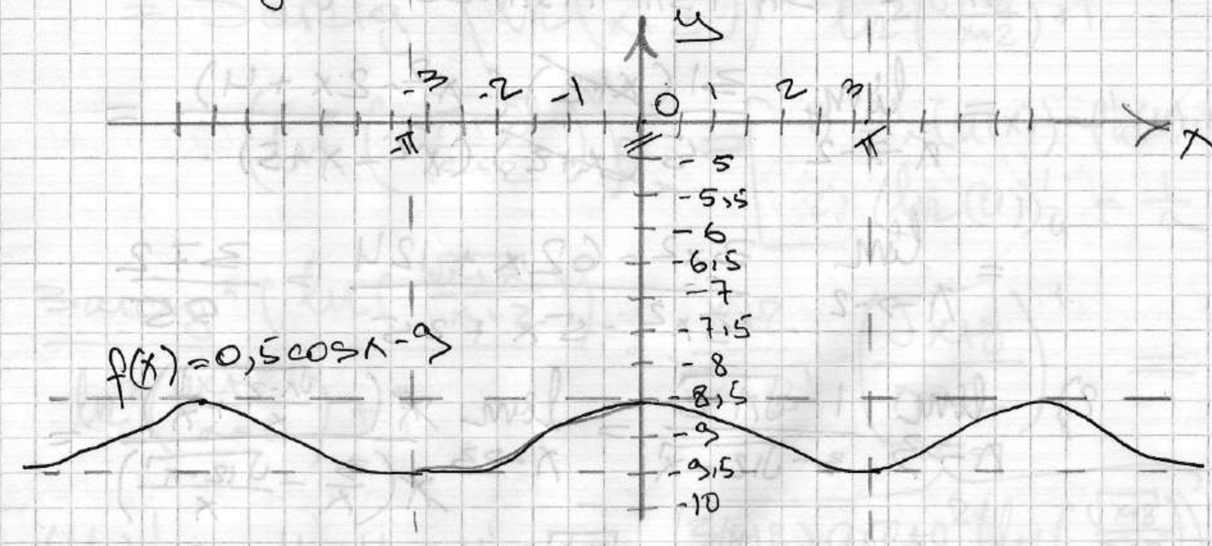
ИИТИТО ИВТ 1к. 2гр. 3пг.
07.07.2021. Вариант 16.

① $f(x) = 0{,}5\cos x - 9$; $E(f(x))$ − ?

1) $E(\cos x) = [-1; 1]$; $f = \cancel{\sin}\ \cos x$

2) $E(0{,}5\cos x) = [-0{,}5; 0{,}5]$; $f = 0{,}5\cos x$
график сжимается по вертикали в 2 раза

3) $E(0{,}5\cos x - 9) = [-9{,}5; -8{,}5]$; $f = 0{,}5\cos x - 9$
график спускается на 9 ед. вдоль оси OY



② $x_n = \dfrac{3n^2 - 5n + 16}{(-1)^n} + n$; $x_{1\ldots5}$ − ?

$x_1 = \dfrac{3\cdot1^2 - 5\cdot1 + 16}{(-1)^1} + 1 = -14 + 1 = -13$

$x_2 = \dfrac{3\cdot2^2 - 5\cdot2 + 16}{(-1)^2} + 2 = 18 + 2 = 20$

$x_3 = \dfrac{3\cdot3^2 - 5\cdot3 + 16}{(-1)^3} + 3 = -28 + 3 = -25$

$$x_4 = \frac{3\cdot 4^2 - 5\cdot 4 + 16}{(-1)^4} + 4 = 44 + 4 = 48$$

$$x_5 = \frac{3\cdot 5^2 - 5\cdot 5 + 16}{(-1)^5} + 5 = -66 + 5 = -61$$

<u>Ответ</u>: $\left\{ x_n : x_n = \frac{3n^2 - 5n + 16}{(-1)^n} + n ;\ \forall n : \begin{matrix} n \in Z ; \\ 1 \le n \le 5 \end{matrix} \right\} =$

$$= \{-13,\ 20,\ -25,\ 48,\ -61\}.$$

③ 1) $\lim\limits_{x \to -2} \frac{31x^3 + 248}{5x^3 + 5x^2 + 15x + 50} = \left\{\frac{0}{0}\right\} =$

$$= \lim_{x \to -2} \frac{31\cancel{(x+2)}(x^2 - 2x + 4)}{5\cancel{(x+2)}(x^2 - x + 5)} =$$

$$= \lim_{x \to -2} \frac{3x^2 - 62x + 124}{5x^2 - 5x + 25} = \frac{372}{55}$$

2) $\lim\limits_{x \to 3} \frac{1 - \sqrt{x-2}}{3 - \sqrt{12 - x}} = \lim\limits_{x \to 3} \frac{\cancel{x}\left(-\frac{\sqrt{x-2}}{x} + \frac{1}{x}\right)}{\cancel{x}\left(\frac{3}{x} - \frac{\sqrt{12-x}}{x}\right)} =$

$$= \lim_{x \to 3} \frac{\frac{1}{x} - \frac{\sqrt{x-2}}{x}}{\frac{3}{x} - \frac{\sqrt{12-x}}{x}} = \lim_{x \to 3} \frac{0}{0} = -3$$

3) см. на др. странице

(11) 1) $\left(y = \operatorname{arctg}^3 \ln \frac{\sqrt{8+x}}{x+2}\right)' - ?$

$$y' = \left(\operatorname{arctg}^3 \ln \frac{\sqrt{8+x}}{x+2}\right)' \quad \left[(u=v)' \Leftrightarrow u' = v' \right]$$

$$y' = \left(\operatorname{arctg}^3 \ln \frac{\sqrt{8+x}}{x+2}\right)' = \left[u^n(x)'_x = n \cdot u^{n-1}(x) \cdot u'_x(x) \right] =$$

$$= 3 \cdot \operatorname{arctg}^2 \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right) \cdot \left(\operatorname{arctg} \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right) \right)' =$$

$$= \left[\begin{array}{l} 1)\ f'_x(u(x)) = f'_u(u) \cdot u'_x(x) \\ 2)\ (\operatorname{arctg}(u))'_u = \frac{1}{u^2+1} \end{array} \right] =$$

$$= 3 \operatorname{arctg}^2 \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right) \cdot \frac{1}{\ln^2 \left(\frac{\sqrt{x+8}}{x+2} \right) + 1} \cdot$$

$$\cdot \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right)'_x = \left[\begin{array}{l} 1)\ f'_x(u(x)) = f'_u(u) \cdot u'_x(x) \\ 2)\ (\ln(u))'_u = \frac{1}{u} \end{array} \right] =$$

$$= \frac{3 \operatorname{arctg}^2 \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right)}{\ln^2 \left(\frac{\sqrt{x+8}}{x+2} \right) + 1} \cdot \frac{x+2}{\sqrt{x+8}} \cdot \underbrace{\left(\frac{\sqrt{x+8}}{x+2} \right)'_x}_{\exists t =} =$$

$$= \left[\left(\frac{u}{v} \right)' = \frac{u' \cdot v - v' \cdot u}{v^2} \right] = \overbrace{\frac{3(x+2) \operatorname{arctg}^2 \left(\ln \left(\frac{\sqrt{x+8}}{x+2} \right) \right)}{\sqrt{x+8} \left(\ln^2 \left(\frac{\sqrt{x+8}}{x+2} \right) + 1 \right)}} \cdot$$

$$\cdot \frac{(\sqrt{x+8})'_x \cdot (x+2) - (x+2)'_x \cdot \sqrt{x+8}}{(x+2)^2} = \left[\begin{array}{l} 1)\ (u+v)' = u' + v' \\ 2)\ u^n(x)'_x = n \cdot u^{n-1}(x) \cdot \\ \quad \cdot u'_x(x) \end{array} \right] =$$

$$= t \cdot \frac{\frac{1}{2} \cdot \frac{1}{\sqrt{x+8}} \cdot (x+8)'_x \cdot x+2 - ((x)'_x + (2)'_x) \cdot \sqrt{x+8}}{(x+2)^2} =$$

$$= \left[(u+v)' = u' + v' \right] = t \cdot \frac{\frac{x+2}{2\sqrt{x+8}} \cdot \left((x)'_x + (8)'_x \right) - (1+0) \cdot \sqrt{x+8}}{(x+2)^2} =$$

$$= t \cdot \frac{\frac{x+2}{2\sqrt{x+8}} \cdot (1+0) - \sqrt{x+8}}{(x+2)^2} =$$

$$= \frac{3(x+2)\operatorname{arctg}^2\left(\ln\left(\frac{\sqrt{x+8}}{x+2}\right)\right)}{\sqrt{x+8}\cdot\left(\ln^2\left(\frac{\sqrt{x+8}}{x+2}\right)+1\right)} \cdot \frac{\frac{x+2}{2\sqrt{x+8}} - \sqrt{x+8}}{(x+2)^2} =$$

$$= \frac{3\left(\frac{x+2}{2\sqrt{x+8}} - \sqrt{x+8}\right)\operatorname{arctg}^2\left(\ln\left(\frac{\sqrt{x+8}}{x+2}\right)\right)}{(x+2)\sqrt{x+8}\left(\ln^2\left(\frac{\sqrt{x+8}}{x+2}\right)+1\right)}$$

2) — — — — — — — — — — — —

$$y' = \left(\left(\sqrt{x^2+10x}\right)^{\operatorname{arcctg}(x+x^2)} \right)' = \left[u^{f(x)} = e^{f(x)\ln(u)} \right] =$$

$$= \left(e^{\frac{\operatorname{arcctg}(x^2+x)\ln(x^2+10x)}{2}} \right)' = \left[e^{u(x)}{}'_x = e^{u(x)} \cdot u'_x(x) \right] =$$

$$= \underbrace{e^{\frac{\operatorname{arcctg}(x^2+x)\ln(x^2+10x)}{2}}}_{t} \cdot \left(\frac{\operatorname{arcctg}(x^2+x)\ln(x^2+10x)}{2} \right)' =$$

$$= \frac{t}{2} \cdot \left(\operatorname{arcctg}(x^2+x) \ln(x^2+10x) \right)' = \left[u \cdot v = u' \cdot v + v' \cdot u \right] =$$

$$= \frac{t}{2} \cdot \left((\operatorname{arcctg}(x^2+x))'_x \cdot \ln(x^2+10x) + (\ln(x^2+10x))'_x \cdot \operatorname{arcctg}(x^2+x) \right) = \left[f'_x(u(x)) = f'_u(u) \cdot u'_x(x) \right] =$$

$$= \frac{t}{2} \cdot \left(-\frac{\ln(x^2+10x)}{(x^2+x)^2+1} \cdot (x^2+x)'_x + \frac{\operatorname{arcctg}(x^2+x)}{x^2+10x} \cdot (x^2+10x)'_x \right) =$$

$$= \begin{bmatrix} 1)\ (u+v)' = u' + v' \\ 2)\ (au + bv)' = a\cdot u' + b\cdot v' \end{bmatrix} =$$

$$= \frac{t}{2}\left(-\frac{\ln(x^2+10x)}{(x^2+x)^2+1}\cdot\left((x^2)'_x + (x)'_x\right) + \frac{\operatorname{arcctg}(x^2+x)}{x^2+10x}\cdot\right.$$

$$\left.\cdot\left((x^2)'_x + 10\cdot(x)'_x\right)\right) = \left[(x^n)' = n\cdot x^{n-1}\right] =$$

$$= \frac{1}{2}\cdot(x^2+10x)^{\frac{\operatorname{arcctg}(x^2+x)}{2}}\cdot\left(\frac{(2x+10)\operatorname{arcctg}(x^2+x)}{x^2+10x} - \right.$$

$$\left. - \frac{(2x+1)\ln(x^2+10x)}{(x^2+x)^2+1}\right)$$

$$(3) \Rightarrow \lim_{x\to 0}\frac{\sin(x^3) - \operatorname{tg}(x)\cdot\sin(x^2)}{x^3\ \sin(x^2)} =$$

$$= \left[\begin{array}{l}\text{вспомним эк-вые преобр-ния:} \\ \sin(x^2) = x^2;\ \sin(x^3) = x^3;\ \text{подставим}\end{array}\right] =$$

$$= \lim_{x\to 0}\frac{x^3 - \operatorname{tg}(x)\cdot x^2}{x^5} = \left[\begin{array}{l}\text{1ый замечат.} \\ \text{предел: } \lim_{x\to 0}\operatorname{tg}(x) = 0\end{array}\right] =$$

$$= \lim_{x\to 0}\frac{x^2(x - \operatorname{tg}(x))}{x^5} = x^{-2} \Rightarrow$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin(x^3) - \operatorname{tg}(x)\cdot\sin(x^2)}{x^3\cdot\sin(x^2)} = \infty$$