

```

1  Algorithm NextValue( $k$ )
2  //  $x[1], \dots, x[k-1]$  have been assigned integer values in
3  // the range  $[1, m]$  such that adjacent vertices have distinct
4  // integers. A value for  $x[k]$  is determined in the range
5  //  $[0, m]$ .  $x[k]$  is assigned the next highest numbered color
6  // while maintaining distinctness from the adjacent vertices
7  // of vertex  $k$ . If no such color exists, then  $x[k]$  is 0.
8  {
9      repeat
10     {
11          $x[k] := (x[k] + 1) \bmod (m + 1)$ ; // Next highest color.
12         if ( $x[k] = 0$ ) then return; // All colors have been used.
13         for  $j := 1$  to  $n$  do
14         { // Check if this color is
15             // distinct from adjacent colors.
16             if ( $(G[k, j] \neq 0)$  and ( $x[k] = x[j]$ ))
17                 // If  $(k, j)$  is an edge and if adj.
18                 // vertices have the same color.
19                 then break;
20         }
21         if ( $j = n + 1$ ) then return; // New color found
22     } until (false); // Otherwise try to find another color.
23 }

```

```
1  Algorithm mColoring( $k$ )
2  // This algorithm was formed using the recursive backtracking
3  // schema. The graph is represented by its boolean adjacency
4  // matrix  $G[1 : n, 1 : n]$ . All assignments of  $1, 2, \dots, m$  to the
5  // vertices of the graph such that adjacent vertices are
6  // assigned distinct integers are printed.  $k$  is the index
7  // of the next vertex to color.
8  {
9      repeat
10     { // Generate all legal assignments for  $x[k]$ .
11         NextValue( $k$ ); // Assign to  $x[k]$  a legal color.
12         if ( $x[k] = 0$ ) then return; // No new color possible
13         if ( $k = n$ ) then // At most  $m$  colors have been
14                         // used to color the  $n$  vertices.
15             write ( $x[1 : n]$ );
16             else mColoring( $k + 1$ );
17     } until (false);
18 }
```

```

1  Algorithm HeapSort( $a, n$ )
2  //  $a[1 : n]$  contains  $n$  elements to be sorted. HeapSort
3  // rearranges them inplace into nondecreasing order.
4  {
5      Heapify( $a, n$ ); // Transform the array into a heap.
6      // Interchange the new maximum with the element
7      // at the end of the array.
8      for  $i := n$  to 2 step -1 do
9      {
10          $t := a[i]; a[i] := a[1]; a[1] := t;$ 
11         Adjust( $a, 1, i - 1$ );
12     }
13 }

```

```
1 Algorithm Heapify( $a, n$ )  
2 // Readjust the elements in  $a[1 : n]$  to form a heap.  
3 {  
4     for  $i := \lfloor n/2 \rfloor$  to 1 step  $-1$  do Adjust( $a, i, n$ );  
5 }
```

```

1  Algorithm Adjust( $a, i, n$ )
2  // The complete binary trees with roots  $2i$  and  $2i + 1$  are
3  // combined with node  $i$  to form a heap rooted at  $i$ . No
4  // node has an address greater than  $n$  or less than 1.
5  {
6       $j := 2i$ ;  $item := a[i]$ ;
7      while ( $j \leq n$ ) do
8      {
9          if ( $(j < n)$  and ( $a[j] < a[j + 1]$ )) then  $j := j + 1$ ;
10         // Compare left and right child
11         // and let  $j$  be the larger child.
12         if ( $item \geq a[j]$ ) then break;
13         // A position for  $item$  is found.
14          $a[\lfloor j/2 \rfloor] := a[j]$ ;  $j := 2j$ ;
15     }
16      $a[\lfloor j/2 \rfloor] := item$ ;
17 }

```

```

1  Algorithm DelMax( $a, n, x$ )
2  // Delete the maximum from the heap  $a[1 : n]$  and store it in  $x$ .
3  {
4      if ( $n = 0$ ) then
5      {
6          write ("heap is empty"); return false;
7      }
8       $x := a[1]$ ;  $a[1] := a[n]$ ;
9      Adjust( $a, 1, n - 1$ ); return true;
10 }

```

```
1  Algorithm Insert( $a, n$ )
2  {
3      // Inserts  $a[n]$  into the heap which is stored in  $a[1 : n - 1]$ .
4       $i := n$ ;  $item := a[n]$ ;
5      while  $((i > 1) \text{ and } (a[\lfloor i/2 \rfloor] < item))$  do
6      {
7           $a[i] := a[\lfloor i/2 \rfloor]$ ;  $i := \lfloor i/2 \rfloor$ ;
8      }
9       $a[i] := item$ ; return true;
10 }
```

```
1  Algorithm Sort( $a, n$ )
2  // Sort the elements  $a[1 : n]$ .
3  {
4      for  $i := 1$  to  $n$  do Insert( $a, i$ );
5      for  $i := n$  to  $1$  step  $-1$  do
6          {
7              DelMax( $a, i, x$ );  $a[i] := x$ ;
8          }
9  }
```

```
1  Algorithm MergeSort(low, high)
2  // a[low : high] is a global array to be sorted.
3  // Small(P) is true if there is only one element
4  // to sort. In this case the list is already sorted.
5  {
6      if (low < high) then // If there are more than one element
7      {
8          // Divide P into subproblems.
9          // Find where to split the set.
10         mid :=  $\lfloor (low + high)/2 \rfloor$ ;
11         // Solve the subproblems.
12         MergeSort(low, mid);
13         MergeSort(mid + 1, high);
14         // Combine the solutions.
15         Merge(low, mid, high);
16     }
17 }
```



```

1  Algorithm Merge(low, mid, high)
2  // a[low : high] is a global array containing two sorted
3  // subsets in a[low : mid] and in a[mid + 1 : high]. The goal
4  // is to merge these two sets into a single set residing
5  // in a[low : high]. b[ ] is an auxiliary global array.
6  {
7      h := low; i := low; j := mid + 1;
8      while ((h ≤ mid) and (j ≤ high)) do
9      {
10         if (a[h] ≤ a[j]) then
11         {
12             b[i] := a[h]; h := h + 1;
13         }
14         else
15         {
16             b[i] := a[j]; j := j + 1;
17         }
18         i := i + 1;
19     }
20     if (h > mid) then
21     for k := j to high do
22     {
23         b[i] := a[k]; i := i + 1;
24     }
25     else
26     for k := h to mid do
27     {
28         b[i] := a[k]; i := i + 1;
29     }
30     for k := low to high do a[k] := b[k];
31 }

```

```

1  Algorithm NQueens( $k, n$ )
2  // Using backtracking, this procedure prints all
3  // possible placements of  $n$  queens on an  $n \times n$ 
4  // chessboard so that they are nonattacking.
5  {
6      for  $i := 1$  to  $n$  do
7      {
8          if Place( $k, i$ ) then
9          {
10              $x[k] := i$ ;
11             if ( $k = n$ ) then write ( $x[1 : n]$ );
12             else NQueens( $k + 1, n$ );
13         }
14     }
15 }

```

```

1  Algorithm Place( $k, i$ )
2  // Returns true if a queen can be placed in  $k$ th row and
3  //  $i$ th column. Otherwise it returns false.  $x[ ]$  is a
4  // global array whose first  $(k - 1)$  values have been set.
5  // Abs( $r$ ) returns the absolute value of  $r$ .
6  {
7      for  $j := 1$  to  $k - 1$  do
8          if  $((x[j] = i) // \text{Two in the same column}$ 
9              or  $(\text{Abs}(x[j] - i) = \text{Abs}(j - k)))$ 
10             // or in the same diagonal
11             then return false;
12      return true;
13  }

```

```
1  Algorithm Partition( $a, m, p$ )
2  // Within  $a[m], a[m + 1], \dots, a[p - 1]$  the elements are
3  // rearranged in such a manner that if initially  $t = a[m]$ ,
4  // then after completion  $a[q] = t$  for some  $q$  between  $m$ 
5  // and  $p - 1$ ,  $a[k] \leq t$  for  $m \leq k < q$ , and  $a[k] \geq t$ 
6  // for  $q < k < p$ .  $q$  is returned. Set  $a[p] = \infty$ .
7  {
8       $v := a[m]; i := m; j := p;$ 
9      repeat
10     {
11         repeat
12              $i := i + 1;$ 
13         until ( $a[i] \geq v$ );
14
15         repeat
16              $j := j - 1;$ 
17         until ( $a[j] \leq v$ );
18
19         if ( $i < j$ ) then Interchange( $a, i, j$ );
20     } until ( $i \geq j$ );
21
22      $a[m] := a[j]; a[j] := v;$  return  $j$ ;
23 }

1  Algorithm Interchange( $a, i, j$ )
2  // Exchange  $a[i]$  with  $a[j]$ .
3  {
4       $p := a[i];$ 
5       $a[i] := a[j]; a[j] := p;$ 
6  }
```

```
1  Algorithm QuickSort( $p, q$ )
2  // Sorts the elements  $a[p], \dots, a[q]$  which reside in the global
3  // array  $a[1 : n]$  into ascending order;  $a[n + 1]$  is considered to
4  // be defined and must be  $\geq$  all the elements in  $a[1 : n]$ .
5  {
6      if ( $p < q$ ) then // If there are more than one element
7      {
8          // divide  $P$  into two subproblems.
9           $j := \text{Partition}(a, p, q + 1)$ ;
10         //  $j$  is the position of the partitioning element.
11         // Solve the subproblems.
12         QuickSort( $p, j - 1$ );
13         QuickSort( $j + 1, q$ );
14         // There is no need for combining solutions.
15     }
16 }
```

```
1  Algorithm RQuickSort( $p, q$ )
2  // Sorts the elements  $a[p], \dots, a[q]$  which reside in the global
3  // array  $a[1 : n]$  into ascending order.  $a[n + 1]$  is considered to
4  // be defined and must be  $\geq$  all the elements in  $a[1 : n]$ .
5  {
6      if ( $p < q$ ) then
7      {
8          if ( $(q - p) > 5$ ) then
9              Interchange( $a, \text{Random}() \bmod (q - p + 1) + p, p$ );
10              $j := \text{Partition}(a, p, q + 1)$ ;
11             //  $j$  is the position of the partitioning element.
12             RQuickSort( $p, j - 1$ );
13             RQuickSort( $j + 1, q$ );
14     }
15 }
```

```

1  Algorithm SumOfSub( $s, k, r$ )
2  // Find all subsets of  $w[1 : n]$  that sum to  $m$ . The values of  $x[j]$ ,
3  //  $1 \leq j < k$ , have already been determined.  $s = \sum_{j=1}^{k-1} w[j] * x[j]$ 
4  // and  $r = \sum_{j=k}^n w[j]$ . The  $w[j]$ 's are in nondecreasing order.
5  // It is assumed that  $w[1] \leq m$  and  $\sum_{i=1}^n w[i] \geq m$ .
6  {
7      // Generate left child. Note:  $s + w[k] \leq m$  since  $B_{k-1}$  is true.
8       $x[k] := 1$ ;
9      if ( $s + w[k] = m$ ) then write ( $x[1 : k]$ ); // Subset found
10     // There is no recursive call here as  $w[j] > 0$ ,  $1 \leq j \leq n$ .
11     else if ( $s + w[k] + w[k + 1] \leq m$ )
12         then SumOfSub( $s + w[k], k + 1, r - w[k]$ );
13     // Generate right child and evaluate  $B_k$ .
14     if ( $(s + r - w[k] \geq m)$  and ( $s + w[k + 1] \leq m$ )) then
15     {
16          $x[k] := 0$ ;
17         SumOfSub( $s, k + 1, r - w[k]$ );
18     }
19 }

```
