Lab 202: Numerical Verification of Gauss's Law

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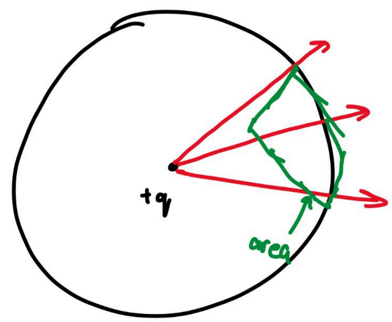
1. INTRODUCTION

1.1 OBJECTIVES

Using Matlab to apply Gauss's law under different conditions. We will confirm that net charge enclosed by a surface is equal to the integral of the electric field acting over the surface multiplied by ε_0.

1.2 THEORETICAL BACKGROUND

The subject today revolves around Gauss's Law, which is applied regarding "flux", area of surfaces, and many of the previously explored variables. When a charge is placed inside of an object (either a physical or theoretical shape), we are aware it emits an electric field that interacts with other charged objects: this is what Coulomb's law and the subsequent electric field formulas represent. However, we are introduced to flux, which is defined as the amount of force lines that pass through a surface. By the definition of flux, we have to find the dot product of the electric field and the area vector in the following equation:

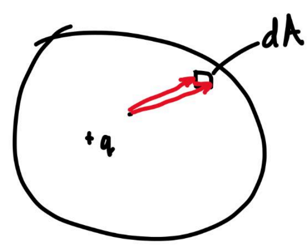


$$\phi = \overline{E} \cdot \overline{A} = |E|\,|A|\,cos\theta$$

The value of A represents the area of the object surrounding the charge, but the vector of $\widehat{An}$ points directly outward from the surface. This vector is the one that is multiplied by E.

When dealing with parts of the whole area, we take the derivative of the full area and put in terms of dA, giving us the following equation:

$$d\phi = E \cdot dA$$



This can be integrated to get the full value of ϕ across the whole of the area as follows:

$$\phi = \oint E \cdot dA$$

Coming away from the standard definition of flux is Gauss's Law, which helps us construct a Gaussian surface (which is any theoretical surface that allows us to calculate the flux, electric field, or charge of a given object. The definition of flux is as follows:

$$\phi = \frac{Q}{\varepsilon_0}$$

Q in this circumstance represents the total amount of charge that is enclosed within the Gaussian surface. Q enclosed is a net value, meaning that positives and negatives cancel each other out, whereas positive+positive or negative+negative add onto one another. Furthermore, the charge enclosed provides the sign (positive or negative) in front of the flux, which determines the direction of the charge lines. Sources release a flux, have a positively enclosed charge, and the field lines point outward, while sinks have the opposite effect for negatively enclosed charges. ε_0 is the permittivity of a vacuum, a constant value. We can use both the definition of Flux and Gauss's Law as equal as they both are equivalent to flux.

2. **EXPERIMENTAL PROCEDURE**

- The direction of E varies based on the point of penetration of the upper plane by the field while determining the flux Φ_e through a cube's upper side. For this reason, you must choose a tiny region and figure out the flow through it. The procedures were very similar to the last lab.
- We had to find the charge of q1 at the cube's center and verify that the same flux passes through each surface. We deduce that the charges flow in an identical manner around the entire cube when the charge q1 is situated in its center. It is dispersed uniformly. Find the same charge (q1) at x = ½*a on the x axis. Obtain the flux values. In contrast to the others, "Phifront" has excellent Flux. whereas "Phiback" has the lowest flux discovered. However, as they all still had the same value, the fluxes Phitop, Phibottom, Phiright, and Phileft produced findings that were comparable.
- When load Q1 is positioned at y=1/2 on the y-axis, the outcomes will be comparable compared to x=1.2 on the x-axis. When compared to the others, the Flux is "Phiright" and excellent. However, "Phileft" has the lowest flux discovered. However, as they all still have the same value, the fluxes Phitop, Phibottom, Phifront, and Phiback showed comparable findings.

3. **RESULTS**

3.0 **CODE**

```
% added parameters for charge position (qPos) and cube side half-length (a)
% replaced constants with parameters based on equations present in lab manual
q = 9e-9;
a = 1;
qPos = [0.1, 0.2, 0.3];

eps0 = 8.85e-12; k = 1/(4*pi*eps0);
I = [1,0,0]; J = [0,1,0]; K = [0,0,1];

syms x y z

% pulled position vector out into separate function to improve readability
```

```
r = @(x,y,z) [x, y, z] - qPos;

% used 'norm' to calculate magnitude in place of lengthy expression
E = @(x,y,z) k*q/norm(r(x,y,z))^3 * r(x,y,z);

Etop    = dot( K, E( x,  y,  a));
Ebottom = dot(-K, E( x,  y, -a));
Eright  = dot( J, E( x,  a,  z));
Eleft   = dot(-J, E( x, -a,  z));
Efront  = dot( I, E( a,  y,  z));
Eback   = dot(-I, E(-a,  y,  z));

% added 'vpa' so all outputs display when file is run
phiTop    = vpa(int(int(Etop   , x, -a, a), y, -a, a), 10)
phiBottom = vpa(int(int(Ebottom, x, -a, a), y, -a, a), 10)
phiRight  = vpa(int(int(Eright , x, -a, a), z, -a, a), 10)
phiLeft   = vpa(int(int(Eleft  , x, -a, a), z, -a, a), 10)
phiFront  = vpa(int(int(Efront , y, -a, a), z, -a, a), 10)
phiBack   = vpa(int(int(Eback  , y, -a, a), z, -a, a), 10)

phiTotal = vpa(phiTop + phiBottom + phiRight + phiLeft + phiFront + phiBack,
10)

expectedCharge = vpa(phiTotal * eps0, 5)
```

3.1 **EXPERIMENTAL DATA**

Table 1. Charge Placed in Center of Cube (qPos = [0, 0, 0];)

Φ_{top} = 169.4915254	Φ_{right} = 169.4915254	Φ_{front} = 169.4915254
Φ_{bottom} = 169.4915254	Φ_{left} = 169.4915254	Φ_{back} = 169.4915254
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 2. Charge Placed on x-axis (qPos = [0.5*a, 0, 0];)

Φ_{top} = 153.8837352	Φ_{right} = 153.8837352	Φ_{front} = 300.1700698
Φ_{bottom} = 153.8837352	Φ_{left} = 153.8837352	Φ_{back} = 101.244142
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 3. Charge Placed on y-axis (qPos = [0, 0.5*a, 0];)

Φ_{top} = 153.8837352	Φ_{right} = 300.1700698	Φ_{front} = 153.8837352

Φ_{bottom} = 153.8837352	Φ_{left} = 101.244142	Φ_{back} = 153.8837352
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 4. Charge Placed Arbitrarily Inside Cube (qPos = [0.5*a, -0.25*a, 0.33*a];)

Φ_{top} = 218.6480315	Φ_{right} = 114.5866429	Φ_{front} = 284.4814913
Φ_{bottom} = 107.7511885	Φ_{left} = 195.3860483	Φ_{back} = 96.09575005
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 5. Charge Magnitude Doubled (qPos = [0,0,0]; q = 18e-9;)

Φ_{top} = 338.9830508	Φ_{right} = 338.9830508	Φ_{front} = 338.9830508
Φ_{bottom} = 338.9830508	Φ_{left} = 338.9830508	Φ_{back} = 338.9830508
Φ_{total} = 2033.898305		$q_{expected}$ = 1.8e-8

Table 6a. Cube Size Increased (qPos = [0,0,0]; a=2;)

Φ_{top} = 169.4915254	Φ_{right} = 169.4915254	Φ_{front} = 169.4915254
Φ_{bottom} = 169.4915254	Φ_{left} = 169.4915254	Φ_{back} = 169.4915254
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 6b. Cube Size Increased (qPos = [0,0,0]; a=10;)

Φ_{top} = 169.4915254	Φ_{right} = 169.4915254	Φ_{front} = 169.4915254
Φ_{bottom} = 169.4915254	Φ_{left} = 169.4915254	Φ_{back} = 169.4915254
Φ_{total} = 1016.949153		$q_{expected}$ = 9.0e-9

Table 6c. Cube Size Increased (qPos = [0,0,0]; a=100;)

Φ_{top} = 169.4915254	Φ_{right} = 169.4915254	Φ_{front} = 169.4915254
Φ_{bottom} = 169.4915254	Φ_{left} = 169.4915254	Φ_{back} = 169.4915254

Φ_{total} = 1016.949153	$q_{expected}$ = 9.0e-9

Table 7. Charge Placed Outside of Cube (qPos = [0,0,2*a];)

Φ_{top} = -169.4915254	Φ_{right} = 34.26670837	Φ_{front} = 34.26670837
Φ_{bottom} = 32.42469196	Φ_{left} = 34.26670837	Φ_{back} = 34.26670837
Φ_{total} = 6.661338148e-16		$q_{expected}$ = 5.8953e-27

3.2 CALCULATION

This lab has no calculations because the work was done through MatLab code.

4. ANALYSIS and DISCUSSION

It was necessary for us to comprehend the goal of the code and the expected outcome before we could enter it into MATLAB and alter the settings. Additionally, in order to correctly develop, modify, and print the figures, we needed to grasp how MATLAB functions. Each program clearly generates a coordinate space when the code structure is compared to the electric field formula and the formulations of Coulomb's law. The electric flux values were equivalent in the first set of calculations where x,y,or z were either a or -a. Since the value of electric flux is equivalent to k multiplied by q divided by r squared, the magnitude for the electric flux for the top, bottom, left, right, front, and back portions were all the same. Also, r is equivalent to the sum of x squared, y squared, and x squared raised to the three-halves power and multiplied by r to the one-half power. Essentially, the formula for electric flux of multiplying the electric constant k by q divided by r squared is constant. x equaled half a in the second set of computations. The cube's top and bottom faces were the greatest in magnitude and carried the same flux values. The cube's left and right faces were the next greatest in magnitude and held the same flux values. The individual electric flux values of the left and right faces were smaller because a value of an is only applied once, at the y position. As it grows in magnitude, the electric flux value will also increase. The electric flux was greater in the first example because it had the biggest magnitude. The electric flux will thus rely on the location of an if the magnitude of an is less. The greatest electric flux is reached when the distance or radius of separation between charged particles is the shortest, hence the position chosen inside the cube will not affect the electric flux. Since the radius of separation (r) between charged particles and electric flux are inversely correlated, electric flux drops as r increases. In contrast, electric flux rises as r decreases. There will be a twofold increase in the electric flux when the charge q1 is doubled. As the charge of the particle grows, the electric flux will also increase by the same factor since the electric flux is exactly proportional to the charge of the item occupying an enclosed surface with regard to the Gaussian distribution.Since the distances would differ even if the charges would be the same, there would be no equilibrium at any other place.

5. **CONCLUSIONS**

In conducting this experiment we learned about Gauss's Law and electric flux. We were able to determine the magnitude and direction of the electrical fields due to the charges and forces and get the surface area of the shapes needed to get the flux. We learned about the enclosed charges and how they work.The distance between charged particles and the charge of the particle itself determine the electric flux in an enclosed surface. The electric flow is inversely proportional to the square of the space between charged particles and directly relates to the particle's charge. According to this physics phenomena, the electric flow doubles with the magnitude of charge, and the electric flux decreases by a factor of four with the twofold distance of separation between charged particles. MatLab is a great way to code and graph Electric fields talking about fluxes and point charges. The field's magnitude may be determined using functions, and MatLab's graphing capabilities let users plot the lines of the electric field. The electric field's flow is depicted by these field lines. We verified our understanding of electric charge and force from the prior lab. Although this lab is more about coding and the last lab was more like real-world touchable experiments, they both work well together. We investigated the fluxes produced by charges and regions. We learned how conduction and induction operate to electrically charge an object, and we demonstrated many of the qualitative features of electrostatics using the lab's conduction.

6. **RAW DATA**

This lab does not have any raw data since the work was done through Matlab.