calculus

12 May 2025 16:06

Lec 1

What is derivative:

Geomtric:
The derivative of a function at a point is the slope of the tangent line to the graph at that point.
Slope = $\frac{y1 - y0}{x1 - xo}$,
{How much function chnages if we move 1 unit}

Physical:
It tells how a function changes

If f(x) is a function then f'(x) is how that function changes
f'(x) maps changes in slope of f(x)

Exmple:
- Velocity = derivative of position w.r.t. time
- Acceleration = derivative of velocity w.r.t. time
- Current: derivative of charge with respect to time

Notaion:
Limit x -> 0 means it approaches to zero, very very close to zero but not zero exactly

Formula:

f'(x) = lim d->0 : ($\frac{f(x+d) - f(x)}{d}$)

Derivative of 1/x = -1/(x**2)
Derivative pf x^n = n * x ^(n-1)

LEC 2

Average Vs instanetous rate of change:

- Average rate of change: change over in a interval
  Geometrically: It's the **slope of the secant line** connecting two points on the graph.
  {A **secant line** is a **straight line** that passes through **two points** on a curve.}

- Instantenous rate of chane:
  Change at specific instnace, very very small interval
  Geometrically: It's the **slope of the tangent line** at a single point.

- **not all derivatives (rates of change) are with respect to time**.
  - Example: error rate when calculating distance with GPS
  - H is measured, L is deduced from H

  This is the **rate of change of leghth** with respect to **height**.
  “ $\frac{dL}{dH}$, How sensitive is theLenghthg to small changes in the height ?”

Right hand limit , Left Hand limit

- Left hand limit:

Lim x->a- : f(x)
What value is the function trying to become **just before** hitting a from left side

- Right hand limit:
  Lim x->a+ : f(x)
  What value is the function trying to become **just before** hitting a from right side

limit at x=a exists **only if both one-sided limits exist and are equal**

Continuity at a Point
For a function to be **continuous** at x=ax = ax=a, 3 things must be true:
1. ☑ f(a) is defined (no hole or gap)
2. ☑ limit exists, (LHL = RHL)
3. ☑ f(a) = lim x->a f(x)

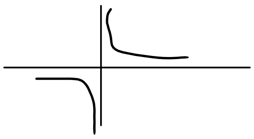
Limit exists AND equals actual value⇒Continuous

1) Jump discontinuity, both LHL and RHL exists, but they are not equal

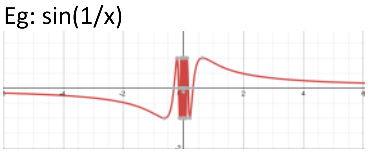
2) Removable discontinuity: LHL = RHL but functionis undefined at a , that is f(a) is undefined

(singularity is a point where a function becomes **undefined**, **infinite**, or **not smooth**.)

3) Infite discontinuity:
   Lim x→0^+ (1/x) = ∞
   Lim x→0^- (1/x) = -∞



   Lim x→0 (1/x) = ∞ , is simply wrong, because which side limit we talkking about?
4) Ugly discontinuity:
   it refers to a situation where a function exhibits wild behavior near a point, such as rapid oscillations, making the limit at that point difficult to handle
   Eg: sin(1/x)



Odd function: f(−x) = −f(x)
Even function f(−x) = f(x)
Even funtion are symmteic across Y axis

LEC 3:

Some common derivative formulas

Prrof of why lim x->0 : $\frac{sinx}{x} = 1$
{draw a circle, know that if theta is in radian,we can driectly calculate arc length: arc length = theta * radian
If x is very small, arc length == sine, so its 1, (sine is vertical line from base radius to point of intersection of mocing radius of theta