# Introduction to Mathematical Thinking

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## Summary

A large subset of the students entering Berkeley's Computer Science program don't have the mathematical preparation necessary to succeed in Berkeley's highly theoretical curriculum. Students with extracurricular math preparation in high school tend to find courses such as CS 70, CS 170 and Math 55 significantly easier than students without this background.

We present the Introduction to Mathematical Thinking DeCal, a 2 unit DeCal (approval pending) designed to bridge this gap. This course will likely first be run in the Fall 2018 semester.

## Goals

**The primary goal of this course is to have students develop mathematical maturity.** Our curriculum is devoted to exposing students to concepts they may have seen before, but to present these concepts in a more precise, generalized way. By the end of our course, students will be able to:
- comfortably read mathematical language, including notation, definitions and proofs
- generalize concepts and techniques introduced in class to harder novel problems
- concisely and clearly express their ideas
- differentiate between a good proof and a proof with logical gaps

As a result, this course will prepare students for higher-level mathematics courses, such as CS 70 at Berkeley. With that being said, students need not plan on taking CS 70 to take this course; we will allow students to enroll in the course out of interest.

## Format

While much of this is subject to approval, the current planned format is to have two classes a week, with one 1.5 hour "lecture" and one 1 hour "discussion". Consider the following structure, where lectures are on Thursdays and discussions on Tuesdays:

- In Thursday's lecture, a new topic is presented. The lecturer will give the students some time to work on simple examples.
- A problem set is released after lecture.
- On Tuesday, students come to discussion section and put their solutions to the problems on the whiteboard, and will present their solutions to the class.

The percentage breakdown of the course is still TBD, however the three most significant components will be attendance and performance on a take-home midterm assignment and in-class final exam.

## Topics

Below is a week-by-week schedule of the topics to be covered in the course. Each topic will have notes accompanying it, for students to read before that week's lecture / to have as reference.

Formal mathematical notation will be introduced in the notes and in class. Proofs will be mentioned in each topic in both the lecture and problem sets. The idea is that during the final two weeks, students realize that they've been doing proofs all along, and we can formalize a set of proof techniques.

| Week | Topic | Subtopics and Concepts |
|---|---|---|
| 0 | **Introduction** | Why does this class exist? How to effectively learn mathematics for problem solving. Overview of topics covered throughout the course. |
| 1 | **Sets of Numbers** | Natural numbers and integers, how to "generate" the integers given the naturals. A formal definition of rational numbers. Proofs of rationality. Real numbers, and why there are "more" real numbers than rationals. The existence of complex numbers. |
| 2 | **Sets of Numbers, Continued** | Fields and equivalence classes. Operations on complex numbers. (TBD) |
| 3 | **Number Theory** | The theory and techniques behind prime factorization. The division algorithm. The concept of equivalence classes as a precursor to modular arithmetic (eg. how to calculate a day of the week far in advance). Patterns in the last digit of repeated multiplication. |
| 4 | **Properties of Polynomials** | Standard representation as $p(x) = \sum_{i=0}^{n} a_i x^i$. The fact that an $n$-degree polynomial has $n$ roots, with some possibly complex. Vieta's formulas for quadratics. The division algorithm for polynomials – if $p(a) = 0$, then $x - a$ is a factor. Polynomial long division and synthetic division. |
| 5 | **Counting – Permutations and Combinations** | Introduction to thinking about the "number of ways" to do something. Counting when order matters vs. when order doesn't matter. The factorial, the permutation term and the combinatorial term $\binom{n}{k}$. Variety of practice problems. |

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| 6 | **Pascal's Triangle** | Construction of the triangle by recursion. Introduction to the combinatorial definition of a row. Various properties of the triangle (eg. sum of a row, Pascal's identity, Hockey-Stick Theorem, Vandermonde's Identity). |
| 7 | **Binomial Theorem** | Binomial Theorem, posed as a combinatorial problem. Determining coefficients of complicated binomials. Extension to multinomials. |
| 8 | **Introduction to Basic Probability** | Probability, viewed through basic axioms. Introduction to conditional probability. Expectation. Binomial distribution. |
| 9 | **Proof Technqiues** | Formalization of all proof techniques discussed earlier in the course, including induction. Flaws in reasoning. |
| 10 | **More Proof Technqiues** | More proof-based problems. A formal discussion on the concept of infinity. |