Combinatorial Analysis of Virginia State Hunting Bag Limits

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Abstract

The combinatorial analysis of Virginia State hunting bag limits explores the complex rules governing the harvesting of waterfowl, particularly ducks, in the state of Virginia. Bag limits are crucial for wildlife management, ensuring sustainable populations for future generations of hunters. This report delves into the intricate restrictions on bag limits for different duck species, including mallards, wood ducks, black ducks, and others, based on Virginia's regulations.

The analysis involves breaking down the bag limits into individual cases, considering the maximum number of each species that can be harvested. A modified form of the stars and bars theorem is applied to calculate the total number of ways to bag 6 ducks within these limits, resulting in 13,566 possible combinations.

Additionally, an Android app was developed in Kotlin to assist hunters in tracking their bag limits. While the app currently relies on a Python solution for calculations, future development could include a Java or Kotlin alternative to integrate the combinatorial analysis directly into the app.

Overall, this project combines a passion for hunting with an academic interest in combinatorics, highlighting the practical applications of mathematical concepts in wildlife management and hunting regulation.

Contents

Introduction

Brief History of Virginia's Waterfowl Hunting

Virginia has a rich history of waterfowl hunting, dating back to the early days of colonial settlement. The state's diverse habitats, including marshes, rivers, and coastal areas, provide ideal conditions for a wide variety of waterfowl species. Over the years, regulations have been put in place to protect these species and ensure their continued existence. Bag limits are an essential part of these regulations, helping to maintain healthy populations and preserve the sport for future generations.

Importance of Bag Limits

Bag limits play a crucial role in wildlife management by controlling the number of animals that can be harvested during a hunting season. They are based on scientific research and population studies, taking into account factors such as species abundance, reproduction rates, and conservation goals (How the hunting seasons and limits are set for waterfowl: U.S. fish & wildlife service). By adhering to bag limits, hunters help ensure the sustainability of wildlife populations, allowing for continued enjoyment of the sport for years to come.

Personal Interest

Hunting has been a cherished tradition for many, offering a unique connection to nature and providing a means of sustenance for generations. As an avid hunter myself, I have always been fascinated by the intricate rules and regulations that govern the sport, particularly in the realm of bag limits for waterfowl. This project delves into the combinatorial analysis of different duck species that can be harvested to reach a waterfowl bag limit, combining my passion for hunting with my academic interest in combinatorics.

Motivation

The motivation behind this project stems from my love for hunting, especially waterfowl hunting. Understanding the complexities of bag limits and the various combinations of species that can be harvested within those limits is not only intellectually stimulating but also contributes to the

sustainable management of wildlife populations. By analyzing these combinations, we can gain insights into optimal harvesting strategies that ensure the long-term health and balance of waterfowl populations.

Applying to Everyday Use

In addition to the theoretical analysis, I have also developed an Android app in Kotlin to assist waterfowl hunters in tracking their bag limits. The app allows users to input variables such as the date, hunting zone, and the number of waterfowl shot so far, and displays the remaining bag limit information. This practical application of technology enhances the hunting experience and promotes adherence to bag limits, contributing to the conservation efforts of waterfowl populations.

Definitions

Waterfowl Terms

Bag Limit – The maximum number of a specific duck species that one is allowed to "take" in a single day.

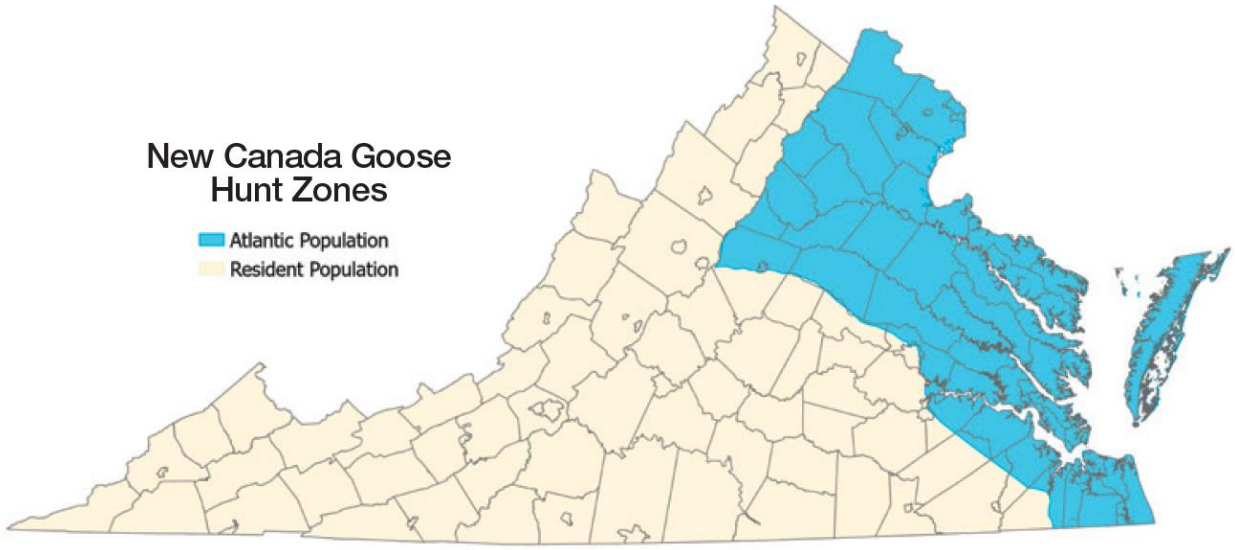
For our purposes "take" refers to harvesting a bird using non-toxic shot in a shotgun, rifle, or handgun (Migratory gamebirds).

Ducks

The appearance of each species of waterfowl discussed in this report is irrelevant, however there are some important terms to help understand why there are so many additional restrictions to this problem when represented combinatorically. For species of ducks a male is referred to as a **Drake** and a female is referred to as a **Hen**.

Canada Goose Hunt Zone

In Virginia depending on what part of the state you are hunting in; the limit and season dates can vary for Canada Geese. This information is referenced in the calculations for how many Geese can be bagged based on the current date and location later on.



(Migratory gamebirds)

September Canada Goose

In Virginia depending on what part of the state you are hunting in the limit and season dates can vary for September Canada Geese. This information is referenced in the calculations for how many Geese can be bagged in September based on the current date and location later on. The Red line is i95 in Virginia, keep this in mind later on when we refer to east and west of i95 for our calculations.

(File:I-95 (VA) map.svg)

Combinatoric Terms

Stars and Bars Theorem – A solution to a frequently occurring problem in combinatorics when "counting the number of ways to group identical objects, such as placing indistinguishable balls into labelled urns." (Integer equations - Stars and bars: Brilliant math & science wiki)

In this report we will use a modified form of stars and bars that has individual maximum partition sizes to represent our max bag limit for each subcategory of duck

Basic Stars and Bars Formula:

The number of ways to place n indistinguishable balls into k labelled urns is.

$$\binom{n+k-1}{n} = \binom{n+k-1}{k-1}$$

(Integer equations - Stars and bars: Brilliant math & science wiki)

Virginia State Waterfowl Bag Limits

With all of the definitions taken care of let's examine what exactly makes Virginia's Waterfowl Bag limits a complicated combinatorial problem. There are many different sub categories within the waterfowl bag limits that are explained in depth at the Virginia website (Migratory gamebirds), including references to changing season dates and limits based on orientation compared to i95 and the Atlantic and Residential Population Zones (refer to Canada Goose Hunt Zone). In this report we will focus on bag limits for Ducks, as there are many restrictions on the ducks that can be bagged, restricting both species and often sex of the ducks.

Below is the official explanation from (Migratory gamebirds):

"Daily Bag Limit: 6 ducks of any species except for the following restrictions: can include no more than 4 mallards (only 2 hen mallards), 3 wood ducks, 2 black ducks (except closed Oct. 6-9), scaup: 1/day for 40 days (October 6-9, November 15-26, December 19-January 11), and 2/day for 20 days (January 12-31), 2 redheads, 2 canvasback, 1 pintail, 1 mottled duck, 1 fulvous whistling duck, 4 total sea ducks (no more than 3 scoters, 3 eiders [only 1 hen], 3 long-tailed ducks)."

For the purpose of this report, we will assume the date is November 15th, meaning black duck season is not closed and 1 scaup can be taken per day.

Case Breakdown

To approach the number of ways to bag 6 ducks given the Virginia State Waterfowl Bag Limits we must break down the species that have sex restrictions or sub species restrictions into cases. Below I have provided an example of how I broke down each sub species of duck according to the restrictions imposed by the state of Virginia, the full case breakdown is available upon request however it is several pages so for the sake of this report I have just included some examples from it:

Mallard Cases:

- 4 Mallards (Drakes, Hens)
    - (4,0), (3,1), (2,2) = 3 Combinations

- 3 Mallards
    - (3,0), (2,1), (1,2) = 3 Combinations
- 2 Mallards
    - (2,0), (1,1), (0,2) = 3 Combinations
- 1 Mallard
    - (1,0), (0,1) = 2 Combinations
- 0 Mallards
    - (0,0) = 1 Combination
- Total: 12 Combinations to bag 0-4 Mallards

Sea Duck:

- 4 Sea Ducks (Scoters, Eider, Long Tails)
    - Group by Same number of Eiders to apply Sex restriction.
    - (0,3,1), (1,3,0) = 2 Combinations
        - 2 Ways to get 3 Eiders (Drakes, Hens): (3,0), (2,1)
        - 2 Combinations * 2 Eider Combinations = 4 Ways
    - (0,2,2), (2,2,0), (1,2,1)
        - 2 Ways to get 2 Eiders: (2,0), (1,1)
        - 3 Combinations * 2 Eider Combinations = 6 Ways
    - (3,1,0), (0,1,3), (2,1,1), (1,1,2)
        - 2 Ways to get 1 Eider: (1,0), (0,1)
        - 4 Combinations * 2 Eider Combinations = 8 Ways
    - (1,0,3), (3,0,1), (2,0,2)
        - 1 Way to get 0 Eider: (0,0)
        - 3 Combinations * 1 Eider Combination = 3 Ways
    - Total = 21 Ways to bag 4 Sea Ducks

To put it all together we must calculate the number of ways we can bag 6 ducks given the limit for each category, essentially we have 10 categories of which the max that can be bagged from each category is [4,3,2,1,2,2,1,1,1,4] (Mallards, wood ducks, black ducks, scaup, redheads, canvasback, pintail, mottled duck, fulvous whistling duck, sea ducks). Then apply the individual number of ways to bag that if it's a sub species with a restriction. For example if the combination was (2,0,0,0,0,0,0,0,0,4), there are 3 ways to bag 2 Mallards according to the case by case breakdown and there are 21 ways to bag 4 sea ducks, so for this one combination there are 3*21 = 63 ways to bag 2 Mallards and 4 Sea ducks to achieve the duck bag limit of 6. Given this example and the length it would take to break this down into cases I found a python program to generate all the possible combinations to achieve a bag limit of 6.

Stars and Bars with Individual Maximum Partition Sizes

Thanks to a similar problem posted on stack overflow (JohanC) I was able to find a solution to this problem in the form of a python program, which I modified to accommodate the further restrictions for this specific problem. Since ducks from the same category are usually identical, except in the cases of Mallards and Sea Ducks, we can apply a modification of stars and bars to this. Using the

variables of 6 total ducks and restrictions for each category being less than or equal to [4,3,2,1,2,2,1,1,1,4] for the corresponding species (Mallards, wood ducks, black ducks, scaup, redheads, canvasback, pintail, mottled duck, fulvous whistling duck, sea ducks), I determined that there are 1863 ways to bag 6 ducks from 10 categories with the provided limits for each species, however this does not yet include the case by case combinations discussed in Case Breakdown. For example if the combination was (2,0,0,0,0,0,0,0,0,4), there are 3 ways to bag 2 Mallards according to the case by case breakdown and there are 21 ways to bag 4 sea ducks, so for this one combination there are 3*21 = 63 ways to bag 2 Mallards and 4 Sea ducks to achieve this particular combination of species. In our current programs state it does not apply this, so I modified the program to account for this.

After modifying the program to accommodate this we get a final answer of 13,566 total number of ways to bag 6 ducks.

Python Program

Now we will break down the parts of the python program to show how it is able to make this calculation.

Step 1 of the program takes an input of x which is the total number or 6 in our case and an input in the form of an integer array which stores the maximum limit each category can have which is [4,3,2,1,2,2,1,1,1,4] in our case. The program then creates an array the size of the number of categories we are picking from and puts all 6 items in the first slot. It then distributes those 6 items into the next slot as long as the current slot has too many items according to its max.

Step 2 of the program goes through and moves surpluses from overfilled categories (species of ducks) to others that are under their limit.

Step 3 has category 1 (index 0) absorb any surpluses left over as long as it stays under its categorical max. This program loops every time a valid combination is found and printed.

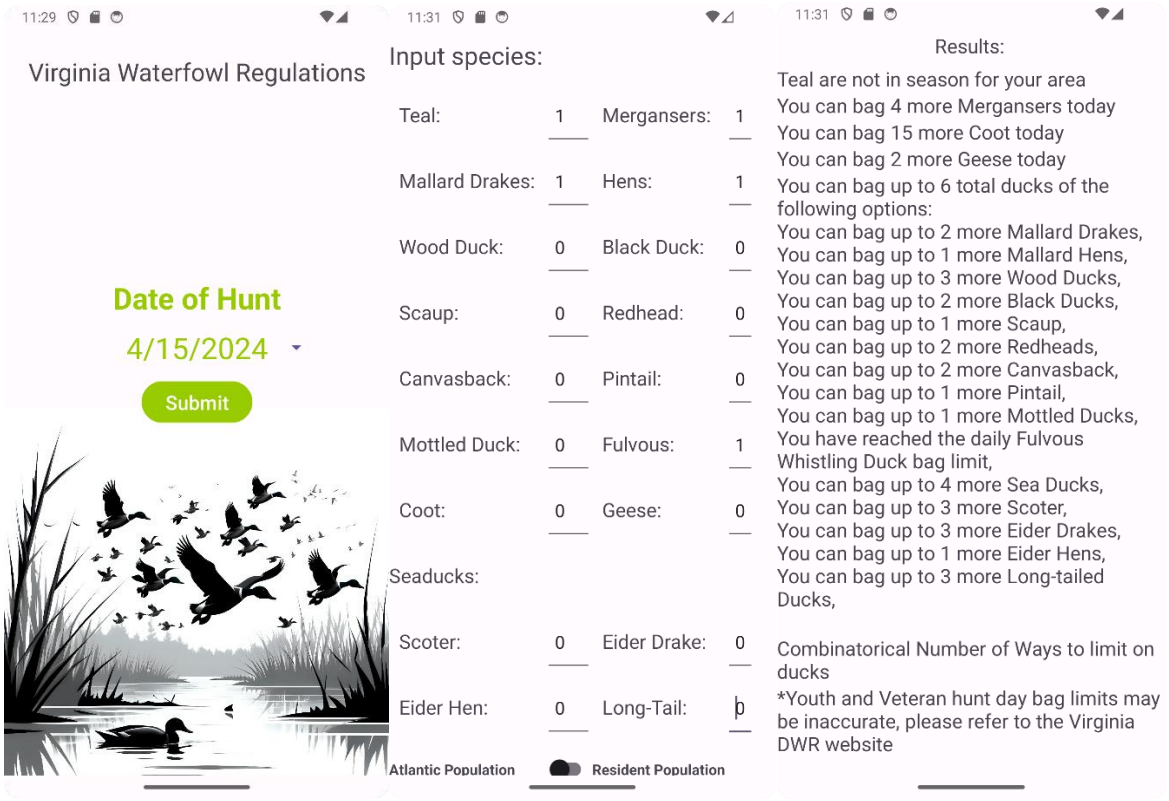
The final step that I implemented is multiplying for sub categorical restrictions, for example, if the specific combination that is being printed out has 4 sea ducks, I know from Case Breakdown there are 21 ways to pick 4 sea ducks, so I multiply the current combination by that amount and add it to the total combination counter. Below is a snippet of this:

```
for i, p in enumerate(find_partitions(x, lims)):
    multiplier = 1
    if p[0]==4 or p[0]==3 or p[0]==2:
        multiplier*=3
    elif p[0]==1:
        multiplier*=2
    if p[9]==4:
        multiplier*=21
    elif p[9]==3:
        multiplier*=16
    elif p[9]==2:
        multiplier*=9
    elif p[9]==1:
        multiplier*=4
    count+=multiplier
    print(f"partition {i+1}: {p} sums to {sum(p)}  lex: { ''.join([str(i) for i in
p[::-1]]) }")
print("There are "+str(count)+" total number of ways to bag 6 ducks")
```

(Python code snippet showing multipliers for different sub species of ducks)

Kotlin App

In addition to the theoretical analysis, I have also developed an Android app in Kotlin to assist waterfowl hunters in tracking their bag limits. The app allows users to input variables such as the date, hunting zone, and the number of waterfowl shot so far, and displays the remaining bag limit information. I took this project as an opportunity to learn Kotlin and android app development, however since my calculations for the combinations is based off a python solution to this problem I have yet to develop a Java or Kotlin alternative that would integrate into my app. I was able to create output based on user input of date and species bagged thus far that informed users of how many more birds they were able to bag in the given day. Below are screenshots of the user interface I designed as well as the accompanying GitHub release in the Bibliography.



(Shaw)

As discussed in Canada Goose Hunt Zone and on the DWR website Canada Geese have a location based restriction for when they can be hunted and how many can be bagged (Migratory gamebirds). To account for this, I added an Atlantic Population and Residential Population as well as an East and West of i95 switch to be used in my calculations. The code for the result screen ended up being close to 500 lines to account for each subcategory and to display different results depending on those calculations. The date selection at the launch of the app determines what season of waterfowl you are in according to the DWR website, and the accompanying if statements match the DWR's season information as of the release of this report.

Conclusion

In conclusion, the combinatorial analysis of Virginia State hunting bag limits presents a complex problem that requires careful consideration of various factors. Bag limits are essential for wildlife management, ensuring the sustainability of waterfowl populations for future generations of hunters. Through this analysis, we explored the intricate rules governing bag limits, particularly for ducks, and developed a Python program to calculate the number of ways to bag 6 ducks based on Virginia's regulations.

The program uses a modified form of stars and bars to account for the restrictions on each duck species, such as the maximum number of mallards or sea ducks that can be harvested. By applying this approach, we found that there are 13,566 total ways to bag 6 ducks within the specified limits.

Additionally, I developed an Android app in Kotlin to assist hunters in tracking their bag limits. While the app currently relies on the Python solution for calculations, future development could include a Java or Kotlin alternative to integrate the combinatorial analysis directly into the app.

Overall, this project combines my passion for hunting with my academic interest in combinatorics, highlighting the practical applications of mathematical concepts in real-world scenarios. Through further research and development, we can continue to improve wildlife management practices and enhance the hunting experience for all.

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