Key Topics in Linear Algebra for AI and Machine Learning

1 Basics of Linear Algebra

- Scalars, Vectors, Matrices, and Tensors
- Vector and Matrix Notation
- Types of Matrices (Square, Diagonal, Identity, Zero, etc.)
- Matrix Operations (Addition, Subtraction, Multiplication)

2 Vector Operations

- Vector Addition and Subtraction
- Scalar Multiplication
- Dot Product and Cross Product
- Norms and Distance Metrics (L_1, L_2 Norms)
- Projection of Vectors

3 Matrix Operations

- Matrix Multiplication (Row-by-Column)
- Transpose of a Matrix
- Inverse of a Matrix
- Determinants and Rank of a Matrix
- Trace of a Matrix

4 Special Matrices and Properties

- Orthogonal Matrices
- Symmetric and Skew-Symmetric Matrices
- Positive Definite Matrices
- Sparse Matrices

5 Systems of Linear Equations

- Solving Linear Systems (Gaussian Elimination, LU Decomposition)
- Homogeneous and Non-Homogeneous Systems
- Cramer's Rule

6 Eigenvalues and Eigenvectors

- Definition and Importance in AI
- Eigenvalue Decomposition
- Spectral Theorem
- Applications in Dimensionality Reduction

7 Singular Value Decomposition (SVD)

- Definition and Computation
- Low-Rank Approximation
- Principal Component Analysis (PCA)

8 Vector Spaces and Basis

- Definition of a Vector Space
- Basis and Dimension
- Column Space, Row Space, and Null Space
- Linear Independence

9 Linear Transformations

- Definition and Examples
- Kernel and Image of a Transformation
- Change of Basis

10 Applications in AI & Machine Learning

- Principal Component Analysis (PCA) for Dimensionality Reduction
- Support Vector Machines (SVM) and Kernel Methods
- Deep Learning: Weight Matrices and Transformations
- Graph Neural Networks (GNNs) and Graph Representations
- Singular Value Decomposition (SVD) in Data Compression