

## 9.4.22

### Quadratic with equal roots

EE25BTECH11010 - Arsh Dhoke

# Question

Find the value of  $k$  such that the quadratic equation  $kx(x - 2) + 6 = 0$  has equal roots. Verify your solution using graph.

# Solution

$$kx^2 - 2kx + 6 = 0 \quad (1)$$

This can be represented as a conic:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2)$$

where

$$\mathbf{V} = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -k \\ 0 \end{pmatrix}, \quad f = 6 \quad (3)$$

# Condition for Equal Roots

For the roots of the quadratic to be equal, the line  $y = 0$  (the  $x$ -axis) must be tangent to the conic. The condition for tangency of a line  $\mathbf{n}^T \mathbf{x} = c$  with a conic  $\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$  is:

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 0 \quad (4)$$

For the given conic:

$$\mathbf{V}^{-1} = \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix} \quad (5)$$

# Finding $k$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 0 \quad (6)$$

$$\Rightarrow \begin{pmatrix} -k & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -k \\ 0 \end{pmatrix} - 6 = 0 \quad (7)$$

$\therefore k = 6$  Thus, the quadratic becomes:  $6x^2 - 12x + 6 = 0$  or equivalently,  $(x - 1)^2 = 0$  which clearly has a double root at  $x = 1$ .

# Verification by Graph

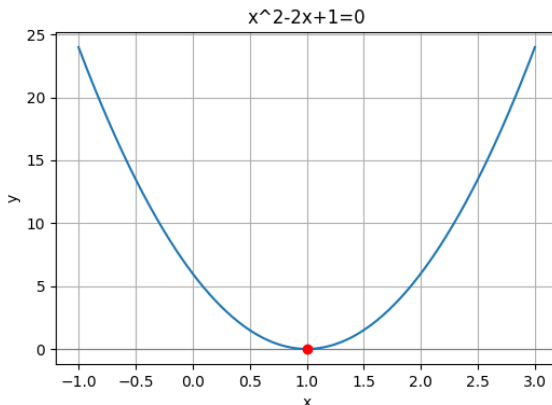


Figure: Graph of  $y = 6x^2 - 12x + 6$  showing a double root at  $x = 1$

```
#include <math.h>

double find_k() {
    double k1, k2;
    double a, b, c;
    double D;

    // Equation:  $4k^2 - 24k = 0$ 
    a = 4;
    b = -24;
    c = 0;
    D = b * b - 4 * a * c;

    k1 = (-b + sqrt(D)) / (2 * a);
    k2 = (-b - sqrt(D)) / (2 * a);
}
```

```
// k = 0 or 6, but k = 0 makes equation invalid
if (k1 != 0)
    return k1;
else
    return k2;
}
```



# Python Code

```
import numpy as np
import matplotlib.pyplot as plt

k = 6
x = np.linspace(-1, 3, 400)
y = k*x*(x-2) + 6 #  $kx^2 - 2kx + 6$ 

plt.figure(figsize=(6,4))
plt.axhline(0, color='gray', linewidth=0.8)
plt.plot(x, y, label=f'k={k} :  $y=kx(x-2)+6$ ')
plt.scatter([1], [0], color='red', zorder=5) # double root at x=1
plt.title('x2-2x+1=0')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True)
plt.savefig("/home/arsh-dhoke/ee1030-2025/ee25btech11010/matgeo/9.4.22/figs/parabola.png")
plt.show()
```

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared C library
lib = ctypes.CDLL("./code.so")

# Define the return type of the C function
lib.find_k.restype = ctypes.c_double

# Call the C function
k = lib.find_k()
print("Value of k for equal roots:", k)

# Define the quadratic function
def f(x, k):
    return k * x * (x - 2) + 6
```

```
# Create x values
x = np.linspace(-1, 3, 200)
y = f(x, k)

# Plot the quadratic
plt.plot(x, y, label=f"k = {k:.2f}")
plt.axhline(0, color="black", linewidth=0.8) # x-axis
plt.axvline(0, color="black", linewidth=0.8) # y-axis
plt.title("Quadratic:  $kx(x - 2) + 6 = 0$  (Equal Roots Condition)")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)
plt.savefig("/home/arsh-dhoke/ee1030-2025/ee25btech11010/matgeo/9.4.22/figs/parabola.png")
plt.show()
```