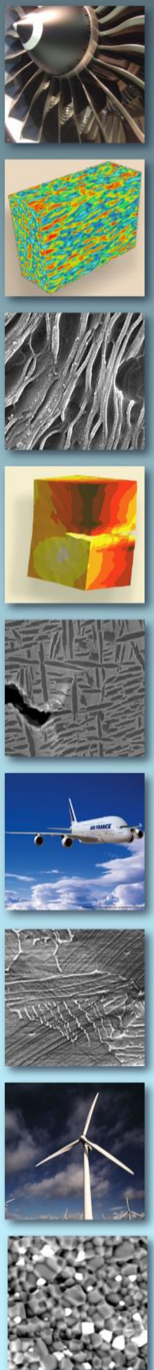

MULTIPHYSIC STUDY OF Nb_3Sn SUPERCONDUCTING STRAND DURING HEAT TREATMENT

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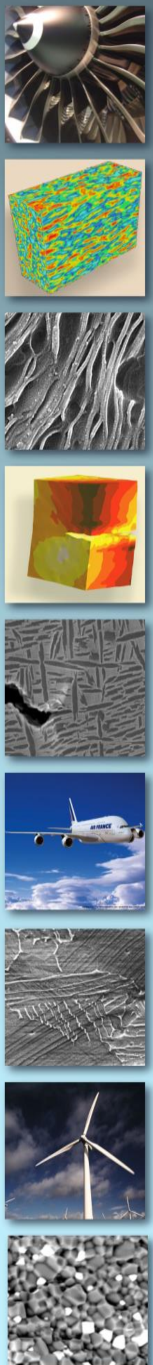

Plan

1. MOTIVATION
2. PROBLEM STATEMENT
3. DISCRETIZATION
4. TESTS, RESULTS
5. FUTURE PLANS

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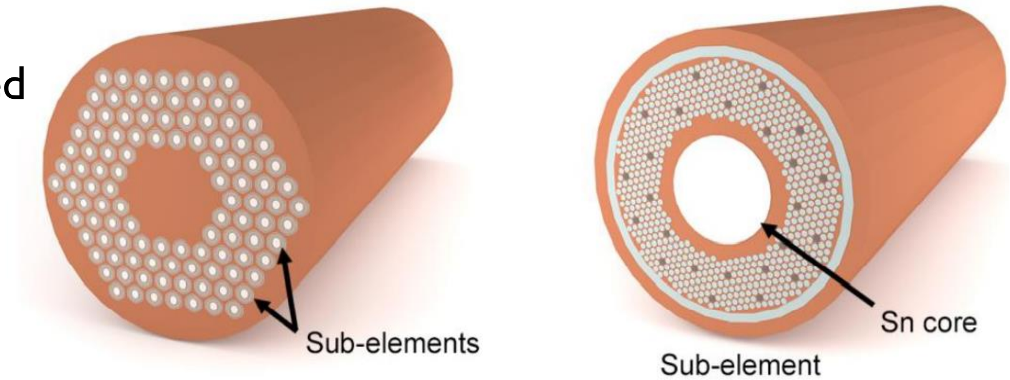
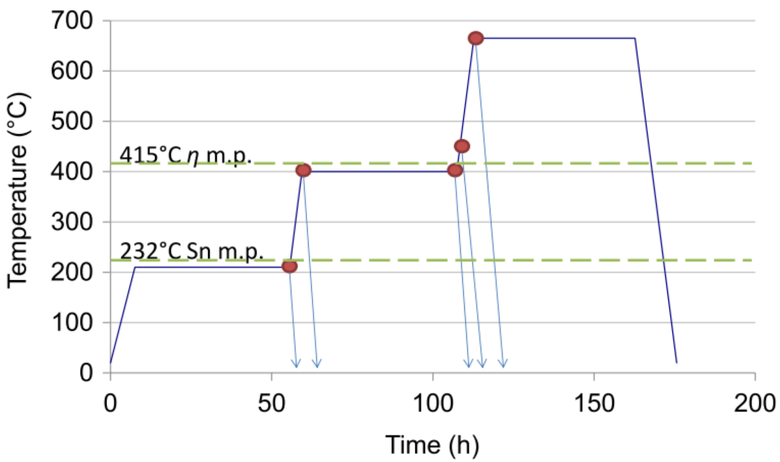
MOTIVATION

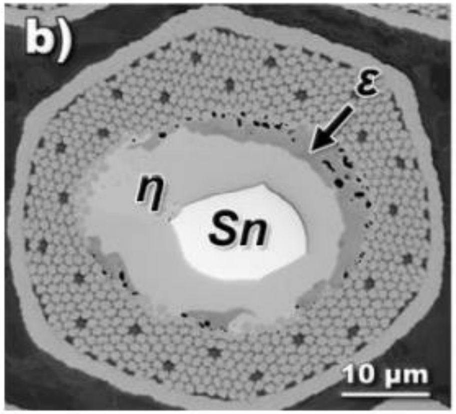
The Nb3Sn-based conductor is produced in rectangular Rutherford type cables, braided from RRP wires





The conductor requires a 3 step heat treatment up to 650°C that lead to the formation of the Nb3Sn superconducting phase in the sub-elements

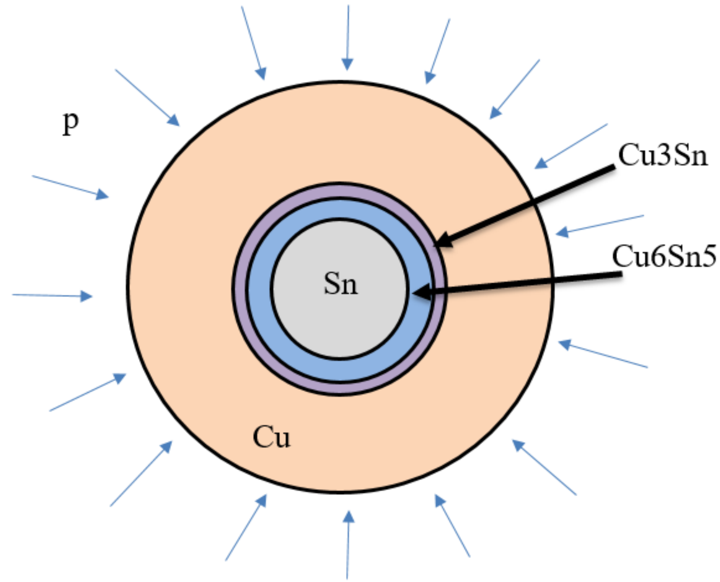
In this study we will focus on the 1 step of Cu-Sn mixing with the formation of intermetallics phases. The growth of phases is coupled with the stress-strain state inside the sub-element.



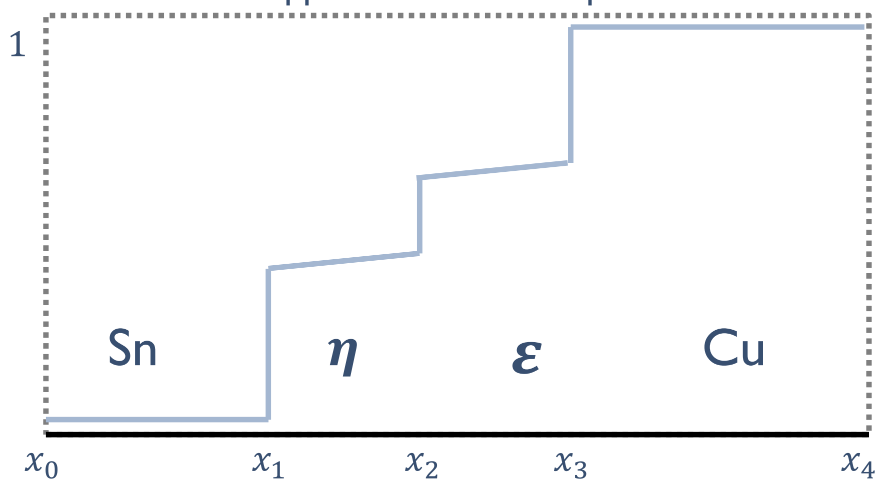
PROBLEM STATEMENT: PHASE KINETICS

Here we use following assumptions:

1. The presence of thin intermetallic layers from the beginning. No account for phase nucleation.
2. Fast rate of chemical reaction. Local chemical equilibrium at the interfaces.
3. Due to the narrow range of the composition in IMCs, the composition profile assumed to be linear.



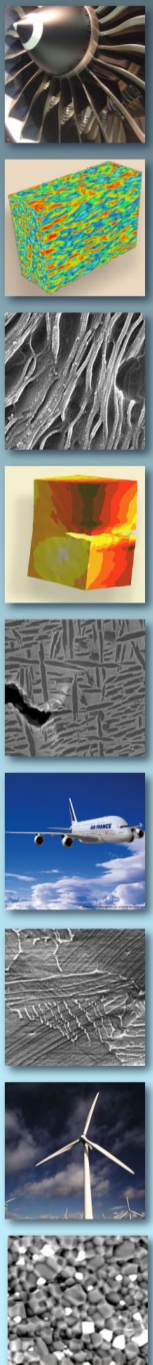
Copper concentration profile



$$\frac{dx_1}{dt} = \frac{1}{c^{Sn} - c^{\eta-}}\left(D^{\eta}\frac{c^{\eta,+} - c^{\eta,-}}{x_2 - x_1}\right),$$

$$\frac{dx_2}{dt} = \frac{1}{c^{\eta,+} - c^{\varepsilon,-}}\left(D^{\varepsilon}\frac{c^{\varepsilon,+} - c^{\varepsilon,-}}{x_3 - x_2} - D^{\eta}\frac{c^{\eta,+} - c^{\eta,-}}{x_2 - x_1}\right),$$

$$\frac{dx_3}{dt} = \frac{1}{c^{Cu} - c^{\varepsilon,+}}\left(-D^{\varepsilon}\frac{c^{\varepsilon,+} - c^{\varepsilon,-}}{x_3 - x_2}\right).$$



PROBLEM STATEMENT: LINEAR ELASTICITY PROBLEM

Equilibrium equations in cylindrical coordinates

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0, \; -> \; \frac{\partial^2 u_r}{\partial r^2} + \frac{\partial u_r}{\partial r}\frac{1}{r} - \frac{u_r}{r^2} = 0$$

$$\frac{\partial \sigma_z}{\partial z} = 0 \quad -> \quad \frac{\partial^2 u_z}{\partial z^2} = 0 \quad -> \quad u_z = \epsilon_z z + b, \quad \epsilon_z^{el} = \epsilon_z - \epsilon_z^*$$

Boundary conditions + Conditions at the interfaces between phases

$$u_r(0) = 0, \quad \int_0^R \sigma_z r dr = 0, \quad -> \quad \epsilon_z = \frac{-\int_0^R \lambda_i(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}) r dr + \int_0^R (3\lambda_i + 2\mu_i)\epsilon_i^* r dr}{\int_0^R (\lambda_i + 2\mu_i) r \, dr},$$

$$\sigma_r(R) = p, \quad -> \quad (\lambda_{Cu} + 2\mu_{Cu})\frac{\partial u_r}{\partial r} + \lambda_{Cu}\frac{u_r}{r} + \lambda_{Cu}\epsilon_z = p + (3\lambda_{Cu} + 2\mu_{Cu})\epsilon_{Cu}^*,$$

$$[\sigma_r(r_i)]_{+,-} = 0, \quad -> \quad \text{continuity of radial stresses at the interfaces.}$$

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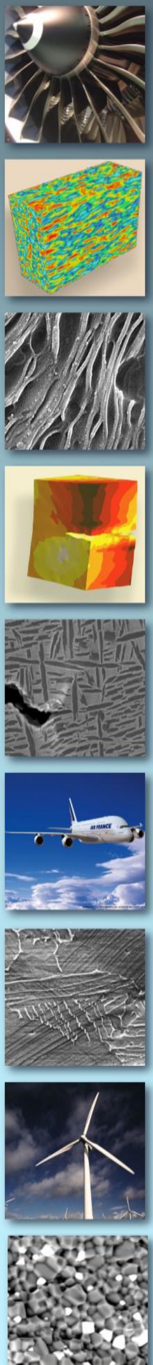

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DIFFUSION-ELASTICITY COUPLING

Diffusion coefficients depends on pressure (trace of stress tensor)

The interdiffusion coefficients are set in the Arrhenius form

$$D^{\varepsilon} = D_0^{\varepsilon} exp(-\frac{Q^{\varepsilon}}{RT}) = 5.48 \times 10^{-9} exp(\frac{-61.86}{RT}),$$

$$D^{\eta} = D_0^{\eta} exp(-\frac{Q^{\eta}}{RT}) = 1.84 \times 10^{-9} exp(\frac{-53.92}{RT}),$$

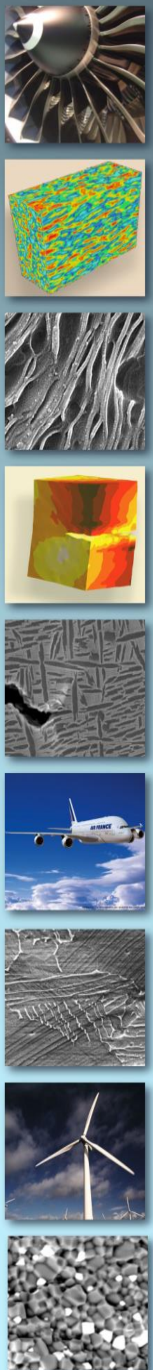
where Q – is activation energy; D_0 – reference diffusion constant.

Pressure effect is introduced through activation energy:

$$Q = Q_0 - k\, tr(\underline{\sigma}), \quad \text{where } k \text{ is a material constant.}$$

Eigenstrains due to phase transformations are assumed to be isotropic

$$\epsilon^*_{Sn} = 0, \quad \epsilon^*_{Cu} = 0, \quad \epsilon^{\eta,*} = -0.00327, \quad \epsilon^{\varepsilon,*} = -0.0279$$



DISCRETIZATION

Explicit scheme for time integration was used:

$$\frac{x_1^{i+1} - x_1^i}{t^{i+1} - t^i} = \frac{1}{c^{Sn} - c^{\eta -}} \left(D^\eta \frac{c^{\eta,+} - c^{\eta,-}}{x_2^i - x_1^i}\right),$$

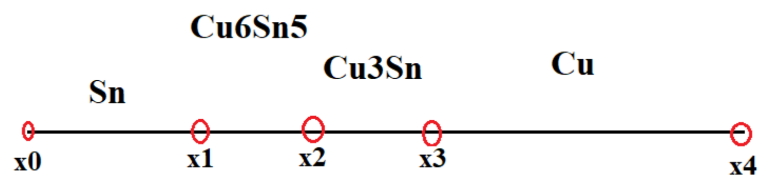
$$\frac{x_2^{i+1} - x_2^i}{t^{i+1} - t^i} = \frac{1}{c^{\eta,+} - c^{\varepsilon,-}} \left(D^\varepsilon \frac{c^{\varepsilon,+} - c^{\varepsilon,-}}{x_3^i - x_2^i} - D^\eta \frac{c^{\eta,+} - c^{\eta,-}}{x_2^i - x_1^i}\right),$$

$$\frac{x_3^{i+1} - x_3^i}{t^{i+1} - t^i} = \frac{1}{c^{Cu} - c^{\varepsilon,+}} \left(-D^\varepsilon \frac{c^{\varepsilon,+} - c^{\varepsilon,-}}{x_3^i - x_2^i}\right).$$

Each phase region is divided by N_i elements. Inside the phase domain we have a regular mesh but size of the elements is different for each phase.

central difference scheme:

$$\frac{u_r^{i+1} - 2u_r^i + u_r^{i-1}}{h_i^2} + \frac{1}{u_r^i} \frac{u_r^{i+1} - u_r i - 1}{2h_i} - \frac{u_r^i}{(r^i)^2} = 0.$$



Boundary conditions:

$$u_r^0 = 0.0 \;, \quad (\lambda_{Cu} + 2\mu_{Cu}) \frac{u_r^N - u_r^{N-1}}{h_N} + \lambda_{Cu} \frac{u_r^N}{r^N} = p + (3\lambda_{Cu} + 2\mu_{Cu})\epsilon_{Cu}^* - \lambda_{Cu}\epsilon_z$$

At the interfaces: $\sigma_r^{-,N_i} = \sigma_r^{+,N_i}, \quad ->$

$$(\lambda^- + 2\mu^-) \frac{u_r^i - u_r^{i-1}}{h^-} - (\lambda^+ + 2\mu^+) \frac{u_r^{i+1} - u_r^i}{h^+} + (\lambda^- - \lambda^+) \frac{u_r^i}{r^i} = (3\lambda^- + 2\mu^-)\epsilon^{*,-} - (3\lambda^+ + 2\mu^+)\epsilon^{*,+} + (\lambda^+ - \lambda^-)\epsilon_z$$

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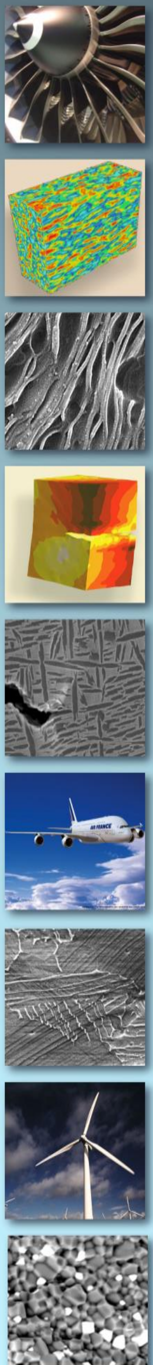

Algorithm

Input values: Initial concentration profile
Mechanical properties, Diffusion coefficients, Eigenstrains

t=0, i=0

While t < T, i <Nt

Solving mechanical part,
Obtain displacements, stresses

Iterations until convergence of ϵ_z
Tridiagonal matrix algorithm at each step

Calculate new interface positions, x1,x2,x3

Explicit scheme for time integration

Update mesh with new interface positions

i=i+1
t=t+dt

Final solution:
Evolution of interfaces with time
Evolution of stress strain state in time

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TESTS

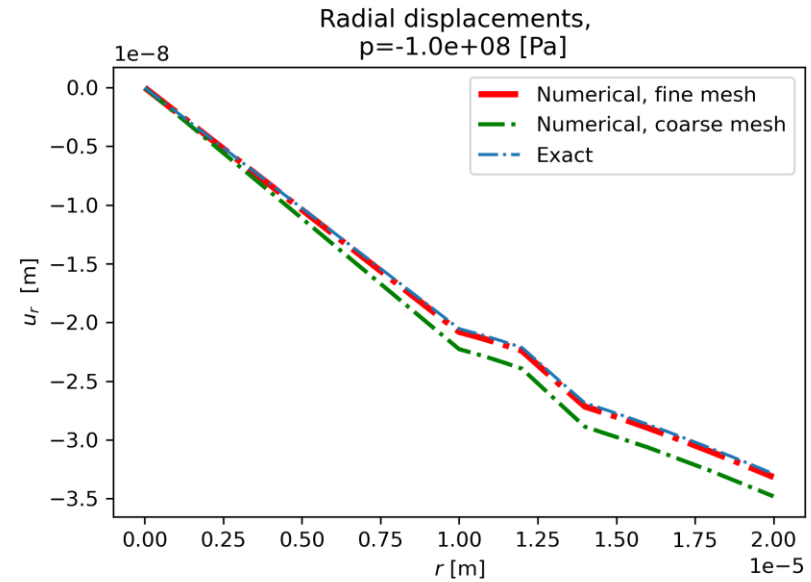
For elasticity problem, the model was verified on couple of analytical solutions

1. Only external pressure applied. No eigenstrains in intermetallics.

2. No pressure applied. Only eigenstrains in intermetallics are presented.

Coarse mesh consists of 4 points in each phase region.
Fine mesh consists of 20 point in each phase region.

Numerical solution converges to the exact one with mesh refinement.



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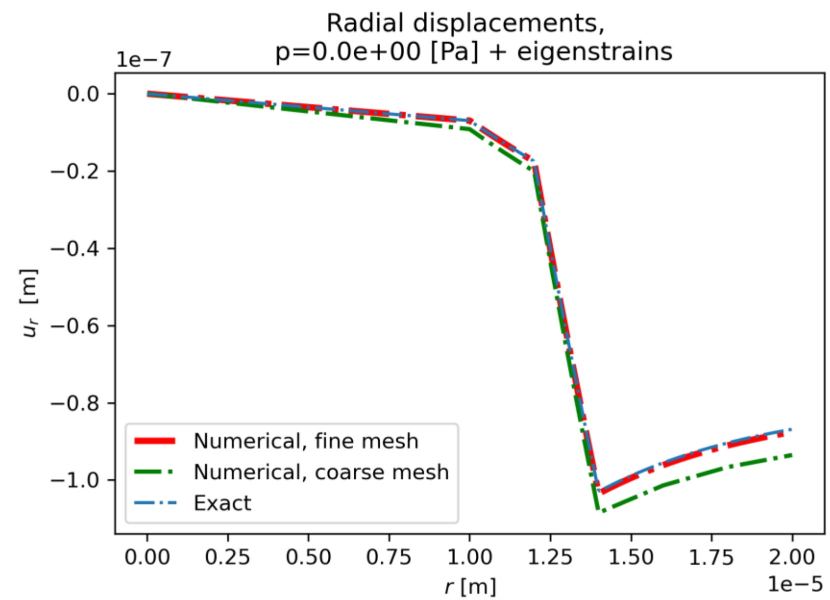

2.

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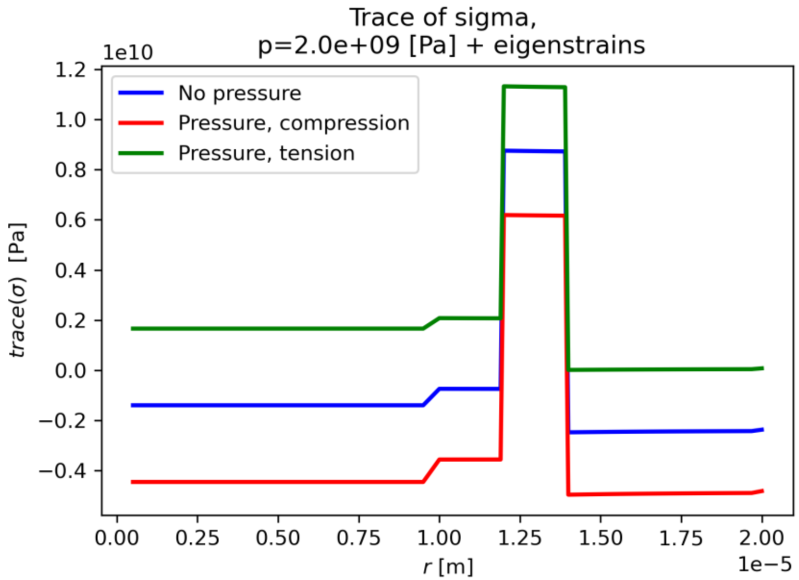
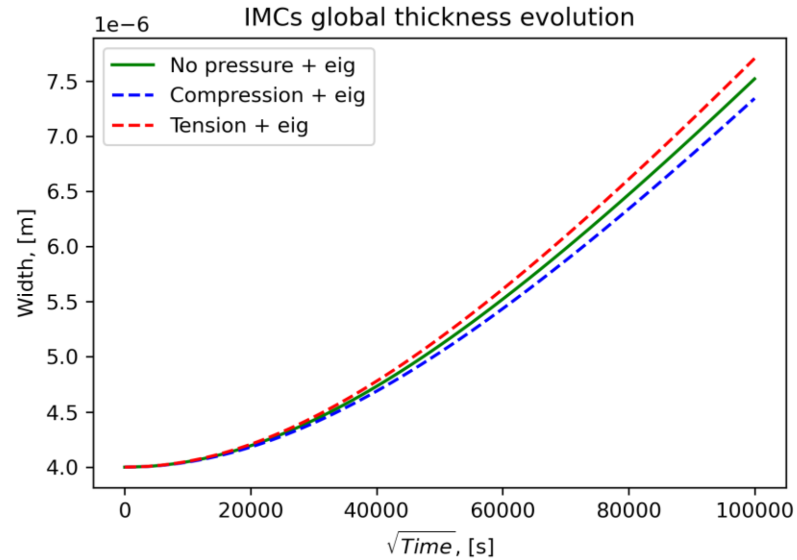
PRESSURE EFFECT ON INTERMETALLICS GROWTH



Trace of stress tensor. Initial distribution.



Total thickness of intermetallics.

As can be seen, hydrostatic pressure is almost constant in each phase. To remain constant, interdiffusion coefficients are assumed to be dependent on average of hydrostatic pressure in each phase.

External pressure decreases the rate of IMCs evolution compared to no pressure state.

In turn, external tension increases the rate of IMCs evolution.

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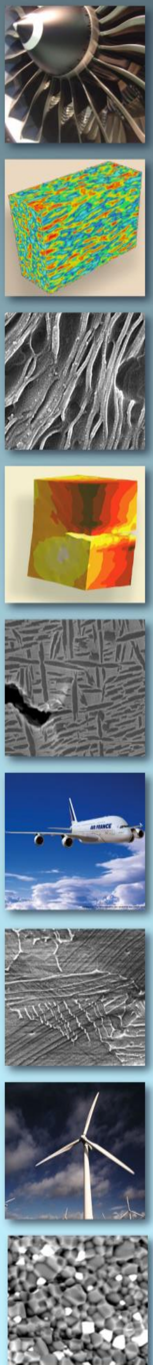

FUTURE PLANS

1. It is necessary to make a comparison with experiment
2. Temperature effect on mechanical properties
3. More accurate accounting of phase kinetics
4. Consider the next heat treatment steps

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