7.4.17

EE25BTECH11010 - Arsh Dhoke

Question:
Find the value of k such that the quadratic equation kx(x-2) + 6 = 0 has equal roots. Verify your solution using graph.
Solution:

$$kx^2 - 2kx + 6 = 0 \tag{0.1}$$

This can be represented as a conic:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.2}$$

where

$$\mathbf{V} = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -k \\ 0 \end{pmatrix}, \quad f = 6 \tag{0.3}$$

For the roots of the quadratic to be equal, the line $y = 0$ (the x-axis) must be a tangent to the conic. The condition for tangency of a line $\mathbf{n}^T\mathbf{x} = c$ with a conic $\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$ is :

$$\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f = 0 \tag{0.4}$$

For the given conic:

$$\mathbf{V}^{-1} = \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix} \tag{0.5}$$

Substituting,

$$\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f = 0 \tag{0.6}$$

$$\implies \begin{pmatrix} -k & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -k \\ 0 \end{pmatrix} - 6 = 0 \tag{0.7}$$

$$\therefore k = 6 \tag{0.8}$$

Thus, the quadratic is

$$6x^2 - 12x + 6 = 0 \tag{0.9}$$

or

$$(x - 1)^2 = 0 \tag{0.10}$$

which clearly has a double root at $x = 1$.

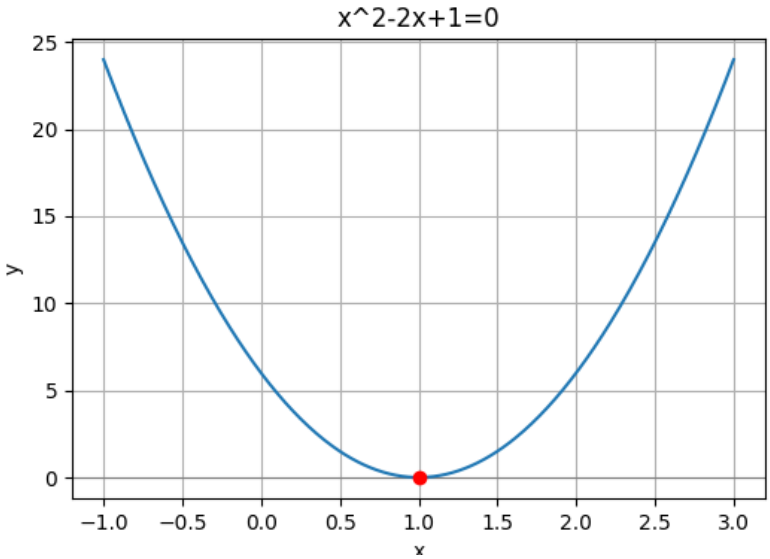

Fig. 0.1: Graph