Relativistic Wave Mechanics and The Zitter Model

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***Abstract*—This paper presents an approach to special relativity which is more in line with electrical engineering, namely as the harmonic analysis of a linear system. The approach is derived from Hestenes' *Zitter* model for the electron[1], [2], which assumes an internal structure of a light-like helix in spacetime. Several fundamental quantum mechanical equations are generated, one of which is a concise form of the Dirac equation. In addition, the grade (dimension) of the quantities involved match their units. Since the model presented is based on a classical physics, it is a 'local hidden-variable' theory which is not consistent with Bell's theorem.**

I. Introduction

A. Context

A few years ago we applied Conformal Geometric Algebra to transmission line theory[3], and found that this produced many relativistic-like relationships. However, at it's core, transmission line theory takes for granted several relations which are derived from the time-harmonic analysis of Maxwell's equations. In order to provide an appropriate foundation for the transmission-line model, we began an effort to derive the transmission line equations directly from Spacetime Algebra[4]. During this time, David Hestenes had shared with us his Zitter electron model[1]. The combination of these two ideas is what led to the current approach.

II. Spacetime Kinematics and Waves

This section introduces spacetime algebra[4] and interprets kinematics through the differential geometry of a worldline in a simplified way. A more comprehensive introduction can be found elsewhere[1], [5], [6], but we provide a minimal setup needed for the rest of the paper.

A. Spacetime Algebra

Spacetime algebra is the geometric algebra of four dimensional minkowski space. It can be defined in terms of an ortho-normal vector basis with a time direction γ_0 which squares to $+1$, and three spacial directions $\gamma_{1,2,3}$ which square to -1,

$$\begin{aligned}-\gamma_0^2 = \gamma_1^2 = \gamma_2^2 = \gamma_3^2 =& -1\\ \gamma_j \cdot \gamma_k =& \delta_{jk}.\end{aligned}$$

Through the use of the outer product between vectors denoted with the wedge symbol $\wedge$, higher order basis elements representing planes, volumes, etc are generated. The bivector $\gamma_1 \wedge \gamma_2$ is a mathematical element which represents the plane spaned by γ_1 and γ_2, for instance. Simple bivectors (planes) and trivectors (volumes) are denoted with subscripts for brevity as such,

$$\begin{aligned}\gamma_{12} &\equiv \gamma_1 \wedge \gamma_2,\\ \gamma_{123} &\equiv \gamma_1 \wedge \gamma_2 \wedge \gamma_3.\end{aligned}$$

There is a single unit *psuedoscalar*, represented by I, which is a 4-volume that squares to -1.

$$I \equiv \gamma_{0123}, \qquad I^2 = -1 \tag{1}$$

Two other basis blades frequently used also square to -1, the trivector γ_{012} and the bivector γ_{12}.

In summary, the Dirac algebra can be made up from the following blades,

$$\begin{gathered}1\\ \gamma_0, \gamma_1, \gamma_2, \gamma_3\\ \gamma_{01}, \gamma_{12}, \gamma_{23}, \gamma_{03}, \gamma_{13}, \gamma_{02},\\ \gamma_{123}, \gamma_{230}, \gamma_{301}, \gamma_{012}\\ \gamma_{0123}\end{gathered}$$

B. Velocity

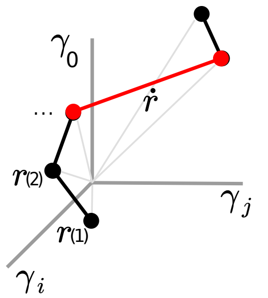
A point-like object's history in spacetime as a curve in four dimensional spacetime, known as a 'worldline'. All the particle kinematics can be interpreted as geometric properties of the curve itself. The curve can be thought of as a discrete set of spacetime 'events' which are represented by vectors, denoted by $r(\tau)$, where τ is some independent variable. In the limit of infinitesimal τ, the curve becomes smooth and we assume this is possible. By taking the difference between two adjacent points $r(a)$ and $r(b)$, we construct a vector differential, denoted with the overdot.

$$\dot{r}(a) \equiv r(b) - r(a). \tag{2}$$

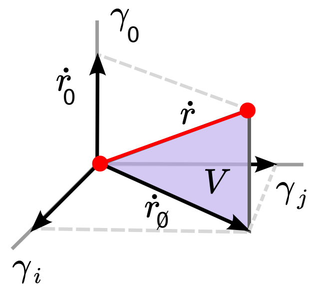
This differential represents the discrete arc-length of the path, as illustrated in Figure 1a. Going from the differentials to positions requires a summation,

$$r(\tau) = \sum_0^\tau \dot{r} + r(0). \tag{3}$$

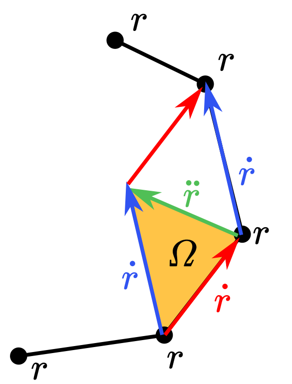
From here on, τ-indexing is implied where appropriate, so we can drop the parenthesis and just refer to $\dot{r}$. To compute the velocity at any point along the path, first decompose the differential into space and time components,



(a) A discrete curve in spacetime highlighting a single differential.



(b) The velocity bivector V created from the spacetime split of the differential $\dot{r}$. ($\dot{r}_0$ should be inverted, which alters the magnitude, but not the direction of V)



(c) The position r, first differential $\dot{r}$, and second differential $\ddot{r}$ for a path. The centripetal acceleration bivector shown as Ω. Indexing on all quantities is implied.

Figure 1: A spacetime path projected into 3D, with differentials.

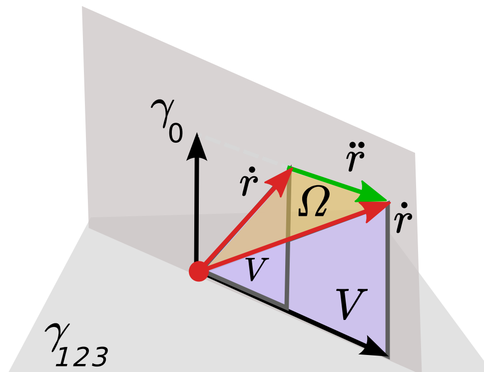


Figure 2: A pair of differentials related by linear acceleration. The subspace γ_{123} is actually a volume, but is depicted as a plane.

$$\begin{aligned} \dot{r} &= \dot{r}\gamma_0\gamma_0^{-1} && (4)\\ &= \dot{r}\cdot\gamma_0\gamma_0^{-1} + \dot{r}\wedge\gamma_0\gamma_0^{-1} && (5)\\ &= \dot{r}_0 + \dot{r}_\emptyset, && (6) \end{aligned}$$

where $\dot{r}_0$ is the vector projection along γ_0, and $\dot{r}_\emptyset$ is the vector rejection from γ_0. The velocity is the ratio of space-like component to the time-like component, as shown in Figure 1b,

$$V = \frac{\dot{r}\wedge\gamma_0}{\dot{r}\cdot\gamma_0} = \frac{\dot{r}_\emptyset}{\dot{r}_0} \tag{7}$$

This is a projective relation between $\dot{r}$ and V, and it results in a bivector V. Since time and space have opposite signatures, V is a minkowski bivector meaning it squares to a positive number. Since Geometric Algebra is non-commutative, division is interpreted as right division, ie $a/b \equiv ab^{-1}$.

C. Acceleration

Given velocity we can define acceleration as a change in velocity. From the projective nature of V, we can see that changing V requires a change in the *direction* of $\dot{r}$, because $\dot{r}$ and $\lambda\dot{r}$ produce the same V. One way to quantify the change in $\dot{r}$-direction would be to compute the bivector between two neighboring differentials, as shown in Figure 1c. Let us define the acceleration bivector Ω to be,

$$\Omega \equiv \dot{r}(a)\wedge\dot{r}(b) \tag{8}$$

Ω can also be formulated with the second differential. Given three neighboring position points, there are two neighboring differentials, $\dot{r}(a)$ and $\dot{r}(b)$. The differential between these points yields the second differential, denoted $\ddot{r}$,

$$\ddot{r}(a) \equiv \dot{r}(b) - \dot{r}(a) \tag{9}$$

Substituting this into 8 allows us to write

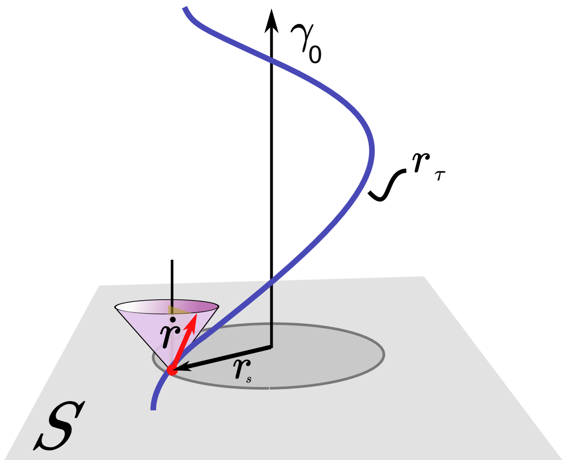
$$\begin{aligned} \Omega &= \dot{r}(a)\wedge\dot{r}(b) && (10)\\ &= \dot{r}(a)\wedge(\ddot{r}(a)+\dot{r}(a)) && (11)\\ &= \dot{r}\wedge\ddot{r} && (12) \end{aligned}$$

Both forms of Ω are visually obvious in Figure 1c. The advantage of using $\Omega = \dot{r}\wedge\ddot{r}$ is that it is indexed to the same τ.

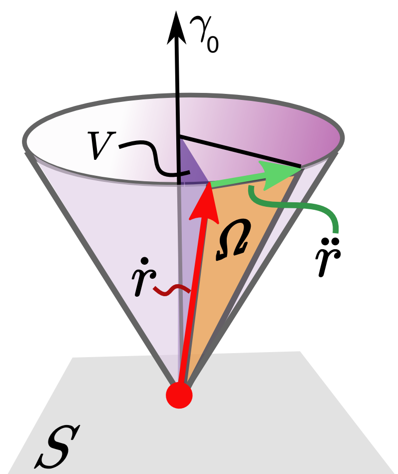
Acceleration can be decomposed into changes in the magnitude of V, which corresponds to *linear acceleration*, or a change in direction of V which corresponds *centripetal acceleration*. Figure 2 shows two differentials related by linear acceleration. In this figure V must change in magnitude only, which is only possible it $\dot{r}$ rotates in the plane of V. This type of rotation is a hyperbolic rotation (aka a *boost*) since V is minkowskian. Rotational motion takes a bit more visualization, this is tackled next.

D. Rotational Motion

Consider the simplest rotation motion possible; a particle moving in a circle at constant speed in the plane S. It's path in spacetime will look like a helix oriented with its axis along γ_0, as illustrated in Figure 3a. In this case our illustration captures the entirety of the path since it



(a) Spacetime path for rotational motion in the S-plane, which sweeps out a helix along γ_0. (figure adapted from [1])



(b) The differential $\dot{r}$ sweeps out a cone, as the curve is followed. The acceleration bivector $\Omega = \dot{r} \wedge \ddot{r}$ is tangent to the cone.

Figure 3: Simple rotational motion in spacetime. (Note: in GA, no specific shape information is associated with blades, but choosing a shape is needed for visualization. For example, a bivector could equally be drawn as a disk, or rectangle or triangle)

exists within the subspace γ_{012}. As we slide along the path, the vector differential $\dot{r}$ will sweep out a cone, as shown in detail in Figure 3b. At any point on the helix, the bivector between γ_0 and $\dot{r}$ yields the particle's velocity V_s, which is unchanging in magnitude but rotating around the cone. The differential can be decomposed into components along time and components within the plane S.

$$\dot{r} = \dot{r_0} + \dot{r}_{\emptyset} = \dot{r_0} + \dot{r}_s$$

Rotational motion introduces the concept of angular velocity, which is given by the pitch of the helix with respect to time. The angular velocity will be a trivector, as it has units of radians/time. Illustrated in Figure 4, angular velocity W can be computed by the trivector swept out by the velocity and the inverse radial position component,

$$W \equiv V_s \wedge r_s^{-1} = \frac{\dot{r}_s}{\dot{r}_0}\frac{1}{r_s} = -\frac{\dot{\theta}}{\dot{r}_0} \tag{13}$$

where,

$$\dot{\theta} \equiv \frac{\dot{r_s}}{r_s} \tag{14}$$

The euclidean bivector $\dot{\theta}$ is interpreted as the angular differential, with units of radians. Note that W squares to -1. The momentum p of the particle in the spin plane is given by

$$p_s = mV_s, \tag{15}$$

So that the angular momentum can be defined as the trivector made from the spin momentum and the radial position vector.

$$L = p_s \wedge r_s = mV_s \wedge r_s = -mr_s^2 W. \tag{16}$$

The kinetic energy in the system is a scalar,

$$E = mV_s^2 = -LW. \tag{17}$$

(The minus sign is needed due to the multivector signatures, but the result is a positive scalar.) Given that $\dot{r}$ rotates around the cone, the second differential $\ddot{r}$ with lay completely in the S-plane; $\ddot{r} \wedge S = 0$, while being perpendicular to the first differential; $\ddot{r} \cdot \dot{r} = 0$. This means the angular acceleration $\Omega = \dot{r} \wedge \ddot{r} = \dot{r}\ddot{r}$, has a signature determined by $\dot{r}$. It is worth pointing out that so far, each quantity's unit dimensions match the grade of their multivector representations. For example, velocity is m/s, so is a bivector, while angular frequency is radians/s which is a trivector. This is very useful, both conceptually and for error checking.

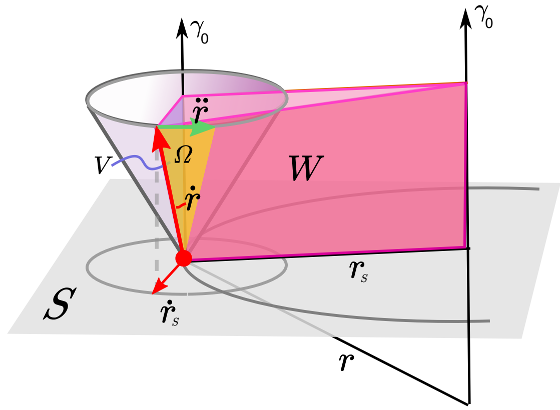
E. Helical Motion and Waves

The linear and rotational kinematics of the last two sections can be combined to create helical motion, which requires the full four dimensions of the Spacetime. Consider a particle that is rotating in a plane S just as before, but is also moving at constant speed in a direction orthogonal to S, denoted γ_k.

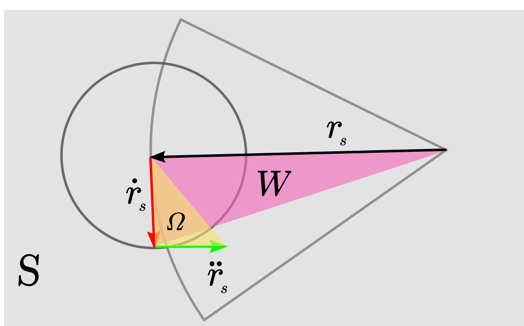
$$\gamma_k \equiv \gamma_{123} S \qquad \gamma_k \wedge \gamma_{123} = 0 \tag{18}$$

We can interpret γ_k as the *direction of propagation*. The particle's path in spacetime will be a helix as well, but the four dimensional helix is no longer fully visualizable in 3-dimenions. A labeling of the relevant geometry is shown in figure 5, which serves as a visual guide, albeit an inaccurate one. It is useful to separate the differential into time, spin, and propagation components,

$$\begin{aligned} \dot{r} &= \dot{r}_0 + \dot{r}_{\emptyset} \\ &= \dot{r}_0 + \dot{r}_s + \dot{r}_k. \end{aligned}$$



(a) Angular velocity trivector W, for planar rotation in S. Note r_s should be inversed, but we omit this for visual clarity. (it only affect magnitude of W)



(b) Planar projection onto euclidean plane S, showing angular velocity trivector W, and centripetal acceleration bivector Ω.

Figure 4: Simple rotational motion in spacetime and space.

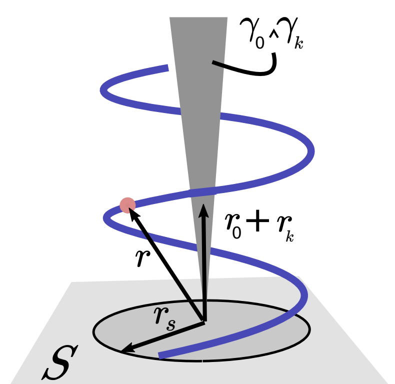


Figure 5: Approximate geometry of a spacetime Helix. The propagation direction is orthogonal to spin plane S, which is somewhere in the subspace of $\gamma_k \wedge \gamma_0$, (depicted as a wedge). (In spacetime, the particle is the path itself, but we draw it distinct for ease of presentation).

So that the velocity can be decomposed,

$$V = \frac{\dot{r}_s}{\dot{r}_0} + \frac{\dot{r}_k}{\dot{r}_0} = V_s + V_k. \tag{19}$$

Which introduces V_k,

$$V_k \equiv \frac{\dot{r}_k}{\dot{r}_0}.$$

V_s is the velocity in the spin plane, and V_k is the velocity along γ_k, which is called the *phase* or *propagation velocity*. Both V_s and V_k are minkowskian. Recall that the angular velocity could be written $W = -\frac{\dot{\theta}}{\dot{r}_0}$, which is more precisely called the *temporal* angular velocity. Since the temporal angular velocity gives the helix's pitch along the time-axis, there must be an analogous quantity for its pitch along the space-axis, call this K.

$$K \equiv -\frac{\dot{\theta}}{\dot{r}_k} = \frac{\dot{r}_s}{\dot{r}_k}\frac{1}{r_s} \tag{20}$$

K is also a trivector like W, but K has positive square, while W has negative square. Since there is only one spacial trivector, γ_{123}, the reader might have noticed that the direction of propagation is lost in this model of K. However it is not clear if this is a feature or a bug. The direction of propagation can be recovered from combining W and K, or by knowing S. The total angular velocity, or *pitch* P is the sum of space and time pitches,

$$P \equiv W + K. \tag{21}$$

The phase velocity V_k now has a geometric interpretation as the ratio of time-pitch to space-pitch, which is an essential result in wave mechanics. This can now be computed by the spacetime-split of P,

$$V_k = \frac{W}{K} = \frac{P \cdot \gamma_0}{P \wedge \gamma_0}. \tag{22}$$

This explains our use of the variables W and K, chosen match the conventional scalar result, $v_k = \frac{\omega}{k}$. Since phase velocity is defined, it is useful to identify some wavelengths, which can be done by unwrapping the helix in space as shown in Figure 6. Since K is space pitch, the distance traveled by the particle per-turn along k can be computed with the help of eq 20,

$$\lambda_k = \frac{2\pi}{K} = -2\pi\frac{V_k}{W}. \tag{23}$$

This is the *propagation wavelength*, and its analogous to the guide wavelength in a waveguide[7]. To keep the units consistent, the factor of 2π in the numerator can be multiplied by the spin plane S. To do this without having a lot of S's everywhere, we define

$$\pi_s \equiv \pi S, \tag{24}$$

which is read as; π in the S-plane (which makes dimensional sense). The propagation wavelength can be related to the *helical wavelength* λ, defined as the path-length

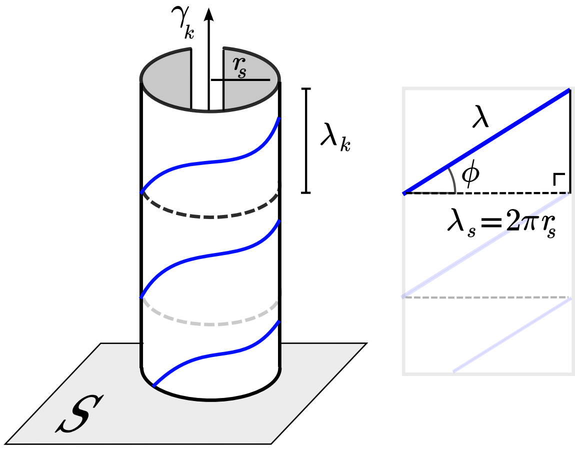


Figure 6: Path of helical motion in space, with various wavelengths identified by unwrapping the cylindrical symmetry.

along a single helical turn. This can derived by dividing the velocity by the temporal frequency,

$$\frac{V}{W} = \frac{V_s + V_k}{W} = -\left(r_s + K^{-1}\right) \tag{25}$$

and multiplying by $2\pi_s$,

$$2\pi_s \frac{V}{W} = 2\pi_s r_s + 2\pi_s K^{-1} \tag{26}$$
$$\lambda = \lambda_s + \lambda_k. \tag{27}$$

Where we have defined,

$$\lambda \equiv 2\pi_s \frac{V}{W} \tag{28}$$
$$\lambda_s \equiv 2\pi_s \frac{V_s}{W} = 2\pi_s r_s \tag{29}$$
$$\lambda_k \equiv 2\pi_s \frac{V_k}{W} = 2\pi_s K^{-1} \tag{30}$$

While λ_s is circumference and not a wavelength, labeling it λ_s is convenient, although slightly abusive. The relation between the $\lambda's$ magnitudes, can be identified in Figure 6 as the constraints of a right triangle. From this figure, the space pitch angle ϕ can be identified as,

$$\tan\phi = \frac{|\lambda_k|}{|\lambda_s|} = \frac{|V_k|}{|V_s|}. \tag{31}$$

The ratio of spin velocity to propagation velocity comes up enough to justify a variable, label this bivector T,

$$T \equiv \frac{V_k}{V_s} = \frac{\dot{r_k}}{\dot{r_s}} = \frac{1}{Kr_s} \tag{32}$$

so that

$$|T| = \tan\phi \tag{33}$$

The total momentum for the particle can also be decomposed into rotation and translational components,

$$p = mV = mV_s + mV_k = p_s + p_k \tag{34}$$

so can the total kinetic energy,

$$\begin{aligned} E = mV^2 &= mV_s^2 + mV_k^2 \\ &= -LW + p_k V_k. \end{aligned} \tag{35}$$

This is a multivector version of the *energy momentum equation*. De Broglie's relations can be seen in a few different ways, the most obvious being,

$$\lambda_s = \frac{2\pi L}{p_s}. \tag{36}$$

While the entire description of the helix has been within a single frame, it is anticipated that any other helix can be related to this reference helix through a lorentz transformation.. This requires an object-level description of the helix, which we have yet to define. The general approach of objectifying the helix, and considering it as a 'reference helix', is analogous to linear system analysis, in that there is always a reference sinusoid in mind.

In conclusion, this section has given a description of helical motion and identified some wave mechanics characteristics. The next section applies the simple restriction that the particle always moves at the speed of light, which results in the Zitter Electron Model.

III. The Zitter Model

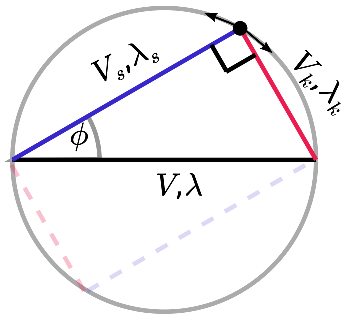
The zitter model of the electron requires the sub-electron particle's worldline have at all times a null vector differential, $\dot{r}^2 = 0$. This implies $r_0^2 = r_\emptyset^2$, and thus $|V| = 1 = c$ (i.e. we have normalized to lightspeed). This constraint creates a helix in spacetime which has a fixed relation between its radius and pitch. A graphical representation constraint can be made by relating the magnitudes of the various wavelength and velocity components, as shown in figure 7. In this diagram the circle's diameter is λ, which is fixed, while the corner of the triangle slides around the perimeter adjusting λ_s and λ_k. The same relation holds for the velocity components, which can also depicted as vectors in the diagram (but keep in mind this is just to relate their magnitudes, they are really bivectors).

A. *At rest-ness*

The special state of the electron *at rest* is defined by $V_k = 0$, $V_s = V$. Since many conventional equations are expressed in reference to this particular state, it is helpful to denote any quantity *at rest* with a subscript e. So, for example, the energy of the system *at rest* is,

$$E = mV^2 = -L_e W, \tag{37}$$

since there is no propagation velocity. This is interpreted as a multivector form of the "Plank-Einstein" relation $E = \hbar\omega$, which leads us to identify the magnitude of L_e as the reduced planks constant.



(a) Constraints on velocity and wavelength components visualized as a right triangle inscribed in a circle.

Figure 7: Constraints on velocity and wavelengths components. While drawn as vectors, this is only done to represent the constraints on the multivector magnitudes.

$$L_e = \gamma_{012}|L_e| = \gamma_0 \underbrace{\gamma_{12}}_{\boldsymbol{i}} \underbrace{|L_e|}_{\hbar} \rightarrow \gamma_0 \boldsymbol{i}\hbar \tag{38}$$

This gives geometric meaning to the factors of $\boldsymbol{i}\hbar$ in Quantum mechanics, as Hestenes has promoted zealously. Conventionally, the mass of a particle is allows to change, while the angular momentum is kept constant. In this model, having a variable mass is unnecessary.

B. *The Lorentz Factor*

If the spin velocity was ignored, all equations involving V_s would have to be expressed with some correction factor. Given the decomposition of velocity

$$V = V_s + V_k \tag{39}$$

V_k is,

$$\begin{aligned} V_k &= V - V_s \\ &= (1 - V_s V^{-1})V \\ &= \left(1 - \Gamma^{-1}\right) V \end{aligned}$$

where

$$\Gamma \equiv V V_s^{-1} = \frac{1}{1 - \frac{V_k}{V}}, \tag{40}$$

which is a spinor. The magnitude of this spinor is equivalent to the *lorentz factor.*

$$\gamma \equiv |\Gamma| = \frac{1}{\sqrt{1 - \frac{V_k^2}{V^2}}} = \frac{1}{\cos\phi}. \tag{41}$$

The scalar ϕ is the space pitch angle as shown in figure 6. The lorentz factor can be used to express equations involving V_s without acknowledging V_s, à la $|V_s| = |V|/\gamma$. It appears as though the accepted interpretation of special relativity absorbs this factor into the *relativistic mass*, which is conceptually confusing.

Many conventional equations contain $\hbar$, which is an at-rest quantity. To compare equations with $\hbar$ to multivector equations containing L, we need to relate $L_e$to L.

$$\frac{L_e}{L} = \frac{-mV^2/W}{-mV_s^2/W} = \frac{V^2}{V_s^2} = \gamma^2, \tag{42}$$

so,

$$L_e = \gamma^2 L \tag{43}$$

The *total* energy (eq 35) can be given in terms γ^2,

$$E = -L_e W = -\gamma^2 LW = -LW + p_k V_k \tag{44}$$

C. *Dirac equation*

The Dirac equation can be written,

$$\gamma^\mu \partial_\mu \psi \boldsymbol{i}\hbar - \frac{q}{c}\gamma^\mu A_\mu \psi = mc\psi. \tag{45}$$

This equation can be produced with the current model. Start with the assumption that the wave function has an analogous form to all wave mechanics,

$$\psi(x) = e^{Px} \tag{46}$$

[NOT CORRECT IN GENERAL. NEEDS WORK] Where P is the trivector-valued, spacetime-pitch given by eq 21 (but more generally an odd multivector), and x is some event (ie vector) in spacetime, which is an independent variable. The product Px produces an even grade multivector, which generates a spinor. This function solves the simple differential equation,

$$\nabla\psi = P\psi. \tag{47}$$

Eq 45 can be produced from 47. To do this, eq 45 is rewritten with the following substitutions given in previous sections,

$$\begin{aligned} |V| &\rightarrow c \\ V_k &\rightarrow |V_k|\gamma_{0k} \\ L_e &\rightarrow \gamma_{012}\hbar = \gamma_0 \boldsymbol{i}\hbar. \end{aligned} \tag{48}$$

The algebra is shown below.

$$\nabla\psi = P\psi \tag{49}$$
$$\nabla\psi = (W+K)\psi \tag{50}$$
$$\nabla\psi = W(1-\frac{1}{V_k})\psi \tag{51}$$
$$\nabla\psi = -\frac{E}{L_e}(1-\frac{1}{V_k})\psi \tag{52}$$
$$L_e\nabla\psi - \frac{E}{V_k}\psi = -E\psi \tag{53}$$
$$\gamma_0 \boldsymbol{i}\hbar\nabla\psi - q\frac{E}{q\gamma_0|V_k|\gamma_k}\psi = -mc^2\psi \tag{54}$$
$$\frac{\gamma_0}{c}\boldsymbol{i}\hbar\nabla\psi - \frac{q}{c\gamma_0}\underbrace{\left(\frac{E}{q|V_k|\gamma_k}\right)}_{\boldsymbol{A}}\psi = -mc\psi \tag{55}$$
$$\frac{1}{c}\boldsymbol{i}\hbar\nabla\psi - \frac{q}{c}\boldsymbol{A}\psi = -\gamma_0 mc\psi \tag{56}$$

Which is close to eq 45. Importantly, it provides geometric meaning for all quantities, and all grades match the units. A major insight is that the vector potential $\boldsymbol{A}$, is given by

$$\boldsymbol{A} = \frac{E}{q|V_k|\gamma_k} \tag{57}$$

However, A is more naturally re-defined as the bivector,

$$A \equiv \frac{E}{qV_k}, \tag{58}$$

We have used a bold-faced to differentiate the 'vector' $\boldsymbol{A}$ from our new definition of the bivector A. So that the units are correct (volts·s/m). It can be expressed in a few other forms.

$$A = \frac{E}{qV_k} = \frac{mc^2}{q}\frac{1}{V_k}. \tag{59}$$

There is a singularity at rest ; as $V_k \to 0$, $A \to \infty$. [INTERPRET]

Reflecting the form for the Dirac eqation in eq 47, we it implies that we can model various phenomenon by creating an odd multivector P-field, which then generates a spinor-field at every point in space time.

D. Lorentz Invariance

[IS THIS TRUE? IS IT USEFUL]

Given a Lorentz transform can be modeled as a rotor V, Lorentz invariance of the Dirac Equation states:

$$\begin{aligned}\psi' &\equiv V\psi V^{-1}\\ &= Ve^{Px}V^{-1}\\ &= e^{VPxV^{-1}}\\ &= e^{VPV^{-1}VxV^{-1}}\\ \psi' &= e^{P'x'}\end{aligned}$$

E. Dirac equation as written by Dirac

The Dirac equation for a free particle as given by Dirac, is

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\beta mc^2 + c\sum_{n=1}^{3}\alpha_n\boldsymbol{p}_n\right)\psi \tag{60}$$

If we only take the temporal derivative, of eq (46), and do some algebra,

$$\begin{aligned}\frac{\partial\psi}{\partial t} &= W\psi\\ \frac{\partial\psi}{\partial t} &= -\frac{E}{L_e}\psi\\ -L_e\frac{\partial\psi}{\partial t} &= (-LW + V_k p_k)\psi\\ -L_e\frac{\partial\psi}{\partial t} &= \left(\frac{1}{\gamma^2}m|V|^2 + |V|\sqrt{1-\frac{1}{\gamma^2}}\gamma_{0k}p_k\right)\psi.\end{aligned}$$

Making similar substitutions as eq (48)above, and moving $\gamma_0$to the right side,

$$-\boldsymbol{i}\hbar\frac{\partial\psi}{\partial t} = \left(\underbrace{\frac{\gamma_0}{\gamma^2}}_{\beta}mc^2 + c\underbrace{\sqrt{1-\frac{1}{\gamma^2}}\gamma_k}_{\alpha_n}p_k\right)\psi. \tag{61}$$

F. Discussion of the equation

[NOT CORRECT NEEDS WORK]

Re-examining eq (47), the generator Px can be broken up into inner and outer parts,

$$\begin{aligned}\psi = e^{Px} &= e^{(P\cdot x + P\wedge x)}\\ &= e^{P\cdot x}e^{P\wedge x}\end{aligned}$$

The inner product $P\cdot x$ produces a bivector, which generates a spinor field $\psi(x)$. The outer product $P\wedge x$ produces a psuedo-scalar part, which generates a duality rotation. It can be recovered by taking

$$\psi\tilde{\psi} = e^{2P\wedge x} = e^{2\vartheta I} = \alpha + \beta I \tag{62}$$

G. [TODO] Cut-off and Complex V_k

We need a scalar part for A for the 'scalar[electric] potential'. so that $\gamma_0 A = \boldsymbol{A} = \phi\gamma_0 + A_i$. I think we need $\gamma_0\cdot\gamma_k \neq 0$.

H. Currents

If the sub-electron particle has an electric charge q, then by moving along the helical path electric current will be generated. Defining the electric current to be number of times the charged particle passes by some distance in a single period, various current components can be computed. Recall the definitions of various wavelengths given in eq 27,

$$\lambda \equiv 2\pi_s \frac{V}{W} \qquad \lambda_s \equiv 2\pi_s \frac{V_s}{W} \qquad \lambda_k \equiv 2\pi_s \frac{V_k}{W}. \tag{63}$$

Referring back to figure 6, the *spin current* is computed as charge times the spin velocity divided by the helix circumference,

$$J_s \equiv \frac{qV_s}{\lambda_s} = -\frac{qW}{2\pi_s}. \tag{64}$$

For the *propagation current* the equation is analogous, but with the period set to the propagation wavelength,

$$J_k \equiv \frac{qV_k}{\lambda_k} = \frac{qW}{2\pi_s}. \tag{65}$$

So that

$$J_s = -J_k. \tag{66}$$

The total current per period is given by the total velocity divided by helical wavelength.

$$\begin{aligned} J \equiv \frac{qV}{\lambda} &= q\left(\frac{V_s + V_k}{\lambda_s + \lambda_k}\right) && (67)\\ &= -\frac{qW}{2\pi_s}\left(\frac{1+T}{1-T}\right) && (68)\\ &= -\frac{qW}{2\pi_s}\frac{V}{-\gamma_k V \gamma_k} && (69)\end{aligned}$$

Which can be interpreted as a time-like vector $(\frac{qW}{2\pi_s})$ multiplied by rotor $(\frac{V}{-\gamma_k V \gamma_k})$. Note that the form $\frac{1+T}{1-T}$ is known as the cayley form for a rotor[6]. Given these definitions, all current magnitudes are equal ,

$$|J_s| = |J_k| = |J|, \tag{70}$$

and constant with respect to the electron's speed V_k, meaning they are lorentz-invariant. While these results may seem odd, the limits work. For example, as $V_k \to 0$, $\lambda_k \to 0$, which can be interpreted as the length of the current element going to 0 at the same rate as the current goes to 0.

I. De Broglie's relations

Interpreting de Broglie's relations can be done by replacing all occurrences of $\hbar$ with L_e. First, the de Broglie momentum p is usually defined as the ratio of energy to the speed of light.

$$p = \frac{E}{c} = \frac{h}{\lambda} \tag{71}$$

This translates into,

$$p = \frac{E}{V} = \frac{-L_e W}{V} = \frac{2\pi L_e}{\lambda}.$$

Which is equivalent. Note that this is total momentum, which is not the same as the momentum used in the energy momentum equation, p_k. The other deBroglie equation,

$$f = \frac{E}{h} \tag{72}$$

translates to,

$$\frac{W}{2\pi} = \frac{E}{2\pi L_e}$$

which was already given in eq 44.

IV. Conclusion

This paper provides an approach to special relativity which is based on adapting the zitter electron model to a time-harmonic analysis. The results are self-consistent and the grades of the quantities match their units. A driving theme throughout this theory is the demand that all scalars be related to multivectors, which essentially requires all quantities to have geometric meaning. The relation between the time-harmonic model presented here, and the conventional relativity theory is to be worked out in the future. It appears as through it may be related through the Kustaanheimo–Stiefel transformation[5]. Again, since the models presented are based on a classical physics, they are 'local hidden-variable' theories which are not consistent with Bell's theorem.

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