Comparison of Multi-Channel EEG Based Graph Learning Methods

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Hello!

I am Arshak Rezvani

I am here because I love to give presentations.

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What is Multi-Channel EEG Signals? [1]

- Electrode placement systems.
  - The 10-20 System of Electrode Placement
- Electroencephalogram Time Series and Channels.
- Like Noise!

Pre-Processing [1]

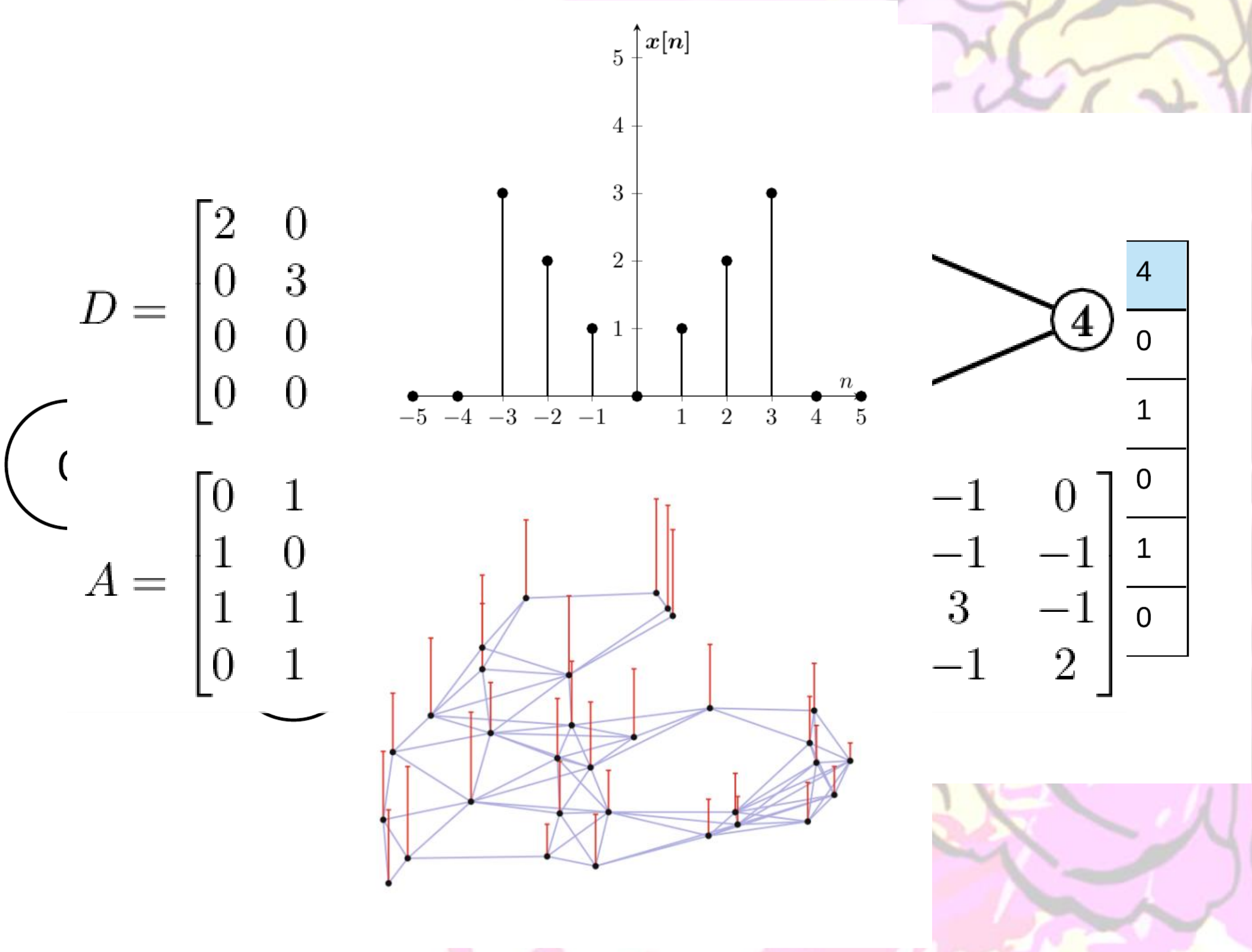
- Artifact Rejection
  - Physiologic
    - EMG
    - ECG
    - EOG
    - Blinking
    - Sweat
  - Non-Physiologic
    - AC electrical and electromagnetic interferences
    - Electrode pop and Movement

EEG as a Random Process [1]

- Ergodicity
  - Mean Ergodic
    - $E\{X(t)\} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)dt$
  - Correlation Ergodic
    - $E\{X(t)X^*(t-\tau)\} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x^*(t-\tau)dt$
- Averaging between trials
  - Cause indeterministic

But What is a Graph Signal? [2]

- A Graph!
  - Adjacency Matrix
  - Laplacian Matrix
- Graph Signal
  - Definition and Example
    - It's a Domain
      - Time
      - Frequency
      - Vertices
    - **No Order!**
      - A Vector

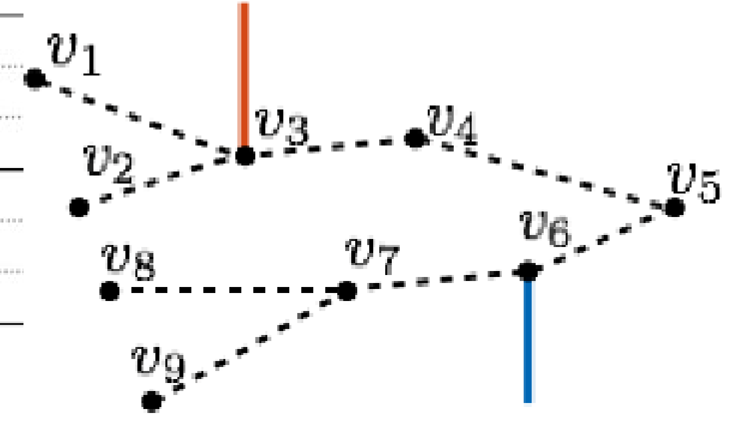


Graph Operations [2]

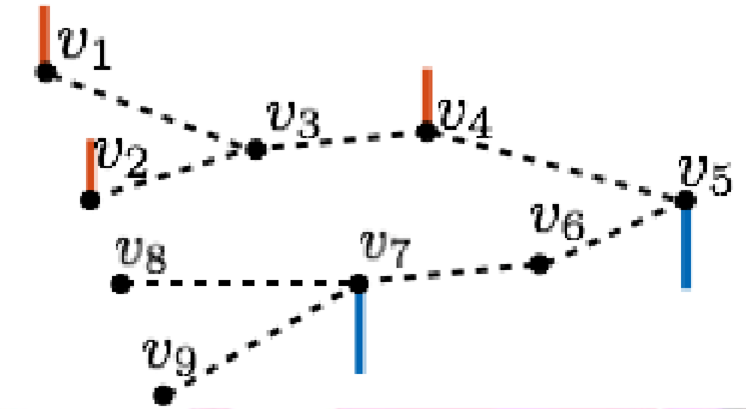
- Graph Shift
  - Multiplication by Laplacian Matrix
    - Shift with weights like time series
- Graph Fourier Transform(GDFT)
  - Linear Shift Invariant Systems(LSI)
  - Eigen Function
    - $e^{j\omega t}$ for time series
    - Eigen Vectors of Laplacian Matrix
      - $Lx = \lambda x \rightarrow x \text{ is a eigen vector for } L$
- Graph Filtering and Convolution

$$y = h(L)x = \sum_i u_i h(\lambda_i) u_i^\top x = \sum_i u_i h(\lambda_i) \hat{x}_i$$

- Sampling Over Graphs
- Graph Wavelet Transform



Initial Signal



Shifted Signal

Graph Learning [3]

- Estimating Functional Connectivity
  - Connectivity Measures
    - Regression Methods
      - Pearson Correlation

$$R^2 = \max_{\tau} \frac{cov^2(x(t), y(t+\tau))}{var(x(t)).var(y(t+\tau))}$$

      - The Magnitude-Squared Coherence Function

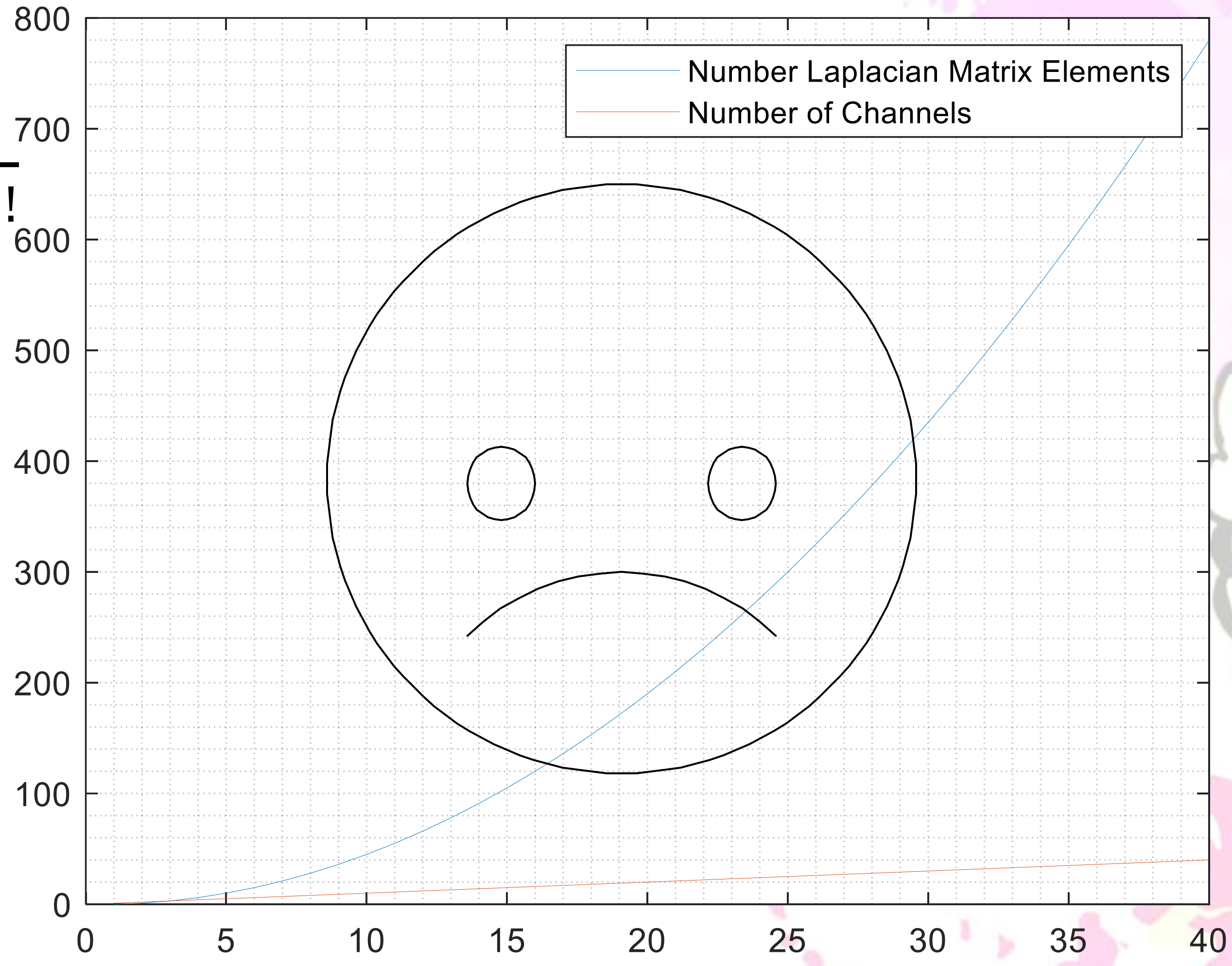
$$\left|\rho_{xy}(f)\right|^2 = \frac{\left|S_{xy}(f)\right|^2}{S_{xx}(f).S_{yy}(f)}$$

Graph Learning [3]

- Estimating Functional Connectivity
  - Connectivity Measures
    - Phase Synchronization Methods
      - Using Hilbert Transform:
        $Z_x(t) = x(t) + iH[x(t)] = A_x^H(t).\, e^{i\phi_x^H(t)}$
      - Using Wavelet Transform:
        $W_x(t) = (\psi * x)(t) = \int \psi(t')x(t-t')dt' = A_x^W(t).\, e^{i\phi_x^W(t)}$
      - Getting Mod.
        $\phi = \left(\phi_x - \phi_y\right) mod\ 2\pi$
      - Shannon Entropy
        $\rho = -\sum p(\phi) log_2 p(\phi)$
      - Mean Phase Coherence
        $\mathrm{R} = |E[e^{i\phi}]|$

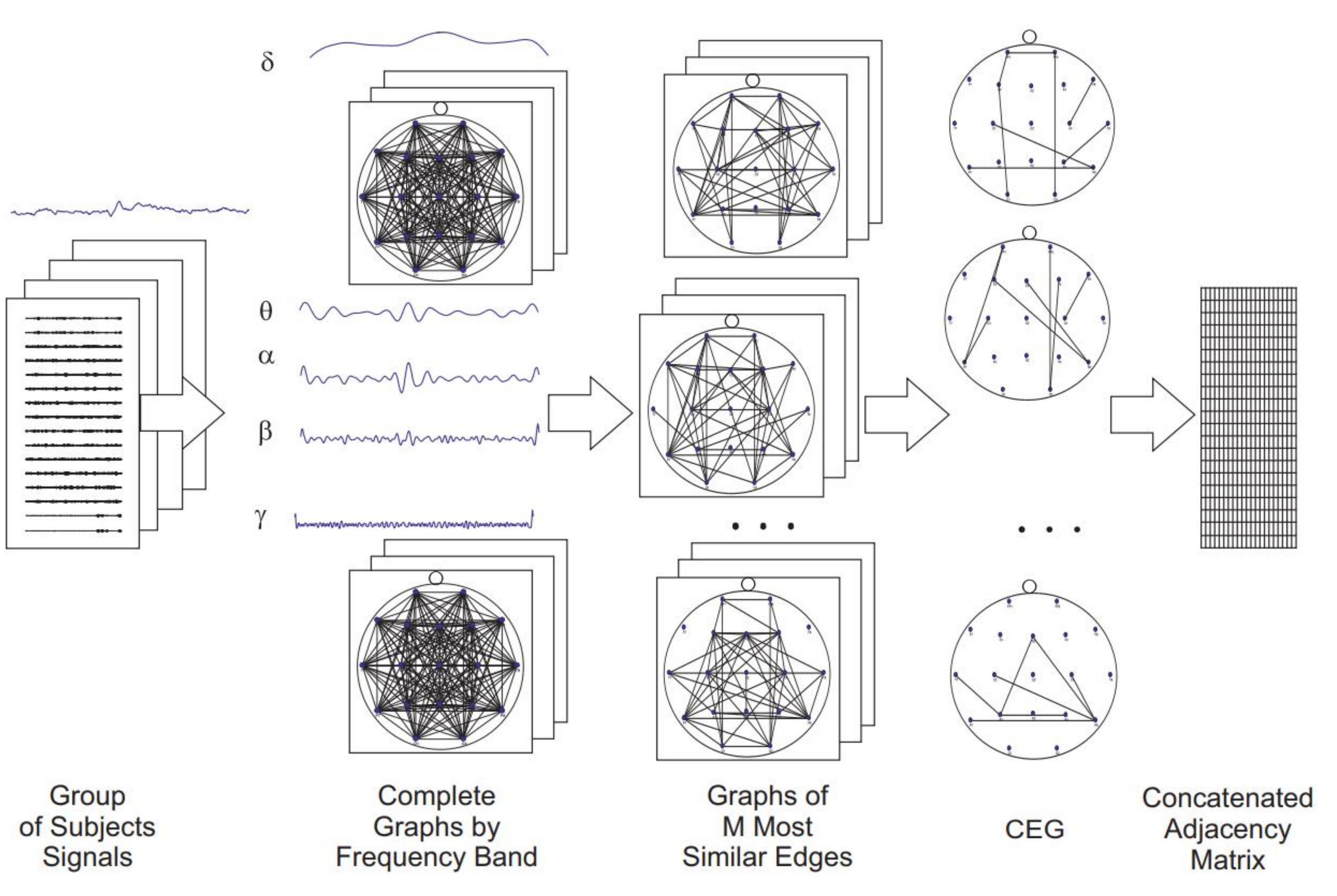
Very Complex Laplacian Matrix

- $\binom{n}{2} = \frac{n!}{2!(n-2)!}$



How To Make It Less Complex [4]

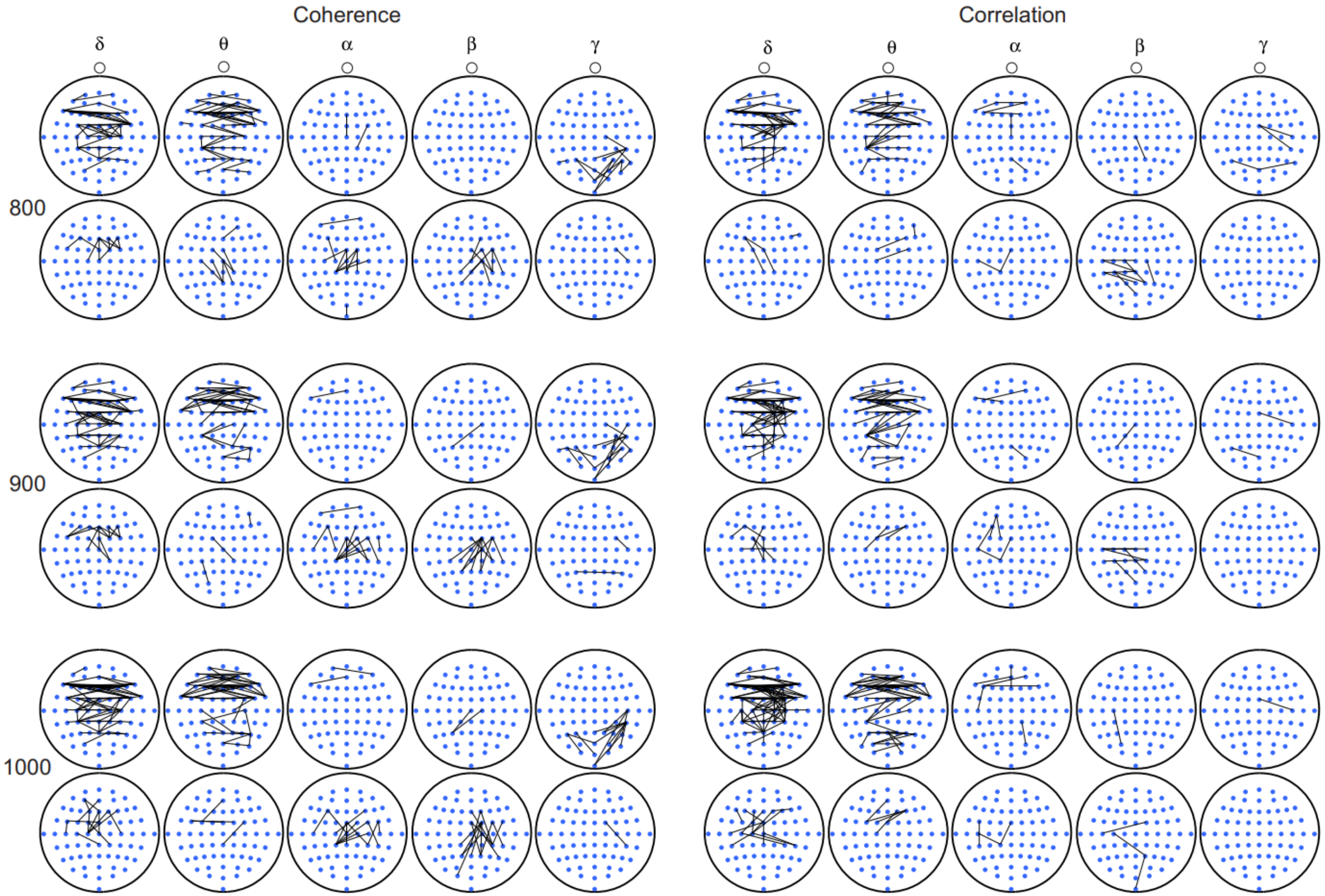
- Fail to Represent Discrim
  - Graphs vary significantly
- Common Edge Graph



CEG(Common Edge Graph) [4]

- Frequency Bands
  - $\delta, \theta, \alpha, \beta, \gamma$
- Compute Correlation and Coherence
- Define a graph with the M most similar edges
  - Sorts The Edges of The Complete Graph
    - $V_{K_N} = \{e_1, e_2, \ldots, e_m\}$
    - Where $\mathrm{m} = \binom{n}{2}$
    - $ch_j[e_1](\omega) \leq \cdots \leq ch_j[e_m](\omega)$
    - Nodes: $V\left(G_j^M(\omega)\right) = V(K_n)$
    - Edges: $E\left(G_j^M(\omega)\right) = \left\{e \in K_n : ch_j[\omega](e) \geq ch_j[\omega](e_M)\right\}$
    - The CEG is defined with same nodes and

      Edges: $E\left(G_S^M(\omega)\right) = \bigcap_j^{|S|} E\left(G_j^M(\omega)\right)$



. CEG for foot (odd rows) vs hand (even rows) motion for five frequency bands from 26 subjects when 800, 900 and 1000 highest coherence electrode pairs are used to build the CEG using Coherence and Correlation similarity metrics.

Learning Graphs From A Smooth Signal

- Not Using the smoothness and time series properties
- Learning the Laplacian as a whole

Smoothness On a Graph Definition [5]

- How to quantify how smooth a set of vectors are on a given weighted undirected graph?
- Vectors: $x_1, \dots, x_m \in \mathbb{R}^n$ so we have $\mathrm{X} \in \mathbb{R}^{m \times n} = [x_1, \dots, x_m]^T$
- Smoothness on the Graph:

$$\frac{1}{2} \sum_{i,j} W_{ij} \left|\left| x_i - x_j \right|\right|^2 = tr(X^T L X)$$

- The Optimization Problem:

$$\min_{L \in \mathcal{L}} tr(\mathrm{X}^{\mathrm{T}} \mathrm{L} \mathrm{X}) + \mathrm{f}(\mathrm{L})$$

Pairwise Distance Matrix [5]

- Definition:

$$Z_{ij} = \left|\left|x_i - x_j\right|\right|^2$$

- Some math:

$$tr\left(X^T L X\right) = \frac{1}{2} tr(WZ) = \frac{1}{2} \left|\left|W \circ Z\right|\right|_{1,1}$$

- The smoothness term is a weighted l-1 norm of W, encoding weighted sparsity.
- But…

Building a Sparse Laplacian [5]

- We Explicitly add a sparsity term:

$$\gamma\left||W|\right|_{1,1}$$

- It is free!

$$tr\left(X^T L X\right) + \gamma\left||W|\right|_{1,1} = \frac{1}{2}\left||W \circ Z|\right|_{1,1} + \gamma\left||W|\right|_{1,1} = \frac{1}{2}\left|\left|W \circ (2\gamma + Z)\right|\right|_{1,1}$$

- All info. about signal(X) is in Z and we have the sparsity.
- So we can use any connectivity measures(Pairwise Distance)

Other Constraint [5]

- Make sure that each node has at least one edge with another node
- And control sparsity
- New Optimization Problem:

$$\min_{W \in \mathcal{W}_m} \left|\left|W \circ Z\right|\right|_{1,1} - \alpha 1^T \log(W1) + \frac{\beta}{2} \left|\left|W\right|\right|_F^2$$

  - The log controls node degree vector(No zero or negative degrees)
  - However, log makes it very sparse
    - Can not use the technique before
    - So we use Frobenius Norm

Convex Optimization [5][6]

- Blah, Blah, B

Algorithm 1 Primal dual algorithm for model (12).

Input: z, α, β, $w^0 \in \mathcal{W}_v$, $d^0 \in \mathbb{R}^m_+$, γ, tolerance ϵ
for $i = 1, \ldots, i_{max}$ **do**
  $y^i = w^i - \gamma(2\beta w^i + S^\top d^i)$
  $\bar{y}^i = d^i + \gamma(Sw^i)$
  $p^i = \max(0, y^i - 2\gamma z)$
  $\bar{p}^i = (\bar{y}^i - \sqrt{(\bar{y}^i)^2 + 4\alpha\gamma})/2$ ▷ elementwise
  $q^i = p^i - \gamma(2\beta p^i + S^\top p^i)$
  $\bar{q}^i = \bar{p}^i + \gamma(Sp^i)$
  $w^i = w^i - y^i + p^i$;
  $d^i = d^i - \bar{y}^i + \bar{q}^i$;
  if $\|w^i - w^{i-1}\|/\|w^{i-1}\| < \epsilon$ **and**
    $\|d^i - d^{i-1}\|/\|d^{i-1}\| < \epsilon$ **then**
    break
  end if
end for

Testing Criteria[8][9]

- Accuracy of a Classifier Using Cross Validation
  - SVM
  - Neural Network
  - Decision Tree
  - Etc.

Graph Features [10]

- Laplacian Matrix Elements
- Two Norm Total Variation of Eigen Vector(TNTV)

$$\mathbf{E}_{\triangle\mathbf{u}} = \left\| \mathbf{u} - \frac{\lambda \mathbf{u}}{\lambda_{max}} \right\|_2^2 = \left\| 1 - \frac{\lambda}{\lambda_{max}} \right\|^2$$

- Graph Laplacian Energy (GLE)

$$LE(G) = \sum_{i=1}^{M} |\ \omega_i - \widetilde{\mathbf{D}}\ |$$

D = 2E/M, E is number of edges and M being the number of nodes of graph.

- Joint Total Variation Energy (JTVE)

$$JTVE(\mathcal{G}_i, \mathcal{G}_j) = \rho \times TNTV + (1 - \rho)GLE$$

Graph Features

- Much more! [7][8]
  - Degree: number of links connected to a node
  - Shortest path length: a basis for measuring integration
  - Global efficiency
  - Transitivity
  - Participation coefficient
  - Average neighbor degree
  - Etc.
- PCA and Dimensionality Reduction!

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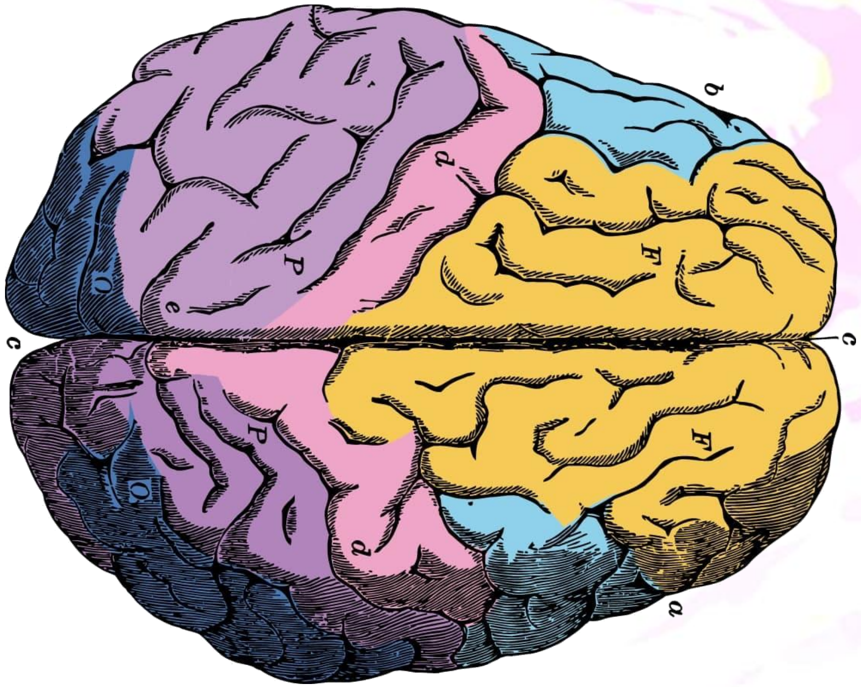

Thanks for Your Attention!