Sixth Edition

Fundamentals of

Electric Circuits

Charles K. Alexander | Matthew N.O. Sadiku

McGraw Hill Education

sixth edition

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Dedicated to our wives, Kikelomo and Hannah, whose understanding and support have truly made this book possible.

Matthew
and
Chuck

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(downloads: Problem Solving Workbook; Multisim Files; PSpice & MATLAB Tutorials)

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Contents

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Preface

In keeping with our focus on space for covers for our book, we have chosen the NASA Voyager spacecraft for the sixth edition. The reason for this is that like any spacecraft there are many circuits that play critical roles in their functionality. The beginning of the Voyager 1 and 2 odyssey began on August 20, 1977, for Voyager 2 and on September 5, 1977, for Voyager 1. Both were launched from NASA's Kennedy Space Center in Florida. The Voyager 1 was launched on a faster orbit so it eventually became the first man-made object to leave our solar system. There is some debate over whether it has actually left the solar system, but it certainly will at some point in time. Voyager 2 and two Pioneer spacecraft will also leave the solar system at some point in time.

Voyager 1 is still functioning and sending back data, a truly significant achievement for NASA engineers. The design processes that make the Voyager operate so reliably are based on the fundamentals discussed in this textbook. Finally, space is vast so that Voyager 1 will fly past other solar systems; the odds of actually coming into contact with something are so remote that it may virtually fly through the universe forever! For more about Voyager 1, go to NASA's website: www.nasa.gov/.

Features

New to This Edition

We have added learning objectives to each chapter to reflect what we believe are the most important items to learn from each chapter. These should help you focus more carefully on what you should be learning.

There are more than 580 revised end-of-chapter problems, new end-of-chapter problems, and revised practice problems. We continue to try and make our problems as practical as possible.

In addition, we have improved Connect for this edition by increasing the number of problems available substantially. Now, professors may select from more than a thousand problems as they build thier online homework assignments.

We have also built SmartBook for this edition. With SmartBook, students get the same text as the print version, along with personalized tips on what to study next, thanks to SmartBook's adaptive technology.

Retained from Previous Editions

A course in circuit analysis is perhaps the first exposure students have to electrical engineering. This is also a place where we can enhance some of the skills that they will later need as they learn how to design. An important part of this book is our 121 ***design a problem*** problems. These problems were developed to enhance skills that are an important part of the design process. We know it is not possible to fully develop a student's design skills in a fundamental course like circuits. To fully develop design skills a student needs a design experience normally reserved for their senior year. This does not mean that some of those skills cannot be developed and exercised in a circuits course. The text already included open-ended questions that help students use creativity, which is an important part of learning how to design. We already have some questions that are open-ended but we desired to add much more into our text in this important area and have developed an approach to do just that. When we develop problems for the student to solve our goal is that in solving the problem the student learns more about the theory and the problem solving process. Why not have the students design problems like we do? That is exactly what we do in each chapter. Within the normal problem set, we have a set of problems where we ask the student to design a problem to help other students better understand an important concept. This has two very important results. The first will be a better understanding of the basic theory and the second will be the enhancement of some of the student's basic design skills. We are making effective use of the principle of learning by teaching. Essentially we all learn better when we teach a subject. Designing effective problems is a key part of the teaching process. Students should also be encouraged to develop problems, when appropriate, which have nice numbers and do not necessarily overemphasize complicated mathematical manipulations.

A very important advantage to our textbook, we have a total of 2,481 Examples, Practice Problems, Review Questions, and End-of-Chapter Problems! Answers are provided for all practice problems and the odd numbered end-of-chapter problems.

The main objective of the sixth edition of this book remains the same as the previous editions—to present circuit analysis in a manner that is clearer, more interesting, and easier to understand than other circuit textbooks, and to assist the student in beginning to see the "fun" in engineering. This objective is achieved in the following ways:

- **Chapter Openers and Summaries**
  Each chapter opens with a discussion about how to enhance skills which contribute to successful problem solving as well as successful careers or a career-oriented talk on a subdiscipline of electrical engineering. This is followed by an introduction that links the chapter with the previous chapters and states the chapter objectives. The chapter ends with a summary of key points and formulas.
- **Problem-Solving Methodology**
  Chapter 1 introduces a six-step method for solving circuit problems which is used consistently throughout the book and media supplements to promote best-practice problem-solving procedures.
- **Student-Friendly Writing Style**
  All principles are presented in a lucid, logical, step-by-step manner. As much as possible, we avoid wordiness and giving too much detail that could hide concepts and impede overall understanding of the material.
- **Boxed Formulas and Key Terms**
  Important formulas are boxed as a means of helping students sort out what is essential from what is not. Also, to ensure that students clearly understand the key elements of the subject matter, key terms are defined and highlighted.

- **Margin Notes**
  Marginal notes are used as a pedagogical aid. They serve multiple uses such as hints, cross-references, more exposition, warnings, reminders not to make some particular common mistakes, and problem-solving insights.
- **Worked Examples**
  Thoroughly worked examples are liberally given at the end of every section. The examples are regarded as a part of the text and are clearly explained without asking the reader to fill in missing steps. Thoroughly worked examples give students a good understanding of the solution process and the confidence to solve problems themselves. Some of the problems are solved in two or three different ways to facilitate a substantial comprehension of the subject material as well as a comparison of different approaches.
- **Practice Problems**
  To give students practice opportunity, each illustrative example is immediately followed by a practice problem with the answer. The student can follow the example step-by-step to aid in the solution of the practice problem without flipping pages or looking at the end of the book for answers. The practice problem is also intended to test a student's understanding of the preceding example. It will reinforce their grasp of the material before the student can move on to the next section. Complete solutions to the practice problems are available to students on the website.
- **Application Sections**
  The last section in each chapter is devoted to practical application aspects of the concepts covered in the chapter. The material covered in the chapter is applied to at least one or two practical problems or devices. This helps students see how the concepts are applied to real-life situations.
- **Review Questions**
  Ten review questions in the form of multiple-choice objective items are provided at the end of each chapter with answers. The review questions are intended to cover the little "tricks" that the examples and end-of-chapter problems may not cover. They serve as a self test device and help students determine how well they have mastered the chapter.
- **Computer Tools**
  In recognition of the requirements by ABET® on integrating computer tools, the use of *PSpice, Multisim, MATLAB, KCIDE for Circuits*, and developing design skills are encouraged in a student-friendly manner. *PSpice* is covered early on in the text so that students can become familiar and use it throughout the text. Tutorials on all of these are available on Connect. *MATLAB* is also introduced early in the book.
- **Design a Problem Problems**
  Finally, *design a problem* problems are meant to help the student develop skills that will be needed in the design process.
- **Historical Tidbits**
  Historical sketches throughout the text provide profiles of important pioneers and events relevant to the study of electrical engineering.

- **Early Op Amp Discussion**
  The operational amplifier (op amp) as a basic element is introduced early in the text.
- **Fourier and Laplace Transforms Coverage**
  To ease the transition between the circuit course and signals and systems courses, Fourier and Laplace transforms are covered lucidly and thoroughly. The chapters are developed in a manner that the interested instructor can go from solutions of first-order circuits to Chapter 15. This then allows a very natural progression from Laplace to Fourier to AC.
- **Four-Color Art Program**
  An interior design and four-color art program bring circuit drawings to life and enhance key pedagogical elements throughout the text.
- **Extended Examples**
  Examples worked in detail according to the six-step problem solving method provide a road map for students to solve problems in a consistent fashion. At least one example in each chapter is developed in this manner.
- **EC 2000 Chapter Openers**
  Based on ABET's skill-based CRITERION 3, these chapter openers are devoted to discussions as to how students can acquire the skills that will lead to a significantly enhanced career as an engineer. Because these skills are so very important to the student while still in college as well after graduation, we use the heading, "*Enhancing your Skills and your Career.*"
- **Homework Problems**
  There are 580 new or revised end-of-chapter problems and changed practice problems which will provide students with plenty of practice as well as reinforce key concepts.
- **Homework Problem Icons**
  Icons are used to highlight problems that relate to engineering design as well as problems that can be solved using *PSpice, Multisim, KCIDE,* or *MATLAB*.

Organization

This book was written for a two-semester or three-quarter course in linear circuit analysis. The book may also be used for a one-semester course by a proper selection of chapters and sections by the instructor. It is broadly divided into three parts.

- Part 1, consisting of Chapters 1 to 8, is devoted to dc circuits. It covers the fundamental laws and theorems, circuits techniques, and passive and active elements.
- Part 2, which contains Chapter 9 to 14, deals with ac circuits. It introduces phasors, sinusoidal steady-state analysis, ac power, rms values, three-phase systems, and frequency response.
- Part 3, consisting of Chapters 15 to 19, are devoted to advanced techniques for network analysis. It provides students with a solid introduction to the Laplace transform, Fourier series, Fourier transform, and two-port network analysis.

The material in the three parts is more than sufficient for a two-semester course, so the instructor must select which chapters or sections to cover. Sections marked with the dagger sign (†) may be skipped, explained briefly, or assigned as homework. They can be omitted without loss of continuity. Each chapter has plenty of problems grouped according to the sections of the related material and diverse enough that the instructor can choose some as examples and assign some as homework. As stated earlier, we are using three icons with this edition. We are using to denote problems that either require *PSpice* in the solution process, where the circuit complexity is such that *PSpice* or *Multisim* would make the solution process easier, and where *PSpice* or *Multisim* makes a good check to see if the problem has been solved correctly. We are using to denote problems where *MATLAB* is required in the solution process, where *MATLAB* makes sense because of the problem makeup and its complexity, and where *MATLAB* makes a good check to see if the problem has been solved correctly. Finally, we use to identify problems that help the student develop skills that are needed for engineering design. More difficult problems are marked with an asterisk (*).

Comprehensive problems follow the end-of-chapter problems. They are mostly applications problems that require skills learned from that particular chapter.

Prerequisites

As with most introductory circuit courses, the main prerequisites, for a course using this textbook, are physics and calculus. Although familiarity with complex numbers is helpful in the later part of the book, it is not required. A very important asset of this text is that ALL the mathematical equations and fundamentals of physics needed by the student, are included in the text.

Acknowledgments

We would like to express our appreciation for the loving support we have received from our wives (Hannah and Kikelomo), daughters (Christina, Tamara, Jennifer, Motunrayo, Ann, and Joyce), son (Baixi), and our extended family members. We sincerely appreciate the invaluable help given us by Richard Rarick in helping us make the sixth edition a significantly more relevant book. He has checked all the new and revised problems and offered advice on making them more accurate and clear.

At McGraw-Hill, we would like to thank the following editorial and production staff: Raghu Srinivasan, global brand manager; Vincent Bradshaw, product developer; Nick McFadden, marketing manager; and Melissa Leick, content project manager.

The sixth edition has benefited greatly from the many outstanding individuals who have offered suggestions for improvements in both the text as well as the various problems. In particular, we thank Nicholas Reeder, Professor of Electronics Engineering Technology, Sinclair Community College, Dayton, Ohio, and Douglas De Boer, Professor of Engineering, Dordt College, Sioux Center, Iowa, for their detailed and careful corrections and suggestions for clarification which have

contributed to making this a better edition. In addition, the following have made important contributions to this edition (in alphabetical order):

Zekeriya Aliyazicioglu, *California State Polytechnic University—Pomona*
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Mohammad Haider, *University of Alabama—Birmingham*
John Heathcote, *Reedley College*
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Aaron Ohta, *University of Hawaii—Manoa*
Salomon Oldak, *California State Polytechnic University—Pomona*
Hesham Shaalan, *U.S. Merchant Marine Academy*
Surendra Singh, *University of Tulsa*

Finally, we sincerely appreciate the feedback received from instructors and students who used the previous editions. We want this to continue, so please keep sending us e-mails or direct them to the publisher. We can be reached at c.alexander@ieee.org for Charles Alexander and sadiku@ieee.org for Matthew Sadiku.

C. K. Alexander and M. N. O. Sadiku

Supplements

Instructor and Student Resources

Available on Connect are a number of additional instructor and student resources to accompany the text. These include complete solutions for all practice and end-of-chapter problems, solutions in *PSpice* and *Multisim* problems, lecture PowerPoints®, and text image files. In addition, instructors can use COSMOS, a complete online solutions manual organization system to create custom homework, quizzes, and tests using end-of-chapter problems from the text.

Knowledge Capturing Integrated Design Environment for Circuits (*KCIDE for Circuits*)

This software, developed at Cleveland State University and funded by NASA, is designed to help the student work through a circuits problem in an organized manner using the six-step problem-solving methodology in the text. *KCIDE for Circuits* allows students to work a circuit problem in *PSpice* and *MATLAB*, track the evolution of their solution, and save a record of their process for future reference. In addition, the software automatically generates a Word document and/or a PowerPoint presentation. The software package can be downloaded for free.

It is hoped that the book and supplemental materials supply the instructor with all the pedagogical tools necessary to effectively present the material.

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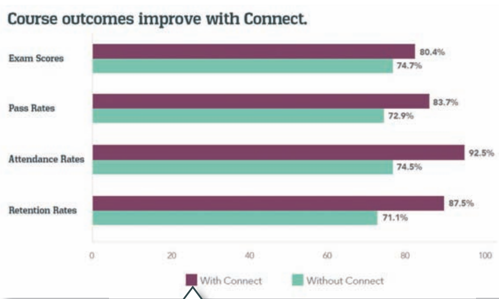

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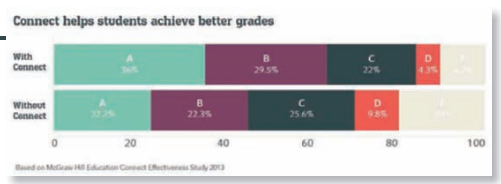

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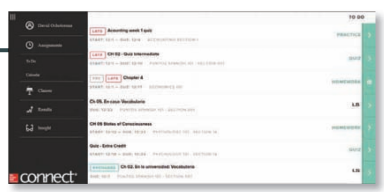

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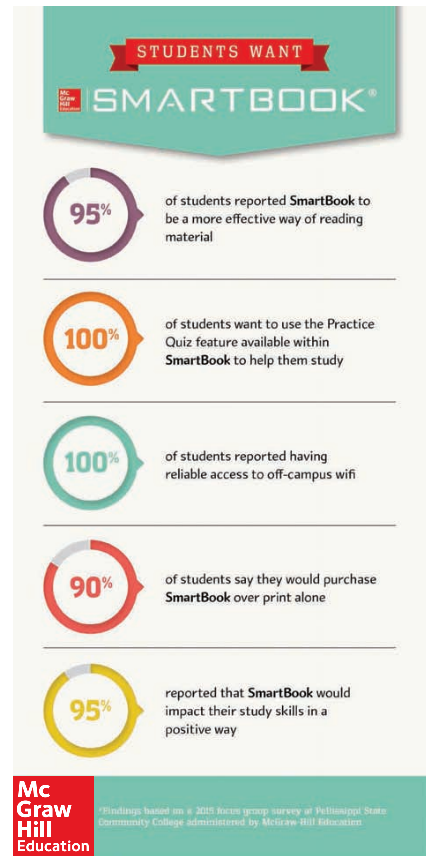

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About the Authors

Charles K. Alexander is professor of electrical and computer engineering in the Washkewicz College of Engineering at Cleveland State University, Cleveland, Ohio. He is also the director of the Center for Research in Electronics and Aerospace Technology (CREATE). From 2002 until 2006 he was dean of the Fenn College of Engineering. He has held the position of dean of engineering at Cleveland State University, California State University, Northridge, and Temple University (acting dean for six years). He has held the position of department chair at Temple University and Tennessee Technological University as well as the position of Stocker Visiting Professor (an endowed chair) at Ohio University. He has held faculty status at all of the aforementioned universities.

Charles K. Alexander

Dr. Alexander has secured funding for two centers of research at Ohio University and Cleveland State University. He has been the director of three additional research centers at Temple and Tennessee Tech and has obtained research funding of approximately $100 million (in today's dollars). He has served as a consultant to 23 private and governmental organizations including the Air Force and the Navy.

He received the honorary Dr. Eng. from Ohio Northern University (2009), his PhD (1971) and M.S.E.E. (1967) from Ohio University, and the B.S.E.E. (1965) from Ohio Northern University.

Dr. Alexander has authored many publications, including a workbook and a videotape lecture series, and is coauthor of *Fundamentals of Electric Circuits*, currently in its fifth edition, *Engineering Skills for Career Success, Problem Solving Made* ALMOST *Easy*, the fifth edition of the *Standard Handbook of Electronic Engineering,* and *Applied Circuit Analysis,* all with McGraw-Hill. He has delivered more than 500 paper, professional, and technical presentations.

Dr. Alexander is a Life Fellow of the IEEE and served as its president and CEO in 1997. In addition he has held several volunteer positions within the IEEE during his more than 45 years of service. This includes serving from 1991 to 1999 on the IEEE board of directors.

He has received several local, regional, national, and international awards for teaching and research, includingan honorary Doctor of Engineering degree, Fellow of the IEEE, the IEEE-USA Jim Watson Student Professional Awareness Achievement Award, the IEEE Undergraduate Teaching Award, the Distinguished Professor Award, the Distinguished Engineering Education Achievement Award, the Distinguished Engineering Education Leadership Award, the IEEE Centennial Medal, and the IEEE/RAB Innovation Award.

Matthew N. O. Sadiku

Matthew N. O. Sadiku received his PhD from Tennessee Technological University, Cookeville. From 1984 to 1988, he was an assistant professor at Florida Atlantic University, where he did graduate work in computer science. From 1988 to 2000, he was at Temple University, Philadelphia, Pennsylvania, where he became a full professor. From 2000 to 2002, he was with Lucent/Avaya, Holmdel, New Jersey, as a system engineer and with Boeing Satellite Systems as a senior scientist. He is currently a professor at Prairie View A&M University.

Dr. Sadiku is the author of more than 240 professional papers and over 60 books, including *Elements of Electromagnetics* (Oxford University Press, 6th ed., 2015), *Numerical Techniques in Electromagnetics with MATLAB* (CRC, 3rd ed., 2009), and *Metropolitan Area Networks* (CRC Press, 1995). Some of his books have been translated into French, Korean, Chinese (and Chinese Long Form in Taiwan), Italian, Portuguese, and Spanish. He was the recipient of the 2000 McGraw-Hill/Jacob Millman Award for outstanding contributions in the field of electrical engineering. He was also the recipient of Regents Professor award for 2012 to 2013 by the Texas A&M University System.

His current research interests are in the areas of numerical modeling of electromagnetic systems and computer communication networks. He is a registered professional engineer and a fellow of the Institute of Electrical and Electronics Engineers (IEEE) "for contributions to computational electromagnetics and engineering education." He was the IEEE Region 2 Student Activities Committee Chairman. He was an associate editor for *IEEE Transactions on Education* and is a member of the Association for Computing Machinery (ACM).

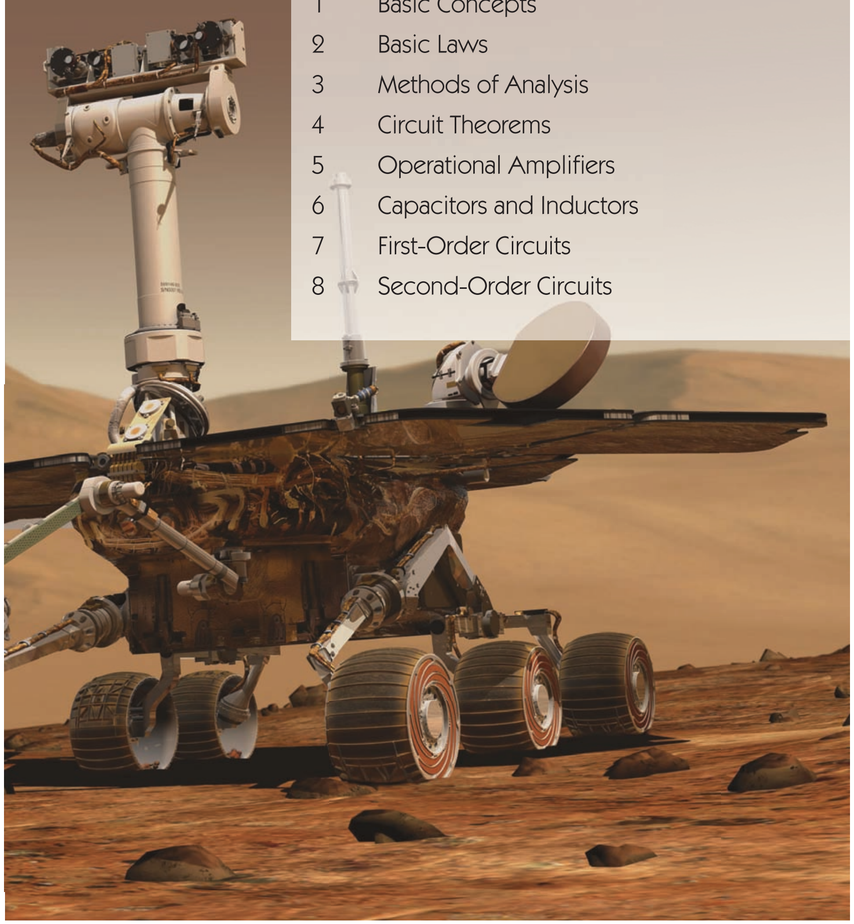
Fundamentals of Electric Circuits

PART ONE

DCCi rcuits

DC Circuits

OUTLINE



NASA

chapter

1

Basic Concepts

Some books are to be tasted, others to be swallowed, and some few to be chewed and digested.

—Francis Bacon

Enhancing Your Skills and Your Career

Photo by Charles Alexander

ABET EC 2000 criteria (3.a), "*an ability to apply knowledge of mathematics, science, and engineering.*"

As students, you are required to study mathematics, science, and engineering with the purpose of being able to apply that knowledge to the solution of engineering problems. The skill here is the ability to apply the fundamentals of these areas in the solution of a problem. So how do you develop and enhance this skill?

The best approach is to work as many problems as possible in all of your courses. However, if you are really going to be successful with this, you must spend time analyzing where and when and why you have difficulty in easily arriving at successful solutions. You may be surprised to learn that most of your problem-solving problems are with mathematics rather than your understanding of theory. You may also learn that you start working the problem too soon. Taking time to think about the problem and how you should solve it will always save you time and frustration in the end.

What I have found that works best for me is to apply our six-step problem-solving technique. Then I carefully identify the areas where I have difficulty solving the problem. Many times, my actual deficiencies are in my understanding and ability to use correctly certain mathematical principles. I then return to my fundamental math texts and carefully review the appropriate sections, and in some cases, work some example problems in that text. This brings me to another important thing you should always do: Keep nearby all your basic mathematics, science, and engineering textbooks.

This process of continually looking up material you thought you had acquired in earlier courses may seem very tedious at first; however, as your skills develop and your knowledge increases, this process will become easier and easier. On a personal note, it is this very process that led me from being a much less than average student to someone who could earn a Ph.D. and become a successful researcher.

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Understand the different units with which engineers work.
2. Understand the relationship between charge and current and how to use both in a variety of applications.
3. Understand voltage and how it can be used in a variety of applications.
4. Develop an understanding of power and energy and their relationship with current and voltage.
5. Begin to understand the volt-amp characteristics of a variety of circuit elements.
6. Begin to understand an organized approach to problem solving and how it can be used to assist in your efforts to solve circuit problems.

1.1 Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element.*

An **electric circuit** is an interconnection of electrical elements.

A simple electric circuit is shown in Fig. 1.1. It consists of three basic elements: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a flashlight, a search light, and so forth.

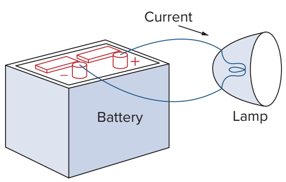


Figure 1.1
A simple electric circuit.

A complicated real circuit is displayed in Fig. 1.2, representing the schematic diagram for a radio receiver. Although it seems complicated, this circuit can be analyzed using the techniques we cover in this book. Our goal in this text is to learn various analytical techniques and computer software applications for describing the behavior of a circuit like this.

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather, our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior of the

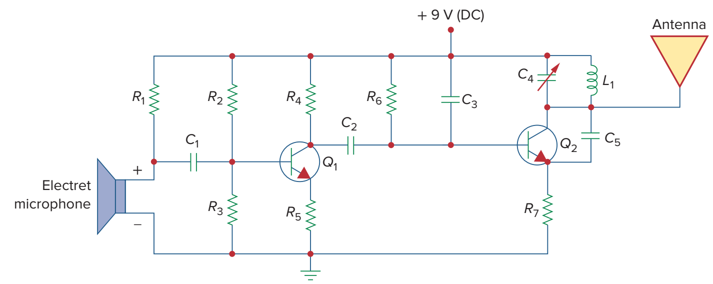


Figure 1.2
Electric circuit of a radio transmitter.

circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

1.2 Systems of Units

As electrical engineers, we must deal with measurable quantities. Our measurements, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country in which the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system, there are seven base units from which the units of all other physical quantities can be derived. Table 1.1 shows six base units and one derived unit (the coulomb) that are related to this text. SI units are commonly used in electrical engineering.

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

600,000,000 mm   600,000 m   600 km

TABLE 1.1
Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

TABLE 1.2
The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 Charge and Current

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge.* We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

> **Charge** is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.[1]
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.

We now consider the flow of electric charges. A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.

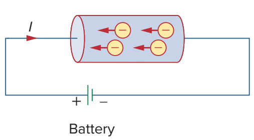


Figure 1.3
Electric current due to flow of electronic charge in a conductor.

> A convention is a standard way of describing something so that others in the profession can understand what we mean. We will be using IEEE conventions throughout this book.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges. That is, opposite to the flow of negative charges, as Fig. 1.3 illustrates. This convention was introduced by Benjamin Franklin (1706–1790), the American scientist and inventor. Although we now know that current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges. Thus,

> **Electric current** is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i, charge q, and time t is

$$\boxed{i \triangleq \frac{dq}{dt}} \tag{1.1}$$

[1] However, a large power supply capacitor can store up to 0.5 C of charge.

Historical

© Apic/Getty Images

Andre-Marie Ampere (1775–1836), a French mathematician and physicist, laid the foundation of electrodynamics. He defined the electric current and developed a way to measure it in the 1820s.

Born in Lyons, France, Ampere at age 12 mastered Latin in a few weeks, as he was intensely interested in mathematics and many of the best mathematical works were in Latin. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and the ammeter. The unit of electric current, the ampere, was named after him.

where current is measured in amperes (A), and

1 ampere = 1 coulomb/second

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$\boxed{Q \triangleq \int_{t_0}^{t} i\, dt} \tag{1.2}$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways.

There are different ways of looking at direct current and alternating current. The best definition is that there are two ways that current can flow: It can always flow in the same direction, where it does not reverse direction, in which case we have *direct current* (dc). These currents can be constant or time varying. If the current flows in both directions, then we have *alternating current* (ac).

> A **direct current** (dc) flows only in one direction and can be constant or time varying.

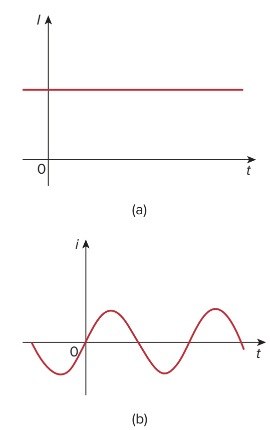


Figure 1.4
Two common types of current: (a) direct current (dc), (b) alternating current (ac).

By convention, we will use the symbol I to represent a constant current. If the current varies with respect to time (either dc or ac) we will use the symbol i. A common use of this would be the output of a rectifier (dc) such as $i(t) = |5 \sin(377t)|$ amps or a sinusoidal current (ac) such as $i(t) = 160 \sin(377t)$ amps.

> An **alternating current** (ac) is a current that changes direction with respect to time.

An example of alternating current (ac) is the current you use in your house to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4 depicts two common examples of

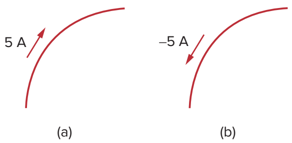


Figure 1.5
Conventional current flow: (a) positive current flow, (b) negative current flow.

dc (coming from a battery) and ac (coming from your home outlets). We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of −5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of +5 A flowing in the opposite direction.

Example 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have -1.602×10^{-19} C/electron $\times$ 4,600 electrons $= -7.369 \times 10^{-16}$ C

Practice Problem 1.1

Calculate the amount of charge represented by 6.667 billion protons.

Answer: 1.0681×10^{-9} C.

Example 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem 1.2

If in Example 1.2, $q = (10 - 10e^{-2t})$ mC, find the current at $t = 1.0$ s.

Answer: 2.707 mA.

Example 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$Q = \int_{t=1}^{2} i\, dt = \int_{1}^{2} (3t^2 - t)\, dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

Practice Problem 1.3

The current flowing through an element is

$$i = \begin{cases} 4 \text{ A}, & 0 < t < 1 \\ 4t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 13.333 C.

1.4 Voltage

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from b to a; mathematically,

$$v_{ab} \triangleq \frac{dw}{dq} \tag{1.3}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Thus,

> **Voltage** (or **potential difference**) is the energy required to move a unit charge from a reference point (−) to another point (+), measured in volts (V).

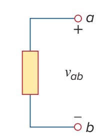


Figure 1.6
Polarity of voltage v_{ab}.

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (−) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) Point a is at a potential of v_{ab} volts higher than point b, or (2) the potential at point a with respect to point b is v_{ab}. It follows logically that in general

$$v_{ab} = -v_{ba} \tag{1.4}$$

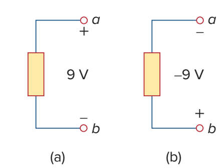


Figure 1.7
Two equivalent representations of the same voltage v_{ab}: (a) Point a is 9 V above point b; (b) point b is −9 V above point a.

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b; in Fig. 1.7(b), point b is −9 V above point a. We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a. In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a.

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for conveying

Historical

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Alessandro Antonio Volta (1745–1827), an Italian physicist, invented the electric battery—which provided the first continuous flow of electricity—and the capacitor.

Born into a noble family in Como, Italy, Volta was performing electrical experiments at age 18. His invention of the battery in 1796 revolutionized the use of electricity. The publication of his work in 1800 marked the beginning of electric circuit theory. Volta received many honors during his lifetime. The unit of voltage or potential difference, the volt, was named in his honor.

Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V, whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v. A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

1.5 Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus, power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$\boxed{p \triangleq \frac{dw}{dt}} \qquad (1.5)$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq}\cdot\frac{dq}{dt} = vi \qquad (1.6)$$

or

$$\boxed{p = vi} \qquad (1.7)$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a + sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a − sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention*. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

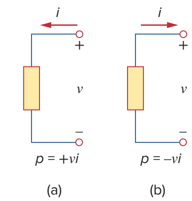


Figure 1.8
Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

When the voltage and current directions conform to Fig. 1.8(b), we have the *active sign convention* and $p = +vi$.

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of +12 W because a positive current enters the negative terminal. Of course, an absorbing power of −12 W is equivalent to a supplying power of +12 W. In general,

$$+\text{Power absorbed} = -\text{Power supplied}$$

In fact, the *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\boxed{\sum p = 0} \qquad (1.8)$$

This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.

From Eq. (1.6), the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt \qquad (1.9)$$

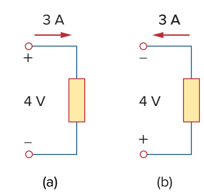


Figure 1.9
Two cases of an element with an absorbing power of 12 W: (a) $p = 4 \times 3 = 12$ W, (b) $p = 4 \times 3 = 12$ W.

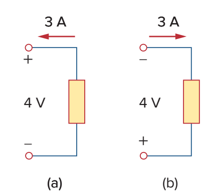


Figure 1.10
Two cases of an element with a supplying power of 12 W: (a) $p = -4 \times 3 = -12$ W, (b) $p = -4 \times 3 = -12$ W.

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3{,}600 \text{ J}$$

Example 1.4

An energy source forces a constant current of 2A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:
The total charge is

$$\Delta q = i\,\Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

Practice Problem 1.4

To move charge q from point b to point a requires 25 J. Find the voltage drop v_{ab} (the voltage at a positive with respect to b) if: (a) $q = 5$ C, (b) $q = -10$ C.

Answer: (a) 5 V, (b) −2.5 V.

Example 1.5

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

Solution:
(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3$ ms,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3\frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3$ ms,

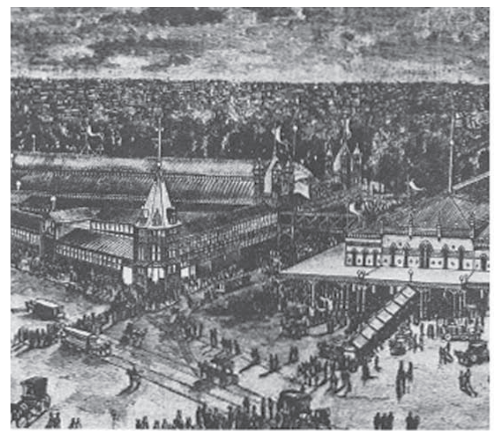
$$\begin{aligned} p &= -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W} \\ &= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW} \end{aligned}$$

Historical

1884 Exhibition In the United States, nothing promoted the future of electricity like the 1884 International Electrical Exhibition. Just imagine a world without electricity, a world illuminated by candles and gaslights, a world where the most common transportation was by walking and riding on horseback or by horse-drawn carriage. Into this world an exhibition was created that highlighted Thomas Edison and reflected his highly developed ability to promote his inventions and products. His exhibit featured spectacular lighting displays powered by an impressive 100-kW "Jumbo" generator.

Edward Weston's dynamos and lamps were featured in the United States Electric Lighting Company's display. Weston's well known collection of scientific instruments was also shown.

Other prominent exhibitors included Frank Sprague, Elihu Thompson, and the Brush Electric Company of Cleveland. The American Institute of Electrical Engineers (AIEE) held its first technical meeting on October 7–8 at the Franklin Institute during the exhibit. AIEE merged with the Institute of Radio Engineers (IRE) in 1964 to form the Institute of Electrical and Electronics Engineers (IEEE).



Source: IEEE History Center

Practice Problem 1.5

Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V,

(b) $v = \left(10 + 5\int_0^t i\,dt\right)$ V.

Answer: (a) 17.27 W, (b) 29.7 W.

Example 1.6

How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$w = pt = 100\text{ (W)} \times 2\text{ (h)} \times 60\text{ (min/h)} \times 60\text{ (s/min)}$$
$$= 720{,}000\text{ J} = 720\text{ kJ}$$

This is the same as

$$w = pt = 100\text{ W} \times 2\text{ h} = 200\text{ Wh}$$

Practice Problem 1.6

A home electric heater draws 10 A when connected to a 115 V outlet. How much energy is consumed by the heater over a period of 6 hours?

Answer: 6.9 k watt-hours

1.6 Circuit Elements

As we discussed in Section 1.1, an element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers. Our aim in this section is to gain familiarity with some important active elements.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

> An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

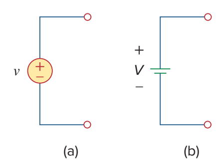


Figure 1.11
Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1.11 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.11(a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.11(a) can be used for a time-varying voltage source. Similarly, an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol for an independent current source is displayed in Fig. 1.12, where the arrow indicates the direction of current i.

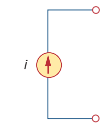


Figure 1.12
Symbol for independent current source.

> An **ideal dependent** (or **controlled**) **source** is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

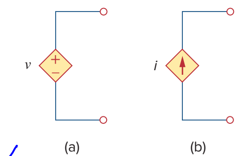


Figure 1.13
Symbols for: (a) dependent voltage source, (b) dependent current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.14, where the voltage $10i$ of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ −) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

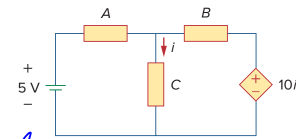


Figure 1.14
The source on the right-hand side is a current-controlled voltage source.

It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus, an ideal source could in theory supply an infinite amount of energy. It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

Example 1.7

Calculate the power supplied or absorbed by each element in Fig. 1.15.

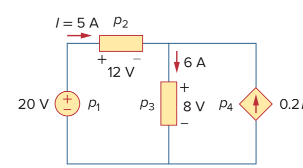


Figure 1.15
For Example 1.7.

Solution:

We apply the sign convention for power shown in Figs. 1.8 and 1.9. For p_1, the 5-A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100\text{ W} \qquad \text{Supplied power}$$

For p_2 and p_3, the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60\text{ W} \qquad \text{Absorbed power}$$
$$p_3 = 8(6) = 48\text{ W} \qquad \text{Absorbed power}$$

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3 since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \qquad \text{Supplied power}$$

We should observe that the 20-V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the two passive elements are absorbing power. Also,

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

In agreement with Eq. (1.8), the total power supplied equals the total power absorbed.

Practice Problem 1.7

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

Answer: $p_1 = -45$ W, $p_2 = 18$ W, $p_3 = 12$ W, $p_4 = 15$ W.

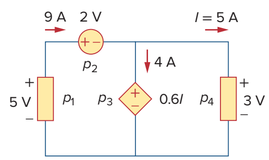


Figure 1.16
For Practice Prob. 1.7.

1.7 †Applications[2]

In this section, we will consider two practical applications of the concepts developed in this chapter. The first one deals with the TV picture tube and the other with how electric utilities determine your electric bill.

1.7.1 TV Picture Tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver.[3] The CRT is depicted in Fig. 1.17. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun, maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus, the beam can be made to "paint" a picture on the TV screen.

[2] The dagger sign preceding a section heading indicates the section that may be skipped, explained briefly, or assigned as homework.

[3] Modern TV tubes use a different technology.

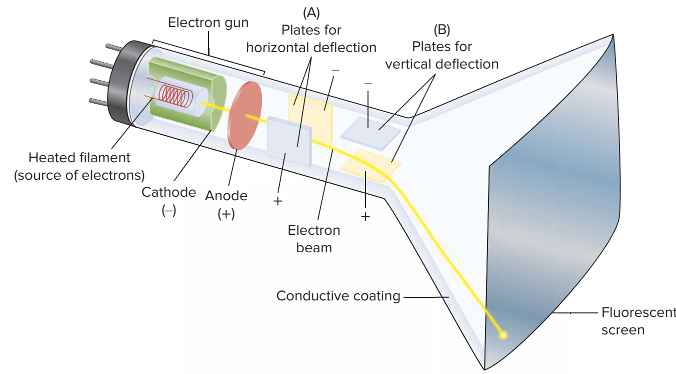


Figure 1.17
Cathode-ray tube.

Historical

Karl Ferdinand Braun and Vladimir K. Zworykin

Karl Ferdinand Braun (1850–1918), of the University of Strasbourg, invented the Braun cathode-ray tube in 1879. This then became the basis for the picture tube used for so many years for televisions. It is still the most economical device today, although the price of flat-screen systems is rapidly becoming competitive. Before the Braun tube could be used in television, it took the inventiveness of **Vladimir K. Zworykin** (1889–1982) to develop the iconoscope so that the modern television would become a reality. The iconoscope developed into the orthicon and the image orthicon, which allowed images to be captured and converted into signals that could be sent to the television receiver. Thus, the television camera was born.

Example 1.8

The electron beam in a TV picture tube carries 10^{15} electrons per second. As a design engineer, determine the voltage V_o needed to accelerate the electron beam to achieve 4 W.

Solution:
The charge on an electron is

$$e = -1.6 \times 10^{-19} \text{ C}$$

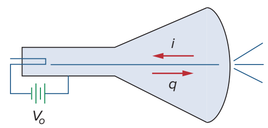


Figure 1.18
A simplified diagram of the cathode-ray tube; for Example 1.8.

If the number of electrons is n, then $q = ne$ and

$$i = \frac{dq}{dt} = e\frac{dn}{dt} = (-1.6 \times 10^{-19})(10^{15}) = -1.6 \times 10^{-4} \text{ A}$$

The negative sign indicates that the current flows in a direction opposite to electron flow as shown in Fig. 1.18, which is a simplified diagram of the CRT for the case when the vertical deflection plates carry no charge. The beam power is

$$p = V_o i \qquad \text{or} \qquad V_o = \frac{p}{i} = \frac{4}{1.6 \times 10^{-4}} = 25{,}000 \text{ V}$$

Thus, the required voltage is 25 kV.

Practice Problem 1.8

If an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 30 kV, calculate the power in the beam.

Answer: 48 mW.

1.7.2 Electricity Bills

The second application deals with how an electric utility company charges their customers. The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh). (Other factors that affect the cost include demand and power factors; we will ignore these for now.) However, even if a consumer uses no energy at all, there is a minimum service charge the customer must pay because it costs money to stay connected to the power line. As energy consumption increases, the cost per kWh drops. It is interesting to note the average monthly consumption of household appliances for a family of five, shown in Table 1.3.

TABLE 1.3

Typical average monthly consumption of household appliances.

Appliance	kWh consumed	Appliance	kWh consumed
Water heater	500	Washing machine	120
Freezer	100	Stove	100
Lighting	100	Dryer	80
Dishwasher	35	Microwave oven	25
Electric iron	15	Personal computer	12
TV	10	Radio	8
Toaster	4	Clock	2

Example 1.9

A homeowner consumes 700 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

Base monthly charge of $12.00.
First 100 kWh per month at 16 cents/kWh.
Next 200 kWh per month at 10 cents/kWh.
Over 300 kWh per month at 6 cents/kWh.

Solution:
We calculate the electricity bill as follows.

$$\begin{aligned}
\text{Base monthly charge} &= \$12.00\\
\text{First 100 kWh @ \$0.16/k Wh} &= \$16.00\\
\text{Next 200 kWh @ \$0.10/k Wh} &= \$20.00\\
\text{Remaining 400 kWh @ \$0.06/k Wh} &= \$24.00\\
\text{Total charge} &= \$72.00
\end{aligned}$$

$$\text{Average cost} = \frac{\$72}{100 + 200 + 400} = 10.2 \text{ cents/kWh}$$

Practice Problem 1.9

Referring to the residential rate schedule in Example 1.9, calculate the average cost per kWh if only 350 kWh are consumed in July when the family is on vacation most of the time.

Answer: 14.571 cents/kWh.

1.8 †Problem Solving

Although the problems to be solved during one's career will vary in complexity and magnitude, the basic principles to be followed remain the same. The process outlined here is the one developed by the authors over many years of problem solving with students, for the solution of engineering problems in industry, and for problem solving in research.

We will list the steps simply and then elaborate on them.

1. Carefully **define** the problem.
2. **Present** everything you know about the problem.
3. Establish a set of **alternative** solutions and determine the one that promises the greatest likelihood of success.
4. **Attempt** a problem solution.
5. **Evaluate** the solution and check for accuracy.
6. Has the problem been solved **satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.

1. *Carefully **define** the problem*. This may be the most important part of the process, because it becomes the foundation for all the rest of the steps. In general, the presentation of engineering problems is

somewhat incomplete. You must do all you can to make sure you understand the problem as thoroughly as the presenter of the problem understands it. Time spent at this point clearly identifying the problem will save you considerable time and frustration later. As a student, you can clarify a problem statement in a textbook by asking your professor. A problem presented to you in industry may require that you consult several individuals. At this step, it is important to develop questions that need to be addressed before continuing the solution process. If you have such questions, you need to consult with the appropriate individuals or resources to obtain the answers to those questions. With those answers, you can now refine the problem, and use that refinement as the problem statement for the rest of the solution process.

2. ***Present*** *everything you know about the problem.* You are now ready to write down everything you know about the problem and its possible solutions. This important step will save you time and frustration later.

3. *Establish a set of* ***alternative*** *solutions and determine the one that promises the greatest likelihood of success.* Almost every problem will have a number of possible paths that can lead to a solution. It is highly desirable to identify as many of those paths as possible. At this point, you also need to determine what tools are available to you, such as *PSpice* and *MATLAB* and other software packages that can greatly reduce effort and increase accuracy. Again, we want to stress that time spent carefully defining the problem and investigating alternative approaches to its solution will pay big dividends later. Evaluating the alternatives and determining which promises the greatest likelihood of success may be difficult but will be well worth the effort. Document this process well since you will want to come back to it if the first approach does not work.

4. ***Attempt*** *a problem solution.* Now is the time to actually begin solving the problem. The process you follow must be well documented in order to present a detailed solution if successful, and to evaluate the process if you are not successful. This detailed evaluation may lead to corrections that can then lead to a successful solution. It can also lead to new alternatives to try. Many times, it is wise to fully set up a solution before putting numbers into equations. This will help in checking your results.

5. ***Evaluate*** *the solution and check for accuracy.* You now thoroughly evaluate what you have accomplished. Decide if you have an acceptable solution, one that you want to present to your team, boss, or professor.

6. *Has the problem been solved* ***satisfactorily****? If so, present the solution; if not, then return to step 3 and continue through the process again.* Now you need to present your solution or try another alternative. At this point, presenting your solution may bring closure to the process. Often, however, presentation of a solution leads to further refinement of the problem definition, and the process continues. Following this process will eventually lead to a satisfactory conclusion.

Now let us look at this process for a student taking an electrical and computer engineering foundations course. (The basic process also applies to almost every engineering course.) Keep in mind that although the steps have been simplified to apply to academic types of problems, the process as stated always needs to be followed. We consider a simple example.

Example 1.10

Solve for the current flowing through the 8-Ω resistor in Fig. 1.19.

Solution:

1. *Carefully* ***define*** *the problem.* This is only a simple example, but we can already see that we do not know the polarity on the 3-V source. We have the following options. We can ask the professor what the polarity should be. If we cannot ask, then we need to make a decision on what to do next. If we have time to work the problem both ways, we can solve for the current when the 3-V source is plus on top and then plus on the bottom. If we do not have the time to work it both ways, assume a polarity and then carefully document your decision. Let us assume that the professor tells us that the source is plus on the bottom as shown in Fig. 1.20.

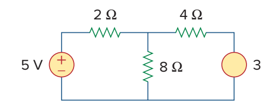


Figure 1.19
Illustrative example.

2. ***Present*** *everything you know about the problem.* Presenting all that we know about the problem involves labeling the circuit clearly so that we define what we seek.

Given the circuit shown in Fig. 1.20, solve for $i_{8\Omega}$.

We now check with the professor, if reasonable, to see if the problem is properly defined.

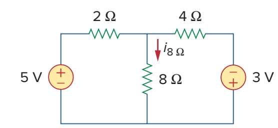


Figure 1.20
Problem definition.

3. *Establish a set of* ***alternative*** *solutions and determine the one that promises the greatest likelihood of success.* There are essentially three techniques that can be used to solve this problem. Later in the text you will see that you can use circuit analysis (using Kirchhoff's laws and Ohm's law), nodal analysis, and mesh analysis.

To solve for $i_{8\Omega}$ using circuit analysis will eventually lead to a solution, but it will likely take more work than either nodal or mesh analysis. To solve for $i_{8\Omega}$ using mesh analysis will require writing two simultaneous equations to find the two loop currents indicated in Fig. 1.21. Using nodal analysis requires solving for only one unknown. This is the easiest approach.

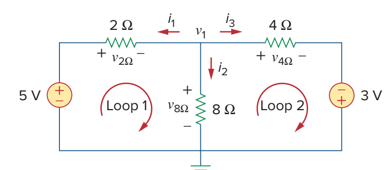


Figure 1.21
Using nodal analysis.

Therefore, we will solve for $i_{8\Omega}$ using nodal analysis.

4. ***Attempt*** *a problem solution.* We first write down all of the equations we will need in order to find $i_{8\Omega}$.

$$i_{8\Omega} = i_2, \qquad i_2 = \frac{v_1}{8}, \qquad i_{8\Omega} = \frac{v_1}{8}$$

$$\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} = 0$$

Now we can solve for v_1.

$$8\left[\frac{v_1-5}{2}+\frac{v_1-0}{8}+\frac{v_1+3}{4}\right]=0$$

leads to $(4v_1-20)+(v_1)+(2v_1+6)=0$

$$7v_1=+14, \qquad v_1=+2\text{ V}, \qquad i_{8\Omega}=\frac{v_1}{8}=\frac{2}{8}=\mathbf{0.25\ A}$$

5. ***Evaluate** the solution and check for accuracy*. We can now use Kirchhoff's voltage law (KVL) to check the results.

$$i_1=\frac{v_1-5}{2}=\frac{2-5}{2}=-\frac{3}{2}=-1.5\text{ A}$$

$$i_2=i_{8\Omega}=0.25\text{ A}$$

$$i_3=\frac{v_1+3}{4}=\frac{2+3}{4}=\frac{5}{4}=1.25\text{ A}$$

$$i_1+i_2+i_3=\mathbf{-1.5+0.25+1.25=0} \qquad \text{(Checks.)}$$

Applying KVL to loop 1,

$$\begin{aligned}-5+v_{2\Omega}+v_{8\Omega}&=-5+(-i_1\times 2)+(i_2\times 8)\\&=-5+[-(-1.5)2]+(0.25\times 8)\\&=\mathbf{-5+3+2=0} \qquad \text{(Checks.)}\end{aligned}$$

Applying KVL to loop 2,

$$\begin{aligned}-v_{8\Omega}+v_{4\Omega}-3&=-(i_2\times 8)+(i_3\times 4)-3\\&=-(0.25\times 8)+(1.25\times 4)-3\\&=\mathbf{-2+5-3}=0 \qquad \text{(Checks.)}\end{aligned}$$

So we now have a very high degree of confidence in the accuracy of our answer.

6. *Has the problem been solved **satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.* This problem has been solved satisfactorily.

The current through the 8-Ω resistor is 0.25 A flowing down through the 8-Ω resistor.

Practice Problem 1.10

Try applying this process to some of the more difficult problems at the end of the chapter.

1.9 Summary

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the seven principal units, the units of other physical quantities can be derived.

3. Current is the rate of charge flow past a given point in a given direction.

$$i=\frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge from a reference point (−) to another point (+).

$$v_{ab}=\frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p=\frac{dw}{dt}=vi$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
9. Two areas of application of the concepts covered in this chapter are the TV picture tube and electricity billing procedure.

Review Questions

1.1 One millivolt is one millionth of a volt.

(a) True (b) False

1.2 The prefix *micro* stands for:

(a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}

1.3 The voltage 2,000,000 V can be expressed in powers of 10 as:

(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV

1.4 A charge of 2 C flowing past a given point each second is a current of 2 A.

(a) True (b) False

1.5 The unit of current is:

(a) coulomb (b) ampere
(c) volt (d) joule

1.6 Voltage is measured in:

(a) watts (b) amperes
(c) volts (d) joules per second

1.7 A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.

(a) True (b) False

1.8 The voltage across a 1.1-kW toaster that produces a current of 10 A is:

(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V

1.9 Which of these is not an electrical quantity?

(a) charge (b) time (c) voltage
(d) current (e) power

1.10 The dependent source in Fig. 1.22 is:

(a) voltage-controlled current source
(b) voltage-controlled voltage source
(c) current-controlled voltage source
(d) current-controlled current source

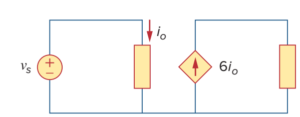


Figure 1.22
For Review Question 1.10.

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5b, 1.6c, 1.7a, 1.8c, 1.9b, 1.10d.

Problems

Section 1.3 Charge and Current

1.1 How much charge is represented by these number of electrons?

(a) 6.482×10^{17}

(b) 1.24×10^{18}

(c) 2.46×10^{19}

(d) 1.628×10^{20}

1.2 Determine the current flowing through an element if the charge flow is given by

(a) $q(t) = (3)$ mC

(b) $q(t) = (4t^2 + 20t - 4)$ C

(c) $q(t) = (15e^{-3t} - 2e^{-18t})$ nC

(d) $q(t) = 5t^2(3t^3 + 4)$ pC

(e) $q(t) = 2e^{-3t} \sin(20\pi t)$ μC

1.3 Find the charge $q(t)$ flowing through a device if the current is:

(a) $i(t) = 3$ A, $q(0) = 1$ C

(b) $i(t) = (2t + 5)$ mA, $q(0) = 0$

(c) $i(t) = 20\cos(10t + \pi/6)$ μA, $q(0) = 2$ μC

(d) $i(t) = 10e^{-30t} \sin 40t$ A, $q(0) = 0$

1.4 A total charge of 300 C flows past a given cross section of a conductor in 30 seconds. What is the value of the current?

1.5 Determine the total charge transferred over the time interval of $0 \leq t \leq 10$ s when $i(t) = \frac{1}{2}t$ A.

1.6 The charge entering a certain element is shown in Fig. 1.23. Find the current at:

(a) $t = 1$ ms (b) $t = 6$ ms (c) $t = 10$ ms

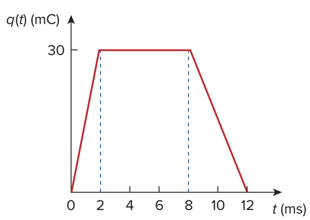


Figure 1.23
For Prob. 1.6.

1.7 The charge flowing in a wire is plotted in Fig. 1.24. Sketch the corresponding current.

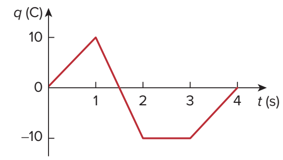


Figure 1.24
For Prob. 1.7.

1.8 The current flowing past a point in a device is shown in Fig. 1.25. Calculate the total charge through the point.

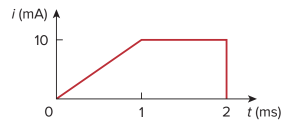


Figure 1.25
For Prob. 1.8.

1.9 The current through an element is shown in Fig. 1.26. Determine the total charge that passed through the element at:

(a) $t = 1$ s (b) $t = 3$ s (c) $t = 5$ s

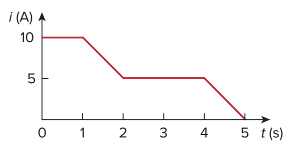


Figure 1.26
For Prob. 1.9.

Sections 1.4 and 1.5 Voltage, Power, and Energy

1.10 A lightning bolt with 10 kA strikes an object for 15 μs. How much charge is deposited on the object?

1.11 A rechargeable flashlight battery is capable of delivering 90 mA for about 12 h. How much charge can it release at that rate? If its terminal voltage is 1.5 V, how much energy can the battery deliver?

1.12 If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t\text{A}, & 0 \leq t < 6 \text{ s} \\ 18\text{A}, & 6 \leq t < 10 \text{ s} \\ -12\text{A}, & 10 \leq t < 15 \text{ s} \\ 0, & t \geq 15 \text{ s} \end{cases}$$

Plot the charge stored in the element over $0 < t < 20$ s.

1.13 The charge entering the positive terminal of an element is

$$q = 5 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 3 \cos 4\pi t \text{ V}$$

(a) Find the power delivered to the element at $t = 0.3$ s.

(b) Calculate the energy delivered to the element between 0 and 0.6 s.

1.14 The voltage $v(t)$ across a device and the current $i(t)$ through it are

$$v(t) = 20 \sin(4t) \text{ V and } i(t) = 10(1 + e^{-2t}) \text{ mA}$$

Calculate:

(a) the total charge in the device at $t = 1$ s, $q(0) = 0$.

(b) the power consumed by the device at $t = 1$ s.

1.15 The current entering the positive terminal of a device is $i(t) = 6e^{-2t}$ mA and the voltage across the device is $v(t) = 10di/dt$V.

(a) Find the charge delivered to the device between $t = 0$ and $t = 2$ s.

(b) Calculate the power absorbed.

(c) Determine the energy absorbed in 3 s.

Section 1.6 Circuit Elements

1.16 Figure 1.27 shows the current through and the voltage across an element.

(a) Sketch the power delivered to the element for $t > 0$.

(b) Fnd the total energy absorbed by the element for the period of $0 < t < 4$s.

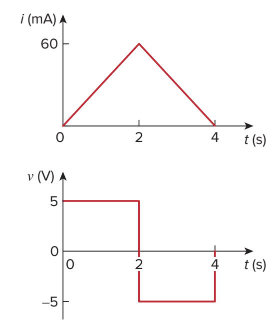


Figure 1.27
For Prob. 1.16.

1.17 Figure 1.28 shows a circuit with four elements, $p_1 = 60$ W absorbed, $p_3 = -145$ W absorbed, and $p_4 = 75$ W absorbed. How many watts does element 2 absorb?

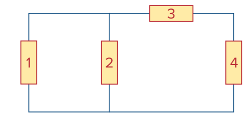


Figure 1.28
For Prob. 1.17.

1.18 Find the power absorbed by each of the elements in Fig. 1.29.

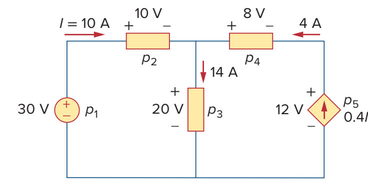


Figure 1.29
For Prob. 1.18.

1.19 Find I and the power absorbed by each element in the network of Fig. 1.30.

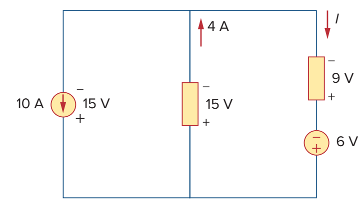


Figure 1.30
For Prob. 1.19.

1.20 Find V_o and the power absorbed by each element in the circuit of Fig. 1.31.

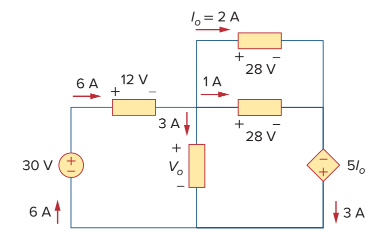


Figure 1.31
For Prob. 1.20.

Section 1.7 Applications

1.21 A 60-W incandescent bulb operates at 120 V. How many electrons and coulombs flow through the bulb in one day?

1.22 A lightning bolt strikes an airplane with 40 kA for 1.7 ms. How many coulombs of charge are deposited on the plane?

1.23 A 1.8-kW electric heater takes 15 min to boil a quantity of water. If this is done once a day and power costs 10 cents/kWh, what is the cost of its operation for 30 days?

1.24 A utility company charges 8.2 cents/kWh. If a consumer operates a 60-W light bulb continuously for one day, how much is the consumer charged?

1.25 A 1.2-kW toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster twice per day for 2 weeks (14 days). Assume energy costs 9 cents/kWh.

1.26 A cell phone battery is rated at 3.85 V and can store 10.78 watt-hours of energy.

(a) How much average current can it deliver over a period of 3 hours if it is fully discharged at the end of that time?

(b) How much average power is delivered in part (a)?

(c) What is the ampere-hour rating of the battery?

1.27 A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$V, where t is in hours,

(a) how much charge is transported as a result of the charging?

(b) how much energy is expended?

(c) how much does the charging cost? Assume electricity costs 9 cents/kWh.

1.28 A 150-W incandescent outdoor lamp is connected to a 120-V source and is left burning continuously for an average of 12 hours per day. Determine:

(a) the current through the lamp when it is lit.

(b) the cost of operating the light for one non-leap year if electricity costs 9.5 cents per kWh.

1.29 An electric stove with four burners and an oven is used in preparing a meal as follows.

Burner 1: 20 minutes   Burner 2: 40 minutes

Burner 3: 15 minutes   Burner 4: 45 minutes

Oven: 30 minutes

If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.

1.30 Reliant Energy (the electric company in Houston, Texas) charges customers as follows:

Monthly charge $6

First 250 kWh @ $0.02/kWh

All additional kWh @ $0.07/kWh

If a customer uses 2,436 kWh in one month, how much will Reliant Energy charge?

1.31 In a household, a business is run for an average of 6 h/day. The total power consumed by the computer and its printer is 230 W. In addition, a 75-W light runs during the same 6 h. If their utility charges 11.75 cents per kWh, how much do the owners pay every 30 days?

Comprehensive Problems

1.32 A telephone wire has a current of 20μA flowing through it. How long does it take for a charge of 15 C to pass through the wire?

1.33 A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?

1.34 Figure 1.32 shows the power consumption of a certain household in 1 day. Calculate:

(a) the total energy consumed in kWh,

(b) the average power over the total 24 hour period.

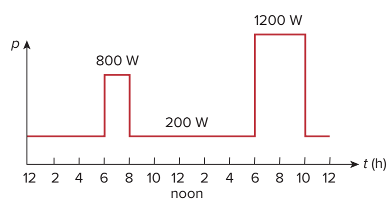


Figure 1.32
For Prob. 1.34.

1.35 The graph in Fig. 1.33 represents the power drawn by an industrial plant between 8:00 and 8:30 a.m. Calculate the total energy in MWh consumed by the plant.

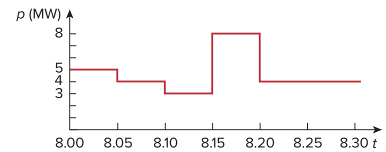


Figure 1.33
For Prob. 1.35.

1.36 A battery can be rated in ampere-hours (Ah) or watt hours (Wh). The ampere hours can be obtained from the watt hours by dividing watt hours by a nominal voltage of 12 V. If an automobile battery is rated at 20 Ah:

(a) What is the maximum current that can be supplied for 15 minutes?

(b) How many days will it last if it is discharged at a rate of 2 mA?

1.37 A total of 2 MJ are delivered to an automobile battery (assume 12 V) giving it an additional charge. How much is that additional charge? Express your answer in ampere-hours.

1.38 How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.

1.39 A 600-W TV receiver is turned on for 4 h with nobody watching it. If electricity costs 10 cents/kWh, how much money is wasted?

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chapter

2

Basic Laws

There are too many people praying for mountains of difficulty to be removed, when what they really need is the courage to climb them!

—Unknown

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.b), *"an ability to design and conduct experiments, as well as to analyze and interpret data."*

Engineers must be able to design and conduct experiments, as well as analyze and interpret data. Most students have spent many hours performing experiments in high school and in college. During this time, you have been asked to analyze the data and to interpret the data. Therefore, you should already be skilled in these two activities. My recommendation is that, in the process of performing experiments in the future, you spend more time in analyzing and interpreting the data in the context of the experiment. What does this mean?

If you are looking at a plot of voltage versus resistance or current versus resistance or power versus resistance, what do you actually see? Does the curve make sense? Does it agree with what the theory tells you? Does it differ from expectation, and, if so, why? Clearly, practice with analyzing and interpreting data will enhance this skill.

Since most, if not all, the experiments you are required to do as a student involve little or no practice in designing the experiment, how can you develop and enhance this skill?

Actually, developing this skill under this constraint is not as difficult as it seems. What you need to do is to take the experiment and analyze it. Just break it down into its simplest parts, reconstruct it trying to understand why each element is there, and finally, determine what the author of the experiment is trying to teach you. Even though it may not always seem so, every experiment you do was designed by someone who was sincerely motivated to teach you something.

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Know and understand the voltage current relationship of resistors (Ohm's law).
2. Understand the basic structure of electrical circuits, essentially nodes, loops, and branches.
3. Understand Kirchhoff's voltage and current laws and their importance in analyzing electrical circuits.
4. Understand series resistances and voltage division, and parallel resistances and current division.
5. Know how to convert delta-connected circuits to wye-connected circuits and how to convert wye-connected circuits to delta-connected circuits.

2.1 Introduction

Chapter 1 introduced basic concepts such as current, voltage, and power in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as Ohm's law and Kirchhoff's laws, form the foundation upon which electric circuit analysis is built.

In this chapter, in addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis. These techniques include combining resistors in series or parallel, voltage division, current division, and delta-to-wye and wye-to-delta transformations. The application of these laws and techniques will be restricted to resistive circuits in this chapter. We will finally apply the laws and techniques to real-life problems of electrical lighting and the design of dc meters.

2.2 Ohm's Law

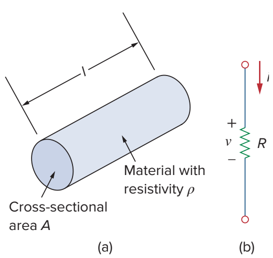

Figure 2.1
(a) Resistor, (b) Circuit symbol for resistance.

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol R. The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ, as shown in Fig. 2.1(a). We can represent resistance (as measured in the laboratory), in mathematical form,

$$R = \rho \frac{\ell}{A} \tag{2.1}$$

where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

The circuit element used to model the current-resisting behavior of a material is the *resistor*. For the purpose of constructing circuits, resistors are

TABLE 2.1

Resistivities of common materials.

Material	Resistivity (Ω•m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

usually made from metallic alloys and carbon compounds. The circuit symbol for the resistor is shown in Fig. 2.1(b), where R stands for the resistance of the resistor. The resistor is the simplest passive element.

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as *Ohm's law*.

> **Ohm's law** states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \tag{2.2}$$

Ohm defined the constant of proportionality for a resistor to be the resistance, R. (The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus, Eq. (2.2) becomes

$$\boxed{v = iR} \tag{2.3}$$

Historical

Georg Simon Ohm (1787–1854), a German physicist, in 1826 experimentally determined the most basic law relating voltage and current for a resistor. Ohm's work was initially denied by critics.

Born of humble beginnings in Erlangen, Bavaria, Ohm threw himself into electrical research. His efforts resulted in his famous law. He was awarded the Copley Medal in 1841 by the Royal Society of London. In 1849, he was given the Professor of Physics chair by the University of Munich. To honor him, the unit of resistance was named the ohm.

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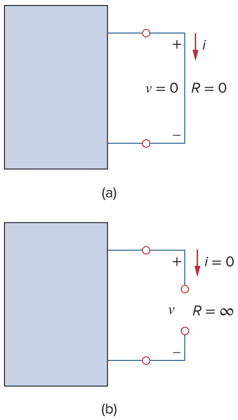

Figure 2.2
(a) Short circuit ($R = 0$), (b) Open circuit ($R = \infty$).

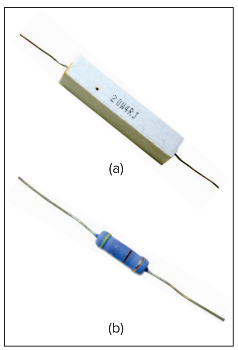

Figure 2.3
Fixed resistors: (a) wirewound type, (b) carbon film type.

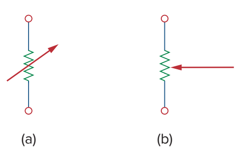

Figure 2.4
Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

which is the mathematical form of Ohm's law. R in Eq. (2.3) is measured in the unit of ohms, designated Ω. Thus,

> The **resistance** R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

We may deduce from Eq. (2.3) that

$$R = \frac{v}{i} \tag{2.4}$$

so that

$$1\ \Omega = 1\ \text{V/A}$$

To apply Ohm's law as stated in Eq. (2.3), we must pay careful attention to the current direction and voltage polarity. The direction of current i and the polarity of voltage v must conform with the passive sign convention, as shown in Fig. 2.1(b). This implies that current flows from a higher potential to a lower potential in order for $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R. An element with $R = 0$ is called a *short circuit*, as shown in Fig. 2.2(a). For a short circuit,

$$v = iR = 0 \tag{2.5}$$

showing that the voltage is zero but the current could be anything. In practice, a short circuit is usually a connecting wire assumed to be a perfect conductor. Thus,

> A **short circuit** is a circuit element with resistance approaching zero.

Similarly, an element with $R = \infty$ is known as an *open circuit*, as shown in Fig. 2.2(b). For an open circuit,

$$i = \lim_{R \to \infty} \frac{v}{R} = 0 \tag{2.6}$$

indicating that the current is zero though the voltage could be anything. Thus,

> An **open circuit** is a circuit element with resistance approaching infinity.

A resistor is either fixed or variable. Most resistors are of the fixed type, meaning their resistance remains constant. The two common types of fixed resistors (wirewound and composition) are shown in Fig. 2.3. The composition resistors are used when large resistance is needed. The circuit symbol in Fig. 2.1(b) is for a fixed resistor. Variable resistors have adjustable resistance. The symbol for a variable resistor is shown in Fig. 2.4(a). A common variable resistor is known as a *potentiometer* or *pot* for short, with the symbol shown in Fig. 2.4(b). The pot is a three-terminal element with a sliding contact or wiper. By sliding the wiper, the resistances between the wiper terminal and the fixed terminals vary. Like fixed resistors, variable resistors can be of either wirewound or composition type, as shown in Fig. 2.5. Although resistors like those in Figs. 2.3 and 2.5 are used in circuit designs, today most circuit components including resistors are either surface mounted or integrated, as typically shown in Fig. 2.6.

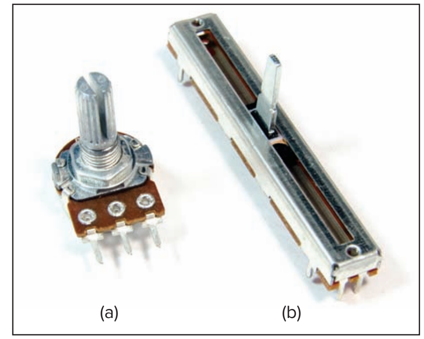

Figure 2.5
Variable resistors: (a) composition type, (b) slider pot.

It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a *linear* resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in Fig. 2.7(a): Its i-v graph is a straight line passing through the origin. A *nonlinear* resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in Fig. 2.7(b). Examples of devices with nonlinear resistance are the light bulb and the diode. Although all practical resistors may exhibit nonlinear behavior under certain conditions, we will assume in this book that all elements actually designated as resistors are linear.

A useful quantity in circuit analysis is the reciprocal of resistance R, known as *conductance* and denoted by G:

$$\boxed{G = \frac{1}{R} = \frac{i}{v}} \tag{2.7}$$

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the *mho* (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mho, in this book we prefer to use the siemens (S), the SI unit of conductance:

$$1\ \text{S} = 1\ \mho = 1\ \text{A/V} \tag{2.8}$$

Thus,

> **Conductance** is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

The same resistance can be expressed in ohms or siemens. For example, 10 Ω is the same as 0.1 S. From Eq. (2.7), we may write

$$i = Gv \tag{2.9}$$

The power dissipated by a resistor can be expressed in terms of R. Using Eqs. (1.7) and (2.3),

$$p = vi = i^2R = \frac{v^2}{R} \tag{2.10}$$

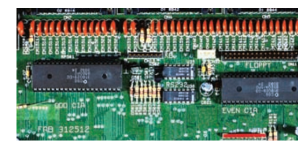

Figure 2.6
Resistors in an integrated circuit board.

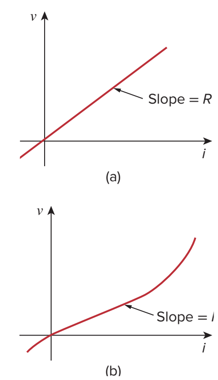

Figure 2.7
The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

The power dissipated by a resistor may also be expressed in terms of G as

$$p = vi = v^2G = \frac{i^2}{G} \quad \textbf{(2.11)}$$

We should note two things from Eqs. (2.10) and (2.11):

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since R and G are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

Example 2.1

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:
From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

Practice Problem 2.1

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

Answer: 7.333 A.

Example 2.2

In the circuit shown in Fig. 2.8, calculate the current i, the conductance G, and the power p.

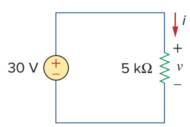

Figure 2.8
For Example 2.2.

Solution:
The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Practice Problem 2.2

For the circuit shown in Fig. 2.9, calculate the voltage v, the conductance G, and the power p.

Answer: 30 V, 100 μS, 90 mW.

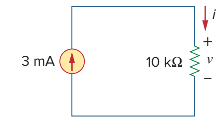

Figure 2.9
For Practice Prob. 2.2

Example 2.3

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

Practice Problem 2.3

A resistor absorbs an instantaneous power of $30 \cos^2 t$ mW when connected to a voltage source $v = 15 \cos t$ V. Find i and R.

Answer: $2 \cos t$ mA, 7.5 kΩ.

2.3 †Nodes, Branches, and Loops

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. To differentiate between a circuit and a network, we may regard a network as an interconnection of elements or devices, whereas a circuit is a network providing one or more closed paths. The convention, when addressing network topology, is to use the word network rather than circuit. We do this even though the word network and circuit mean the same thing when used in this context. In network topology, we study the properties relating to the placement of elements in the network and the geometric configuration of the network. Such elements include branches, nodes, and loops.

> A **branch** represents a single element such as a voltage source or a resistor.

In other words, a branch represents any two-terminal element. The circuit in Fig. 2.10 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

> A **node** is the point of connection between two or more branches.

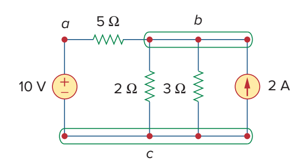

Figure 2.10
Nodes, branches, and loops.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 2.10 has three nodes a, b, and c. Notice that

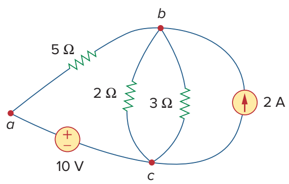

Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

the three points that form node b are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node c. We demonstrate that the circuit in Fig. 2.10 has only three nodes by redrawing the circuit in Fig. 2.11. The two circuits in Figs. 2.10 and 2.11 are identical. However, for the sake of clarity, nodes b and c are spread out with perfect conductors as in Fig. 2.10.

> A **loop** is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once. A loop is said to be *independent* if it contains at least one branch which is not a part of any other independent loop. Independent loops or paths result in independent sets of equations.

It is possible to form an independent set of loops where one of the loops does not contain such a branch. In Fig. 2.11, *abca* with the 2 Ω resistor is independent. A second loop with the 3 Ω resistor and the current source is independent. The third loop could be the one with the 2 Ω resistor in parallel with the 3 Ω resistor. This does form an independent set of loops.

A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$\boxed{b = l + n - 1} \tag{2.12}$$

As the next two definitions show, circuit topology is of great value to the study of voltages and currents in an electric circuit.

> Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.
> Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

Elements are in series when they are chain-connected or connected sequentially, end to end. For example, two elements are in series if they share one common node and no other element is connected to that common node. Elements in parallel are connected to the same pair of terminals. Elements may be connected in a way that they are neither in series nor in parallel. In the circuit shown in Fig. 2.10, the voltage source and the 5-Ω resistor are in series because the same current will flow through them. The 2-Ω resistor, the 3-Ω resistor, and the current source are in parallel because they are connected to the same two nodes b and c and consequently have the same voltage across them. The 5-Ω and 2-Ω resistors are neither in series nor in parallel with each other.

Example 2.4

Determine the number of branches and nodes in the circuit shown in Fig. 2.12. Identify which elements are in series and which are in parallel.

Solution:
Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in Fig. 2.13. The 5-Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6-Ω resistor is in parallel with the 2-A current source because both are connected to the same nodes 2 and 3.

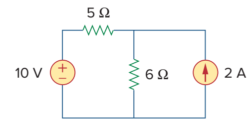

Figure 2.12
For Example 2.4.

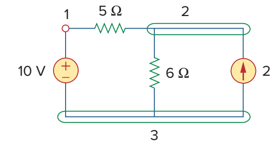

Figure 2.13
The three nodes in the circuit of Fig. 2.12.

Practice Problem 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

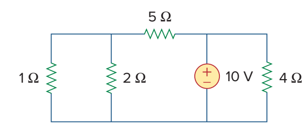

Figure 2.14
For Practice Prob. 2.4.

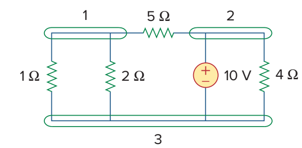

Figure 2.15
Answer for Practice Prob. 2.4.

2.4 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

> **Kirchhoff's current law (KCL)** states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\boxed{\sum_{n=1}^{N} i_n = 0} \tag{2.13}$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

Historical

Gustav Robert Kirchhoff (1824–1887), a German physicist, stated two basic laws in 1847 concerning the relationship between the currents and voltages in an electrical network. Kirchhoff's laws, along with Ohm's law, form the basis of circuit theory.

Born the son of a lawyer in Konigsberg, East Prussia, Kirchhoff entered the University of Konigsberg at age 18 and later became a lecturer in Berlin. His collaborative work in spectroscopy with German chemist Robert Bunsen led to the discovery of cesium in 1860 and rubidium in 1861. Kirchhoff was also credited with the Kirchhoff law of radiation. Thus, Kirchhoff is famous among engineers, chemists, and physicists.

To prove KCL, assume a set of currents $i_k(t)$, $k = 1, 2, \ldots$, flow into a node. The algebraic sum of currents at the node is

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \cdots \tag{2.14}$$

Integrating both sides of Eq. (2.14) gives

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \cdots \tag{2.15}$$

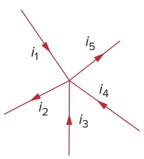

Figure 2.16
Currents at a node illustrating KCL.

where $q_k(t) = \int i_k(t)\,dt$ and $q_T(t) = \int i_T(t)\,dt$. But the law of conservation of electric charge requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus, $q_T(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

Consider the node in Fig. 2.16. Applying KCL gives

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \tag{2.16}$$

since currents i_1, i_3, and i_4 are entering the node, while currents i_2 and i_5 are leaving it. By rearranging the terms, we get

$$i_1 + i_3 + i_4 = i_2 + i_5 \tag{2.17}$$

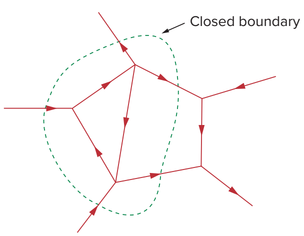

Figure 2.17
Applying KCL to a closed boundary.

Equation (2.17) is an alternative form of KCL:

> The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Note that KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point. In two dimensions, a closed boundary is the same as a closed path. As typically illustrated in the circuit of Fig. 2.17, the total current entering the closed surface is equal to the total current leaving the surface.

> Two sources (or circuits in general) are said to be equivalent if they have the same *i-v* relationship at a pair of terminals.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in

Fig. 2.18(a) can be combined as in Fig. 2.18(b). The combined or equivalent current source can be found by applying KCL to node a.

$$I_T + I_2 = I_1 + I_3$$

or

$$I_T = I_1 - I_2 + I_3 \tag{2.18}$$

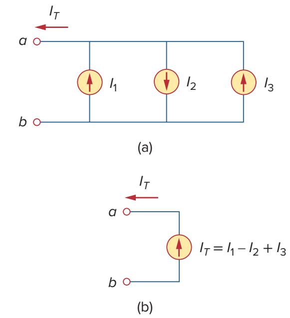

Figure 2.18
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

A circuit cannot contain two different currents, I_1 and I_2, in series, unless $I_1 = I_2$; otherwise KCL will be violated.

Kirchhoff's second law is based on the principle of conservation of energy:

> **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\boxed{\sum_{m=1}^{M} v_m = 0} \tag{2.19}$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

> KVL can be applied in two ways: by taking either a clockwise or a counterclockwise trip around the loop. Either way, the algebraic sum of voltages around the loop is zero.

To illustrate KVL, consider the circuit in Fig. 2.19. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $+v_4$, and $-v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \tag{2.20}$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.21}$$

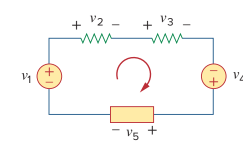

Figure 2.19
A single-loop circuit illustrating KVL.

which may be interpreted as

$$\text{Sum of voltage drops} = \text{Sum of voltage rises} \tag{2.22}$$

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1$, $-v_5$, $+v_4$, $-v_3$, and $-v_2$, which is the same as before except that the signs are reversed. Hence, Eqs. (2.20) and (2.21) remain the same.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 2.20(a), the combined or equivalent voltage source in Fig. 2.20(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

or

$$V_{ab} = V_1 + V_2 - V_3 \tag{2.23}$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.

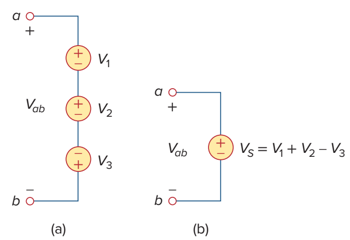

Figure 2.20
Voltage sources in series: (a) original circuit, (b) equivalent circuit.

Example 2.5

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

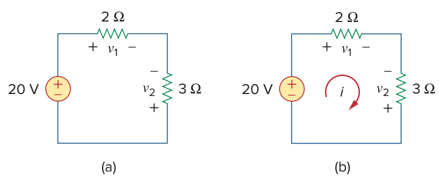

Figure 2.21
For Example 2.5.

Solution:
To find v_1 and v_2 we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \quad \text{or} \quad 5i = 20 \quad \Rightarrow \quad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Practice Problem 2.5

Find v_1 and v_2 in the circuit of Fig. 2.22.

Answer: 16 V, −8 V.

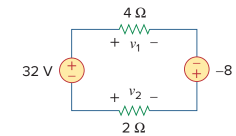

Figure 2.22
For Practice Prob. 2.5.

Example 2.6

Determine v_o and i in the circuit shown in Fig. 2.23(a).

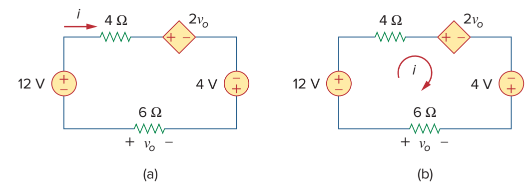

Figure 2.23
For Example 2.6.

Solution:
We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \tag{2.6.1}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \tag{2.6.2}$$

Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Practice Problem 2.6

Find v_x and v_o in the circuit of Fig. 2.24.

Answer: 20 V, −10 V.

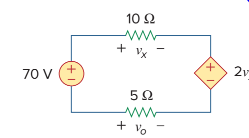

Figure 2.24
For Practice Prob. 2.6.

Example 2.7

Find current i_o and voltage v_o in the circuit shown in Fig. 2.25.

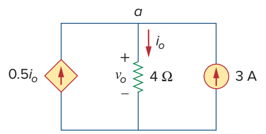

Figure 2.25
For Example 2.7.

Solution:
Applying KCL to node a, we obtain

$$3 + 0.5i_o = i_o \qquad \Rightarrow \qquad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

Practice Problem 2.7

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 20 V, 10 A.

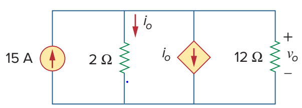

Figure 2.26
For Practice Prob. 2.7.

Example 2.8

Find currents and voltages in the circuit shown in Fig. 2.27(a).

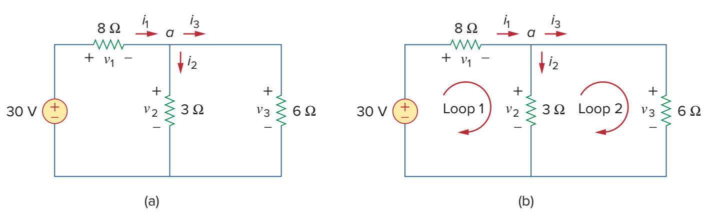

Figure 2.27
For Example 2.8.

Solution:
We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_2 \tag{2.8.4}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Rightarrow \qquad i_3 = \frac{i_2}{2} \tag{2.8.5}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the value of i_2, we now use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3 \text{ A}, \qquad i_3 = 1 \text{ A}, \qquad v_1 = 24 \text{ V}, \qquad v_2 = 6 \text{ V}, \qquad v_3 = 6 \text{ V}$$

Practice Problem 2.8

Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 2.5$ A.

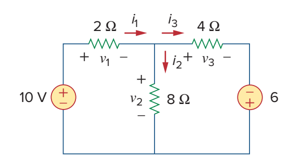

Figure 2.28
For Practice Prob. 2.8.

2.5 Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 2.29. The two resistors are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \qquad v_2 = iR_2 \tag{2.24}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \tag{2.25}$$

Combining Eqs. (2.24) and (2.25), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \tag{2.26}$$

or

$$i = \frac{v}{R_1 + R_2} \tag{2.27}$$

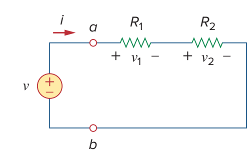

Figure 2.29
A single-loop circuit with two resistors in series.

Notice that Eq. (2.26) can be written as

$$v = iR_{eq} \tag{2.28}$$

implying that the two resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$R_{eq} = R_1 + R_2 \tag{2.29}$$

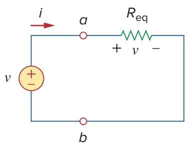

Figure 2.30
Equivalent circuit of the Fig. 2.29 circuit.

Thus, Fig. 2.29 can be replaced by the equivalent circuit in Fig. 2.30. The two circuits in Figs. 2.29 and 2.30 are equivalent because they exhibit the same voltage-current relationships at the terminals *a-b*. An equivalent circuit such as the one in Fig. 2.30 is useful in simplifying the analysis of a circuit. In general,

> The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

> Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

For N resistors in series then,

$$\boxed{R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n} \tag{2.30}$$

To determine the voltage across each resistor in Fig. 2.29, we substitute Eq. (2.26) into Eq. (2.24) and obtain

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v} \tag{2.31}$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the *principle of voltage division*, and the circuit in Fig. 2.29 is called a *voltage divider*. In general, if a voltage divider has N resistors ($R_1, R_2, \ldots, R_N$) in series with the source voltage v, the nth resistor (R_n) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v \tag{2.32}$$

2.6 Parallel Resistors and Current Division

Consider the circuit in Fig. 2.31, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

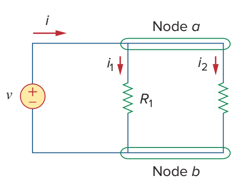

Figure 2.31
Two resistors in parallel.

$$v = i_1 R_1 = i_2 R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \tag{2.33}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \tag{2.34}$$

Substituting Eq. (2.33) into Eq. (2.34), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \tag{2.35}$$

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2.36}$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \tag{2.37}$$

Thus,

> The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (2.37), if $R_1 = R_2$ then $R_{eq} = R_1/R_2$.

We can extend the result in Eq. (2.36) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \tag{2.38}$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \tag{2.39}$$

> Conductances in parallel behave as a single conductance whose value is equal to the sum of the individual conductances.

For example, if four 100-Ω resistors are connected in parallel, their equivalent resistance is 25 Ω.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (2.38), the equivalent conductance for N resistors in parallel is

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N} \tag{2.40}$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, …, $G_N = 1/R_N$. Equation (2.40) states:

> The **equivalent conductance** of resistors connected in parallel is the sum of their individual conductances.

This means that we may replace the circuit in Fig. 2.31 with that in Fig. 2.32. Notice the similarity between Eqs. (2.30) and (2.40). The equivalent conductance of parallel resistors is obtained the same way as the equivalent resistance of series resistors. In the same manner, the equivalent conductance of resistors in series is obtained just the same

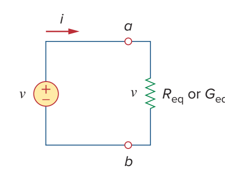

Figure 2.32
Equivalent circuit to Fig. 2.31.

way as the resistance of resistors in parallel. Thus, the equivalent conductance G_{eq} of N resistors in series (such as shown in Fig. 2.29) is

$$\boxed{\frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \cdots + \frac{1}{G_N}} \tag{2.41}$$

Given the total current i entering node a in Fig. 2.31, how do we obtain current i_1 and i_2? We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2} \tag{2.42}$$

Combining Eqs. (2.33) and (2.42) results in

$$\boxed{i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}} \tag{2.43}$$

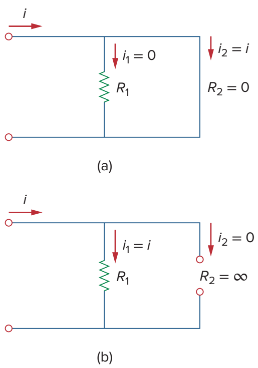

Figure 2.33
(a) A shorted circuit, (b) an open circuit.

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the *principle of current division*, and the circuit in Fig. 2.31 is known as a *current divider*. Notice that the larger current flows through the smaller resistance.

As an extreme case, suppose one of the resistors in Fig. 2.31 is zero, say $R_2 = 0$; that is, R_2 is a short circuit, as shown in Fig. 2.33(a). From Eq. (2.43), $R_2 = 0$ implies that $i_1 = 0$, $i_2 = i$. This means that the entire current i bypasses R_1 and flows through the short circuit $R_2 = 0$, the path of least resistance. Thus when a circuit is short circuited, as shown in Fig. 2.33(a), two things should be kept in mind:

1. The equivalent resistance $R_{eq} = 0$. [See what happens when $R_2 = 0$ in Eq. (2.37).]
2. The entire current flows through the short circuit.

As another extreme case, suppose $R_2 = \infty$, that is, R_2 is an open circuit, as shown in Fig. 2.33(b). The current still flows through the path of least resistance, R_1. By taking the limit of Eq. (2.37) as $R_2 \to \infty$, we obtain $R_{eq} = R_1$ in this case.

If we divide both the numerator and denominator by R_1R_2, Eq. (2.43) becomes

$$i_1 = \frac{G_1}{G_1 + G_2} i \tag{2.44a}$$

$$i_2 = \frac{G_2}{G_1 + G_2} i \tag{2.44b}$$

Thus, in general, if a current divider has N conductors ($G_1, G_2, \ldots, G_N$) in parallel with the source current i, the nth conductor (G_n) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i \tag{2.45}$$

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single *equivalent resistance* R_{eq}. Such an equivalent resistance is the resistance between the designated terminals of the network and must exhibit the same i-v characteristics as the original network at the terminals.

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

Solution:
To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

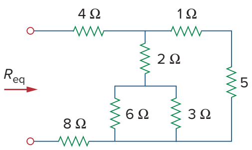

Figure 2.34
For Example 2.9.

(The symbol $\parallel$ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

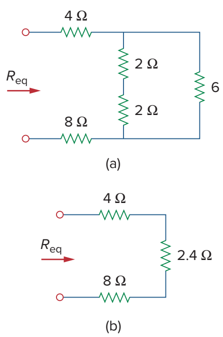

Figure 2.35
Equivalent circuits for Example 2.9.

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 11 Ω.

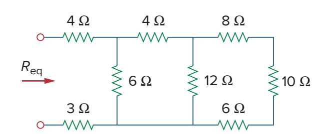

Figure 2.36
For Practice Prob. 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

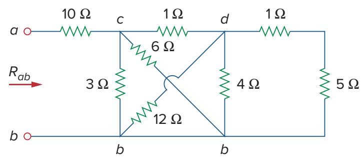

Figure 2.37
For Example 2.10.

Solution:
The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3+6} = 2\ \Omega \tag{2.10.1}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since the y are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12+4} = 3\ \Omega \tag{2.10.2}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \tag{2.10.3}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\,\Omega + 2\,\Omega = 3\,\Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

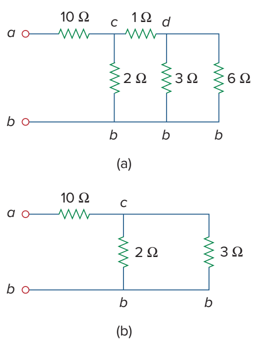

Figure 2.38
Equivalent circuits for Example 2.10.

Practice Problem 2.10

Find R_{ab} for the circuit in Fig. 2.39.

Answer: 19 Ω.

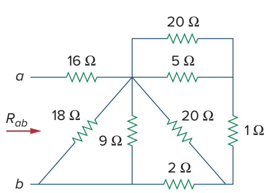

Figure 2.39
For Practice Prob. 2.10.

Example 2.11

Find the equivalent conductance G_{eq} for the circuit in Fig. 2.40(a).

Solution:
The 8-S and 12-S resistors are in parallel, so their conductance is

$$8\ \mathrm{S} + 12\ \mathrm{S} = 20\ \mathrm{S}$$

This 20-S resistor is now in series with 5 S as shown in Fig. 2.40(b) so that the combined conductance is

$$\frac{20 \times 5}{20+5} = 4\ \mathrm{S}$$

This is in parallel with the 6-S resistor. Hence,

$$G_{eq} = 6 + 4 = 10\ \mathrm{S}$$

We should note that the circuit in Fig. 2.40(a) is the same as that in Fig. 2.40(c). While the resistors in Fig. 2.40(a) are expressed in siemens, those in Fig. 2.40(c) are expressed in ohms. To show that the circuits are the same, we find R_{eq} for the circuit in Fig. 2.40(c).

$$R_{eq} = \frac{1}{6} \Big\| \left(\frac{1}{5} + \frac{1}{8} \Big\| \frac{1}{12}\right) = \frac{1}{6} \Big\| \left(\frac{1}{5} + \frac{1}{20}\right) = \frac{1}{6} \Big\| \frac{1}{4}$$

$$= \frac{\frac{1}{6} \times \frac{1}{4}}{\frac{1}{6} + \frac{1}{4}} = \frac{1}{10}\ \Omega$$

$$G_{eq} = \frac{1}{R_{eq}} = 10\ \mathrm{S}$$

This is the same as we obtained previously.

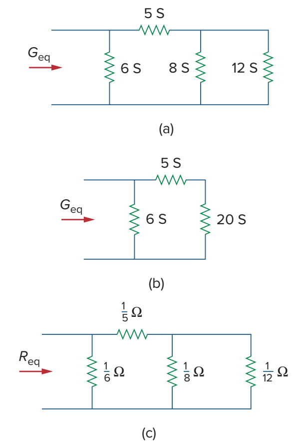

Figure 2.40
For Example 2.11: (a) original circuit, (b) its equivalent circuit, (c) same circuit as in (a) but resistors are expressed in ohms.

Practice Problem 2.11

Calculate G_{eq} in the circuit of Fig. 2.41.

Answer: 8 S.

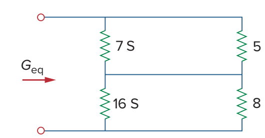

Figure 2.41
For Practice Prob. 2.11.

Example 2.12

Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.

Solution:
The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = \frac{6 \times 3}{6+3} = 2\ \Omega$$

Thus, our circuit reduces to that shown in Fig. 2.42(b). Notice that v_o is not affected by the combination of the resistors because the resistors are

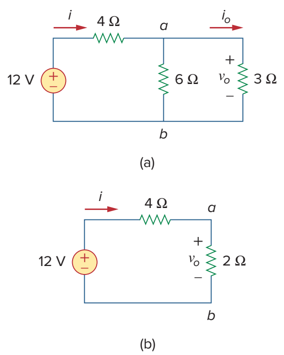

Figure 2.42
For Example 2.12: (a) original circuit, (b) its equivalent circuit.

in parallel and therefore have the same voltage v_o. From Fig. 2.42(b), we can obtain v_o in two ways. One way is to apply Ohm's law to get

$$i = \frac{12}{4+2} = 2 \text{ A}$$

and hence, $v_o = 2i = 2 \times 2 = 4$ V. Another way is to apply voltage division, since the 12 V in Fig. 2.42(b) is divided between the 4-Ω and 2-Ω resistors. Hence,

$$v_o = \frac{2}{2+4}(12 \text{ V}) = 4 \text{ V}$$

Similarly, i_o can be obtained in two ways. One approach is to apply Ohm's law to the 3-Ω resistor in Fig. 2.42(a) now that we know v_o; thus,

$$v_o = 3i_o = 4 \quad \Rightarrow \quad i_o = \frac{4}{3} \text{ A}$$

Another approach is to apply current division to the circuit in Fig. 2.42(a) now that we know i, by writing

$$i_o = \frac{6}{6+3} i = \frac{2}{3}(2 \text{ A}) = \frac{4}{3} \text{ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333 \text{ W}$$

Practice Problem 2.12

Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 = 500$ mA, $p_2 = 10$ W.

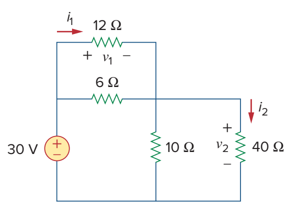

Figure 2.43
For Practice Prob. 2.12.

Example 2.13

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.

Solution:
(a) The 6-k Ω and 12-k Ω resistors are in series so that their combined value is 6 + 12 = 18 kΩ. Thus the circuit in Fig. 2.44(a) reduces to that shown in Fig. 2.44(b). We now apply the current division technique to find i_1 and i_2.

$$i_1 = \frac{18{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 20 \text{ mA}$$

$$i_2 = \frac{9{,}000}{9{,}000 + 18{,}000}(30 \text{ mA}) = 10 \text{ mA}$$

Notice that the voltage across the 9-kΩ and 18-kΩ resistors is the same, and $v_o = 9{,}000 i_1 = 18{,}000 i_2 = 180$ V, as expected.
(b) Power supplied by the source is

$$p_o = v_o i_o = 180(30) \text{ mW} = 5.4 \text{ W}$$

(c) Power absorbed by the 12-kΩ resistor is

$$p = iv = i_2(i_2 R) = i_2^2 R = (10 \times 10^{-3})^2 (12{,}000) = 1.2 \text{ W}$$

Power absorbed by the 6-kΩ resistor is

$$p = i_2^2 R = (10 \times 10^{-3})^2 (6{,}000) = 0.6 \text{ W}$$

Power absorbed by the 9-kΩ resistor is

$$p = \frac{v_o^2}{R} = \frac{(180)^2}{9{,}000} = 3.6 \text{ W}$$

or

$$p = v_o i_1 = 180(20) \text{ mW} = 3.6 \text{ W}$$

Notice that the power supplied (5.4 W) equals the power absorbed (1.2 + 0.6 + 3.6 = 5.4 W). This is one way of checking results.

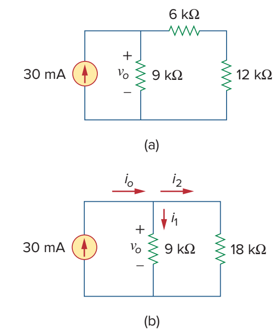

Figure 2.44
For Example 2.13: (a) original circuit, (b) its equivalent circuit.

Practice Problem 2.13

For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2, (b) the power dissipated in the 3-k Ω and 20-k Ω resistors, and (c) the power supplied by the current source.

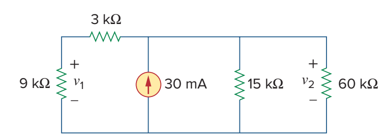

Figure 2.45
For Practice Prob. 2.13.

Answer: (a) 135 V, 180 V, (b) 2.025 W, 540 mW, (c) 5.4 W.

2.7 †Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta (Δ) or pi (Π) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest

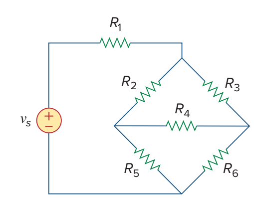

Figure 2.46
The bridge network.

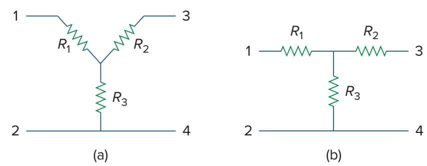

Figure 2.47
Two forms of the same network: (a) Y, (b) T.

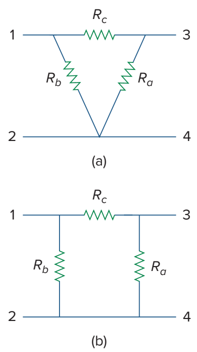

Figure 2.48
Two forms of the same network: (a) Δ, (b) Π.

here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the Δ (or Π) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

$$R_{12}(\mathrm{Y}) = R_1 + R_3 \tag{2.46}$$

$$R_{12}(\Delta) = R_b \| (R_a + R_c)$$

Setting $R_{12}(\mathrm{Y}) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.47a}$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.47b}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.47c}$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \tag{2.48}$$

Adding Eqs. (2.47b) and (2.48) gives

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}} \tag{2.49}$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}} \tag{2.50}$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}} \tag{2.51}$$

We do not need to memorize Eqs. (2.49) to (2.51). To transform a Δ network to Y, we create an extra node n as shown in Fig. 2.49 and follow this conversion rule:

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

One can follow this rule and obtain Eqs. (2.49) to (2.51) from Fig. 2.49.

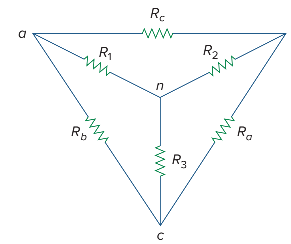

Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$\begin{aligned} R_1R_2 + R_2R_3 + R_3R_1 &= \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \\ &= \frac{R_aR_bR_c}{R_a + R_b + R_c} \end{aligned} \tag{2.52}$$

Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$\boxed{R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}} \tag{2.53}$$

$$\boxed{R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}} \tag{2.54}$$

$$\boxed{R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}} \tag{2.55}$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to Δ is as follows:

> Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and Δ networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_\mathrm{Y}, \qquad R_a = R_b = R_c = R_\Delta \tag{2.56}$$

Under these conditions, conversion formulas become

$$\boxed{R_\mathrm{Y} = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_\mathrm{Y}} \tag{2.57}$$

One may wonder why R_Y is less than R_Δ. Well, we notice that the Y-connection is like a "series" connection while the Δ-connection is like a "parallel" connection.

Note that in making the transformation, we do not take anything out of the circuit or put in anything new. We are merely substituting different but mathematically equivalent three-terminal network patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate R_{eq} if necessary.

Example 2.14

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

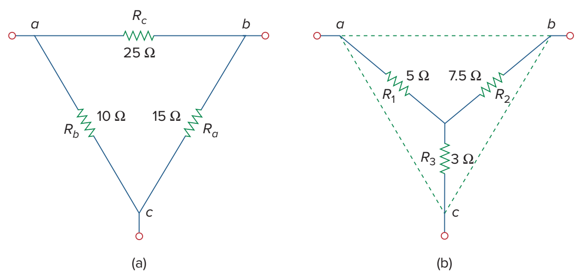

Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

Solution:
Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

Practice Problem 2.14

Transform the wye network in Fig. 2.51 to a delta network.

Answer: $R_a = 140\ \Omega$, $R_b = 70\ \Omega$, $R_c = 35\ \Omega$.

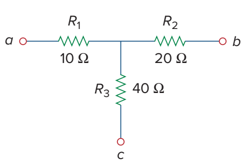

Figure 2.51
For Practice Prob. 2.14.

Only see

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

Solution:

1. **Define.** The problem is clearly defined. Please note, this part normally will deservedly take much more time.
2. **Present.** Clearly, when we remove the voltage source, we end up with a purely resistive circuit. Since it is composed of deltas and wyes, we have a more complex process of combining the elements together. We can use wye-delta transformations as one approach to find a solution. It is useful to locate the wyes (there are two of them, one at n and the other at c) and the deltas (there are three: *can, abn, cnb*).
3. **Alternative.** There are different approaches that can be used to solve this problem. Since the focus of Sec. 2.7 is the wye-delta transformation, this should be the technique to use. Another approach would be to solve for the equivalent resistance by injecting one amp into the circuit and finding the voltage between a and b; we will learn about this approach in Chap. 4.

 The approach we can apply here as a check would be to use a wye-delta transformation as the first solution to the problem. Later we can check the solution by starting with a delta-wye transformation.
4. **Attempt.** In this circuit, there are two Y networks and three Δ networks. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

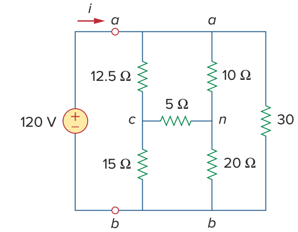

Figure 2.52
For Example 2.15.

Only check not Assignment task same only see

$$R_1 = 10\ \Omega, \qquad R_2 = 20\ \Omega, \qquad R_3 = 5\ \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35\ \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5\ \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70\ \Omega$$

With the Y converted to Δ, the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

$$70 \| 30 = \frac{70 \times 30}{70 + 30} = 21\ \Omega$$

$$12.5 \| 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.292\ \Omega$$

$$15 \| 35 = \frac{15 \times 35}{15 + 35} = 10.5\ \Omega$$

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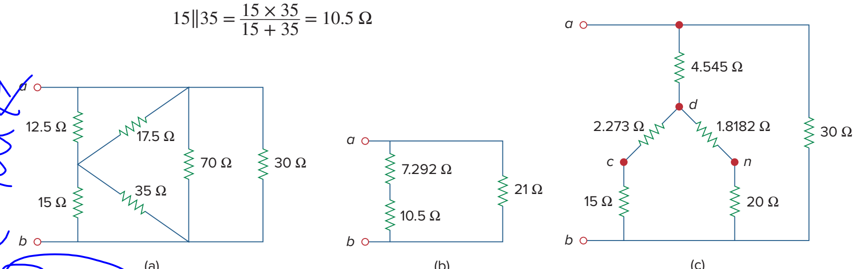


Figure 2.53
Equivalent circuits to Fig. 2.52, with the voltage source removed.

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = \mathbf{9.632\ \Omega}$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = \mathbf{12.458\ A}$$

We observe that we have successfully solved the problem. Now we must evaluate the solution.

5. **Evaluate.** Now we must determine if the answer is correct and then evaluate the final solution.

   It is relatively easy to check the answer; we do this by solving the problem starting with a delta-wye transformation. Let us transform the delta, *can*, into a wye.

   Let $R_c = 10\ \Omega$, $R_a = 5\ \Omega$, and $R_n = 12.5\ \Omega$. This will lead to (let d represent the middle of the wye):

$$R_{ad} = \frac{R_c R_n}{R_a + R_c + R_n} = \frac{10 \times 12.5}{5 + 10 + 12.5} = 4.545\ \Omega$$

$$R_{cd} = \frac{R_a R_n}{27.5} = \frac{5 \times 12.5}{27.5} = 2.273\ \Omega$$

$$R_{nd} = \frac{R_a R_c}{27.5} = \frac{5 \times 10}{27.5} = 1.8182\ \Omega$$

This now leads to the circuit shown in Figure 2.53(c). Looking at the resistance between d and b, we have two series combination in parallel, giving us

$$R_{db} = \frac{(2.273 + 15)(1.8182 + 20)}{2.273 + 15 + 1.8182 + 20} = \frac{376.9}{39.09} = 9.642\ \Omega$$

This is in series with the 4.545-Ω resistor, both of which are in parallel with the 30-Ω resistor. This then gives us the equivalent resistance of the circuit.

$$R_{ab} = \frac{(9.642 + 4.545)30}{9.642 + 4.545 + 30} = \frac{425.6}{44.19} = \mathbf{9.631\ \Omega}$$

This now leads to

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.631} = \mathbf{12.46\ A}$$

We note that using two variations on the wye-delta transformation leads to the same results. This represents a very good check.

6. **Satisfactory?** Since we have found the desired answer by determining the equivalent resistance of the circuit first and the answer checks, then we clearly have a satisfactory solution. This represents what can be presented to the individual assigning the problem.

Practice Problem 2.15

For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer: 60 Ω, 4 A.

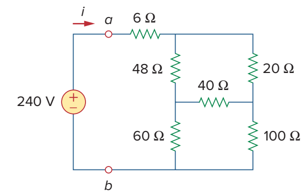


Figure 2.54
For Practice Prob. 2.15.

2.8 †Applications

Resistors are often used to model devices that convert electrical energy into heat or other forms of energy. Such devices include conducting wire, light bulbs, electric heaters, stoves, ovens, and loudspeakers. In this section, we will consider two real-life problems that apply the concepts developed in this chapter: electrical lighting systems and design of dc meters.

2.8.1 Lighting Systems

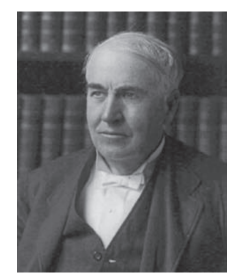
Lighting systems, such as in a house or on a Christmas tree, often consist of N lamps connected either in parallel or in series, as shown in Fig. 2.55. Each lamp is modeled as a resistor. Assuming that all the lamps are identical and V_o is the power-line voltage, the voltage across each lamp is V_o for the parallel connection and V_o/N for the series connection. The series connection is easy to manufacture but is seldom used in practice, for at least two reasons. First, it is less reliable; when a lamp fails, all the lamps go out. Second, it is harder to maintain; when a lamp is bad, one must test all the lamps one by one to detect the faulty one.

So far, we have assumed that connecting wires are perfect conductors (i.e., conductors of zero resistance). In real physical systems, however, the resistance of the connecting wire may be appreciably large, and the modeling of the system must include that resistance.

Historical

Thomas Alva Edison (1847–1931) was perhaps the greatest American inventor. He patented 1093 inventions, including such history-making inventions as the incandescent electric bulb, the phonograph, and the first commercial motion pictures.

Born in Milan, Ohio, the youngest of seven children, Edison received only three months of formal education because he hated school. He was home-schooled by his mother and quickly began to read on his own. In 1868, Edison read one of Faraday's books and found his calling. He moved to Menlo Park, New Jersey, in 1876, where he managed a well-staffed research laboratory. Most of his inventions came out of this laboratory. His laboratory served as a model for modern research organizations. Because of his diverse interests and the overwhelming number of his inventions and patents, Edison began to establish manufacturing companies for making the devices he invented. He designed the first electric power station to supply electric light. Formal electrical engineering education began in the mid-1880s with Edison as a role model and leader.



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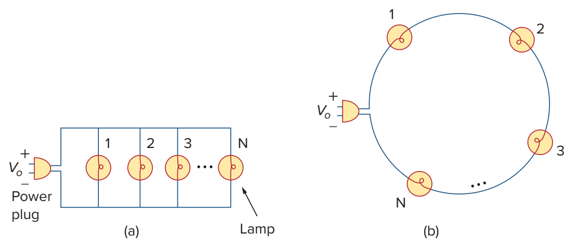

Figure 2.55
(a) Parallel connection of light bulbs, (b) series connection of light bulbs.

Only sel

Example 2.16

Three light bulbs are connected to a 9-V battery as shown in Fig. 2.56(a). Calculate: (a) the total current supplied by the battery, (b) the current through each bulb, (c) the resistance of each bulb.

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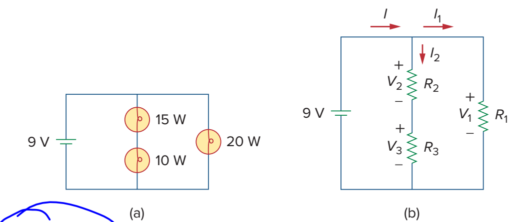


Figure 2.56
(a) Lighting system with three bulbs, (b) resistive circuit equivalent model.

Solution:
(a) The total power supplied by the battery is equal to the total power absorbed by the bulbs; that is,

$$p = 15 + 10 + 20 = 45 \text{ W}$$

Since $p = V\,I$, then the total current supplied by the battery is

$$I = \frac{p}{V} = \frac{45}{9} = 5 \text{ A}$$

(b) The bulbs can be modeled as resistors as shown in Fig. 2.56(b). Since R_1 (20-W bulb) is in parallel with the battery as well as the series combination of R_2 and R_3,

$$V_1 = V_2 + V_3 = 9 \text{ V}$$

The current through R_1 is

$$I_1 = \frac{p_1}{V_1} = \frac{20}{9} = 2.222 \text{ A}$$

By KCL, the current through the series combination of R_2 and R_3 is

$$I_2 = I - I_1 = 5 - 2.222 = 2.778 \text{ A}$$

(c) Since $p = I^2R$,

$$R_1 = \frac{p_1}{I_1^2} = \frac{20}{2.222^2} = 4.05\ \Omega$$

$$R_2 = \frac{p_2}{I_2^2} = \frac{15}{2.777^2} = 1.945\ \Omega$$

$$R_3 = \frac{p_3}{I_3^2} = \frac{10}{2.777^2} = 1.297\ \Omega$$

Practice Problem 2.16

Refer to Fig. 2.55 and assume there are six light bulbs that can be connected in parallel and six different light bulbs that can be connected in series. In either case, each light bulb is to operate at 40 W. If the voltage at the plug is 115 V for the parallel and series connections, calculate the current through and the voltage across each bulb for both cases.

Answer: 115 V and 347.8 mA (parallel), 19.167 V and 2.087 A (series).

2.8.2 Design of DC Meters

By their nature, resistors are used to control the flow of current. We take advantage of this property in several applications, such as in a potentiometer (Fig. 2.57). The word *potentiometer*, derived from the words *potential* and *meter*, implies that potential can be metered out. The potentiometer (or pot for short) is a three-terminal device that operates on the principle of voltage division. It is essentially an adjustable voltage divider. As a voltage regulator, it is used as a volume or level control on radios, TVs, and other devices. In Fig. 2.57,

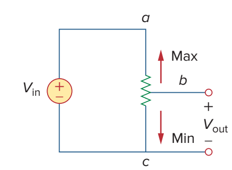


Figure 2.57
The potentiometer controlling potential levels.

$$V_{out} = V_{bc} = \frac{R_{bc}}{R_{ac}} V_{in} \tag{2.58}$$

where $R_{ac} = R_{ab} + R_{bc}$. Thus, V_{out} decreases or increases as the sliding contact of the pot moves toward c or a, respectively.

Another application where resistors are used to control current flow is in the analog dc meters—the ammeter, voltmeter, and ohmmeter, which measure current, voltage, and resistance, respectively. Each of these meters employs the d'Arsonval meter movement, shown in Fig. 2.58. The movement consists essentially of a movable iron-core coil mounted on a pivot between the poles of a permanent magnet. When current flows through the coil, it creates a torque which causes the pointer to deflect. The amount of current through the coil determines the deflection of the pointer, which is registered on a scale attached to the meter movement. For example, if the meter movement is rated 1 mA, 50 Ω, it would take 1 mA to cause a full-scale deflection of the meter movement. By introducing additional circuitry to the d'Arsonval meter movement, an ammeter, voltmeter, or ohmmeter can be constructed.

Consider Fig. 2.59, where an analog voltmeter and ammeter are connected to an element. The voltmeter measures the voltage across a *load* and

An instrument capable of measuring voltage, current, and resistance is called a *multimeter* or a *volt-ohm meter* (VOM).

A load is a component that is receiving energy (an energy sink), as opposed to a generator supplying energy (an energy source). More about loading will be discussed in Section 4.9.1.

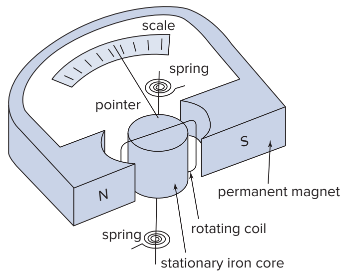


Figure 2.58
A d'Arsonval meter movement.

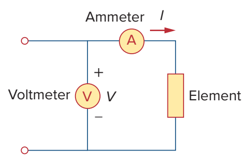


Figure 2.59
Connection of a voltmeter and an ammeter to an element.

is therefore connected in parallel with the element. As shown in Fig. 2.60(a), the voltmeter consists of a d'Arsonval movement in series with a resistor whose resistance R_m is deliberately made very large (theoretically, infinite), to minimize the current drawn from the circuit. To extend the range of voltage that the meter can measure, series multiplier resistors are often connected with the voltmeters, as shown in Fig. 2.60(b). The multiple-range voltmeter in Fig. 2.60(b) can measure voltage from 0 to 1 V, 0 to 10 V, or 0 to 100 V, depending on whether the switch is connected to R_1, R_2, or R_3, respectively.

Let us calculate the multiplier resistor R_n for the single-range voltmeter in Fig. 2.60(a), or $R_n = R_1$, R_2, or R_3 for the multiple-range voltmeter in Fig. 2.60(b). We need to determine the value of R_n to be connected in series with the internal resistance R_m of the voltmeter. In any design, we consider the worst-case condition. In this case, the worst case occurs when the full-scale current $I_{fs} = I_m$ flows through the meter. This should also correspond

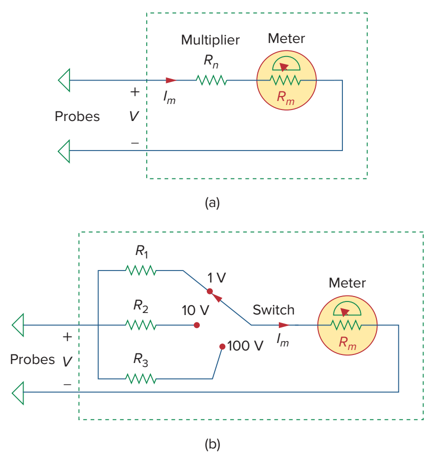


Figure 2.60
Voltmeters: (a) single-range type, (b) multiple-range type.

to the maximum voltage reading or the full-scale voltage V_{fs}. Since the multiplier resistance R_n is in series with the internal resistance R_m,

$$V_{fs} = I_{fs}(R_n + R_m) \tag{2.59}$$

From this, we obtain

$$R_n = \frac{V_{fs}}{I_{fs}} - R_m \tag{2.60}$$

Similarly, the ammeter measures the current through the load and is connected in series with it. As shown in Fig. 2.61(a), the ammeter consists of a d'Arsonval movement in parallel with a resistor whose resistance R_m is deliberately made very small (theoretically, zero) to minimize the voltage drop across it. To allow multiple ranges, shunt resistors are often connected in parallel with R_m as shown in Fig. 2.61(b). The shunt resistors allow the meter to measure in the range 0–10 mA, 0–100 mA, or 0–1 A, depending on whether the switch is connected to R_1, R_2, or R_3, respectively.

Now our objective is to obtain the multiplier shunt R_n for the single-range ammeter in Fig. 2.61(a), or $R_n = R_1$, R_2, or R_3 for the multiple-range ammeter in Fig. 2.61(b). We notice that R_m and R_n are in parallel and that at full-scale reading $I = I_{fs} = I_m + I_n$, where I_n is the current through the shunt resistor R_n. Applying the current division principle yields

$$I_m = \frac{R_n}{R_n + R_m} I_{fs}$$

or

$$R_n = \frac{I_m}{I_{fs} - I_m} R_m \tag{2.61}$$

The resistance R_x of a linear resistor can be measured in two ways. An indirect way is to measure the current I that flows through it by connecting an ammeter in series with it and the voltage V across it by connecting a voltmeter in parallel with it, as shown in Fig. 2.62(a). Then

$$R_x = \frac{V}{I} \tag{2.62}$$

The direct method of measuring resistance is to use an ohmmeter. An ohmmeter consists basically of a d'Arsonval movement, a variable resistor or potentiometer, and a battery, as shown in Fig. 2.62(b). Applying KVL to the circuit in Fig. 2.62(b) gives

$$E = (R + R_m + R_x)I_m$$

or

$$R_x = \frac{E}{I_m} - (R + R_m) \tag{2.63}$$

The resistor R is selected such that the meter gives a full-scale deflection; that is, $I_m = I_{fs}$ when $R_x = 0$. This implies that

$$E = (R + R_m)I_{fs} \tag{2.64}$$

Substituting Eq. (2.64) into Eq. (2.63) leads to

$$R_x = \left(\frac{I_{fs}}{I_m} - 1\right)(R + R_m) \tag{2.65}$$

As mentioned, the types of meters we have discussed are known as *analog* meters and are based on the d'Arsonval meter movement. Another type of meter, called a *digital meter*, is based on active circuit elements

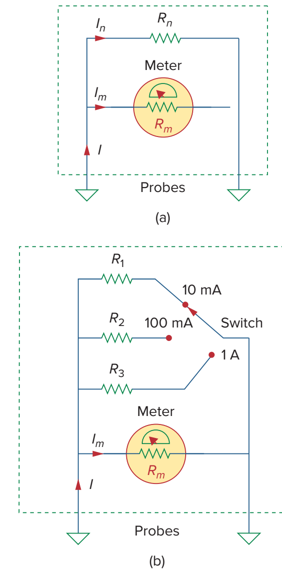


Figure 2.61
Ammeters: (a) single-range type, (b) multiple-range type.

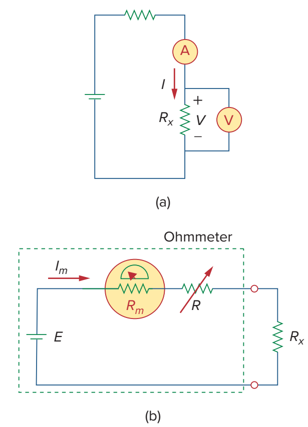
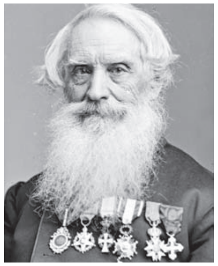


Figure 2.62
Two ways of measuring resistance: (a) using an ammeter and a voltmeter, (b) using an ohmmeter.

Historical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Samuel F. B. Morse (1791–1872), an American painter, invented the telegraph, the first practical, commercialized application of electricity.

Morse was born in Charlestown, Massachusetts, and studied at Yale and the Royal Academy of Arts in London to become an artist. In the 1830s, he became intrigued with developing a telegraph. He had a working model by 1836 and applied for a patent in 1838. The U.S. Senate appropriated funds for Morse to construct a telegraph line between Baltimore and Washington, D.C. On May 24, 1844, he sent the famous first message: "What hath God wrought!" Morse also developed a code of dots and dashes for letters and numbers, for sending messages on the telegraph. The development of the telegraph led to the invention of the telephone.

such as op amps. For example, a digital multimeter displays measurements of dc or ac voltage, current, and resistance as discrete numbers, instead of using a pointer deflection on a continuous scale as in an analog multimeter. Digital meters are what you would most likely use in a modern lab. However, the design of digital meters is beyond the scope of this book.

Example 2.17

Following the voltmeter setup of Fig. 2.60, design a voltmeter for the following multiple ranges:
(a) 0–1 V (b) 0–5 V (c) 0–50 V (d) 0–100 V
Assume that the internal resistance $R_m = 2\ \text{k}\Omega$ and the full-scale current $I_{fs} = 100\ \mu\text{A}$.

Solution:
We apply Eq. (2.60) and assume that R_1, R_2, R_3, and R_4 correspond with ranges 0–1 V, 0–5 V, 0–50 V, and 0–100 V, respectively.
(a) For range 0–1 V,

$$R_1 = \frac{1}{100 \times 10^{-6}} - 2000 = 10{,}000 - 2000 = 8\ \text{k}\Omega$$

(b) For range 0–5 V,

$$R_2 = \frac{5}{100 \times 10^{-6}} - 2000 = 50{,}000 - 2000 = 48\ \text{k}\Omega$$

(c) For range 0–50 V,

$$R_3 = \frac{50}{100 \times 10^{-6}} - 2000 = 500{,}000 - 2000 = 498\ \text{k}\Omega$$

(d) For range 0–100 V,

$$R_4 = \frac{100\ \text{V}}{100 \times 10^{-6}} - 2000 = 1{,}000{,}000 - 2000 = 998\ \text{k}\Omega$$

Note that the ratio of the total resistance ($R_n + R_m$) to the full-scale voltage V_{fs} is constant and equal to $1/I_{fs}$ for the four ranges. This ratio (given in ohms per volt, or Ω/V) is known as the *sensitivity* of the voltmeter. The larger the sensitivity, the better the voltmeter.

Practice Problem 2.17

Following the ammeter setup of Fig. 2.61, design an ammeter for the following multiple ranges:
(a) 0–1 A (b) 0–100 mA (c) 0–10 mA
Take the full-scale meter current as $I_m = 1$ mA and the internal resistance of the ammeter as $R_m = 50\ \Omega$.

Answer: Shunt resistors: 50 mΩ, 505 mΩ, 5.556 Ω.

2.9 Summary

1. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law,
$$v = iR$$
where R is the resistance of the resistor.
2. A short circuit is a resistor (a perfectly, conducting wire) with zero resistance ($R = 0$). An open circuit is a resistor with infinite resistance ($R = \infty$).
3. The conductance G of a resistor is the reciprocal of its resistance:
$$G = \frac{1}{R}$$
4. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit. The number of branches b, the number of nodes n, and the number of independent loops l in a network are related as
$$b = l + n - 1$$
5. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
6. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
7. Two elements are in series when they are connected sequentially, end to end. When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).
8. When two resistors R_1 ($= 1/G_1$) and R_2 ($= 1/G_2$) are in series, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are
$$R_{eq} = R_1 + R_2, \qquad G_{eq} = \frac{G_1 G_2}{G_1 + G_2}$$
9. When two resistors R_1 ($= 1/G_1$) and R_2 ($= 1/G_2$) are in parallel, their equivalent resistance R_{eq} and equivalent conductance G_{eq} are
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \qquad G_{eq} = G_1 + G_2$$

10. The voltage division principle for two resistors in series is

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

11. The current division principle for two resistors in parallel is

$$i_1 = \frac{R_2}{R_1 + R_2} i, \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

12. The formulas for a delta-to-wye transformation are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}, \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

13. The formulas for a wye-to-delta transformation are

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}, \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

14. The basic laws covered in this chapter can be applied to the problems of electrical lighting and design of dc meters.

Review Questions

2.1 The reciprocal of resistance is:

(a) voltage (b) current
(c) conductance (d) coulombs

2.2 An electric heater draws 10 A from a 120-V line. The resistance of the heater is:

(a) 1200 Ω (b) 120 Ω
(c) 12 Ω (d) 1.2 Ω

2.3 The voltage drop across a 1.5-kW toaster that draws 12 A of current is:

(a) 18 kV (b) 125 V
(c) 120 V (d) 10.42 V

2.4 The maximum current that a 2W, 80 kΩ resistor can safely conduct is:

(a) 160 kA (b) 40 kA
(c) 5 mA (d) 25 μA

2.5 A network has 12 branches and 8 independent loops. How many nodes are there in the network?

(a) 19 (b) 17 (c) 5 (d) 4

2.6 The current I in the circuit of Fig. 2.63 is:

(a) −0.8 A (b) −0.2 A
(c) 0.2 A (d) 0.8 A

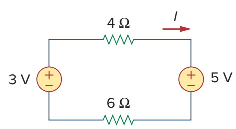


Figure 2.63
For Review Question 2.6.

2.7 The current I_o of Fig. 2.64 is:

(a) −4 A (b) −2 A (c) 4 A (d) 16 A

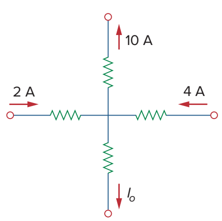


Figure 2.64
For Review Question 2.7.

2.8 In the circuit in Fig. 2.65, V is:

(a) 30 V (b) 14 V (c) 10 V (d) 6 V

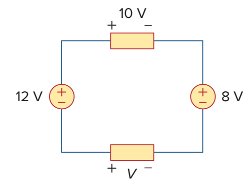


Figure 2.65
For Review Question 2.8.

2.9 Which of the circuits in Fig. 2.66 will give you $V_{ab} = 7$ V?

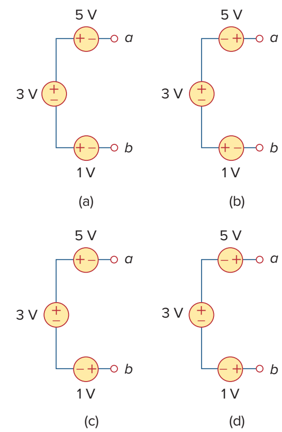


Figure 2.66
For Review Question 2.9.

2.10 In the circuit of Fig. 2.67, a decrease in R_3 leads to a decrease of, select all that apply:

(a) current through R_3

(b) voltage across R_3

(c) voltage across R_1

(d) power dissipated in R_2

(e) none of the above

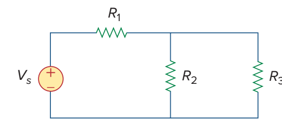


Figure 2.67
For Review Question 2.10.

Answers: 2.1c, 2.2c, 2.3b, 2.4c, 2.5c, 2.6b, 2.7a, 2.8d, 2.9d, 2.10b, d.

Problems

Section 2.2 Ohm's Law

2.1 Design a problem, complete with a solution, to help students to better understand Ohm's law. Use at least two resistors and one voltage source. Hint, you could use both resistors at once or one at a time, it is up to you. Be creative.

e2d

2.2 Find the hot resistance of a light bulb rated 60 W, 120 V.

2.3 A bar of silicon is 4 cm long with a circular cross section. If the resistance of the bar is 240 Ω at room temperature, what is the cross-sectional radius of the bar?

2.4 (a) Calculate current i in Fig. 2.68 when the switch is in position 1.

(b) Find the current when the switch is in position 2.

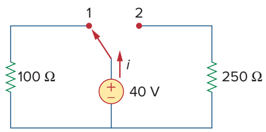


Figure 2.68
For Prob. 2.4.

Section 2.3 Nodes, Branches, and Loops

2.5 For the network graph in Fig. 2.69, find the number of nodes, branches, and loops.

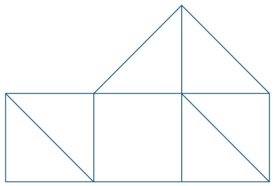


Figure 2.69
For Prob. 2.5.

2.6 In the network graph shown in Fig. 2.70, determine the number of branches and nodes.

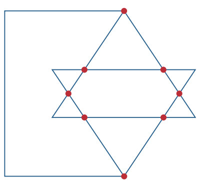


Figure 2.70
For Prob. 2.6.

2.7 Determine the number of branches and nodes in the circuit of Fig. 2.71.

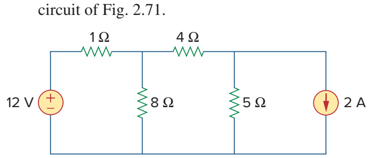


Figure 2.71
For Prob. 2.7.

Section 2.4 Kirchhoff's Laws

2.8 **e⊘d** Design a problem, complete with a solution, to help other students better understand Kirchhoff's Current Law. Design the problem by specifying values of i_a, i_b, and i_c, shown in Fig. 2.72, and asking them to solve for values of i_1, i_2, and i_3. Be careful to specify realistic currents.

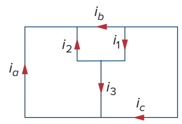


Figure 2.72
For Prob. 2.8.

2.9 Find i_1, i_2, and i_3 in Fig. 2.73.

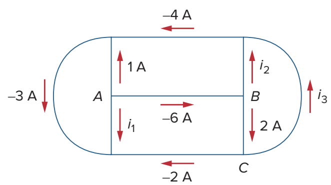


Figure 2.73
For Prob. 2.9.

2.10 Determine i_1 and i_2 in the circuit of Fig. 2.74.

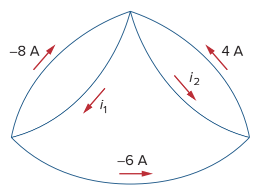


Figure 2.74
For Prob. 2.10.

2.11 In the circuit of Fig. 2.75, calculate V_1 and V_2.

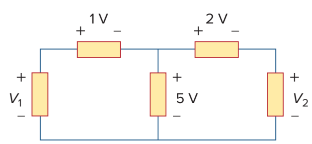


Figure 2.75
For Prob. 2.11.

2.12 In the circuit in Fig. 2.76, obtain v_1, v_2, and v_3.

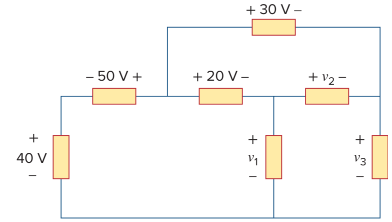


Figure 2.76
For Prob. 2.12.

2.13 For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4.

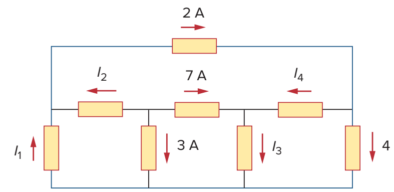


Figure 2.77
For Prob. 2.13.

2.14 Given the circuit in Fig. 2.78, use KVL to find the branch voltages V_1 to V_4.

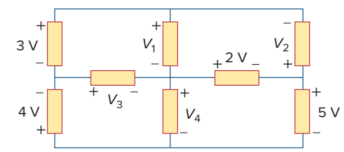


Figure 2.78
For Prob. 2.14.

2.15 Calculate v and i_x in the circuit of Fig. 2.79.

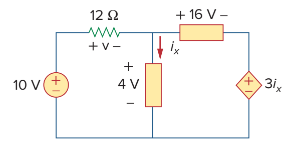


Figure 2.79
For Prob. 2.15.

2.16 Determine V_o in the circuit in Fig. 2.80.

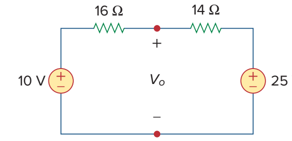


Figure 2.80
For Prob. 2.16.

2.17 Obtain v_1 through v_3 in the circuit of Fig. 2.81.

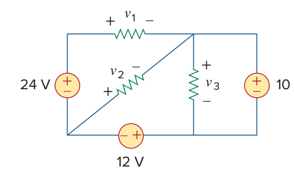


Figure 2.81
For Prob. 2.17.

2.18 Find I and V in the circuit of Fig. 2.82.

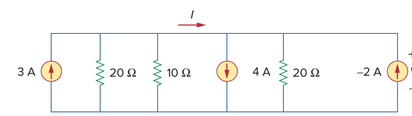


Figure 2.82
For Prob. 2.18.

2.19 From the circuit in Fig. 2.83, find I, the power dissipated by the resistor, and the power supplied by each source.

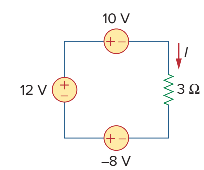


Figure 2.83
For Prob. 2.19.

2.20 Determine i_o in the circuit of Fig. 2.84.

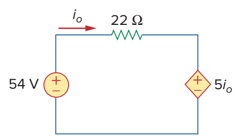


Figure 2.84
For Prob. 2.20.

2.21 Find V_x in the circuit of Fig. 2.85.

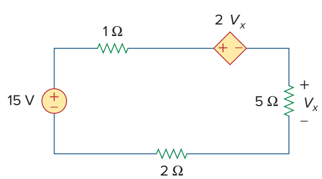


Figure 2.85
For Prob. 2.21.

2.22 Find V_o in the circuit in Fig. 2.86 and the power absorbed by the dependent source.

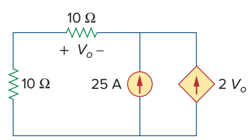


Figure 2.86
For Prob. 2.22.

2.23 In the circuit shown in Fig. 2.87, determine V_x and the power absorbed by the 60-Ω resistor.

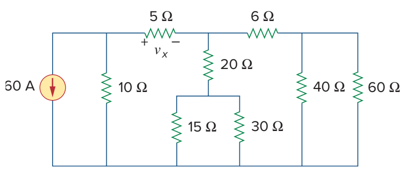


Figure 2.87
For Prob. 2.23.

2.24 For the circuit in Fig. 2.88, find V_o/V_s in terms of α, R_1, R_2, R_3, and R_4. If $R_1 = R_2 = R_3 = R_4$, what value of α will produce $|V_o/V_s| = 10$?

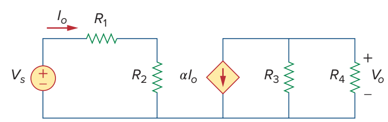


Figure 2.88
For Prob. 2.24.

2.25 For the network in Fig. 2.89, find the current, voltage, and power associated with the 20-kΩ resistor.

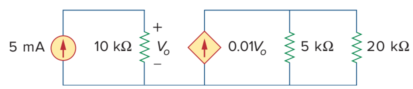


Figure 2.89
For Prob. 2.25.

Sections 2.5 and 2.6 Series and Parallel Resistors

2.26 For the circuit in Fig. 2.90, $i_o = 3$ A. Calculate i_x and the total power absorbed by the entire circuit.

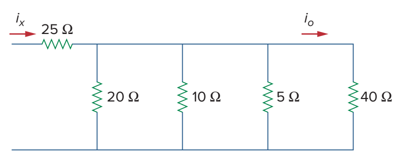


Figure 2.90
For Prob. 2.26.

2.27 Calculate I_o in the circuit of Fig. 2.91.

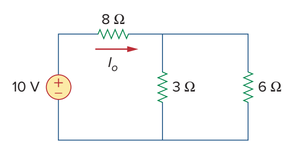


Figure 2.91
For Prob. 2.27.

2.28 Design a problem, using Fig. 2.92, to help other students better understand series and parallel circuits.

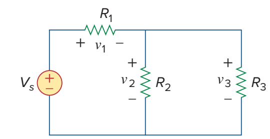


Figure 2.92
For Prob. 2.28.

2.29 All resistors (R) in Fig. 2.93 are 10 Ω each. Find R_{eq}.

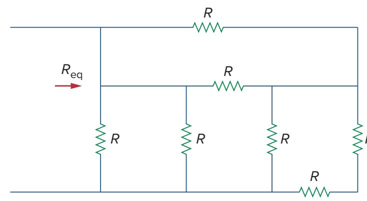


Figure 2.93
For Prob. 2.29.

2.30 Find R_{eq} for the circuit in Fig. 2.94.

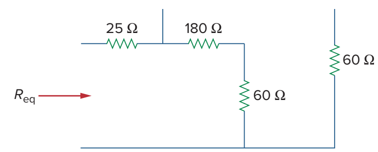


Figure 2.94
For Prob. 2.30.

2.31 For the circuit in Fig. 2.95, determine i_1 to i_5.

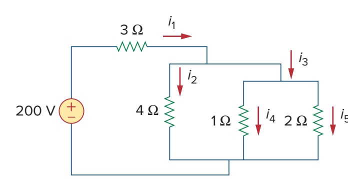


Figure 2.95
For Prob. 2.31.

2.32 Find i_1 through i_4 in the circuit in Fig. 2.96.

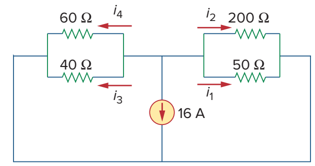


Figure 2.96
For Prob. 2.32.

2.33 Obtain v and i in the circuit of Fig. 2.97.

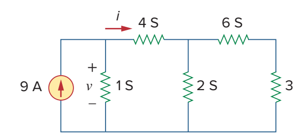


Figure 2.97
For Prob. 2.33.

2.34 Using series/parallel resistance combination, find the equivalent resistance seen by the source in the circuit of Fig. 2.98. Find the overall absorbed power by the resistor network.

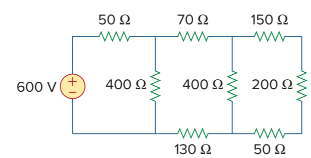


Figure 2.98
For Prob. 2.34.

2.35 Calculate V_o and I_o in the circuit of Fig. 2.99.

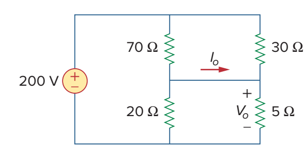


Figure 2.99
For Prob. 2.35.

2.36 Find i and V_o in the circuit of Fig. 2.100.

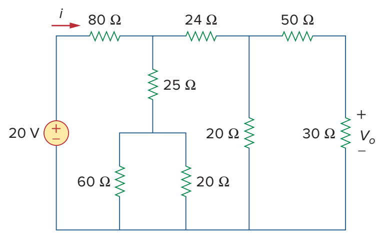


Figure 2.100
For Prob. 2.36.

2.37 Given the circuit in Fig. 2.101 and that the resistance, R_{eq}, looking into the circuit from the left is equal to 100 Ω, determine the value of R_1.

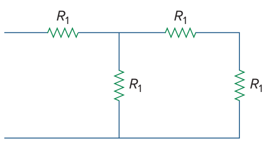


Figure 2.101
For Prob. 2.37.

2.38 Find R_{eq} and i_o in the circuit of Fig. 2.102.

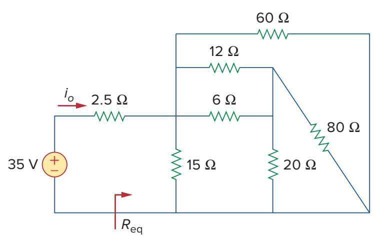


Figure 2.102
For Prob. 2.38.

2.39 Evaluate R_{eq} looking into each set of terminals for each of the circuits shown in Fig. 2.103.

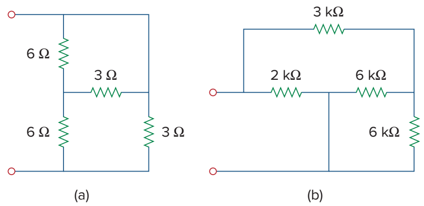


Figure 2.103
For Prob. 2.39.

2.40 For the ladder network in Fig. 2.104, find I and R_{eq}.

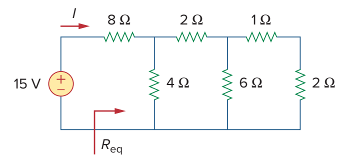


Figure 2.104
For Prob. 2.40.

2.41 If $R_{eq} = 50$ Ω in the circuit of Fig. 2.105, find R.

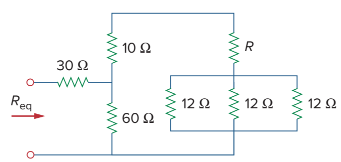
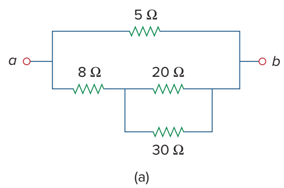


Figure 2.105
For Prob. 2.41.

2.42 Reduce each of the circuits in Fig. 2.106 to a single resistor at terminals a-b.



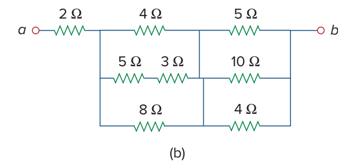


Figure 2.106
For Prob. 2.42.

2.43 Calculate the equivalent resistance R_{ab} at terminals a-b for each of the circuits in Fig. 2.107.

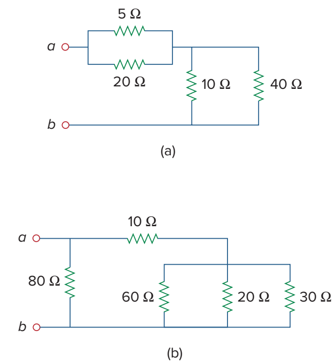


Figure 2.107
For Prob. 2.43.

2.44 For the circuits in Fig. 2.108, obtain the equivalent resistance at terminals a-b.

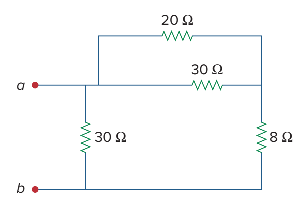


Figure 2.108
For Prob. 2.44.

2.45 Find the equivalent resistance at terminals a-b of each circuit in Fig. 2.109.

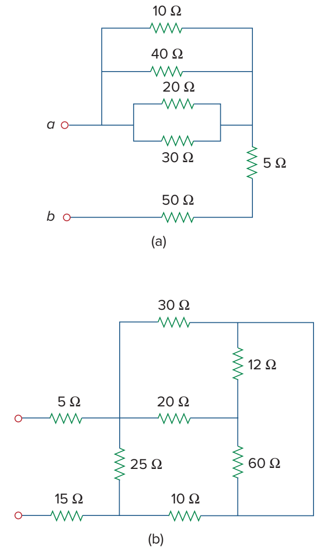


Figure 2.109
For Prob. 2.45.

2.46 Find I in the circuit of Fig. 2.110.

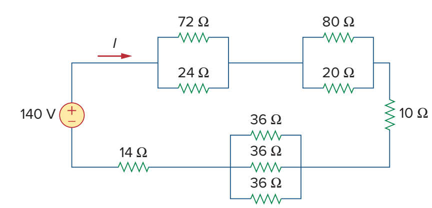


Figure 2.110
For Prob. 2.46.

2.47 Find the equivalent resistance R_{ab} in the circuit of Fig. 2.111.

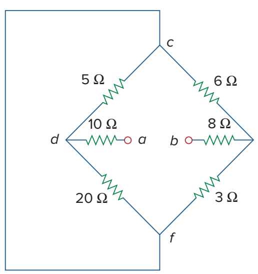


Figure 2.111
For Prob. 2.47.

Section 2.7 Wye-Delta Transformations

2.48 Convert the circuits in Fig. 2.112 from Y to Δ.

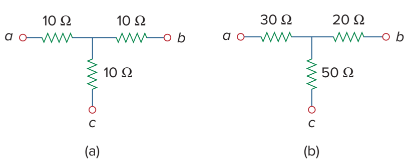


Figure 2.112
For Prob. 2.48.

2.49 Transform the circuits in Fig. 2.113 from Δ to Y.

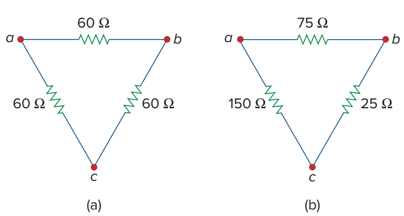


Figure 2.113
For Prob. 2.49.

* An asterisk indicates a challenging problem.

2.50 Design a problem to help other students better understand wye-delta transformations using Fig. 2.114.

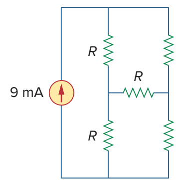


Figure 2.114
For Prob. 2.50.

2.51 Obtain the equivalent resistance at the terminals *a*-*b* for each of the circuits in Fig. 2.115.

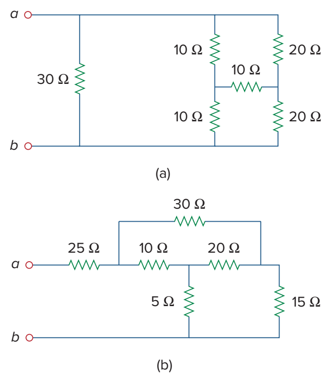


Figure 2.115
For Prob. 2.51.

***2.52** For the circuit shown in Fig. 2.116, find the equivalent resistance. All resistors are 3Ω.

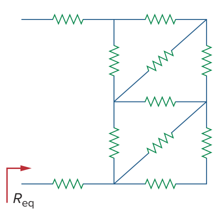


Figure 2.116
For Prob. 2.52.

***2.53** Obtain the equivalent resistance R_{ab} in each of the circuits of Fig. 2.117. In (b), all resistors have a value of 30 Ω.

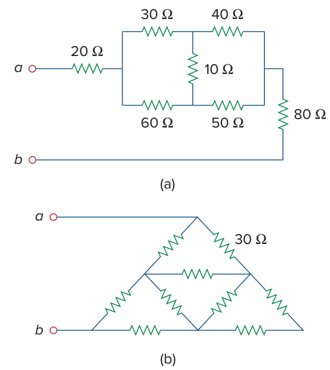


Figure 2.117
For Prob. 2.53.

2.54 Consider the circuit in Fig. 2.118. Find the equivalent resistance at terminals: (a) *a*-*b*, (b) *c*-*d*.

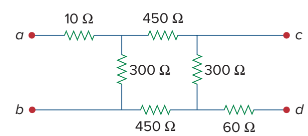


Figure 2.118
For Prob. 2.54.

2.55 Calculate I_o in the circuit of Fig. 2.119.

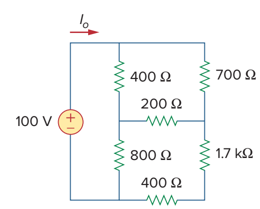


Figure 2.119
For Prob. 2.55.

2.56 Determine V in the circuit of Fig. 2.120.

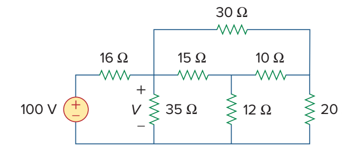


Figure 2.120
For Prob. 2.56.

***2.57** Find R_{eq} and I in the circuit of Fig. 2.121.

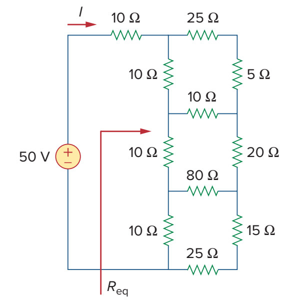


Figure 2.121
For Prob. 2.57.

Section 2.8 Applications

2.58 The 150 W light bulb in Fig. 2.122 is rated at 110 volts. Calculate the value of V_s to make the light bulb operate at its rated conditions.

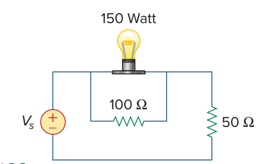


Figure 2.122
For Prob. 2.58.

2.59 An enterprising young man travels to Europe carrying three light bulbs he had purchased in North America. The light bulbs he has are a 100-W light bulb, a 60-W light bulb, and a 40-W light bulb. Each light bulb is rated at 110 V. He wishes to connect these to a 220-V system that is found in Europe. For reasons we are not sure of, he connects the 40-W light bulb in series with a parallel combination of the 60-W light bulb and the 100-W light bulb as shown in Fig. 2.123. How much power is actually being delivered to each light bulb? What does he see when he first turns on the light bulbs?

Is there a better way to connect these light bulbs in order to have them work more effectively?

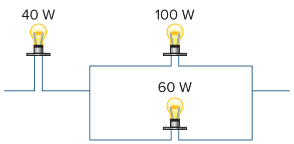


Figure 2.123
For Prob. 2.59.

2.60 If the three bulbs of Prob. 2.59 are connected in parallel to the 120-V source, calculate the current through each bulb.

2.61 As a design engineer, you are asked to design a lighting system consisting of a 70-W power supply and two light bulbs as shown in Fig. 2.124. You must select the two bulbs from the following three available bulbs.

$R_1 = 80\ \Omega$, cost = \$0.60 (standard size)
$R_2 = 90\ \Omega$, cost = \$0.90 (standard size)
$R_3 = 100\ \Omega$, cost = \$0.75 (nonstandard size)

The system should be designed for minimum cost such that I lies within the range $I = 1.2\text{ A} \pm 5$ percent.

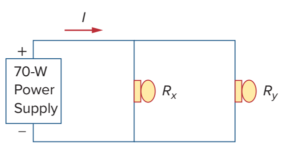


Figure 2.124
For Prob. 2.61.

2.62 A three-wire system supplies two loads A and B as shown in Fig. 2.125. Load A consists of a motor drawing a current of 8 A, while load B is a PC drawing 2 A. Assuming 10 h/day of use for 365 days and 6 cents/kWh, calculate the annual energy cost of the system.

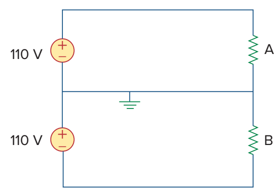


Figure 2.125

2.63 If an ammeter with an internal resistance of 100 Ω and a current capacity of 2 mA is to measure 5 A, determine the value of the resistance needed. Calculate the power dissipated in the shunt resistor.

2.64 The potentiometer (adjustable resistor) R_x in Fig. 2.126 is to be designed to adjust current i_x from 10 mA to 1 A. Calculate the values of R and R_x to achieve this.

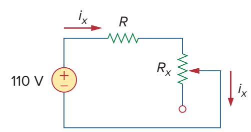


Figure 2.126
For Prob. 2.64.

2.65 Design a circuit that uses a d'Arsonval meter (with an internal resistance of 2 kΩ that requires a current of 5 mA to cause the meter to deflect full scale) to build a voltmeter to read values of voltages up to 100 volts.

2.66 A 20-kΩ/V voltmeter reads 10 V full scale.

(a) What series resistance is required to make the meter read 50 V full scale?

(b) What power will the series resistor dissipate when the meter reads full scale?

2.67 (a) Obtain the voltage V_o in the circuit of Fig. 2.127(a).

(b) Determine the voltage V_o' measured when a voltmeter with 6-kΩ internal resistance is connected as shown in Fig. 2.127(b).

(c) The finite resistance of the meter introduces an error into the measurement. Calculate the percent error as

$$\left|\frac{V_o - V_o'}{V_o}\right| \times 100\ \%$$

(d) Find the percent error if the internal resistance were 36 kΩ.

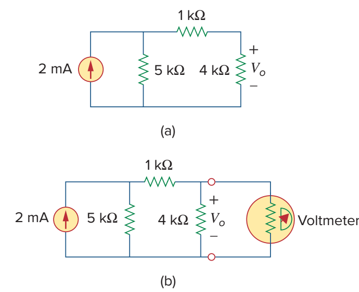


Figure 2.127
For Prob. 2.67.

2.68 (a) Find the current I in the circuit of Fig. 2.128(a).
(b) An ammeter with an internal resistance of 1 Ω is inserted in the network to measure I' as shown in Fig. 2.128(b). What is I'?
(c) Calculate the percent error introduced by the meter as

$$\left|\frac{I - I'}{I}\right| \times 100\%$$

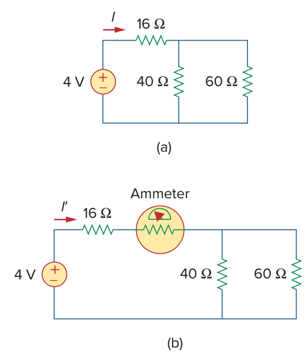


Figure 2.128
For Prob. 2.68.

2.69 A voltmeter is used to measure V_o in the circuit in Fig. 2.129. The voltmeter model consists of an ideal voltmeter in parallel with a 250-kΩ resistor. Let $V_s = 95$ V, $R_s = 25$ kΩ, and $R_1 = 40$ kΩ. Calculate V_o with and without the voltmeter when

(a) $R_2 = 5$ kΩ (b) $R_2 = 25$ kΩ
(c) $R_2 = 250$ kΩ

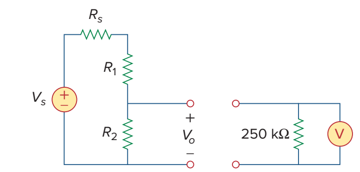


Figure 2.129
For Prob. 2.69.

2.70 (a) Consider the Wheatstone bridge shown in Fig. 2.130. Calculate v_a, v_b, and v_{ab}.

(b) Rework part (a) if the ground is placed at a instead of o.

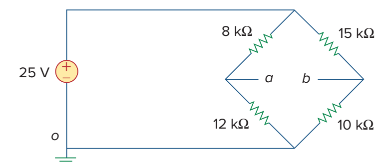


Figure 2.130
For Prob. 2.70.

2.71 Figure 2.131 represents a model of a solar photovoltaic panel. Given that $V_s = 95$ V, $R_1 = 25\ \Omega$, and $i_L = 2$ A, find R_L.

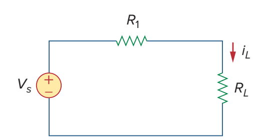


Figure 2.131
For Prob. 2.71.

2.72 Find V_o in the two-way power divider circuit in Fig. 2.132.

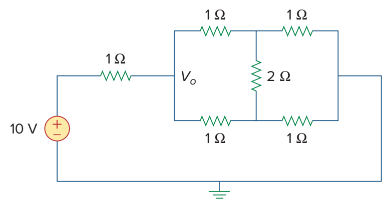


Figure 2.132
For Prob. 2.72.

2.73 An ammeter model consists of an ideal ammeter in series with a 20-Ω resistor. It is connected with a current source and an unknown resistor R_x as shown in Fig. 2.133. The ammeter reading is noted. When a potentiometer R is added and adjusted until the ammeter reading drops to one half its previous reading, then $R = 65\ \Omega$. What is the value of R_x?

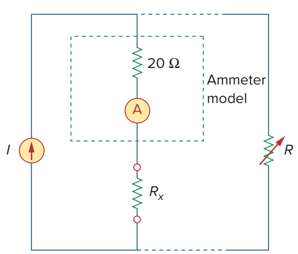


Figure 2.133
For Prob. 2.73.

2.74 The circuit in Fig. 2.134 is to control the speed of a motor such that the motor draws currents 5 A, 3 A, and 1 A when the switch is at high, medium, and low positions, respectively. The motor can be modeled as a load resistance of 20 mΩ. Determine the series dropping resistances R_1, R_2, and R_3.

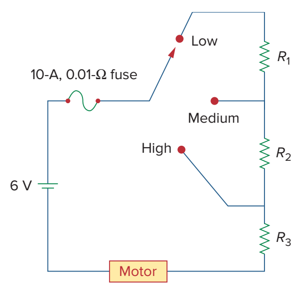


Figure 2.134
For Prob. 2.74.

2.75 Find R_{ab} in the four-way power divider circuit in Fig. 2.135. Assume each $R = 4\ \Omega$.

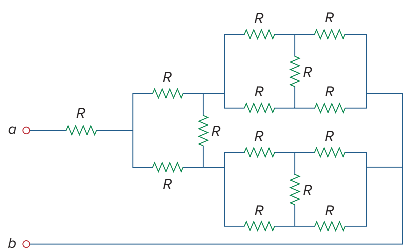


Figure 2.135
For Prob. 2.75.

Comprehensive Problems

2.76 Repeat Prob. 2.75 for the eight-way divider shown in Fig. 2.136.

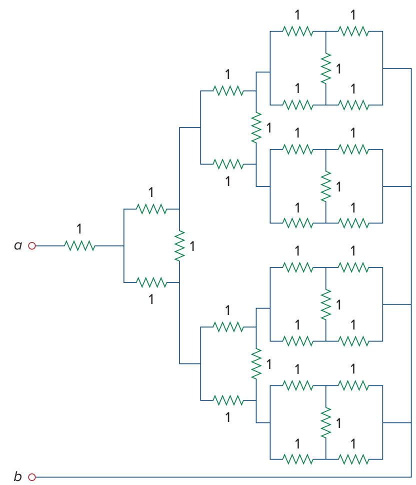


Figure 2.136
For Prob. 2.76.

2.77 Suppose your circuit laboratory has the following standard commercially available resistors in large quantities:

1.8 Ω 20 Ω 300 Ω 24 kΩ 56 kΩ

Using series and parallel combinations and a minimum number of available resistors, how would you obtain the following resistances for an electronic circuit design?

(a) 5 Ω (b) 311.8 Ω
(c) 40 kΩ (d) 52.32 kΩ

2.78 In the circuit in Fig. 2.137, the wiper divides the potentiometer resistance between αR and $(1 - \alpha)R$, $0 \le \alpha \le 1$. Find v_o / v_s.

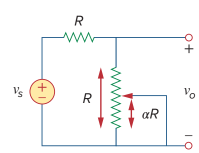


Figure 2.137
For Prob. 2.78.

2.79 An electric pencil sharpener rated 240 mW, 6 V is connected to a 9-V battery as shown in Fig. 2.138. Calculate the value of the series-dropping resistor R_x needed to power the sharpener.

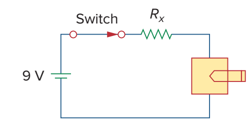


Figure 2.138
For Prob. 2.79.

2.80 A loudspeaker is connected to an amplifier as shown in Fig. 2.139. If a 10-Ω loudspeaker draws the maximum power of 12 W from the amplifier, determine the maximum power a 4-Ω loudspeaker will draw.

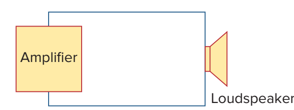


Figure 2.139
For Prob. 2.80.

2.81 For a specific application, the circuit shown in Fig. 2.140 was designed so that $I_L = 83.33$ mA and that $R_{in} = 5$ kΩ. What are the values of R_1 and R_2?

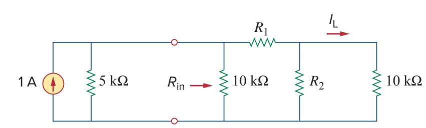


Figure 2.140
For Prob. 2.81.