11.06.2022

Бережной М. А. ИВТ 2 курс 1 группа 1 подгруппа.

① $\int \frac{3x\,dx}{(23-18x^2)^{10}} = [23-18x^2=t \Rightarrow dt=-36x\,dx \Rightarrow x\,dx=-\frac{dt}{36}] \Rightarrow$

$= \int -\frac{3\,dt}{36t^{10}} = -\frac{1}{12}\int t^{-10}dt = -\frac{1}{12}\cdot\frac{t^{-9}}{-9}+C = \frac{1}{108t^9}+C = [t=23-18x^2] =$

$= \frac{1}{108(23-18x^2)^9}+C$ Ответ.

② $\int -2(-3,5x^4+2,4x^3)\cdot\ln(x^3)\cdot dx = -2\int(-3,5x^4+2,4x^3)\cdot\ln(x^3)\,dx =$

$= -2\int(-3,5x^4(\ln(x^3))+2,4x^3(\ln(x^3)))dx = 7\int x^4\ln(x^3)dx - 4,8\int x^3\ln(x^3)dx$

1) $\int x^4\ln(x^3)dx = \frac{x^5}{5}\ln(x^3) - \int\frac{x^5}{5}\cdot\frac{3}{x}dx = \frac{x^5}{5}\ln(x^3) - \frac{3}{5}\int x^4dx =$

$\int\downarrow \quad \downarrow$

$\frac{x^5}{5} \quad \frac{3}{x}$

$= \frac{x^5}{5}\ln(x^3) - \frac{3x^5}{25}+C$

2) $\int x^3\ln(x^3)dx = \frac{x^4}{4}\ln(x^3) - \int\frac{x^4}{4}\cdot\frac{3}{x}dx = \frac{x^4}{4}\ln(x^3) - \frac{3x^4}{16}$

$\int\downarrow \quad \downarrow$

$\frac{x^4}{4} \quad \frac{3}{x}$

общ: $7\cdot\left(\frac{x^5}{5}\ln(x^3)-\frac{3x^5}{25}\right) - 4,8\left(\frac{x^4}{4}\ln(x^3)-\frac{3x^4}{16}\right)+C=$

$=\left(\frac{7x^5}{5}\ln(x^3-\frac{21x^5}{25}\right)-\left(\frac{4,8x^4}{4}\ln(x^3)-\frac{14,4x^4}{16}\right)+C$ Ответ:

③ $\int\frac{x-\frac{17}{5}}{x^3-6x^2+4x-24}dx = \int\frac{x-\frac{17}{5}}{(x-6)(x^2+4)} = [\frac{A}{x-6}+\frac{Bx+C}{x^2+4} = \frac{x-\frac{17}{5}}{(x-6)(x^2+4)}$

$A(x^2+4)+(Bx+C)(x-6) = x-\frac{17}{5}$

$Ax^2+4A+Bx^2-6Bx+Cx-6C = x-\frac{17}{5}$

$x^2(A+B)+x(-6B+C)+(4A-6C) = x-\frac{17}{5}$

$\begin{cases} A+B=0 \\ -6B+C=1 \\ 4A-6C=-\frac{17}{5} \end{cases} \Rightarrow \begin{cases} A=-B \\ C=1+6B \\ 4A-6C=-\frac{17}{5} \end{cases} \Rightarrow$

$-4B-6(1+6B)=-\frac{17}{5}$

$-4B-6-36B=-\frac{17}{5}$

$-40B=6-\frac{17}{5}$

$-40B = \frac{13}{5}$

$B = -\frac{13}{5\cdot 40} = -\frac{13}{200} \Rightarrow A = \frac{13}{200}$

$C = 1 + 6\cdot\left(-\frac{13}{200}\right) = 1 - \frac{3\cdot 13}{100} = 1 - \frac{39}{100} = \frac{61}{100}\Big] =$

$= \frac{13}{200}\int\frac{dx}{x-6} + \int\frac{-\frac{13}{200}x + \frac{61}{100}}{x^2+4}dx = \underbrace{\frac{13}{200}\int\frac{dx}{x-6}}_{1} - \underbrace{\frac{13}{200}\int\frac{x - \frac{122}{13}}{x^2+4}dx}_{2}$

1) $\frac{13}{200}\int\frac{dx}{x-6} = \frac{13}{200}\ln|x-6| + C$

2) $-\frac{13}{200}\int\frac{x-\frac{122}{13}}{x^2+4}dx = -\frac{13}{200}\int\frac{x\,dx}{x^2+4} + \frac{\cancel{13}\cdot \overset{61}{\cancel{122}}}{\underset{100}{\cancel{200}}\cdot\cancel{13}}\int\frac{dx}{x^2+4} = \Big[x^2 = t \Rightarrow dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2}\Big] =$

$= -\frac{13}{200\cdot 2}\int\frac{dt}{t+4} + \frac{122}{200}\cdot\frac{1}{2}\cdot \operatorname{arctg}\frac{x}{2} + C = -\frac{13}{200\cdot 2}\cdot\ln|x^2+4| + \frac{122}{200\cdot 2}\cdot$

$\cdot \operatorname{arctg}\frac{x}{2} + C$

Отв: $\frac{13}{200}\ln|x-6| - \frac{13}{400}\ln|x^2+4| + \frac{122}{400}\operatorname{arctg}\frac{x}{2} + C$ Ответ

④ $\int\frac{2}{13+\sqrt{14x+42}}dx = 2\int\frac{dx}{13+\sqrt{14x+42}} = [14x+42 = t] = 2\int\frac{dt}{13+\sqrt{t}} = \Big[$

$y = 13 + \sqrt{t} \Rightarrow dy = \frac{1}{2\sqrt{t}}dt;\ dt = 2\sqrt{t}\,dy$

$\sqrt{t} = y - 13;\ dt = 2(y-13)dy\Big] = 2\cdot 2\int\frac{y-13}{y}dy = 4\int dy - 4\int\frac{13}{y} =$

$= 4y - 4\cdot 13\ln|y| + C = 4(13+\sqrt{t}) - 52\ln|13+\sqrt{t}| + C = 4(13+\sqrt{14x+42}) -$

$-52\ln|13+\sqrt{14x+42}| + C$ Ответ.

⑤ $\int\frac{2dx}{13\cos^2(x) + 17\sin^2(x)} = \int\frac{3dx}{13(1-\sin^2(x)) + 17\sin^2(x)} =$

$= \int\frac{3dx}{13 - 13\sin^2x + 17\sin^2x} = \int\frac{3dx}{4\sin^2x + 13} = 3\int\frac{1}{\cos^2x}\cdot\frac{1}{4tg^2x + 13}dx =$

$= \Big[t = tg\,x \Rightarrow dt = \frac{dx}{\cos^2x}\Big] = 3\int\frac{dt}{4t^2+13} = \frac{3}{4}\int\frac{dt}{t^2+\frac{13}{4}} = \frac{3}{4}\Big(\frac{1}{\sqrt{\frac{13}{4}}}\cdot$

$\cdot \operatorname{arctg}\frac{x}{\sqrt{\frac{13}{4}}}\Big) + C = \frac{3\sqrt{13}}{\cancel{4\sqrt{4}}\ 8}\cdot\operatorname{arctg}\frac{x\sqrt{13}}{2} + C$ Ответ.

⑥ $\int x\cdot \arcsin(17x)\,dx = \arcsin(17x)\cdot\frac{x^2}{2} - \int\frac{17x^2\,dx}{2\sqrt{1-289x^2}} =$

$\downarrow \frac{x^2}{2} \quad \downarrow \frac{17}{\sqrt{1-289x^2}}$

$= \arcsin(17x)\cdot\frac{x^2}{2} + \frac{17}{2}\int\frac{x^2\,dx}{\sqrt{1-289x^2}} =$

$= \left[\frac{x^2\,dx}{\sqrt{1-289x^2}} = \begin{cases} x = \frac{\sin t}{17} \Rightarrow dx = \frac{\cos t\,dt}{17} \\ t = \arcsin(17x)\end{cases}\right] =$

$= \frac{1}{4913}\int\frac{\sin^2 t\cdot\cos t\,dt}{1-\left(\cancel{289}\cdot\frac{\sin^2 t}{\cancel{289}}\right)} = \frac{1}{4913}\int\frac{\sin^2 t\cos t\,dt}{\underbrace{1-\sin^2 t}_{\cos^2 t}} = \frac{1}{4913}\int\frac{\sin^2 t\,dt}{\cos t} =$

$= \frac{1}{4913}\int \mathrm{tg}\,t\,\sin t\,dt = \mathrm{tg}\,t(-\cos t) - \int\frac{1}{\cos^2 t}\cdot(-\cos t)\,dt =$

$\downarrow \frac{1}{\cos^2 t} \quad \downarrow -\cos t$

$= -\cancel{\mathrm{tg}\,t\cdot\cos t} - \sin t + \int\frac{dt}{\cos t} =$

$= -\sin t + \ln\left|\mathrm{tg}\left(\frac{t}{2}+\frac{\pi}{4}\right)\right| + C \Big] =$

$= -\sin(\arcsin(17x)) + \ln\left|\mathrm{tg}\left(\frac{\arcsin(17x)}{2}+\frac{\pi}{4}\right)\right| + C$

$= \underline{\arcsin(17x)\cdot\frac{x^2}{2} + \frac{17}{2}\left(-17x + \ln\left|\mathrm{tg}\left(\frac{\arcsin(17x)}{2}+\frac{\pi}{4}\right)\right|\right) + C}$

ответ.

⑦ $\int_{-5}^{10}\frac{dx}{\sqrt{45-4x-x^2}} = \int_{-5}^{10}\frac{dx}{\sqrt{49-(x+2)^2}} = \left[\begin{matrix} x+2 = 7t \Rightarrow dt = \frac{dx}{7} \Rightarrow \\ \Rightarrow dx = 7\,dt \quad t = \frac{x+2}{7}\end{matrix}\right] =$

$= \int_{-5}^{10}\frac{7\,dt}{\sqrt{49-(7t)^2}} = \int_{-5}^{10}\frac{\cancel{7}\,dt}{\sqrt{49-49t^2}} = \int_{-5}^{10}\frac{dt}{\sqrt{1-t^2}} = \arcsin(t)\Big|_{-5}^{10} =$

$= \arcsin\left(\frac{x+2}{7}\right)\Big|_{-5}^{10} = \arcsin\left(-\frac{3}{7}\right) - \arcsin\left(\frac{12}{7}\right)$

$\boxed{\text{Задание 2}}$

① $y' = -2y,\ y(0) = 14$

$y' = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -2y \Rightarrow dy = -2y\,dx \;\Big|\cdot\frac{1}{y},\ y\neq 0$

$\frac{dy}{y} = -2\,dx$

$\int\frac{dy}{y} = -\int 2\,dx \Rightarrow \ln|y| = -2x + C \Rightarrow y = e^{C-2x} \Rightarrow \underline{y = \frac{C}{e^{2x}}}$

отв.

Част. реш:

$$14 = \frac{C}{e^{2\cdot 0}} = C \Rightarrow C = 14$$

Ответ: част. реш: $y = \frac{14}{e^{2x}}$

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(2)

$$xy' = 2\sqrt{ax^2+y^2}+y$$

$$y' = \frac{dy}{dx} \Rightarrow x\frac{dy}{dx} = 2\sqrt{\cancel{a}\,15x^2+y^2}+y$$

$$x\,dy = dx\left(2\sqrt{15x^2+y^2}+y\right)$$

$$u = \frac{y}{x} \Rightarrow y = ux \Rightarrow dy = u\,dx + x\,du$$

$$x(u\,dx + x\,du) = \left(2\sqrt{15x^2+u^2x^2}+ux\right)dx$$

$$x(u\,dx + x\,du) = \left(2\sqrt{15+u^2}+u\right)x\,dx$$

$$\cancel{ux\,dx} + x^2du = 2\sqrt{15+u^2}\,(x\,dx) + \cancel{ux\,dx}$$

$$x^2du = 2\sqrt{15+u^2}\,(x\,dx) \Big| \cdot \frac{1}{x^2\sqrt{15+u^2}}$$

$$\frac{du}{\sqrt{u^2+15}} = \frac{2\,dx}{x} \Rightarrow \int\frac{du}{\sqrt{u^2+15}} = \int\frac{2\,dx}{x}$$

$$\ln\left|\sqrt{u^2+15}+u\right| = 2\ln|x| + C$$

$$\sqrt{u^2+15}+u = e^C x^2$$

$$\sqrt{\frac{y^2}{x^2}+15} + \frac{y}{x} = Cx^2$$ Ответ: