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1.1

$$\int\frac{x\,dx}{(11-30x^2)^{12}} = \cancel{\left[m=1,\ n=2,\ p=-12\right]} = \frac{1}{2}\int\frac{d(x^2)}{(11-30x^2)^{12}} = -\frac{1}{60}\int\frac{d(11-30x^2)}{(11-30x^2)^{12}} = \frac{1}{660\,(11-30x^2)^{11}}+C$$

1.2

$$\int(3x^8+20x)\ln(x^3)\,dx = 3\int(3x^8+20x)\ln x\,dx = \left[\begin{array}{l} u=\ln x \Rightarrow u'=\frac{1}{x} \\ v'=(3x^8+20x) \Rightarrow v=\frac{x^9}{3}+10x^2 \end{array}\right] =$$

$$= 3\ln x\left(\frac{x^9}{3}+10x^2\right) - 3\int\left(\frac{x^9}{3}+10x^2\right)\cdot\frac{1}{x}\,dx = (x^9+30x^2)\ln x - \int x^8dx - 30\int x\,dx =$$

$$= (x^9+30x^2)\ln x - \frac{x^9}{9} - 15x^2 + C$$

1.3

$$\int\frac{x-\frac{17}{5}}{x^3-2x^2+3x-6}\,dx = \int\frac{x-\frac{17}{5}}{(x^2+3)(x-2)}\,dx = \left[\begin{array}{l} \frac{Ax+B}{(x^2+3)}+\frac{C}{x-2} = \frac{x-\frac{17}{5}}{(x^2+3)(x-2)} \\ Ax^2-2Ax+Bx-2B+Cx^2+3C = x-\frac{17}{5} \\ \begin{cases} A+C=0 \\ -2A+B=1 \\ -2B+3C=-\frac{17}{5} \end{cases} \begin{cases} A=-C \\ B=1-2C \\ -2+4C+3C=-\frac{17}{5} \end{cases} \begin{cases} A=-C \\ B=1-2C \\ 7C=-\frac{7}{5} \end{cases} \begin{cases} A=\frac{1}{5} \\ B=\frac{7}{5} \\ C=-\frac{1}{5} \end{cases} \end{array}\right] =$$

$$= \int\frac{x+7}{5(x^2+3)}\,dx - \int\frac{x}{5(x-2)}\,dx = \frac{1}{5}\int\frac{x\,dx}{x^2+3} + \frac{7}{5}\int\frac{dx}{x^2+3} - \frac{1}{5}\int\frac{x-2+2}{x-2}\,dx = \frac{1}{10}\int\frac{d(x^2+3)}{x^2+3} + \frac{7}{5}\int\frac{dx}{x^2+(\sqrt{3})^2} - \frac{1}{5}\int dx - \frac{2}{5}\int\frac{dx}{x-2} =$$

$$= \frac{1}{10}\ln(x^2+3) + \frac{7}{5\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}} - \frac{x}{5} - \frac{2}{5}\ln|x-2| + C$$

1.4

$$\int\frac{2\,dx}{1+\sqrt{2x+1}} = \left[\begin{array}{l} 2x+1=t^2 \Rightarrow x=\frac{t^2-1}{2} \\ dx=t\,dt \quad t=\sqrt{2x+1} \end{array}\right] = \int\frac{2t\,dt}{1+t} = 2\int\frac{1+t-1}{1+t}\,dt = 2\int dt - 2\int\frac{dt}{1+t} = 2t - 2\ln|1+t| + C =$$

$$= 2\sqrt{2x+1} - 2\ln|\sqrt{2x+1}| + C = 2\sqrt{2x+1} - \ln|2x+1| + C$$

1.5

$$\int\frac{dx}{2\cos^2x+3\sin^2x} = \int\frac{dx}{1+\cos 2x+\frac{3}{2}-\frac{3}{2}\cos 2x} = \int\frac{2\,dx}{5-\cos 2x} = \int\frac{d(2x)}{5-\cos 2x} = \left[\begin{array}{l} t=tg\,x \quad d(2x)=\frac{2dt}{1+t^2} \\ \cos(2x)=\frac{1-t^2}{1+t^2} \end{array}\right] =$$

$$= \int\frac{\frac{2dt}{1+t^2}}{5-\frac{1-t^2}{1+t^2}} = \int\frac{2dt}{5t^2+5-1+t^2} = \int\frac{2dt}{6t^2+4} = \frac{1}{3}\int\frac{dt}{t^2+\frac{2}{3}} = \frac{3\sqrt{3}}{\sqrt{2}}\cdot\mathrm{arctg}\,\frac{t\sqrt{3}}{\sqrt{2}} + C = 3\sqrt{\frac{3}{2}}\,\mathrm{arctg}\left(\sqrt{\frac{3}{2}}\,tg\,x\right) + C$$

1.6

$$\int x\cdot\arcsin(5x)\,dx = \left[\begin{array}{l} u=\arcsin(5x) \Rightarrow u'=\frac{5}{\sqrt{1-25x^2}} \\ v'=x \Rightarrow v=\frac{x^2}{2} \end{array}\right] = \arcsin(5x)\cdot\frac{x^2}{2} - \int\frac{5x^2\,dx}{2\sqrt{1-25x^2}} =$$

$$\cancel{= \arcsin(5x)\cdot\frac{x^2}{2} - 5\int\frac{x\,dx}{\sqrt{\frac{1}{x^2}-25}} = \left[\begin{array}{l} t^2=\frac{1}{x^2}-25 \quad dx=\left(\frac{1}{\sqrt{t^2+25}}\right)' = -\frac{t\,dt}{(t^2+25)\sqrt{t^2+25}} \\ x=\frac{1}{\sqrt{t^2+25}} \end{array}\right] =}$$

$$\cancel{= \arcsin(5x)\cdot\frac{x^2}{2} - 5\int\frac{t\,dt}{t\sqrt{t^2+25}\sqrt{t^2+25}(t^2+25)} = \arcsin(5x)\cdot\frac{x^2}{2} - 5\int\frac{dt}{(t^2+25)^2} =}$$

$$= \arcsin(5x)\cdot\frac{x^2}{2} - \frac{5}{2}\int\frac{x^2dx}{\sqrt{1-25x^2}} = \left[\int\frac{x^2dx}{\sqrt{1-25x^2}} = \left[\begin{array}{l} x=\frac{\sin t}{5} \Rightarrow dx=\frac{\cos t\,dt}{5} \\ t=\arcsin(5x) \end{array}\right] = \int\frac{\sin^2t\cos t\,dt}{125\sqrt{1-\sin^2t}} = \frac{1}{125}\int\frac{\sin^2t\cos t\,dt}{\cos t} =\right.$$

$$\left. = \frac{1}{125}\int\sin^2t\,dt = \frac{1}{250}\int(1-\cos 2t)\,dt = \frac{1}{250}\int dt - \frac{1}{250}\int\cos 2t\,dt = \frac{t}{250} - \frac{\sin 2t}{500} + C = \frac{\arcsin(5x)}{250} - \frac{5x\cdot\sqrt{1-25x^2}}{250} + C\right] =$$

$$= \arcsin(5x)\cdot\frac{x^2}{2} - \frac{\arcsin(5x)}{100} + \frac{x\sqrt{1-25x^2}}{20} + C$$

1.7

$$\int_{-3}^{8} \frac{dx}{\sqrt{16-6x-x^2}} = \int_{-3}^{8} \frac{dx}{\sqrt{25-(x+3)^2}} = \left[\begin{matrix} x+3=5t, & t=\frac{x+3}{5} \\ dx=5dt \end{matrix}\right] = \int_{\cancel{-3}\ 0}^{\cancel{8}\ \frac{11}{5}} \frac{5dt}{\sqrt{25-25t^2}} = \int_{\cancel{-3}\ 0}^{\cancel{8}\ \frac{11}{5}} \frac{5dt}{5\sqrt{1-t^2}} = \int_{\cancel{-3}\ 0}^{\cancel{8}\ \frac{11}{5}} \frac{dt}{\sqrt{1-t^2}} =$$

$$= \arcsin(t) \Big|_{\cancel{-3}\ 0}^{\cancel{8}\ \frac{11}{5}} = \arcsin\left(\frac{11}{5}\right) - \arcsin(0) = \arcsin\left(\frac{11}{5}\right)$$

2.1

$$y' = -2y \qquad y(0) = 102$$

$$\frac{dy}{dx} = -2y$$

$$\frac{dy}{y} = -2dx$$

$$\int \frac{dy}{y} = -2\int dx$$

$$\ln|y| = -2x + C^*$$

$$y = e^{-2x+C^*} = e^{C^*} \cdot e^{-2x} = C \cdot e^{-2x}$$

$$y(0) = C \cdot e^{-2\cdot 0} = C \cdot 1 = 102$$

$$C = 102$$

Ответ: частное решение: $y = 102 \cdot e^{-2x}$

2.2

$$xy' = 2\sqrt{103x^2+y^2} + y$$

$$\lambda x y' = 2\sqrt{\lambda^2 103x^2 + \lambda^2 y^2} + \lambda y$$

$\cancel{\lambda} x y' = 2\cancel{\lambda}\sqrt{103x^2 + \cancel{\lambda} y^2} + \cancel{\lambda} y$ => однородное

Замена: $y = tx \Rightarrow y' = t'x + t$, $t = \frac{y}{x}$

$$x(t'x+t) = 2\sqrt{103x^2 + \cancel{(tx)^2}\ t^2x^2} + tx$$

$$x(t'x+t) = 2x\sqrt{103+t^2} + tx$$

$$t'x + t = 2\sqrt{103+t^2} + t$$

$$\frac{dt}{dx} = \frac{2\sqrt{103+t^2}}{x}$$

$$\int \frac{dt}{\sqrt{103+t^2}} = 2\int \frac{dx}{x}$$

$$\ln\left|t + \sqrt{t^2+103}\right| = \ln x^2 + \ln|C|$$

$$t + \sqrt{t^2+103} = x^2 + C$$

$$\frac{y}{x} + \sqrt{\frac{y^2}{x^2}+103} = x^2 + C$$

Ответ: общий интеграл: $\frac{y}{x} + \sqrt{\frac{y^2}{x^2}+103} - x^2 = C$, $C = const$