Nave Bayes Classifier

1 Bayes' Theorem

Bayes' Theorem is used to update probabilities when new evidence is introduced. It is mathematically expressed as:

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \tag{1}$$

1.1 Explanation of Terms

- $P(Y|X)$: Posterior Probability
  The probability of event Y occurring given that event X has occurred.
- $P(X|Y)$: Likelihood
  The probability of observing X given that Y is true.
- $P(Y)$: Prior Probability
  The probability of Y occurring before considering X.
- $P(X)$: Marginal Probability (Evidence)
  The total probability of observing X.

2 Example: Medical Diagnosis

Suppose a patient takes a test for a rare disease. We have the following probabilities:

- The disease affects only 0.1% of the population: $P(Y) = 0.001$.
- If a person has the disease, the test detects it correctly 99% of the time: $P(X|Y) = 0.99$.
- If a person does not have the disease, the test gives a false positive 5% of the time: $P(X|\neg Y) = 0.05$.

We want to determine the probability that the patient actually has the disease given that they tested positive:

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \tag{2}$$

First, we calculate $P(X)$ (total probability of a positive test):

$$P(X) = P(X|Y)P(Y) + P(X|\neg Y)P(\neg Y) \tag{3}$$

Substituting values:

$$P(X) = (0.99 \times 0.001) + (0.05 \times 0.999) \tag{4}$$

$$= 0.00099 + 0.04995 = 0.05094 \tag{5}$$

Now, applying Bayes' Theorem:

$$P(Y|X) = \frac{0.99 \times 0.001}{0.05094} \tag{6}$$

$$= \frac{0.00099}{0.05094} \approx 0.0194 \text{ (or } 1.94\%) \tag{7}$$

3 Conclusion

Even though the test is 99% accurate, the probability that a patient actually has the disease after testing positive is only 1.94%. This is because the disease is very rare, and false positives significantly affect the result.

4 Applications of Bayes' Theorem

- **Medical Diagnosis** – Estimating disease probability from test results.
- **Spam Filtering** – Detecting spam emails using probabilistic models.
- **Machine Learning** – Used in Naïve Bayes classifiers.
- **Autonomous Vehicles** – Probabilistic decision-making for obstacle detection.