

9.4.22

EE25BTECH11010 - Arsh Dhoke

Question:

Find the value of k such that the quadratic equation $kx(x-2) + 6 = 0$ has equal roots. Verify your solution using graph.

Solution:

$$kx^2 - 2kx + 6 = 0 \quad (0.1)$$

This can be represented as a conic:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.2)$$

where

$$\mathbf{V} = \begin{pmatrix} k & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -k \\ 0 \end{pmatrix}, \quad f = 6 \quad (0.3)$$

For the roots of the quadratic to be equal, the line $y = 0$ (the x -axis) must be a tangent to the conic. The condition for tangency of a line $\mathbf{n}^T \mathbf{x} = c$ with a conic $\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$ is :

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 0 \quad (0.4)$$

For the given conic:

$$\mathbf{V}^{-1} = \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix} \quad (0.5)$$

Substituting,

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 0 \quad (0.6)$$

$$\Rightarrow \begin{pmatrix} -k & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{k} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -k \\ 0 \end{pmatrix} - 6 = 0 \quad (0.7)$$

$$\therefore k = 6 \quad (0.8)$$

Thus, the quadratic is

$$6x^2 - 12x + 6 = 0 \quad (0.9)$$

or

$$(x - 1)^2 = 0 \quad (0.10)$$

which clearly has a double root at $x = 1$.

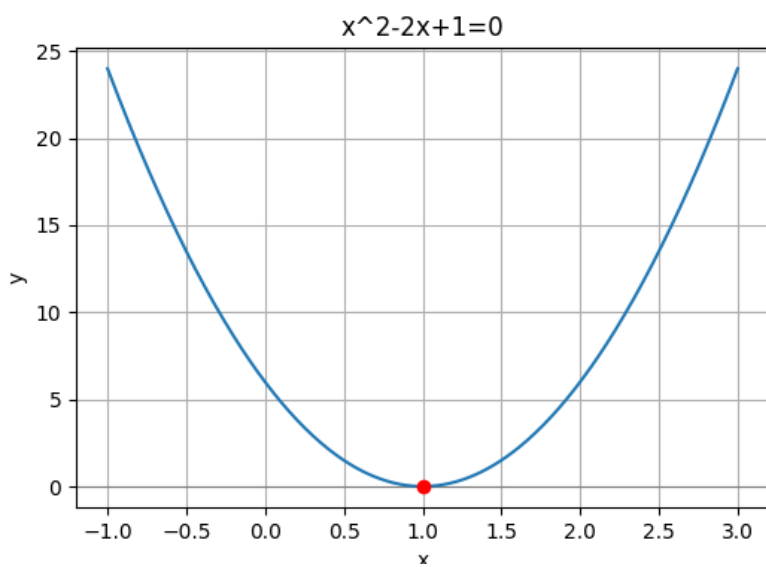


Fig. 0.1: Graph