Time Harmonic Relativistic Wave Mechanics

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***Abstract*—This paper presents an approach to special relativity which is more in line with electrical engineering, namely as the time-harmonic analysis of a linear system. The approach is derived from Hestenes' *Zitter* model for the electron[1], [2], which assumes an internal structure of a light-like helix in spacetime. A time-harmonic model is constructed and combined with classical physical arguments to produce several fundamental multivector equations. In addition, the grade (dimension) of the quantities involved match their units. The Zitter model is then extended to a nested helix, as suggested by Consa[3], and attempts are made to derive the fine structure constant, and vacuum constants μ_0 and ϵ_0. Since the models presented are based on a classical physics, they are 'local hidden-variable' theories which are not consistent with Bell's theorem.**

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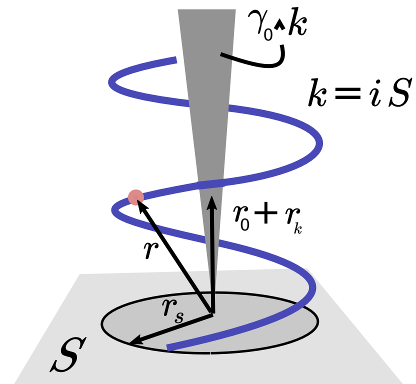


Figure 1: Approximate geometry of a spacetime Helix. The propagation direction is orthogonal to spin plane S, which is somewhere in the subspace of $k \wedge \gamma_0$, (depicted as a wedge). (In spacetime, the particle is the path itself, but we draw it distinct for ease of presentation).

eq#	Multivector	Scalar/Vector
(23)	$V = \frac{W}{K}$	$V = \frac{\omega}{k}$
(36)	$E = -LW + p_k V_k$	$E = \hbar\omega + p_k V_k$
(37)	$\lambda = \frac{2\pi L}{p}$	$\lambda = \frac{2\pi\hbar}{p}$
(71)	$\Phi = -\frac{2\pi_s L}{2q}$	$\Phi = \frac{2\pi\hbar}{2q}$
(81)	$\mu = \frac{4\pi m r_s}{q^2}\frac{a}{A}$	$\mu = \frac{4\pi m r}{q^2}\alpha$
(83)	$M = 2(1+\frac{a}{A})\frac{q}{2m}\frac{L}{2}$	$M = 2(1+\frac{\alpha}{2\pi})\frac{q}{2m}\frac{\hbar}{2}$

Table I: Multivector equations derived in this paper, and their conventional equivalents.

I. Introduction

A. Context

A few years ago we applied Conformal Geometric Algebra to transmission line theory[4], and found that this produced many relativistic-like relationships. However, at it's core, transmission line theory takes for granted several relations which are derived from the time-harmonic analysis of Maxwell's equations. In order to provide an appropriate foundation for the transmission-line model, we began an effort to derive the transmission line equations directly from Hestenes' Spacetime Algebra[5]. During this time, Hestenes had shared with us his Zitter electron model[1]. The combination of these two ideas is what led to the current approach. The relation between the time-harmonic model presented here, and the conventional relativity theory is to be worked out in the future. It appears as through it may be related through the Kustaanheimo–Stiefel transformation[6].

B. Summary and Outline

This paper develops an approach to special relativity which similar to the time-harmonic analysis of a linear system used throughout engineering disciplines. To accomplish this, relativistic kinematics are developed from the differential properties of the world-line of helical motion. The resulting model is self-consistent and the grades of the physical quantities match their units. Self-consistency has been checked numerically using the clifford python package where possible.

Section II introduces the kinematics of the helical motion. This model is combined with some classical physical arguments in sections III-IV to produce some fundamental equations based on Hestene's Zitter model for the electron. Finally, a brief sketch is given of the photon in section V. A guiding theme of this theory is the demand for all scalars be related to multivectors, which requires all quantities to have geometric meaning.

II. Spacetime Kinematics and Waves

A. Spacetime Algebra

This section gives a terse overview of Spacetime algebra used throughout this paper. For those unacquainted with the subject we recommend the original reference[5]. Spacetime algebra is the geometric algebra of four dimensional minkowski space. It can be defined in terms of an orthonormal vector basis with a time direction γ_0 which squares to $+1$, and three spacial directions $\gamma_{1,2,3}$ which square to -1,

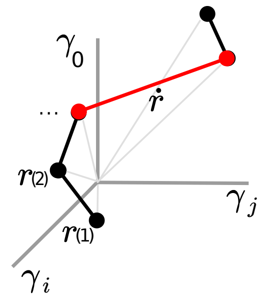
$$
\begin{aligned}
-\gamma_0^2 = \gamma_1^2 = \gamma_2^2 = \gamma_3^2 =& -1 \\
\gamma_j \cdot \gamma_k =& \delta_{jk}.
\end{aligned}
$$

Through the use of the outer product between vectors denoted with the wedge symbol $\wedge$, higher order basis elements representing planes, volumes, etc are generated. The bivector $\gamma_1 \wedge \gamma_2$ is a mathematical element which represents the plane spaned by γ_1 and γ_2, for instance. Simple bivectors (planes) and trivectors (volumes) are denoted with subscripts for brevity as such,

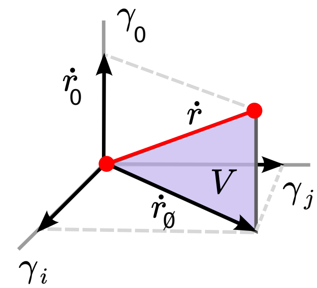
$$
\begin{aligned}
\gamma_{12} &\equiv \gamma_1 \wedge \gamma_2, \\
\gamma_{123} &\equiv \gamma_1 \wedge \gamma_2 \wedge \gamma_3.
\end{aligned}
$$

There is a single unit *psuedoscalar*, represented by I, which is a 4-volume that squares to -1.

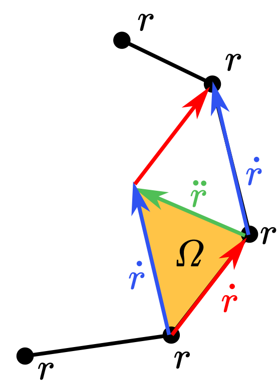
$$
I \equiv \gamma_{0123}, \qquad I^2 = -1 \tag{1}
$$



(a) A discrete curve in spacetime highlighting a single differential.



(b) The velocity bivector V created from the spacetime split of the differential $\dot{r}$. ($\dot{r}_0$ should be inverted, which alters the magnitude, but not the direction of V)



(c) The position r, first differential $\dot{r}$, and second differential $\ddot{r}$ for a path. The centripetal acceleration bivector shown as Ω. Indexing on all quantities is implied.

Figure 2: A spacetime path projected into 3D, with differentials.

The spacial trivector also squares to -1 , and is used frequently so we give it a symbol as well.

$$
i \equiv \gamma_{123} \qquad i^2 = -1 \tag{2}
$$

In summary, the Dirac algebra can be made up from the following blades,

$$
\begin{gathered}
1 \\
\gamma_0, \gamma_1, \gamma_2, \gamma_3 \\
\gamma_{01}, \gamma_{12}, \gamma_{23}, \gamma_{03}, \gamma_{13}, \gamma_{02}, \\
\gamma_{123}, \gamma_{230}, \gamma_{301}, \gamma_{012} \\
\gamma_{0123}
\end{gathered}
$$

B. Velocity And Acceleration

Imagine an object's history in spacetime as a curve in four dimensional spacetime. This construction allows all the particle kinematics to be seen geometric properties of the curve itself. The curve can be thought of as a discrete set of spacetime positions which are represented by vectors, denoted by $r(\tau)$, where τ is some independent variable. [1] In the limit of infinitesimal τ, the curve becomes smooth and we assume this is possible. By taking the difference between two adjacent points $r(a)$ and $r(b)$, we construct a vector differential, denoted with the overdot.

$$\dot{r}(a) \equiv r(b) - r(a)\,. \tag{3}$$

This differential represents the discrete arc-length of the path, as illustrated in Figure 2a. Going from the differentials to positions requires a summation,

$$r(\tau) = \sum_{0}^{\tau} \dot{r} + r(0). \tag{4}$$

From here on, τ-indexing is implied where appropriate, so we can drop the parenthesis and just refer to $\dot{r}$. To compute the velocity at any point along the path, first decompose the differential into space and time components,

$$\begin{aligned} \dot{r} &= \dot{r}\cdot\gamma_0\gamma_0^{-1} + \dot{r}\wedge\gamma_0\gamma_0^{-1} && (5)\\ &= \dot{r}_0 + \dot{r}_{\emptyset}, && (6)\end{aligned}$$

where $\dot{r}_0$ is the vector projection along γ_0, and $\dot{r}_{\emptyset}$ is the vector rejection from γ_0. The velocity is the ratio of space-like component to the time-like component, as shown in Figure 2b,

$$V = \frac{\dot{r}\wedge\gamma_0}{\dot{r}\cdot\gamma_0} = \frac{\dot{r}_{\emptyset}}{\dot{r}_0} \tag{7}$$

This relation is the known as the spacetime split, and the result V is a bivector. Since time and space have opposite signatures, V is a minkowski bivector meaning it squares to a positive number. We can set a scale by requiring an object moving at the speed of light to have a null vector differential, $\dot{r}^2 = 0$. This implies $r_0^2 = r_{\emptyset}^2$, and $|V| = 1$. When this is done, velocity is normalized to units of lightspeed.

Given velocity we can define acceleration as a change in velocity. From the definition of V, we can see that changing V requires a change in the *direction* of $\dot{r}$, because $\dot{r}$ and $\lambda\dot{r}$ produce the same V. One way to quantify the change in $\dot{r}$-direction would be to compute the bivector between two neighboring differentials, as shown in Figure 2c. Let us define the acceleration bivector Ω to be,

$$\Omega \equiv \dot{r}(a)\wedge\dot{r}(b) \tag{8}$$

Ω can also be formulated with the second differential. Given three neighboring position points, there are two neighboring differentials, $\dot{r}(a)$ and $\dot{r}(b)$. The differential between these points yields the second differential, denoted $\ddot{r}$,

$$\ddot{r}(a) \equiv \dot{r}(b) - \dot{r}(a) \tag{9}$$

Substituting this into 8 allows us to write

[1] We use parenthesis to avoid confusion between τ-indexing and coordinate projections, but we dont like it.

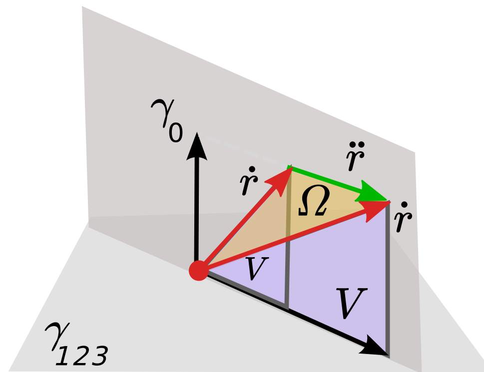


Figure 3: A pair of differentials related by linear acceleration.

$$\begin{aligned} \Omega &= \dot{r}(a)\wedge\dot{r}(b) && (10)\\ &= \dot{r}(a)\wedge(\ddot{r}(a) + \dot{r}(a)) && (11)\\ &= \dot{r}\wedge\ddot{r} && (12)\end{aligned}$$

Both forms of Ω are visually obvious in Figure 2c. The advantage of using $\Omega = \dot{r}\wedge\ddot{r}$ is that it is indexed to the same τ.

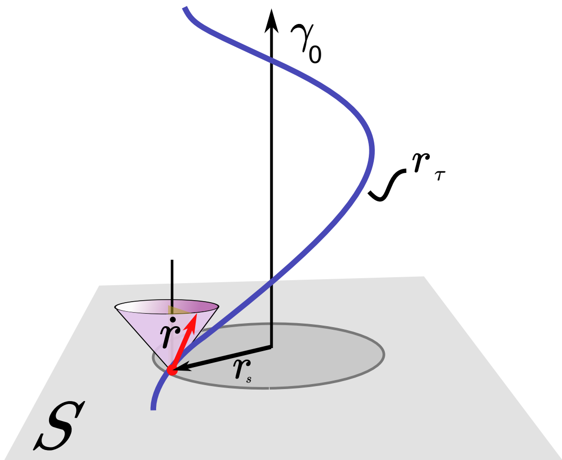
Acceleration can be decomposed into changes in the magnitude of V, which corresponds to *linear acceleration*, or a change in direction of V which corresponds *centripetal acceleration* (translational/rotational would be better terms). Figure 3 shows two differentials related by linear acceleration. In this figure V must change in magnitude only, which is only possible it $\dot{r}$ rotates in the plane of V. This type of rotation is a hyperbolic rotation (aka a *boost*) since V is minkowskian. Rotational motion takes a bit more visualization, this is tackled next.

C. Rotational Motion

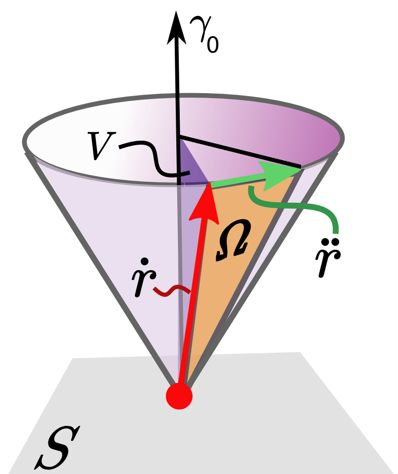
Consider the simplest rotation motion possible; a particle moving in a circle at constant speed in the plane S. It's path in spacetime will look like a helix oriented with its axis along γ_0, as illustrated in Figure 4a. In this case our illustration captures the entirety of the path since it exists within the subspace γ_{012}. As the path is followed by particle, the vector differential $\dot{r}$ will sweep out a cone, as shown in detail in Figure 4b. At any point on the helix, the bivector between γ_0 and $\dot{r}$ yields the particle's velocity V_s, which is unchanging in magnitude but rotating around the cone. The differential can be decomposed into components along time and components within the plane S.

$$\dot{r} = \dot{r}_0 + \dot{r}_{\emptyset} = \dot{r}_0 + \dot{r}_s$$

Rotational motion introduces the concept of angular velocity, which is given by the pitch of the helix with respect to time. The angular velocity will be a trivector, as it has units of radians/time. Illustrated in Figure 5, angular velocity W can be computed by the trivector



(a) Spacetime path for rotational motion in the S-plane, which sweeps out a helix along γ_0. (figure adapted from [1])



(b) The differential $\dot{r}$ sweeps out a cone, as the curve is followed. The acceleration bivector $\Omega = \dot{r} \wedge \ddot{r}$ is tangent to the cone.

Figure 4: Simple rotational motion in spacetime.

swept out by the velocity and the inverse radial position component,

$$W \equiv V_s \wedge r_s^{-1} = \frac{\dot{r}_s}{\dot{r}_0 r_s} = \frac{\dot{\theta}}{\dot{r}_0} \tag{13}$$

where,

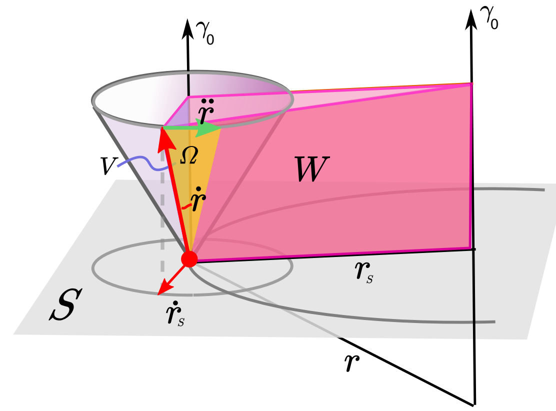
$$\dot{\theta} \equiv \frac{\dot{r}_s}{r_s} \tag{14}$$

The euclidean bivector $\dot{\theta}$ is interpreted as the angular differential, with units of radians. Note that W is analogous to V, although it squares to -1. The momentum p of the particle in the spin plane is given by

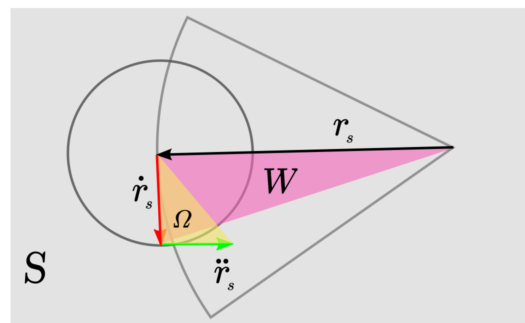
$$p_s = mV_s, \tag{15}$$

So that the angular momentum can be defined as the trivector made from the spin momentum and the radial position vector.

$$L = p_s \wedge r_s = mV_s \wedge r_s = -mr_s^2 W. \tag{16}$$



(a) Angular velocity trivector W, for planar rotation in S. Note r_s should be inversed, but we omit this for visual clarity. (it only affect magnitude of W)



(b) Planar projection onto euclidean plane S, showing angular velocity trivector W, and centripetal acceleration bivector Ω.

Figure 5: Simple rotational motion in spacetime and space.

Which is a multivector version of the classical result. The kinetic energy in the system is a scalar,

$$E = mV_s^2 = -LW. \tag{17}$$

(The minus sign is needed due to the multivector signatures, but the result is a positive scalar.) Given that $\dot{r}$ rotates around the cone, the second differential $\ddot{r}$ with lay completely in the S-plane; $\ddot{r} \wedge S = 0$, while being perpendicular to the first differential; $\ddot{r} \cdot \dot{r} = 0$. This means the angular acceleration $\Omega = \dot{r} \wedge \ddot{r} = \dot{r}\ddot{r}$, has a signature determined by $\dot{r}$. It is worth pointing out that so far, each quantity's unit dimensions match the grade of their multivector representations. For example, velocity is m/s, so is a bivector, while angular frequency is radians/s which is a trivector. We find this to be very useful, both conceptually and for error checking.

D. *Helical Motion and Waves*

The linear and rotational kinematics of the last two sections can be combined to create helical motion, which requires the full four dimensions of the Spacetime. Consider a particle that is rotating in a plane S just as before, but is also moving at constant speed in a direction orthogonal to S. It's path in spacetime will be a helix as well, but the four dimensional helix is no longer fully visualizable in 3-dimenions. A labeling of the relevant geometry is shown in figure 1, which serves as a visual guide, albeit an inaccurate one. Overall, several decompositions of the differential can be made,

$$\begin{aligned} \dot{r} &= \dot{r}_0 + \dot{r}_{\not{0}} \\ &= \dot{r}_0 + \dot{r}_s + \dot{r}_k \\ &= \qquad \dot{r}_s + \dot{r}_{\not{s}} \end{aligned} \tag{18}$$

In the purely rotational case, $\dot{r_k} = 0$, and so $\dot{r}_{\not{s}} = \dot{r}_0$, but this no does not hold true for helical motion. The *direction of propagation* is defined to be spatially orthogonal to the spin plane,

$$\frac{\dot{r_k}}{|\dot{r_k}|} = iS = \gamma_{123} S \tag{19}$$

Decomposing $\dot{r}$ into spin and not-spin components allows us to express the velocity as

$$V = \frac{\dot{r}_s}{\dot{r}_0} + \frac{\dot{r}_k}{\dot{r}_0} = V_s + V_k \tag{20}$$

Here V_s is the velocity in the spin plane, and V_k is the velocity along $\dot{r_k}$, which turns out to be the *phase velocity*. Both V_s and V_k are minkowskian. Recall that the angular velocity was defined to be $W = \frac{\dot{\theta}}{\dot{r_0}}$, which is more precisely called the *temporal* angular velocity. Since the temporal angular velocity gives the helix's pitch along the time-axis, there must be an analogous quantity for its pitch along the space-axis, which we label K.

$$K \equiv \frac{\dot{\theta}}{\dot{r_k}} = \frac{\dot{r_s}}{r_s \dot{r_k}} = \frac{V_s}{V_k} \frac{1}{r_s} \tag{21}$$

K is also a trivector, like W, but K has positive square, while W has negative square. The total angular velocity, or pitch P is the sum of space and time pitches,

$$P \equiv W + K. \tag{22}$$

The phase velocity V_k now has a geometric interpretation as the ratio of time-pitch to space-pitch, which is an essential result in wave mechanics. This can now be computed by the spacetime-split of P,

$$V_k = \frac{W}{K} = \frac{P \cdot \gamma_0}{P \wedge \gamma_0}. \tag{23}$$

This explains our use of the variables W and K, chosen match the conventional scalar result, $v = \frac{\omega}{k}$. Since phase velocity is defined, it is useful to identify some wavelengths, which can be done by unwrapping the helix in space as shown in Figure 6. Since K is space pitch, the distance traveled by the particle per-turn along k can be computed with the help of eq 21,

$$\lambda_k \equiv \frac{2\pi}{K} = 2\pi \frac{V_k}{W}. \tag{24}$$

This is the *propagation wavelength*, and its analogous to the guide wavelength in a waveguide[7]. To be consistent,

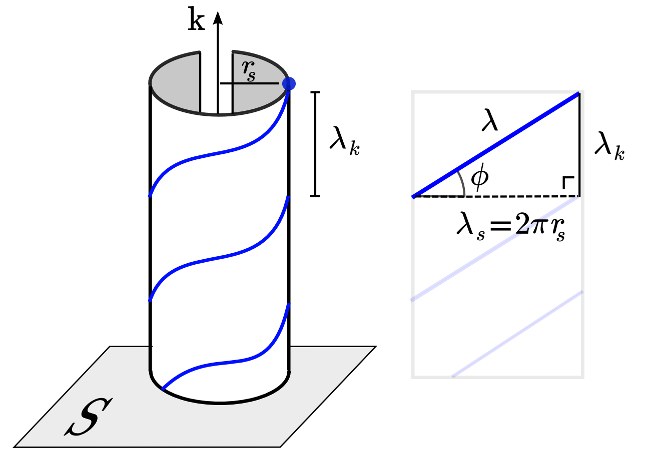


Figure 6: Path of helical motion in space, with various wavelengths identified by unwrapping the cylindrical symmetry.

the factor of 2π in the numerator should be multiplied by the spin plane S to keep the units and grades of λ_k matched. To do this without having a lot of S's everywhere, we define

$$\pi_s \equiv \pi S, \tag{25}$$

which is just a bivector version of π (this makes dimensional sense too). The propagation wavelength can be related to the *helical wavelength* λ, defined as the path-length along a single helical turn. This can derived by dividing the velocity by the temporal frequency,

$$\frac{V}{W} = \frac{V_s + V_k}{W} = r_s + K^{-1} \tag{26}$$

and multiplying by $2\pi_s$,

$$2\pi_s \frac{V}{W} = 2\pi_s r_s + 2\pi_s K^{-1} \tag{27}$$

$$\lambda = \lambda_s + \lambda_k. \tag{28}$$

Where we have defined,

$$\lambda \equiv 2\pi_s \frac{V}{W} \tag{29}$$

$$\lambda_s \equiv 2\pi_s \frac{V_s}{W} = 2\pi_s r_s \tag{30}$$

$$\lambda_k \equiv 2\pi_s \frac{V_k}{W} = 2\pi_s K^{-1} \tag{31}$$

While λ_s is circumference and not a wavelength, labeling it λ_s is just too convenient. The relation between the $\lambda's$ magnitudes, can be identified in Figure 6 as the constraints of a right triangle. From this figure, the space pitch angle ϕ can be identified as,

$$\tan\phi = \frac{|\lambda_k|}{|\lambda_s|} = \frac{|V_k|}{|V_s|}. \tag{32}$$

The ratio of spin velocity to propagation velocity comes up enough to justify a variable, label this bivector T,

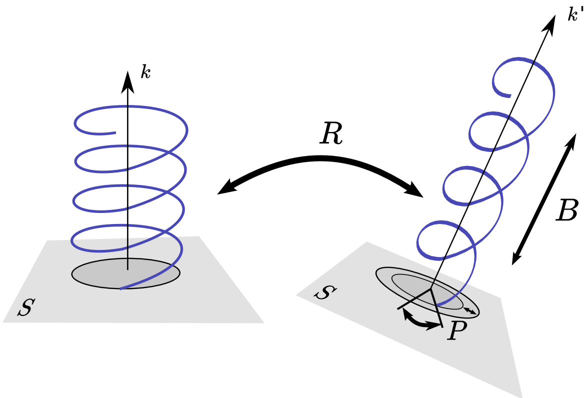


Figure 7: Relation between two helices, broken down into a rotation R , boost along the propagation axis B, and a phase rotation and radius dilation P. The Zitter model in the next section links the radius dilation to the boost, through the light-like constraint the differential.

$$T \equiv \frac{V_k}{V_s} = \frac{\dot{r}_k}{\dot{r}_s} = \frac{1}{K r_s} \tag{33}$$

so that

$$|T| = \tan\phi \tag{34}$$

The total momentum for the particle can also be decomposed into rotation and translational components,

$$p = mV = mV_s + mV_k = p_s + p_k \tag{35}$$

so can the total kinetic energy,

$$\begin{aligned} E = mV^2 &= mV_s^2 + mV_k^2 \\ &= -LW + p_k V_k. \end{aligned} \tag{36}$$

This is a multivector version of the *energy momentum equation*. De Brogloie's relations can be seen in a few different ways, the most obvious being,

$$\lambda_s = \frac{2\pi L}{p_s}. \tag{37}$$

While the entire description of the helix has been within a single frame, it is anticipated that any other helix can be related to this reference helix through a lorentz transformation, as depicted in figure 7. This requires an object-level description of the helix, which we have yet to define. The general approach of objectifying the helix, and considering it as a 'reference helix', is analogous to linear system analysis, in that there is always a reference sinusoid in mind.

In conclusion, this section has given a description of helical motion and identified some wave mechanics characteristics. The next section applies the simple restriction that the particle always moves at the speed of light, which results in the Zitter Electron Model, and things start to get interesting, as well as more speculative.

III. The Zitter Model for The Electron

While the model which follows has been tested for numerical consistency were possible, the physical arguments have not been thoroughly reviewed.

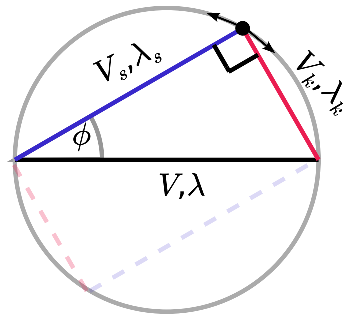
A. *Inaccuracy of a point particle**

The Zitter model for the electron postulates that the electron has an internal structure consisting of a sub-electron particle with mass and charge that is rotating about a center of mass [2], [1], [8], [3], as depicted in figure 1. It is important to realize that while useful, the mental image of a point particle following a path is not very accurate. The correct viewpoint is see things like a Tralfamadorian, as Vonnegut describes in [9]. A curve in spacetime is not followed parametrically, it just exists, and a projection along a given time-location produces a 3D structure. In this way, all the particle kinematics are manifest geometric properties of the curve structure itself. This is more easily visualized with a thought experiment of only two space dimensions. Furthermore, as far as all the kinematics described thus far are concerned, the point particle approximation is *indistinguishable* from an evenly distributed mass/charge along the spacetime curve. It is not clear if the point-particle is a necessary feature. Regardless, we continue to use the point-particle model for ease of presentation, but we keep it's possible inaccuracy in mind.

B. *Why W is constant with respect to V_k*

The Zitter model assumes that the sub-electron particle always moves at the speed of light. So as the electron as a whole starts to move, the particle has a translational velocity which detracts from it's rotational velocity. When the translation velocity V_k approaches the speed of light, the rotational velocity must go to zero, $V_s \to 0$. For the angular momentum $L = m r_s V_s$ to be constant, the radius would have grow to ∞, which is not likely. So, we conclude that L is not constant. If instead the helix radius r_s went to zero at the same rate as the spin velocity went to zero, then the angular frequency W would be constant, as $W = \frac{V_s}{r_s}$. Since this seems like the only possible stable solution, we conclude that W is constant with respect to V_k (ie is lorentz invariant). Thus, the model can be best described as *time-harmonic relativistic wave mechanics*. We follow this conjecture in this section and find that it provides simple interpretations to several fundamental equations.

Applying the kinematics of helical motion laid out in the previous sections to the zitter model of the electron requires the sub-electron particle path to have at all times a null vector differential, $\dot{r}^2 = 0$. As already pointed out, this implies $r_0^2 = r_{\emptyset}^2$, and thus $|V| = 1$. If the sub-electron particle is given a charge, then the zitter model is describing a superconducting helix in spacetime which has a fixed relation between its radius and pitch. A graphical representation of the light-like differential constraint can be made by relating the magnitudes of the various wavelength components. This produces a right



(a) Constraints on velocity and wavelength components visualized as a right triangle inscribed in a circle.

Figure 8: Constraints on velocity and wavelengths components. While drawn as vectors, this is only done to represent the constraints on the multivector magnitudes.

triangle inscribed within a circle, as shown in figure 8. In this diagram the circle's diameter is λ which is fixed, while the corner of the triangle slides around the perimeter adjusting λ_s and λ_k. The same relation holds for the velocity components, which can also depicted as vectors in the diagram (but keep in mind this is just to relate their magnitudes, they are really bivectors).

C. Currents

If the sub-electron particle has an electric charge q, then by moving along the helical path electric current will be generated. Defining the electric current to be number of times the charged particle passes by some distance in a single period, various current components can be computed. Referring back to figure 6, the *spin current* is computed as charge times the spin velocity divided by the helix circumference,

$$J_s \equiv \frac{qV_s}{\lambda_s} = -\frac{qW}{2\pi_s}. \tag{38}$$

For the propagation current the equation is analogous, but with the period set to the propagation wavelength,

$$J_k \equiv \frac{qV_k}{\lambda_k} = \frac{qW}{2\pi_s}. \tag{39}$$

So that

$$J_s = -J_k. \tag{40}$$

The total current per period is given by the total velocity divided by helical wavelength.

$$J \equiv \frac{qV}{\lambda} = q\left(\frac{V_s + V_k}{\lambda_s + \lambda_k}\right) \tag{41}$$

$$= \frac{qW}{2\pi_s}\left(\frac{1+T}{1-T}\right) \tag{42}$$

$$= \frac{qW}{2\pi_s}\frac{V}{-kVk} \tag{43}$$

Which can be interpreted as a vector $(\frac{qW}{2\pi_s})$ multiplied by rotor $(\frac{V}{-kVk})$. Note that the form $\frac{1+T}{1-T}$ is known as the cayley form for a rotor[10]. Given these definitions, all current magnitudes are equal ,

$$|J_s| = |J_k| = |J|, \tag{44}$$

and constant with respect to the electron's speed V_k, meaning they are lorentz-invariant. While these results may seem odd, the limits work. For example, as $V_k \to 0$, $\lambda_k \to 0$, which can be interpreted as the length of the current element going to 0 at the same rate as the current goes to 0.

D. At rest-ness

The special state of the electron *at rest* is defined by $V_k = 0$, $V_s = V$. Since many conventional equations are expressed in reference to this particular state, it is helpful to denote any quantity *at rest* with a subscript e. So, for example, the kinetic energy of the system *at rest* is,

$$E = mV^2 = -L_e W, \tag{45}$$

since there is no propagation velocity. This is interpreted as a multivector form of the "Plank-Einstein" relation $E = \hbar\omega$, which leads us to identify the magnitude of L_e as the reduced planks constant.

$$|L_e| = \hbar. \tag{46}$$

Since $\hat{L}^2 = -1$, L_e can be used to replace factors of $i\hbar$,

$$L_e \to i\hbar \tag{47}$$

This gives geometric meaning to the factors of $i\hbar$ in Quantum mechanics. Conventionally, the mass of a particle is allows to change, while the angular momentum is kept constant. In this model, it is found that having a variable mass is unnecessary (and incoherent).

E. The Lorentz Factor

If the spin velocity was ignored, all equations involving V_s would have to be expressed with some correction factor. This is known as the lorentz factor. Given the decomposition of velocity

$$V = V_s + V_k \tag{48}$$

V_k is,

$$\begin{aligned} V_k &= V - V_s \\ &= (1 - V_s V^{-1})V \\ &= \left(1 - \Gamma^{-1}\right)V \end{aligned}$$

where the ratio of V to V_s to defined to be Γ so that,

$$\Gamma \equiv V V_s^{-1} = \frac{1}{1 - \frac{V_k}{V}}, \tag{49}$$

which is a spinor. The magnitude of this spinor is equivalent to the *lorentz factor.*

$$\gamma \equiv |\Gamma| = \frac{1}{\sqrt{1 - \frac{V_k^2}{V^2}}} = \frac{1}{\cos\phi}, \tag{50}$$

where ϕ is the space pitch angle as shown in figure 6. The lorentz factor can be used to express equations involving V_s without acknowledging V_s, à la $|V_s| = |V|/\gamma$. It appears as though the accepted interpretation of special relativity absorbs this factor into the *relativistic mass*, which is conceptually confusing. γ can also be expressed in terms of helix radius. Since W is a constant,

$$W = \frac{V}{r_e} = \frac{V_s}{r_s}. \tag{51}$$

and so

$$\Gamma = \frac{V}{V_s} = \frac{r_e}{r_s} \tag{52}$$

Many conventional equations contain $\hbar$, which is an at-rest quantity. So, to compare equations with $\hbar$ to multivector equations containing L, we need to relate $L_e$to L.

$$\frac{L_e}{L} = \frac{-mV^2/W}{-mV_s^2/W} = \frac{V^2}{V_s^2} = \gamma^2, \tag{53}$$

so,

$$L_e = \gamma^2 L \tag{54}$$

The *total* kinetic energy (eq 36) can be given in terms γ^2,

$$E = mV^2 = -\gamma^2 LW = -L_e W = -LW + p_k V_k \tag{55}$$

Which is lorentz invariant. This equation is long-winded, but shows how the various equations involving energy are equivalent. Note that since both T and Γ are normalized velocities, and they are related by,

$$\Gamma = 1 + T. \tag{56}$$

F. De Broglie's relations

Interpreting de Broglie's relations can be done by replacing all occurrences of $\hbar$ with L_e. First, the de Broglie momentum p is usually defined as the ratio of energy to the speed of light.

$$p = \frac{E}{c} = \frac{h}{\lambda} \tag{57}$$

This translates into,

$$p = \frac{E}{V} = \frac{-L_e W}{V} = \frac{2\pi L_e}{\lambda}.$$

Which is equivalent. Note that this is total momentum, which is not the same as the momentum used in the energy momentum equation, p_k. The other deBroglie equation,

$$f = \frac{E}{h} \tag{58}$$

translates to,

$$\frac{W}{2\pi} = \frac{E}{2\pi L_e}$$

which was already given in eq 55.

G. Spin Magnetic Moment

The classical magnetic moment M for a circular loop of electric current J and radius r is

$$M = J\pi r^2 \tag{59}$$

Inserting the electron radius r_s, π_s, and spin current J_s we get the classical result for magnetic moment of the electron,

$$M = J_s \pi_s r_s^2 = -\frac{1}{2} q r_s^2 W = \frac{q}{2m} L \tag{60}$$

Which means M is a trivector proportional to W and L. At rest this is,

$$M = 2\frac{q}{2m}\frac{L_e}{2}. \tag{61}$$

The accepted value for the 'spin' magnetic moment for spin-$\frac{1}{2}$ particles is,

$$M = g\frac{q}{2m}\frac{\hbar}{2} \tag{62}$$

Where g is known as the *g-factor*. However, it is known (to great accuracy) that g differs from 2 by a little bit, causing the *anomalous magnetic moment*. One possible explanation for this is that the orbit of the sub-electron particle is not circular. Consa has suggested this is due to a toriodal orbit[3], [1], and further indicated how this feature could also explain the fine structure constant. This idea is entertained in section IV.

Note that the angular momentum and magnetic moment are dual equations in mass and charge, respectively. They can be divided to produce the charge-mass ratio,

$$\frac{L}{M} = \frac{mr^2 W}{\frac{1}{2} q r^2 W} = 2\frac{m}{q}. \tag{63}$$

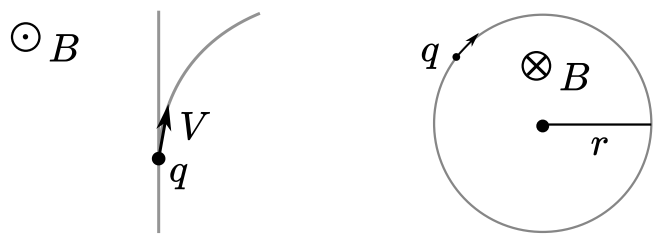
This ratio can be used to translate between electrical and kinematic equations.

H. Vacuum Permeability

The question naturally arises; why is this sub-electron structure stable? A simple answer is that the particle inside the electron is just like an electron in a cyclotron; the centripetal force is balanced with the lorentz force for some radius. Simply put,

$$qV \wedge B = mW^2 r.$$

Where B is the magnetic field part of the faraday bivector[5] (a more rigorous introduction of B is needed to be consistent with grades of the current theory, but likely



(a) A charged particle moving in the presence of a magnetic field will bend with constant curvature.

(b) The resultant motion is a current ring which generates a magnetic field in the center.

Figure 9: Lorentz force balancing centrifugal force for a charged particle moving in a constant magnetic field. Same as in a cyclotron.

the consequences will remain the same up to a grade). Assuming all motion occurs in the spin plane the algebra simplifies to produce,

$$\begin{aligned} qV_sB &= mW^2r_s \\ qB &= mW \qquad (64) \\ \frac{m}{q} &= \frac{B}{W}. \qquad (65) \end{aligned}$$

μ_0 can be computed once a form for B is postulated, but walking through the intuitive explanation is useful. A charge moving in the presence of a constant magnetic field will bend according to the lorentz force, $f = qV \wedge B$. This will create a circular path of radius r. This path itself is a loop of current with $J = \frac{qV}{2\pi r}$, which in turn generates a magnetic field at the center of the loop equal to,

$$B = \frac{\mu J_s}{2r_s} = \mu \frac{qW}{4\pi_s r_s}. \qquad (66)$$

If this self-field further acts on the charge, a stable solution occurs when the lorentz force equals the centripetal force. This is a known effect in a cyclotron.

$$\begin{aligned} -qV_sB &= mW^2r_s \\ \mu\frac{q^2V_sW}{4\pi_s r_s} &= mW^2r_s \\ \mu &= 4\pi_s\frac{mr_s}{q^2}. \end{aligned}$$

This can be expressed in terms of angular momentum,

$$\mu = 4\pi_s\frac{mV_sr_s}{q^2V_s} = \frac{4\pi}{q^2}\frac{L}{V_s} \qquad (67)$$

At-rest this becomes,

$$\mu_e = \frac{2}{q^2}\frac{2\pi L_e}{V} \qquad (68)$$

Compare this to the current accepted value for vacuum permeability,

$$\mu_0 = \frac{2\alpha}{q^2}\frac{2\pi\hbar}{c}. \qquad (69)$$

They are equivalent with the exception of the fine structure constant α. This problem is solved by the toroid model of section IV.

I. Flux Quantum

The magnetic flux of a current ring is defined to be

$$\Phi = B\pi_s r_s^2 \qquad (70)$$

Putting the flux quantum in terms of angular momentum be done by using the relation between mass/charge ratio and W as found in 65.

$$\Phi = \frac{m}{q}W\pi_s r_s^2 = -\frac{\pi_s L}{q} \qquad (71)$$

Which is correct. It also yields,

$$LW = 2BM = 2J_s\Phi, \qquad (72)$$

which shows the equivalence of magnetic and rotational kinetic energy.

J. Dirac-Like equation

The Dirac equation for a free particle is

$$\left(\beta mc^2 + c\sum_{n=1}^{3}\alpha_n p_n\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad (73)$$

If we interpret the term on the left as kinetic energy (eq 36), and the term on the right as the angular momentum of the particle when at rest (eq 47), Dirac's equation for a free particle can be re-written,

$$\begin{aligned} E\psi &= L_e\frac{\partial\psi}{\partial t} \qquad (74) \\ \frac{\partial\psi}{\partial t} &= \frac{E}{L_e}\psi \qquad (75) \\ \frac{\partial\psi}{\partial t} &= -W\psi \qquad (76) \end{aligned}$$

Where the ratio of energy to rest angular momentum was given by eq 55. This equation is a simple wave equation. However, while simple the left-hand term in eq 73 is a scalar, which is overly simplistic. This equation is not used in the rest of the paper.

K. Conclusion

This section has developed some fundamental equations regarding spin-$\frac{1}{2}$ particles by applying the time-harmonic approach to the Zitter electron model. The resulting equations for the vacuum permeability and magnetic moment are close in form, but not in exact agreement with accepted values. The next section introduces an additional orbit to the sub-electron path, and this change brings the relations into better agreement with known results.

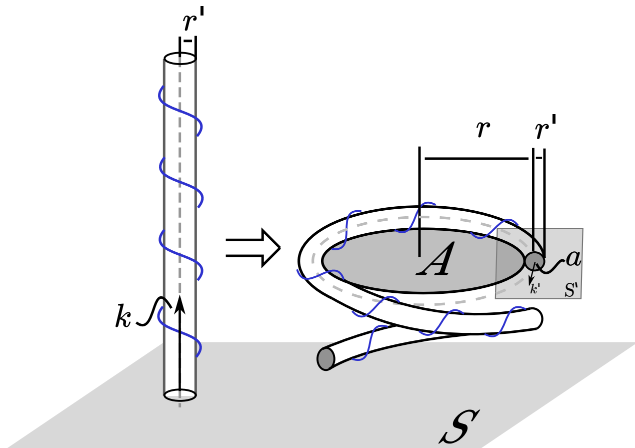


Figure 10: An electron at high speed, bent into a toroidal (nested) helix. (There is likely only one poloidal turn per toroidal turn, but illustrating many poloidal turns makes the idea more clear)

IV. Toroidal electron

Consa has recently proposed a model for the electron in which the path is a helix about a helix[3], [1]. Projecting this path into space, the electron *at rest* looks not like a ring, but like a toroidal solenoid, a.k.a. a *toroid*. Since a helix bent into a ring is a toroid, a recursive application of the existing model can be used to approximate this structure. Let us call it the Zitter^2 model. (We suspect a rigorous treatment will require curvature of spacetime, which will require General Relativity.)

Imagine the electron as a light-like helix traveling at high speed, so that $V_k' > V_s'$ as shown on the left in figure 10. Here we use the primes to separate levels of recursion. At such a high speed, the radius r_s' will be very small. If this helix is bent around onto itself by some force, just as the the lorentz force bent the charged particle into a ring, then it will result in a toroid with toroidal radius r_s and poloidal radius r_s', also shown in figure 10. The translational components of velocity/current/wavelength now becomes the toroid's rotational velocity/current/wavelength.

If r_s' remains fixed, the maximum 'particle' velocity will now be limited to translational velocity before bending, V_k', which is less than V. An illustration of this relationship is nicely shown by comparing the triangle-inscribed-in-a-circle diagram for both systems, shown in figure 11.

A. Vacuum Permeability

Since permeability has units of distributed inductance (henry/meter), the permeability produced by the toroidal model can be found by computing the inductance, and dividing it by the appropriate distance. To compute the inductance, the electrical energy is equated to the rotational energy. Since an ideal toroid looks like an infinite inductor, the magnetic field will be contained within the windings of the toroid. Start with the energy contained within an inductor,

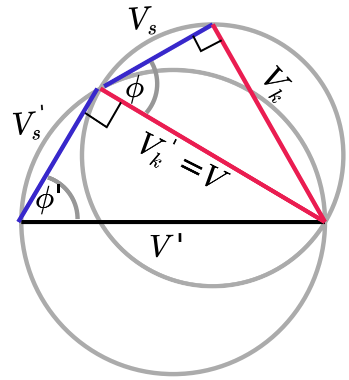


Figure 11: Relation of total V, spin V_s, and propagation V_k velocities for a single iteration of recursion on the toroidal electron model. The translation velocity V_k' appears to be the speed of light in next level up.

$$E_{\text{inductor}} = \frac{1}{2} l |J|^2, \tag{77}$$

where l is the inductance, and J is the current. Equating this to the rotational energy, and solving for the inductance,

$$\begin{aligned} \frac{1}{2} l |J|^2 &= -LW \\ l &= -2\frac{LW}{|J|^2} \\ l &= -\frac{4\pi m r_s'}{q^2}\lambda_s'. \end{aligned}$$

Permeability has units of distributed inductance, which requires a distance d to be defined.

$$\mu = \frac{l}{d} \tag{78}$$

Imagine this electron is moving at high speed and is then curved around upon itself to form a toroidal solenoid, as shown in 10. In this case, the length of the inductor becomes the toroidal circumference, λ_s, and the permeability becomes

$$\mu = \frac{l}{\lambda_s} = -\frac{4\pi m r_s'}{q^2}\frac{\lambda_s'}{\lambda_s} \tag{79}$$

To compare this with expressions containing r_s (since the primed quantities are not observed), throw in a factor of $\frac{r_s'}{r_s}$

$$\mu = -\frac{4\pi m r_s}{q^2}\frac{r_s'^{2'}}{r_s^2} \tag{80}$$

The last term is equal to the ratio of the poloidal area a to toroidal area A, as shown in figure 10.

$$\mu = -\frac{4\pi m r_s}{q^2}\frac{a}{A} \tag{81}$$

To have this agree with accepted value for μ_0, the fine constant must be,

$$\alpha = \frac{a}{A}$$

For the toroid model to work, this value must be consistent with the magnetic moment.

B. Anomalous Magnetic Moment

The total angular momentum for compound rotating systems is the sum of all angular momentum of each subsystem.

$$L_{\text{tot}} = L_{\text{toroidal}} + L_{\text{poloidal}} \tag{82}$$

Likewise, so will the magnetic moments since they are dual equations by 63. Thus we write

$$\begin{aligned} M_{\text{total}} &= M_{\text{toroidal}} + M_{\text{poloidal}} \\ &= J_k' A + J_s' a \\ &= J_s A (1 + \frac{a}{A}) \\ &= M_{\text{ring}} (1 + \alpha) \end{aligned}$$

Using eq 61 for M_{ring}, to write out the full expression in terms of L_e

$$M = 2\,(1 + \alpha)\,\frac{q}{2m}\frac{L}{2} \tag{83}$$

Which agrees up to a factor of 2π with the accepted value of the for the anomalous magnetic moment with the g-factor approximated to the second term, and is consistent with the permeability equation. Unfortunately, we have been unable to locate the missing factor of 2π from α, but hope to. If this model is correct it implies that the speed of light, as we see it, is not the maximum possible velocity since there is some component of velocity in the poloidal rotational energy.

C. Conjecture City

Lets visit conjecture city for a brief moment. A guiding theme of this theory is the desire for all scalars be related to multivectors, which essentially requires all quantities to have geometric meaning. Any scalars left un-associated with a geometric feature, such as charge and mass, are evidence of an incomplete theory. Since general relativity explains mass, what about charge? The toroidal model might provide an answer. If what we perceive as electron spin is the angular momentum of a sub-particle, it appears likely that what we model as *charge* is actually the angular momentum of the sub-sub-particle. But, since the sub-sub-particle can be thought of as a recursive application of the helix-model, charge is the same thing as spin, but down layer in recursion. This makes more sense by looking at figure 10. This could explain why it only has two-values (due to chirality), and if true, should produce the correct charge symmetry in CPT.

It is natural to wonder how far could the recursion go? Could there be an infinite level of recursion leading to a fractal super-helix like that described in[11], [12]? If true, the entire model might be expressible in terms of the differential properties of a fractal helical path in spacetime. Lets now leave conjecture city and move onto applying the time-harmonic model to the photon.

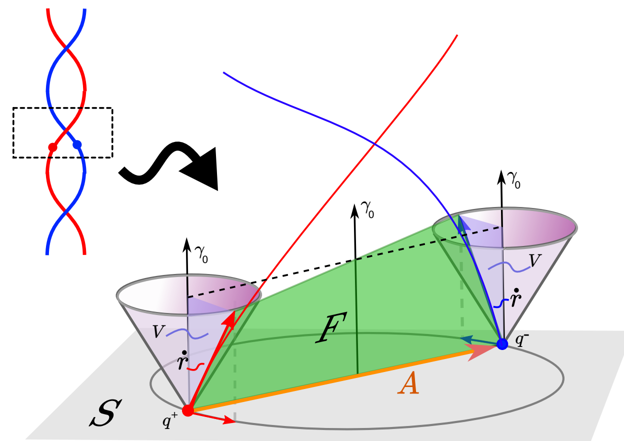


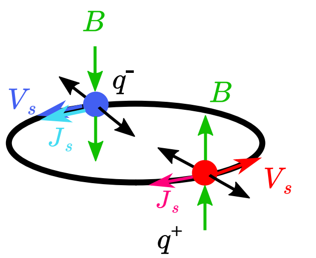
Figure 12: Twisted pair photon model, consisting of an electron and position with sub-particles orbiting a common center. Magnetic vector potential is A and electromagnetic bivector F.

V. Twisted Pair Photon

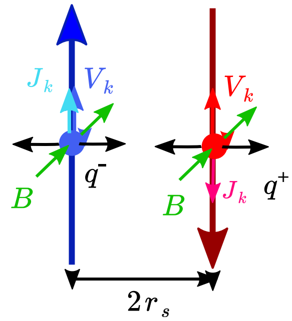
In this section we give a brief sketch of how the time-harmonic analysis might be applied to the zitter-based photon model. Hestenes has recently put forth a model for the photon which consists of a pair of sub-particles with opposite charges orbiting around a common center[1]. An illustration of this for the configuration of antipodal electron/positrons is shown in figure 12 (other configurations might also be possible). Both the electron and positron sub-particles follow light-like helical paths, as shown in blue and red. We conjecture that the particles are connected by the vector potential A which sweeps out the body of a helicoid as the path is followed. (this A is not related the A in last section). The body of the helicoid is the electromagnetic bivector F. If this model is accurate, the photon is like a superconducting twisted pair transmission line flowing DC current, which we can analyze with classical electromagnetics.

The angular momentum of this system is twice that of the electron. To make the system stable, the forces must balance, just as they did in the electron. The twisted pair can be decomposed into a spin current loop and a straight longitudinal current filament as shown in figure 13. Starting with the spin current which is illustrated in subfigure 13a, the magnetic field from one charge felt by the other is attractive. The attractive force is balanced by the centrifugal force for a specific value of μ_0, just as with the electron.

$$qV_s \wedge B = qV_s \frac{\mu_0 J_s}{4\pi r_s} = \frac{\mu_0}{4\pi} q^2 W^2 \tag{84}$$



(a) Forces due to spin current, and centripetal acceleration.



(b) Forces due to propagation current, and Coulomb attraction

Figure 13: Decomposition of forces on the photon subparticles in a) spin and b) propagation components.

This balances the centrifugal force for a specific value of μ_0, just as with the electron in the helix model.

$$\frac{\mu_0}{4\pi} q^2 W^2 = m W^2 r_s$$
$$\mu_0 = 4\pi \frac{m r_s}{q^2} \tag{85}$$

This is consistent with the toroidal model, since at the speed of light the toroid turns back into a helix, and a r_s is needed to prevent $\mu_0 \to 0$ at $V_k = V$. The longitudinal current components shown in figure 13b produce a repulsive Ampere's force, but an attractive Coulomb force. To compute the Coulomb force, the charge density must be defined as charge per helical wavelength (eq 28) ,

$$\sigma \equiv \frac{q}{\lambda} = \frac{qW}{2\pi_s V} = \frac{J_k}{V} \tag{86}$$

The Ampere and Coulomb forces balance as such,

$$\frac{\mu_0 J_k^2}{2\pi r_s} = \frac{\sigma^2}{2\pi r_s \epsilon_0} \tag{87}$$

Which produces the relation between vacuum permativity, permeability, and the speed of light.

$$\frac{J_k^2}{\sigma^2} = \frac{1}{\mu_0 \epsilon_0} = V^2 \tag{88}$$

We hope to expand upon this model in future work.

VI. Conclusion

This paper provides an approach to special relativity which is based on adapting the zitter electron model to a time-harmonic analysis. The results are self-consistent and the grades of the quantities match their units. A driving theme throughout this theory is the demand that all scalars be related to multivectors, which essentially requires all quantities to have geometric meaning. The relation between the time-harmonic model presented here, and the conventional relativity theory is to be worked out in the future. It appears as through it may be related through the Kustaanheimo–Stiefel transformation[6]. Again, since the models presented are based on a classical physics, they are 'local hidden-variable' theories which are not consistent with Bell's theorem.

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