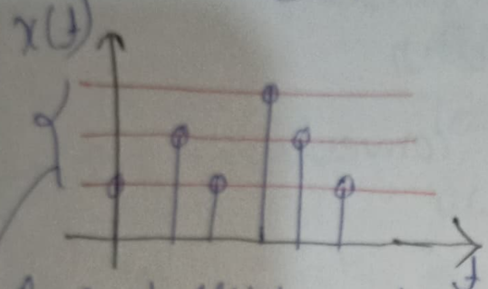
Digital Logic

⇒ Digital Logic :-

- Digital has been derived from the word Digit
- Digits are always finite in number.
- It represents a system with finite number of possibilities.

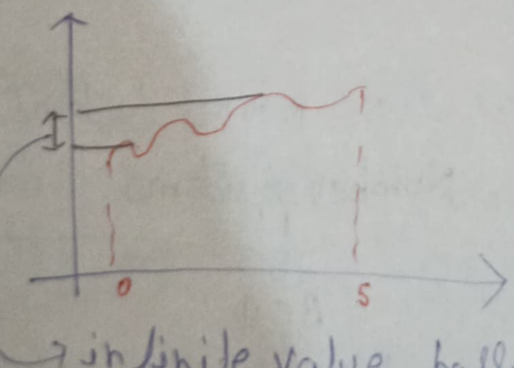
ex :-
Tube light, Traffic lights, switch, Gear Box etc.



- 3 possible value of function : digital

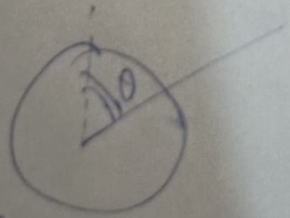
⇒ Analog :-

Analog has been derived from the word Analogous



→ infinite value possible.

- Analog is similar to real world signals.



θ : analog

θ → continous from 0 - 360°

$\theta = 36^\circ$ → than this will be digital.
↳ θ is fixed

analog i/p → ADC → DSP → DAC → analog o/p

ADC → analog to digital converter.
DSP → Digital signal processor
DAC → Digital to analog converter.

⇒ Number systems:-

A system used to represent a number.

1) Decimal Number system
2) Binary
3) octal
4) Hexa decimal

Conversions b/w these number systems.

⇒ Number systems

- Base "n"
  - octal
  - Hexadecimal
  - Binary
  - Decimal
- BCD
  - Binary code Decimal
- Data representations
  - [n-1]'s compliment
  - n's compliment

4 → quatornary.

⇒ Number Systems of different Base :-

Base :- In a positional numeral system the radix or, base is the number of unique digits, including the digit zero, used to represent numbers.

Base	Terminology	Range
1) 2	Binary	0, 1
2) 3	Ternary	0, 1, 2
4) 8	octal	0 - 7
5) 10	Decimal	0 - 9
6) 16	Hexadecimal	0 - 9, A - F

where :-

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

⇒ Conversion :-

1) Decimal to other Base System :-

Q 1) $(619)_{10} \rightarrow (x)_8$

	Q	R
619/8	77	3
77/8	9	5
9/8	1	1
1/8	0	1

$\Rightarrow (1153)_8$ Ans

8) 619 (77
616
3

Q → Quotient
R → Reminder

2) ⇒ Other Base System to Decimal System:-

Q $(11530)_8 \rightarrow (x)_{10}$

$$\begin{array}{ccccc} 1 & 1 & 5 & 3 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ 8^3 & 8^2 & 8^1 & 8^0 & \end{array}$$

$= 1\times8^3 + 1\times8^2 + 5\times8^1 + 3\times8^0$

$= 512 + 64 + 40 + 3$

$= (619)_{10}$ Ans

⇒ Other Base System to Non-Decimal System:-

1) Convert the original number to decimal number (base 10).

2) Convert the decimal number so obtained to the new base number.

Q $(1001)_2 \rightarrow (x)_3$

Binary to decimal →

$$\begin{array}{cccc} 1 & 0 & 0 & 1 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$\Downarrow$

$1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$

$\Rightarrow (9)_{10} \rightarrow (x)_3$

Decimal to ternary ⇒

9/3	3	0
3/3	1	0
1/3	0	1

$(100)_3$ Ans

⇒ Binary to Octal :-

1) – Divide the binary digits into groups of three (starting from the right).

2) – Convert each group of three binary digits to one octal digit.

000 → 0		101 → 5	
001 → 1		110 → 6	
010 → 2		111 → 7	
011 → 3			
100 → 4			

0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

Q1) $00(\underbrace{1}_{1}\,\underbrace{100}_{4}\,\underbrace{101}_{5})_2 \rightarrow (x)_8$

$(145)_8$ Ans

Q2) $0(\underbrace{10}_{2}\,\underbrace{010}_{2}\,\underbrace{001}_{1}\,\underbrace{110}_{6}\,\underbrace{011}_{3}\,\underbrace{010}_{2}\,\underbrace{001}_{1})_2 \rightarrow (x)_8$

2 2 1 6 3 2 1

$(2216321)_8$ Ans

⇒ Octal to Binary :-

1) → Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion).

2) → Combine all the resulting binary groups (of 3 digit each) into a binary number.

Q 1) $(2467531)_8 \rightarrow (X)_2$

$(\underbrace{010}_{2}\,\underbrace{100}_{4}\,\underbrace{110}_{6}\,\underbrace{111}_{7}\,\underbrace{101}_{5}\,\underbrace{011}_{3}\,\underbrace{001}_{1})_2$ Ans

Q 2) $(7542106)_8$

$(\underbrace{111}_{7}\,\underbrace{101}_{5}\,\underbrace{100}_{4}\,\underbrace{010}_{2}\,\underbrace{001}_{1}\,\underbrace{000}_{0}\,\underbrace{110}_{6})_2$ Ans

⇒ Binary to hexadecimal :-

1) Divide the binary digits into groups of four (starting from the right).
2) Convert each group of four binary digits to one hexadecimal symbol.

0000 → 0
0001 → 1
0010 → 2
0011 → 3
0100 → 4
0101 → 5
0110 → 6
0111 → 7
1000 → 8
1001 → 9
1010 → 10 (A)
1011 → 11 (B)
1100 → 12 (C)
1101 → 13 (D)
1110 → 14 (E)
1111 → 15 (F)

Q $(100|0111\,011000100101100)_2 \rightarrow (X)_{16}$

4 $\frac{11}{B}$ $\frac{11}{B}$ 1 2 $\frac{12}{C}$

$(4BB12C)_{16}$ Ans

⇒ Hexadecimal to Binary :-

1 → convert each hexadecimal digit to a 4 digit binary number (the hexa-decimal digits may be treated as decimal for this conversion)

2 → combine all the resulting binary groups (of 4 digits each) into a single binary number

Q $(4BCDA975)_{16} \rightarrow (X)_2$

~~(0100 1100 1101 1110 1011 1010 0111 0101~~

~~$(\underbrace{0100}_{4}\underbrace{1100}_{B}\underbrace{1101}_{C}\underbrace{1110}_{D}\underbrace{1011}_{A}\underbrace{1010}_{9}\underbrace{0111}_{7}\underbrace{0101}_{5})_2$ Ans~~

$(\underbrace{0100}_{4}\ \underbrace{1011}_{B}\ \underbrace{1100}_{C}\ \underbrace{1101}_{D}\ \underbrace{1010}_{A}\ \underbrace{1001}_{9}\ \underbrace{0111}_{7}\ \underbrace{0101}_{5})_2$ Ans

Q Addition :-

1) if $(2.3)_{base 4} + (1.2)_{base 4} = (y)_{base 4}$; what is the value of y?

$$\begin{array}{cc} & 1 \\ 2 & . \ 3 \\ 1 & . \ 2 \\ \hline 4 & \ 5 \\ 10 & \ 1 \end{array} \quad 11$$

$(10.1)_4$ Ans

on base 4 → only $\boxed{0, 1, 2, 3}$

	Q	R
5/4	1	1
1/4	0	1

$5 = (11)_4$

	Q	R
4/4	1	0
1/4	0	1

$4 = (10)_4$

$(43)_x = (y3)_8$

$8 \rightarrow$ that means y must be up to 7

$\rightarrow \boxed{x \geq 5} \rightarrow$ because highest digit is 4

$y = 7 \rightarrow x = 14$
$y = 6 \rightarrow x = 12$
$y = 5 \rightarrow x = 10$
$y = 4 \rightarrow x = 8$
$y = 3 \rightarrow x = 6$
~~$y = 2 \rightarrow x = 4$~~

5 Ans $\rightarrow$ no of Solution

$\Rightarrow$ Decimal to other Base System :-

$(9743.56)_{10} \longrightarrow (X)_b$

Steps :-

1) The integer part is converted to base-b as we did in previous lecture. (regularly divide by base-b & collect all remainders).

2) continuously multiply by base-b & collect all integer parts. (Keep doing it untill we get 0).

Q $(37.625)_{10} \longrightarrow (X)_2$

	Q	R
$\frac{37}{2}$ =	18	1
18/2 =	9	0
9/2 =	4	1
4/2 =	2	0
2/2 =	1	0
1/2 =	0	1

$0.625 \times 2 = 1.25$ → 1
$0.25 \times 2 = 0.50$ → 0
$0.50 \times 2 = 1.00$ → 1
$0.00 \times 2 = 0.$

$(100101.101)_2$ Ans.

Q $(37.625)_{10} \rightarrow (x)_8$

①

	Q	R
37/8	4	5
4/8	0	4

$(37)_{10} \rightarrow (45)_8$

② ~~0.625×8 = 2.600 2~~
~~0.600×8 = 4.800 4~~

$0.625 \times 8 = 5.000$ 5

$0.000 \times 8 = 0$

$(0.625)_{10} = (5)_8$

$(45.5)_8$ Ans

Q $(78.025)_{10} \rightarrow (x)_{16}$

①

	Q	R
78/16	4	14 → E
4/16	0	4

$(78)_{10} \rightarrow (4E)_{16}$

②

$0.025 \times 16 \rightarrow 0.400$ 0

$0.400 \times 16 \rightarrow 6.400$ 6

$0.400 \times 16 \rightarrow 6.400$ 6

$(0.025)_{10} \rightarrow (066\ldots)_{16}$

$(78.025) = (4E.066\ldots)_{16}$ Ans

⇒ Other Base System to Decimal :-

$(d_3 d_2 d_1 d_0 d_{-1} d_{-2})_b \rightarrow (x)_{10}$

$$\boxed{x = (d_3 b^3 + d_2 b^2 + d_1 b + d_0 + d_{-1} b^{-1} + d_{-2} b^{-2})}$$

- Based on position of digit each of the digits carries a weightage so we multiply each digit by corresponding weightage & take sum

Q $(0110110 \cdot 1010)_2 \rightarrow (x)_{10}$

$\Rightarrow 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 + 1 \times 2^5 + 0 = (0110110)$

$\Rightarrow 0 + 2 + 4 + 0 + 16 + 32 = 54$

$(.1010) \rightarrow 1 \times 2^{-1} + 0 + 1 \times 2^{-3} + 0 \Rightarrow \frac{1}{2} + \frac{1}{8}$

$\Rightarrow 0.5 + 0.125 \Rightarrow 0.625$

$(0110110.1010)_2 \rightarrow (54.625)_{10}$ Ans

Q $(7632.15)_8 \rightarrow (x)_{10}$

~~$2 \times 2^0 + 3 \times 2^1 + 6 \times 2^2 + 7 \times 2^3 \Rightarrow 2 + 6 + 24 + 56 = 88$~~

~~$(0.15) \Rightarrow 1 \times 2^{-1} + 5 \times 2^{-2} = 0.5 + \frac{5}{4} = 0.5 + 1.25 = 1.75$~~

~~$(7632.15)_8 = (89.75)_{10}$ Ans~~

$2 \times 8^0 + 3 \times 8^1 + 6 \times 8^2 + 7 \times 8^3 = 2 + 24 + 384 + 3584 = 3994$

$(0.15) \Rightarrow 1 \times 8^{-1} + 5 \times 8^{-2} \Rightarrow \frac{1}{8} + \frac{5}{64} \Rightarrow$

$\Rightarrow 0.078125 + 0.125 \Rightarrow 0.203$

$(7632.15)_8 = (3994.203)_{10}$ Ans

Q $(DC4F.16)_{16} \to (X)_{10}$

$F\times16^0 + 4\times16^1 + C\times16^2 + D\times16^3 = 15\times1 + 4\times16 + 12\times256$
$+ 13\times4096$

$\Rightarrow 15 + 64 + 3072 + 53248 \Rightarrow 56399$

$(0.16) \Rightarrow 1\times16^{-1} + 6\times16^{-2} \Rightarrow \frac{1}{16} + \frac{6}{256}$

$\Rightarrow 0.0625 + 0.0234$

$= 0.0859$

$(DC4F.16)_{16} \Rightarrow (56399.0859)_{10}$ Ans

Q

$00(1110011.1011)_2 00 \to (X)_8$

1 6 3 . 5 4

$(163.54)_8$ Ans

We can add zeroes at begining of a no. or at end of number after decimal point.

$\Rightarrow$ BCD codes (Binary coded decimal):–

arithmetic is complicated need more no. of bit

4 bits $\to$ $\overline{0/1}$ $\overline{0/1}$ $\overline{0/1}$ $\overline{0/1}$

$\Rightarrow 2^4 \Rightarrow 16$ No of combination.

$\Rightarrow$ Weighted codes:- Weighted binary codes are those binary codes which obey the positional weight principle. Each position of the number represents a specific weight. Several systems of the codes are used to express it.