Technical Appendix

1 Loading the libraries

```
library(ggplot2)
library(tidyverse)
library(dplyr)
library(leaps)
library(MASS)
library(foreign)
library(lme4)
library(knitr)
library(HLMdiag)
library(boot)
library(arm)
library(car)
```

```
ratings <- read.csv("ratings.csv")
ratings <- ratings[-c(1, 26)]
```

1.1 Exploring the data set

1.2 Harmony

- Four unique values

```
unique(ratings$Harmony)
```

```
## [1] "I-IV-V" "I-V-IV" "I-V-VI" "IV-I-V"
```

```
summary(ratings$Harmony)
```

```
##    Length     Class      Mode
##      2520 character character
```

1.3 Instrument

- Three unique values

```
unique(ratings$Instrument)
```

```
## [1] "guitar" "piano"  "string"
```

```
summary(ratings$Instrument)
```

```
##    Length     Class      Mode
##      2520 character character
```

1.4 Voice

- Three unique values

```
unique(ratings$Voice)
```

```
## [1] "contrary" "par3rd"   "par5th"
```

```
summary(ratings$Voice)
```

```
##    Length     Class      Mode
##      2520 character character
```

1.5 Selfdeclare

- 6 unique values
- 2 is most common
- Mean value is 2.443

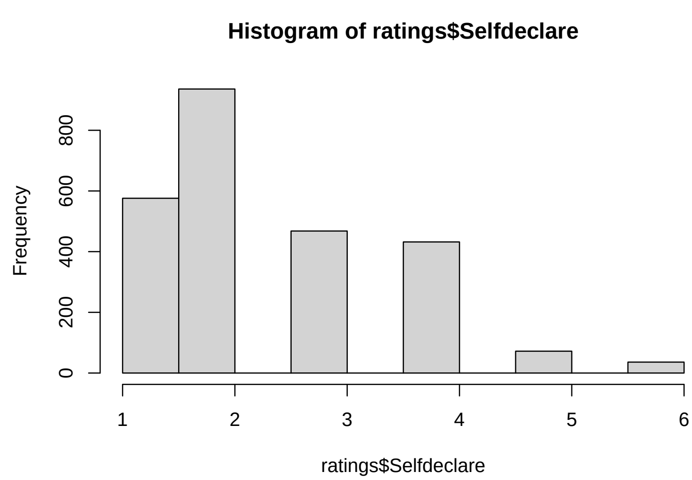
```
unique(ratings$Selfdeclare)
```

```
## [1] 5 1 2 4 3 6
```

```
summary(ratings$Selfdeclare)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.000   2.000   2.000   2.443   3.000   6.000
```

```
hist(ratings$Selfdeclare)
```



1.6 OMSI

- 60 unique values
- Mean is 145.5
- Histogram is right skewed

```
length(sort(unique(ratings$OMSI)))
```

```
## [1] 60
```

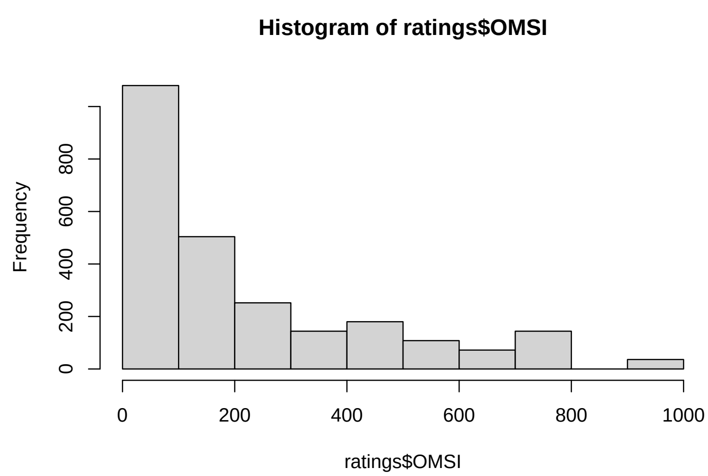
```
sort(unique(ratings$OMSI))
```

```
##  [1]  11  14  15  18  20  21  23  29  30  31  38  40  44  46  49  55  67  68  82
## [20]  88  94  96  97 122 127 142 145 146 147 150 164 179 180 194 199 201 204 233
## [39] 234 259 277 319 323 325 345 421 425 466 481 482 541 567 586 642 649 734 749
## [58] 759 784 970
```

```
summary(ratings$OMSI)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    11.0    49.0   145.5   225.9   323.0   970.0
```

```
hist(ratings$OMSI)
```



1.7 X16.minus.17

- 13 unique values
- Mean is 1.721
- Histogram has slight right skewed

```r
length(sort(unique(ratings$X16.minus.17)))
```

```
## [1] 13
```

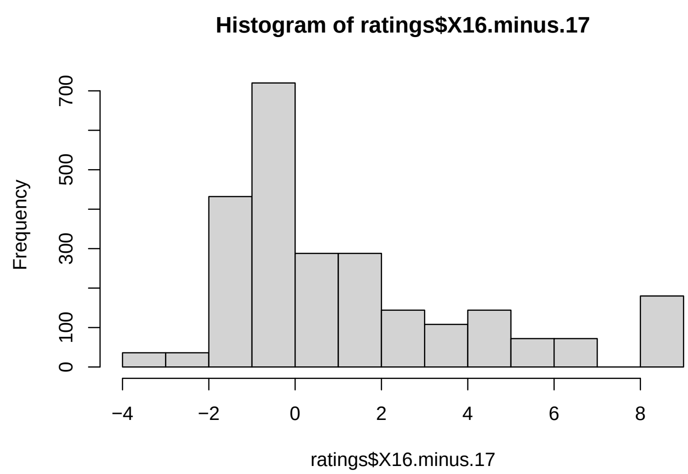
```r
sort(unique(ratings$X16.minus.17))
```

```
##  [1] -4.0 -2.0 -1.0 -0.5  0.0  1.0  2.0  3.0  4.0  5.0  6.0  7.0  9.0
```

```r
summary(ratings$X16.minus.17)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  -4.000   0.000   1.000   1.721   3.000   9.000
```

```r
hist(ratings$X16.minus.17)
```



1.8 ConsInstr

- 14 unique values
- Mean is 2.857
- Histogram is mostly uniformly distributed

```r
length(sort(unique(ratings$ConsInstr)))
```

```
## [1] 14
```

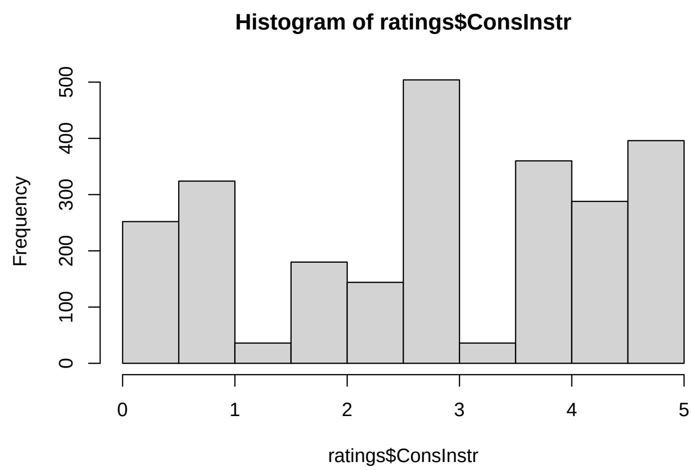
```r
sort(unique(ratings$ConsInstr))
```

```
##  [1] 0.00 0.67 1.00 1.33 1.67 2.00 2.33 2.67 3.00 3.33 3.67 4.00 4.33 5.00
```

```
summary(ratings$ConsInstr)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   1.670   3.000   2.857   4.330   5.000
```

```
hist(ratings$ConsInstr)
```



1.9 ConsNotes

- 5 factor levels with some NA values
- Mean is 2.533
- Histogram has three peaks and values $0, 3$, and 5 are most common

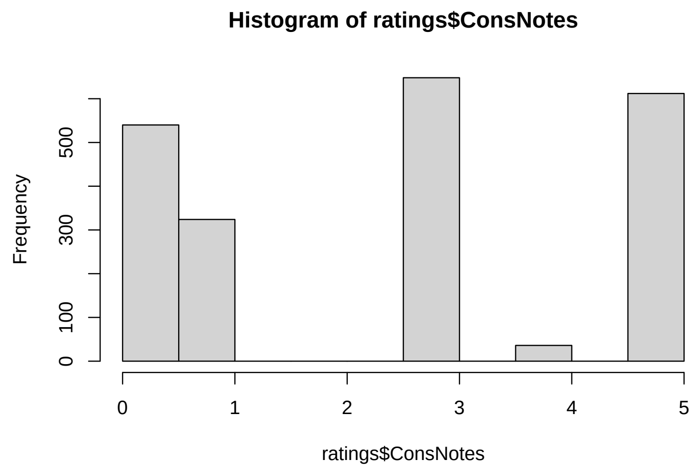
```
unique(ratings$ConsNotes)
```

```
## [1]  5 NA  0  3  1  4
```

```
summary(ratings$ConsNotes)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   0.750   3.000   2.533   5.000   5.000     360
```

```
hist(ratings$ConsNotes)
```



1.10 Instr.minus.Notes

- 20 unique values
- Mean is 0.6857
- Distribution is normal with one peak

```
length(sort(unique(ratings$Instr.minus.Notes)))
```

```
## [1] 20
```

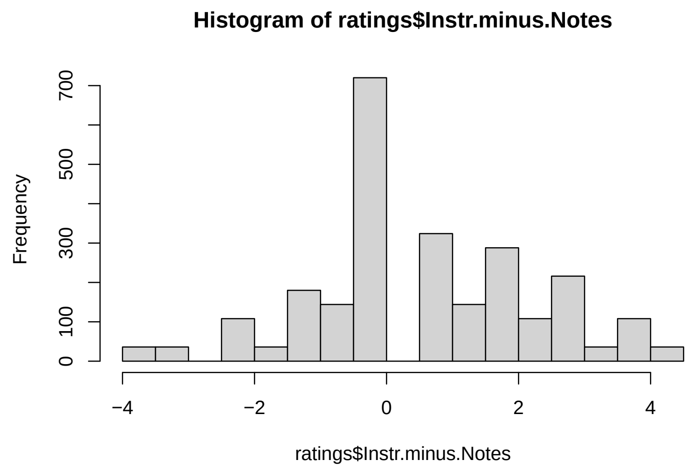
```
sort(unique(ratings$Instr.minus.Notes))
```

```
##  [1] -4.00 -3.00 -2.00 -1.67 -1.33 -1.00 -0.67  0.00  0.67  1.00  1.33  1.67
## [13]  2.00  2.33  2.67  3.00  3.33  3.67  4.00  4.33
```

```
summary(ratings$Instr.minus.Notes)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -4.0000  0.0000  0.3350  0.6857  2.0000  4.3300
```

```
hist(ratings$Instr.minus.Notes)
```



1.11 PachListen

- 6 factor levels with some NA values
- Mean is 4.515
- 5 is most common
- Distribution is highly left skewed

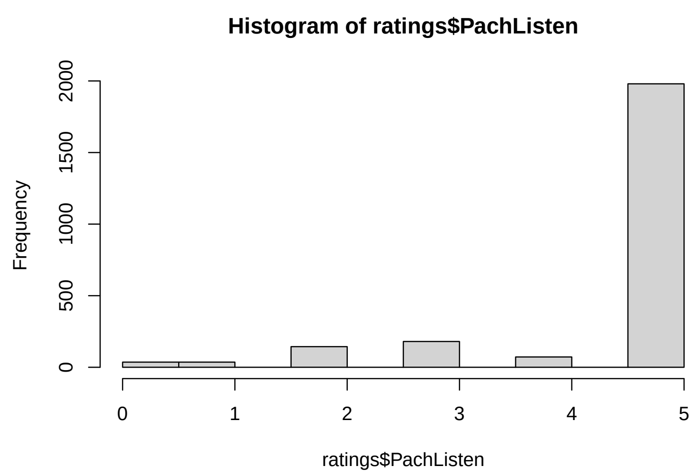
```
unique(ratings$PachListen)
```

```
## [1]  5  3 NA  2  1  4  0
```

```
summary(ratings$PachListen)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   5.000   5.000   4.515   5.000   5.000      72
```

```
hist(ratings$PachListen)
```



1.12 ClsListen

- 6 factor levels with some NA values
- Mean is 2.159
- 1 and 3 are most common
- Histogram has three peaks

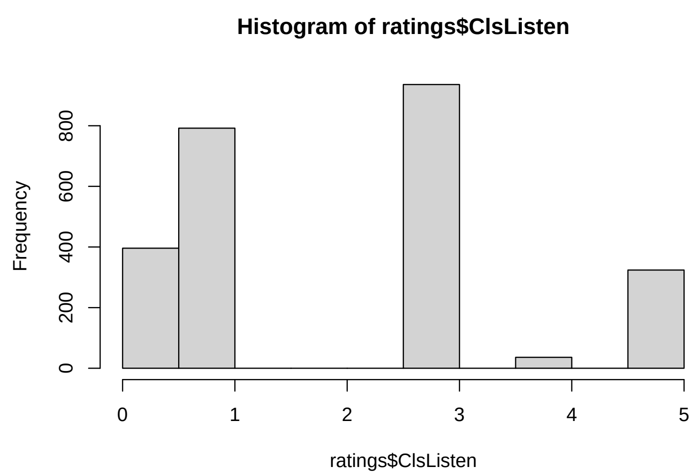
```
unique(ratings$ClsListen)
```

```
## [1]  4  0  1 NA  3  5
```

```
summary(ratings$ClsListen)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   1.000   3.000   2.159   3.000   5.000      36
```

```
hist(ratings$ClsListen)
```



1.13 KnowRob

- 6 factor levels with some NA values
- Mean is 0.7692
- 0 is most common
- Histogram has one peak and is highly right skewed

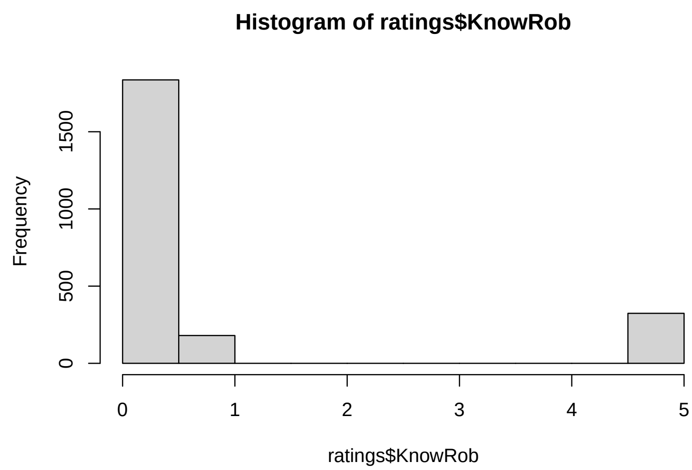
```
unique(ratings$KnowRob)
```

```
## [1]  0 NA  5  1
```

```r
summary(ratings$KnowRob)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##  0.0000  0.0000  0.0000  0.7692  0.0000  5.0000     180
```

```r
hist(ratings$KnowRob)
```



1.14 KnowAxis

- 6 factor levels with some NA values
- Mean is 0.9032
- 0 is most common
- Histogram has one peak and is highly right skewed

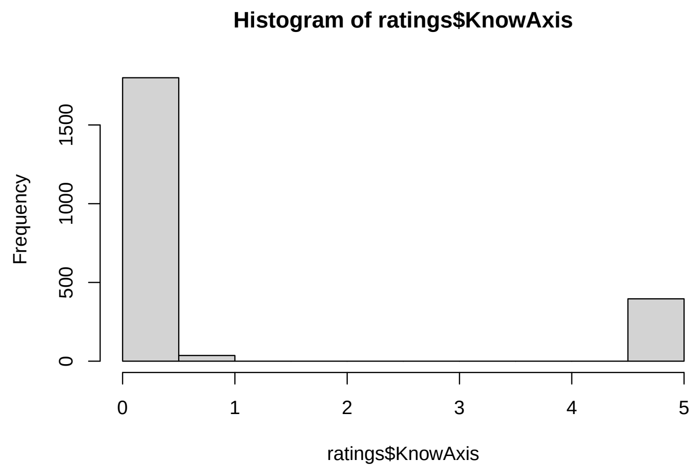
```r
unique(ratings$KnowAxis)
```

```
## [1]  0 NA  5  1
```

```r
summary(ratings$KnowAxis)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##  0.0000  0.0000  0.0000  0.9032  0.0000  5.0000     288
```

```r
hist(ratings$KnowAxis)
```



1.15 X1990s2000s

- 6 factor levels with some NA values
- Mean is 4.061
- 5 is most common
- Histogram has one peak and is highly left skewed

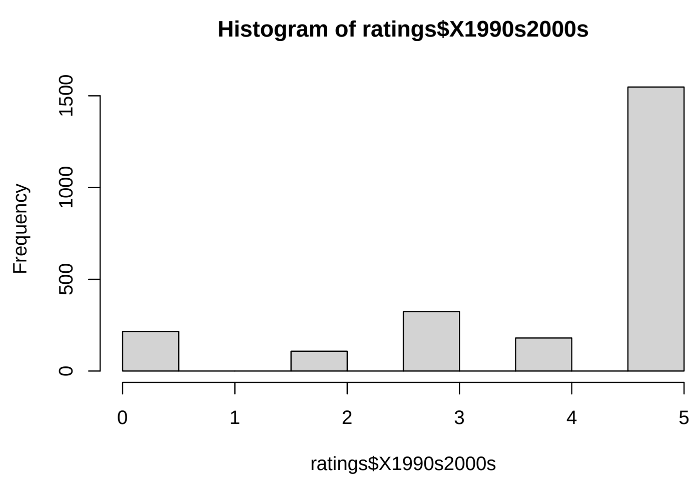
```
unique(ratings$X1990s2000s)
```

```
## [1]  5 NA  0  3  2  4
```

```
summary(ratings$X1990s2000s)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   3.000   5.000   4.061   5.000   5.000     144
```

```
hist(ratings$X1990s2000s)
```



1.16 X1990s2000s.minus.1960s1970s

- 9 unique values with some NA values
- Mean is 2.015
- 0 and 3 are most common
- Histogram has mostly uniformly distributed

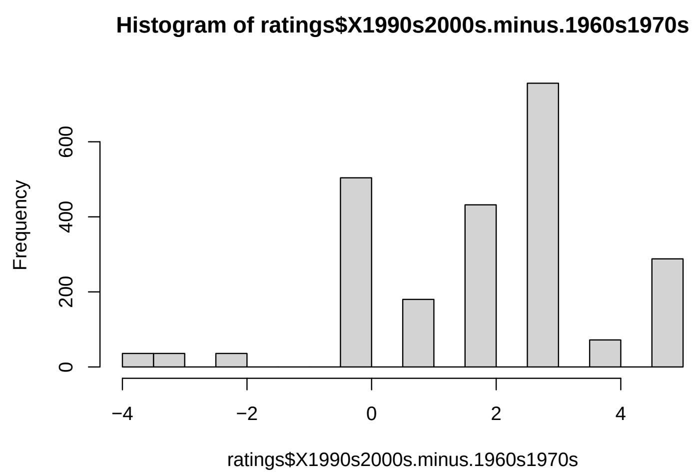
```
unique(ratings$X1990s2000s.minus.1960s1970s)
```

```
## [1]  2  3  5 NA  4  0  1 -2 -4 -3
```

```
summary(ratings$X1990s2000s.minus.1960s1970s)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##  -4.000   0.000   2.000   2.015   3.000   5.000     180
```

```
hist(ratings$X1990s2000s.minus.1960s1970s)
```



1.17 CollegeMusic

- 2 factor levels with some NA values
- Mean is 0.791 > 0.5 meaning 1 is more common than 0

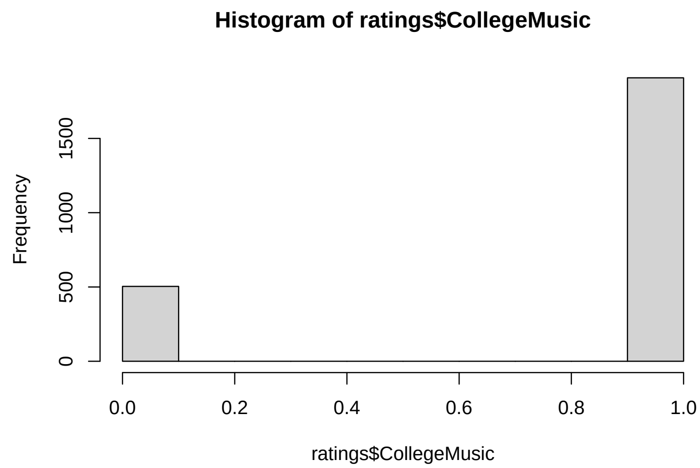
```
unique(ratings$CollegeMusic)
```

```
## [1]  0  1 NA
```

```
summary(ratings$CollegeMusic)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   1.000   1.000   0.791   1.000   1.000     108
```

```r
hist(ratings$CollegeMusic)
```



1.18 `NoClass`

- 6 factor levels with some NA values
- Mean is 0.9194
- Histogram is highly right skewed

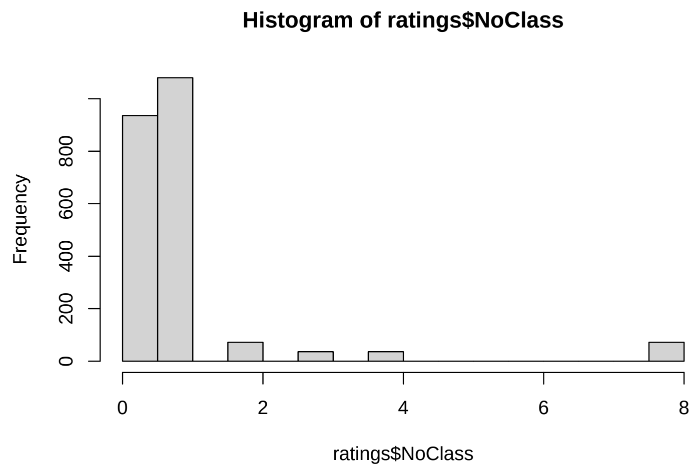
```r
unique(ratings$NoClass)
```

```
## [1]  0  1 NA  4  3  8  2
```

```r
summary(ratings$NoClass)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's 
##  0.0000  0.0000  1.0000  0.9194  1.0000  8.0000     288
```

```r
hist(ratings$NoClass)
```



1.19 APTheory

- 2 factor levels with some NA values
- Mean is 0.2344 < 0.5 meaning 0 is more common than 1

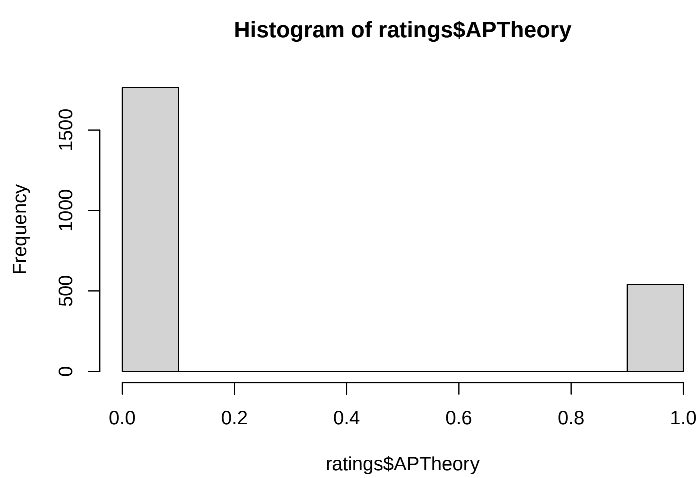
```
unique(ratings$APTheory)
```

```
## [1]  0 NA  1
```

```
summary(ratings$APTheory)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##  0.0000  0.0000  0.0000  0.2344  0.0000  1.0000     216
```

```
hist(ratings$APTheory)
```



1.20 Composing

- 6 factor levels with some NA values
- Mean is 1
- Histogram is highly right skewed

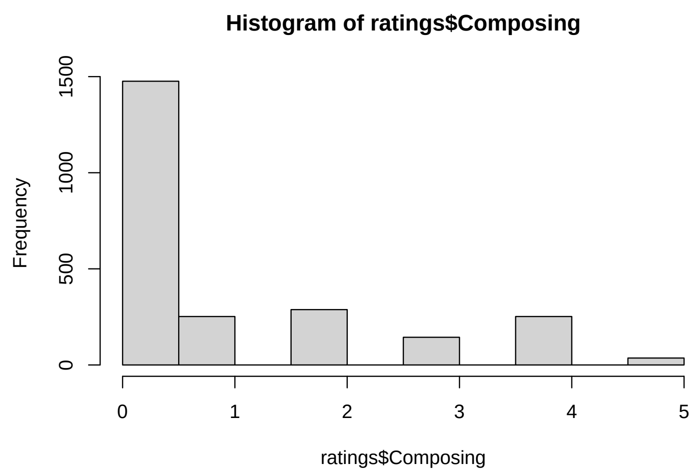
```
unique(ratings$Composing)
```

```
## [1]  4  0  1  2 NA  3  5
```

```
summary(ratings$Composing)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##       0       0       0       1       2       5      72
```

```
hist(ratings$Composing)
```



1.21 PianoPlay

- 6 factor levels with no NA values
- Mean is 1.086
- Histogram is right skewed

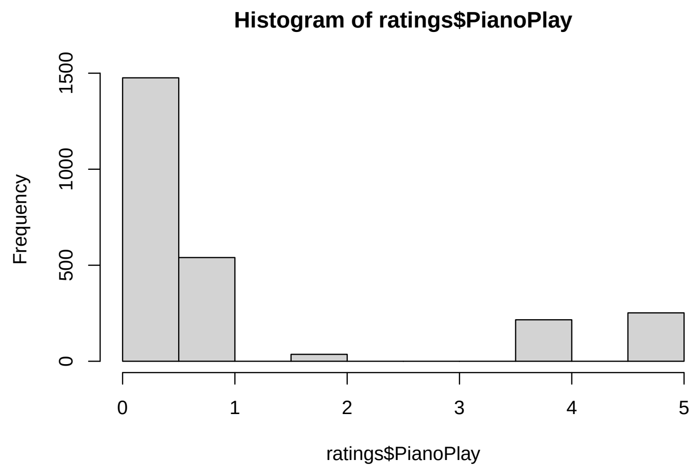
```
unique(ratings$PianoPlay)
```

```
## [1] 1 0 5 4 2
```

```
summary(ratings$PianoPlay)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   0.000   1.086   1.000   5.000
```

```r
hist(ratings$PianoPlay)
```



1.22 GuitarPlay

- 6 factor levels with no NA values
- Mean is 0.6857
- Histogram is highly right skewed

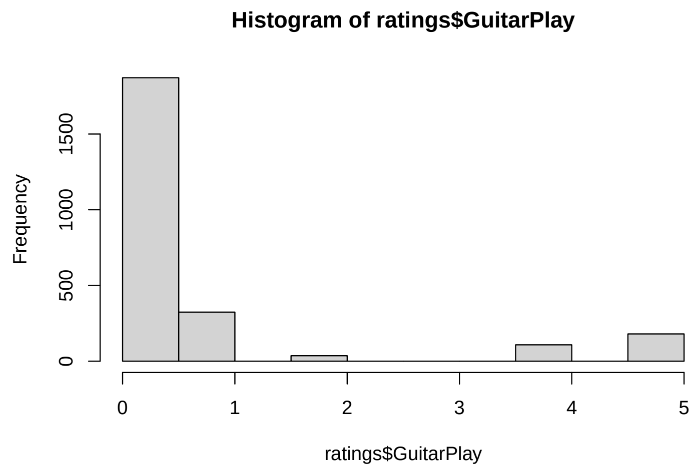
```r
unique(ratings$GuitarPlay)
```

```
## [1] 5 0 1 4 2
```

```r
summary(ratings$GuitarPlay)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.0000  0.0000  0.0000  0.6857  1.0000  5.0000
```

```r
hist(ratings$GuitarPlay)
```



1.23 X1stInstr

- 6 factor levels with some NA values
- Mean is 2.786
- 1 and 4 are most common values
- Histogram is has no skew and two peaks

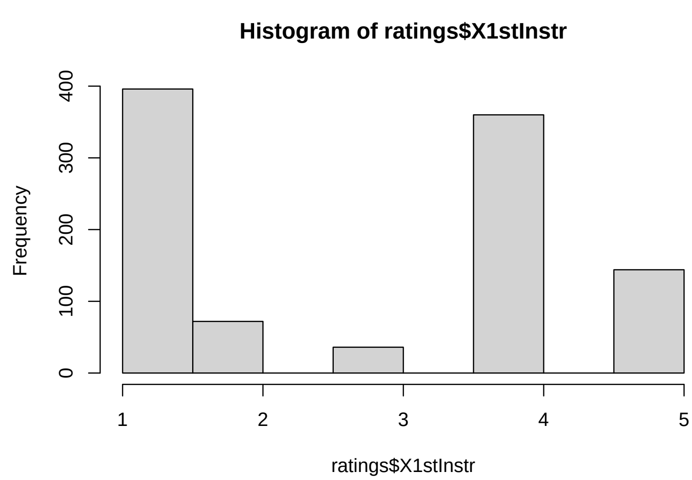
```
unique(ratings$X1stInstr)
```

```
## [1]  4  3 NA  1  5  2
```

```
summary(ratings$X1stInstr)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   1.000   1.000   3.500   2.786   4.000   5.000    1512
```

```
hist(ratings$X1stInstr)
```



1.24 X1stInstr

- 6 factor levels with some NA values
- Mean is 1.556
- 1 is the most common value
- Histogram is right skewed with one peak

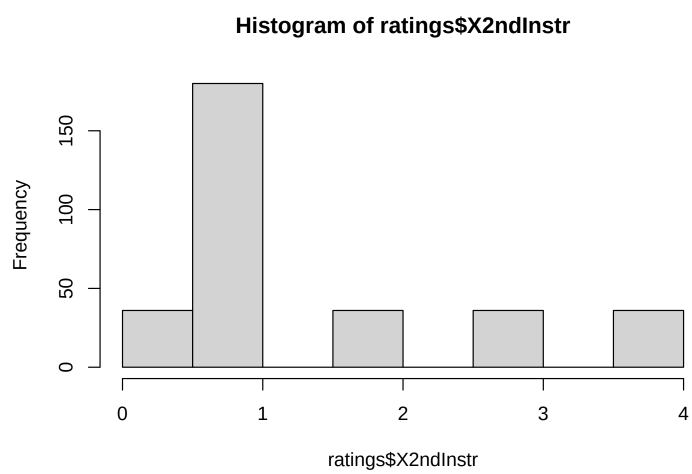
```
unique(ratings$X2ndInstr)
```

```
## [1] NA  1  0  4  2  3
```

```
summary(ratings$X2ndInstr)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   1.000   1.000   1.556   2.000   4.000    2196
```

```
hist(ratings$X2ndInstr)
```



1.25 Classical

- 17 unique values with some NA values
- One of the unique values is 19, which is greater than the scale that the participants were presented with. We see that it occurs only once, so it can be reasonably inferred that this might be an error input. Hence, we will remove it from our data set
- Mean is 5.783
- Distribution is normal

```
length(unique(ratings$Classical))
```

```
## [1] 17
```

```
unique(ratings$Classical)
```

```
##  [1]  3.0  1.0  2.0  8.0 10.0  6.0  5.0  4.0  9.0  7.0   NA  0.0 19.0  9.5  4.6
## [16]  3.5  4.2
```

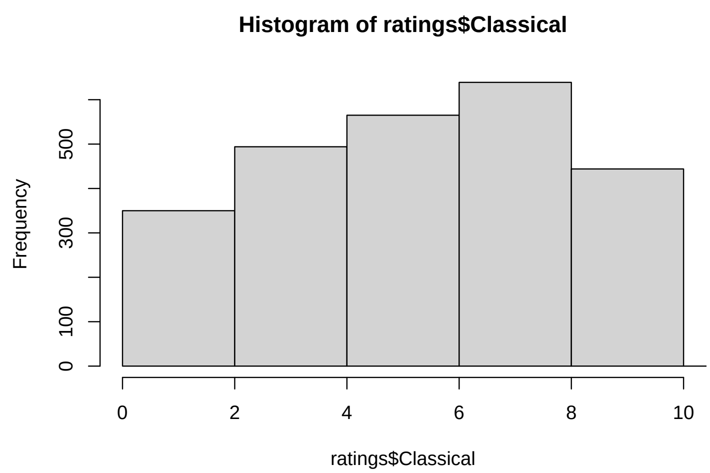
```
summary(ratings$Classical)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
##   0.000   4.000   6.000   5.783   8.000  19.000      27
```

```
count(ratings[which(ratings$Classical > 10), ])
```

```
##   n
## 1 1
```

```
hist(ratings$Classical, xlim = c(0, 10))
```



1.26 Popular

- 17 unique values with some NA values
- One of the unique values is 19, which is greater than the scale that the participants were presented with. We see that it occurs only once, so it can be reasonably inferred that this might be an error input. Hence, we will remove it from our data set
- Mean is 5.381
- Distribution is normal

```
length(unique(ratings$Popular))
```

```
## [1] 17
```

```
unique(ratings$Popular)
```

```
##  [1]  9.0  7.0  8.0  3.0  1.0  4.0  5.0  6.0  2.0 10.0  0.0   NA 19.0  3.5  4.6
## [16]  6.8  4.2
```

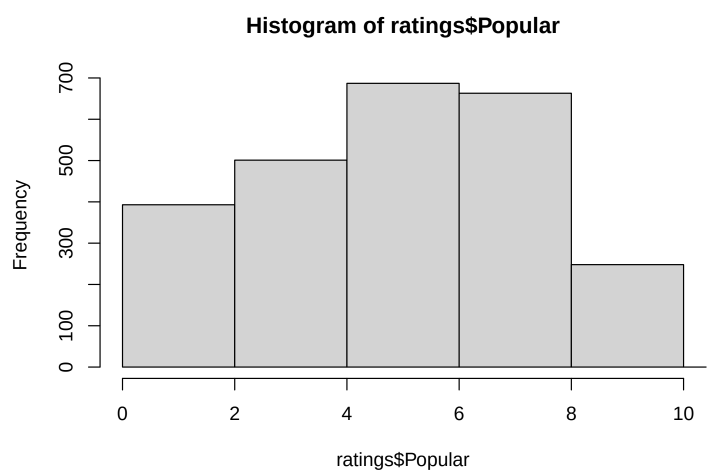
```
summary(ratings$Popular)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's 
##   0.000   4.000   5.000   5.381   7.000  19.000      27
```

```
count(ratings[which(ratings$Popular > 10), ])
```

```
##   n
## 1 1
```

```
hist(ratings$Popular, xlim = c(0, 10))
```



1.27 Creating a preliminary OLS mode

We first create a linear model including three-way interactions between all the predictors `Instrument`, `Harmony`, and `Voice`. Upon running the `anova()` function, we see that the only interaction that is statistically significant at the 5% level of significance is `Harmony:Voice`, we keep only that. Then, we run the `anova()` function between the current and previous model to compare performance, which confirms that the second model is better.

```
model_1 <- lm(Classical ~ Instrument * Harmony * Voice,
              data = ratings)

anova(model_1)
```

```
## Analysis of Variance Table
##
## Response: Classical
##                              Df  Sum Sq Mean Sq  F value    Pr(>F)
## Instrument                    2  4127.9 2063.96 391.7698 < 2.2e-16 ***
## Harmony                       3   273.6   91.20  17.3120  4.01e-11 ***
## Voice                         2    85.6   42.82   8.1278 0.0003032 ***
## Instrument:Harmony            6    10.4    1.74   0.3305 0.9211803
## Instrument:Voice              4     9.5    2.37   0.4504 0.7722123
## Harmony:Voice                 6    81.2   13.53   2.5691 0.0175040 *
## Instrument:Harmony:Voice     12    62.1    5.18   0.9829 0.4626550
## Residuals                  2457 12944.2    5.27
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
model_2 <- lm(Classical ~ Instrument + Harmony + Voice + Harmony:Voice,
              data = ratings)

anova(model_1, model_2)
```

```
## Analysis of Variance Table
##
## Model 1: Classical ~ Instrument * Harmony * Voice
## Model 2: Classical ~ Instrument + Harmony + Voice + Harmony:Voice
##   Res.Df   RSS  Df Sum of Sq      F Pr(>F)
## 1   2457 12944
## 2   2479 13026 -22   -82.005 0.7075 0.8362
```

1.28 Creating a multi-level model

We then create a multi-level model with only a random intercept for each individual.

```
multilevel_model_1 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             (1 | Subject),
                           data = ratings,
                           REML = FALSE)
```

1.29 Examining the performance of MLM

We examine the influence of the three main experimental factors on the initial MLM using couple of approaches:

- **ANOVA:** We run the `anova()` function which confirms that the effect of these three experimental factors is significant.
- **Plotting residuals:** We plot the conditional and marginal residuals as a function of conditional and fitted values respectively. The smooth fitted line for both is almost a horizontal line centered at zero suggesting that the data fits our current model well.

- **Checking normality:** The standardized residuals and standardized random effects are normally distributed suggesting a good fit.
- **Fixed and random effect variances:** From the `summary()` output, we see that $\hat{\tau}_0^2 = 1.678$ and $\hat{\sigma}_0^2 = 3.537$. While these are not as low as we would like, suggesting possible scope of improvement within the model, it is a good starting point.
- **AIC/BIC:** Upon computing the AIC and BIC for the OLS `lm()` model and multi-level `lmer()` model, we see that the `lmer()` model has lower values for both AIC and BIC relative to the `lm()` model.

Based on the above findings, we decide that including the random effects part has improved the original model and thus we will keep it.

```
anova(multilevel_model_1)
```

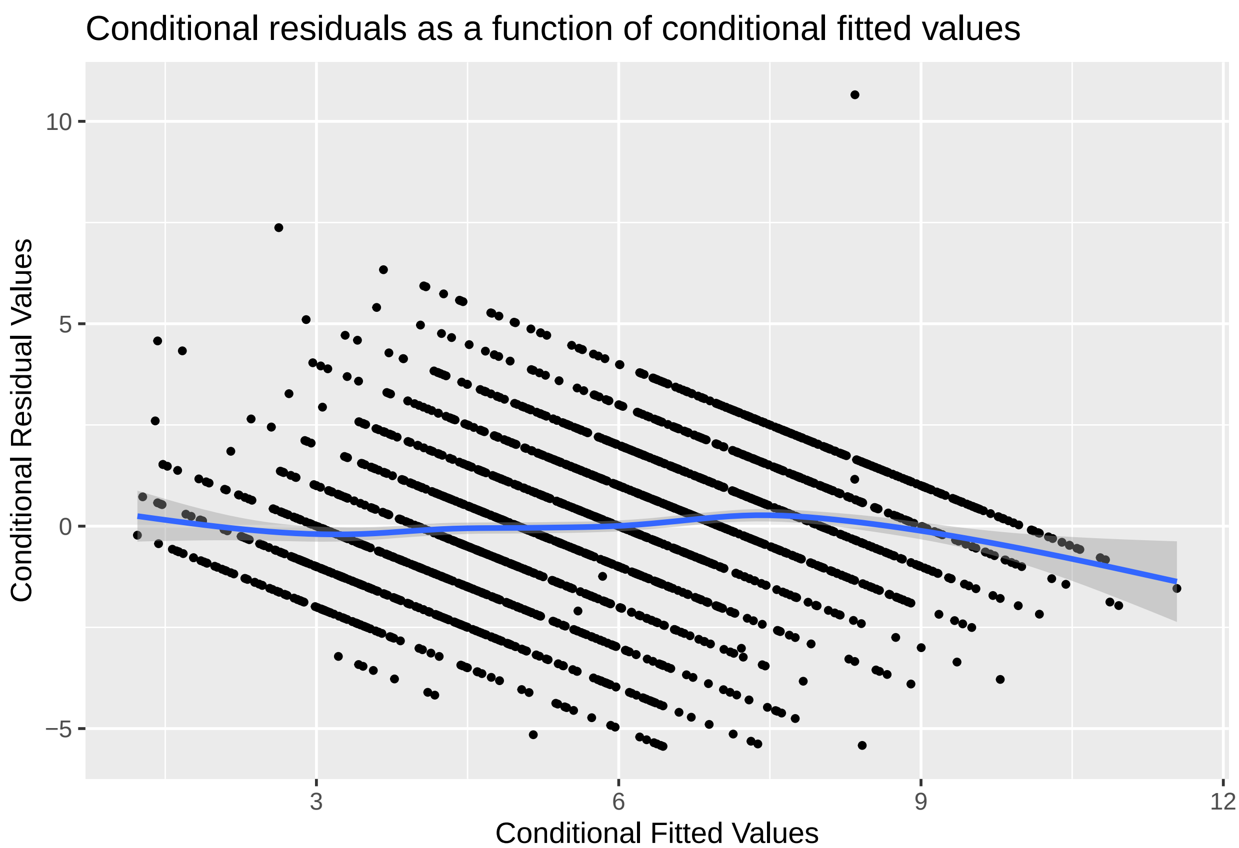
```
## Analysis of Variance Table
##                npar Sum Sq Mean Sq  F value
## Instrument        2 4119.1 2059.53 582.2537
## Harmony           3  275.4   91.79  25.9495
## Voice             2   87.0   43.49  12.2958
## Harmony:Voice     6   80.9   13.48   3.8108
```

```
summary(multilevel_model_1)
```

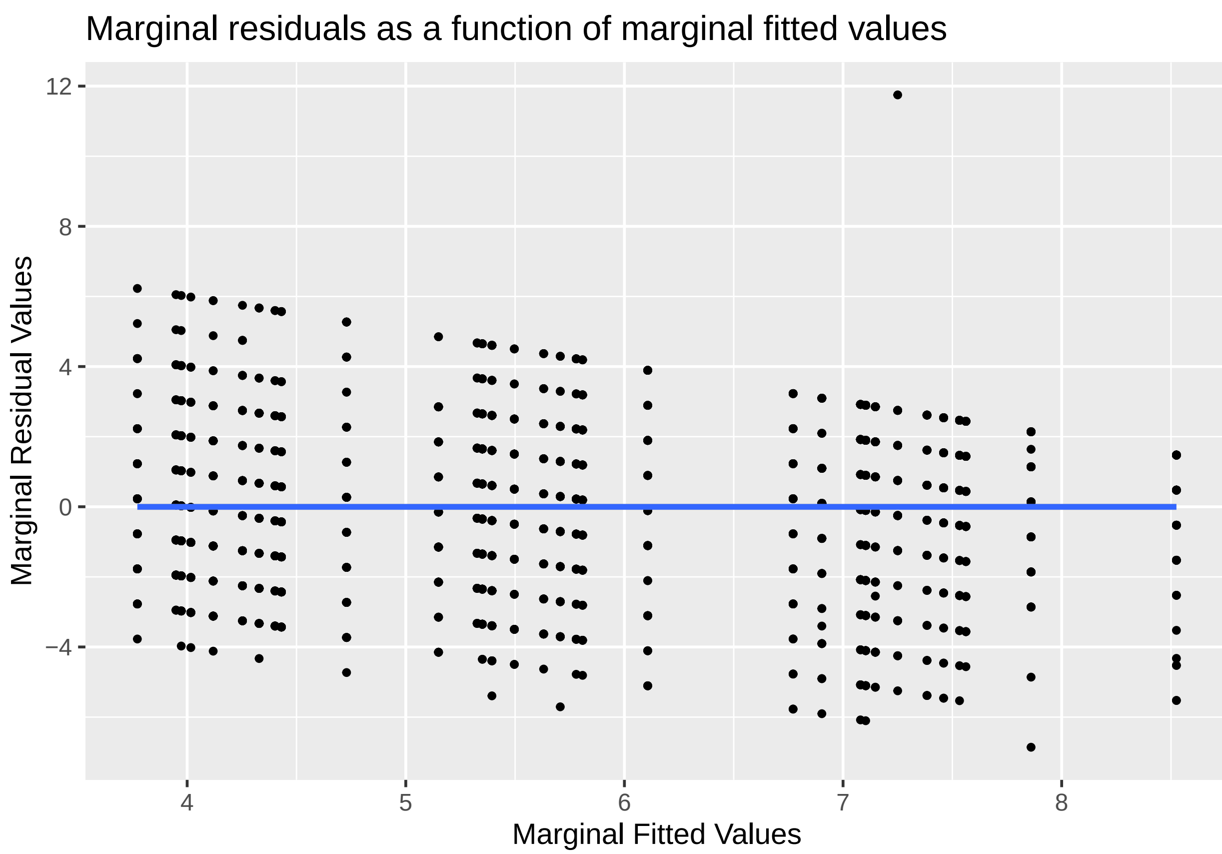
```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Instrument + Harmony + Voice + Harmony:Voice + (1 |
##     Subject)
##    Data: ratings
##
##      AIC      BIC   logLik deviance df.resid
##  10458.1  10551.2  -5213.1  10426.1     2477
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.8924 -0.6212 -0.0165  0.6392  5.6657
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  Subject  (Intercept) 1.678    1.295
##  Residual             3.537    1.881
## Number of obs: 2493, groups:  Subject, 70
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                  4.25306    0.20907  20.342
## Instrumentpiano              1.37746    0.09261  14.873
## Instrumentstring             3.13086    0.09200  34.030
## HarmonyI-V-IV                0.14892    0.18469   0.806
## HarmonyI-V-VI                1.14100    0.18445   6.186
## HarmonyIV-I-V               -0.13397    0.18398  -0.728
## Voicepar3rd                 -0.28018    0.18400  -1.523
## Voicepar5th                 -0.23618    0.18444  -1.281
## HarmonyI-V-IV:Voicepar3rd -0.34960    0.26072  -1.341
```

```
## HarmonyI-V-VI:Voicepar3rd -0.68277    0.26100  -2.616
## HarmonyIV-I-V:Voicepar3rd  0.49026    0.26068   1.881
## HarmonyI-V-IV:Voicepar5th -0.19316    0.26130  -0.739
## HarmonyI-V-VI:Voicepar5th -0.42874    0.26087  -1.644
## HarmonyIV-I-V:Voicepar5th  0.06604    0.26051   0.254
```

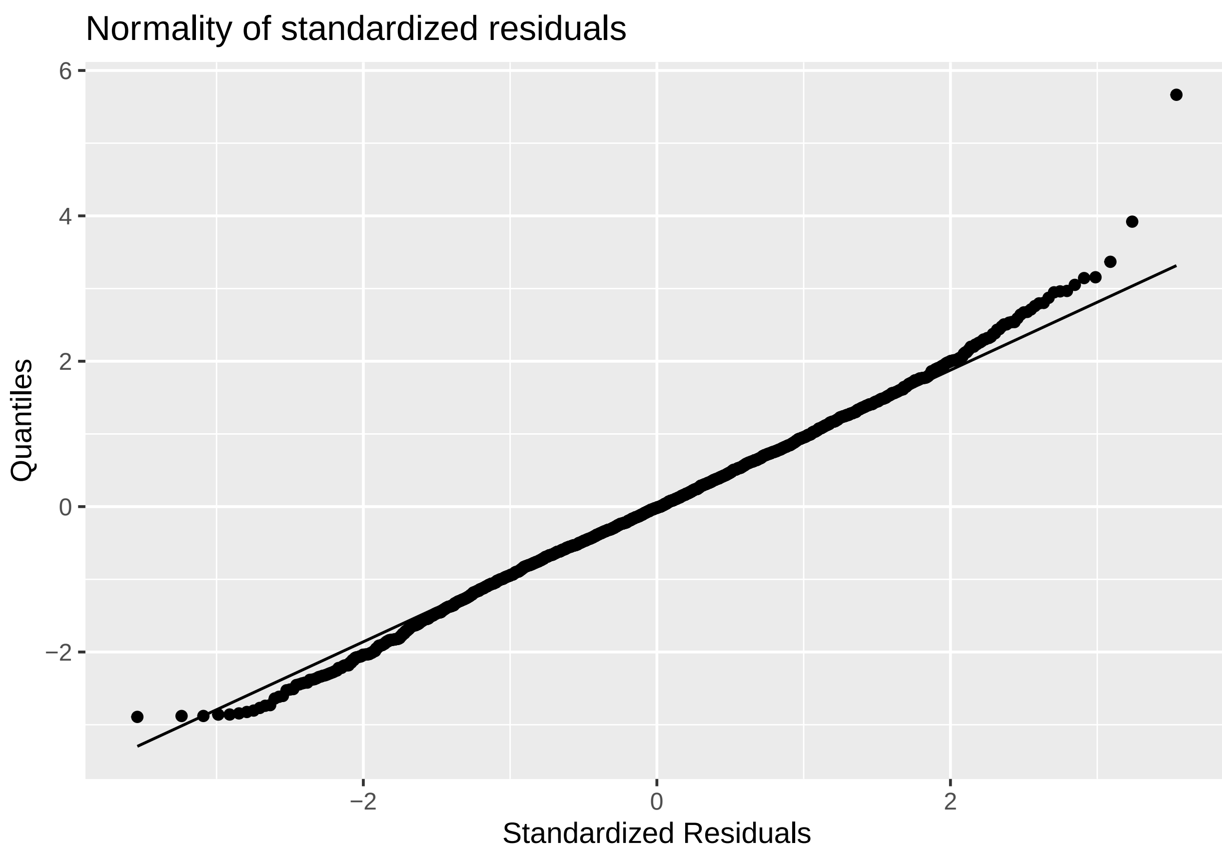
```
residuals_11 <- hlm_resid(multilevel_model_1,
                          level = 1,
                          include.ls = F)

residuals_11_std <- hlm_resid(multilevel_model_1,
                              level = 1,
                              include.ls = F,
                              standardize = T)

residuals_12 <- hlm_resid(multilevel_model_1,
                          level = "Subject",
                          include.ls = F)

residuals_12_std <- hlm_resid(multilevel_model_1,
                              level = "Subject",
                              include.ls = F,
                              standardize = T)

std_resid <- residuals_11_std$.std.resid
std_ranef_intercept <- residuals_12_std$.std.ranef.intercept

conditional_plot_1 <- ggplot(data = as.data.frame(residuals_11),
                             mapping = aes(y = residuals_11$.resid,
                                           residuals_11$.fitted)) +
  xlab("Conditional Fitted Values") +
  ylab("Conditional Residual Values") +
  ggtitle("Conditional residuals as a function of conditional fitted values") +
  geom_point(pch = 20) +
  geom_smooth()

conditional_plot_1
```



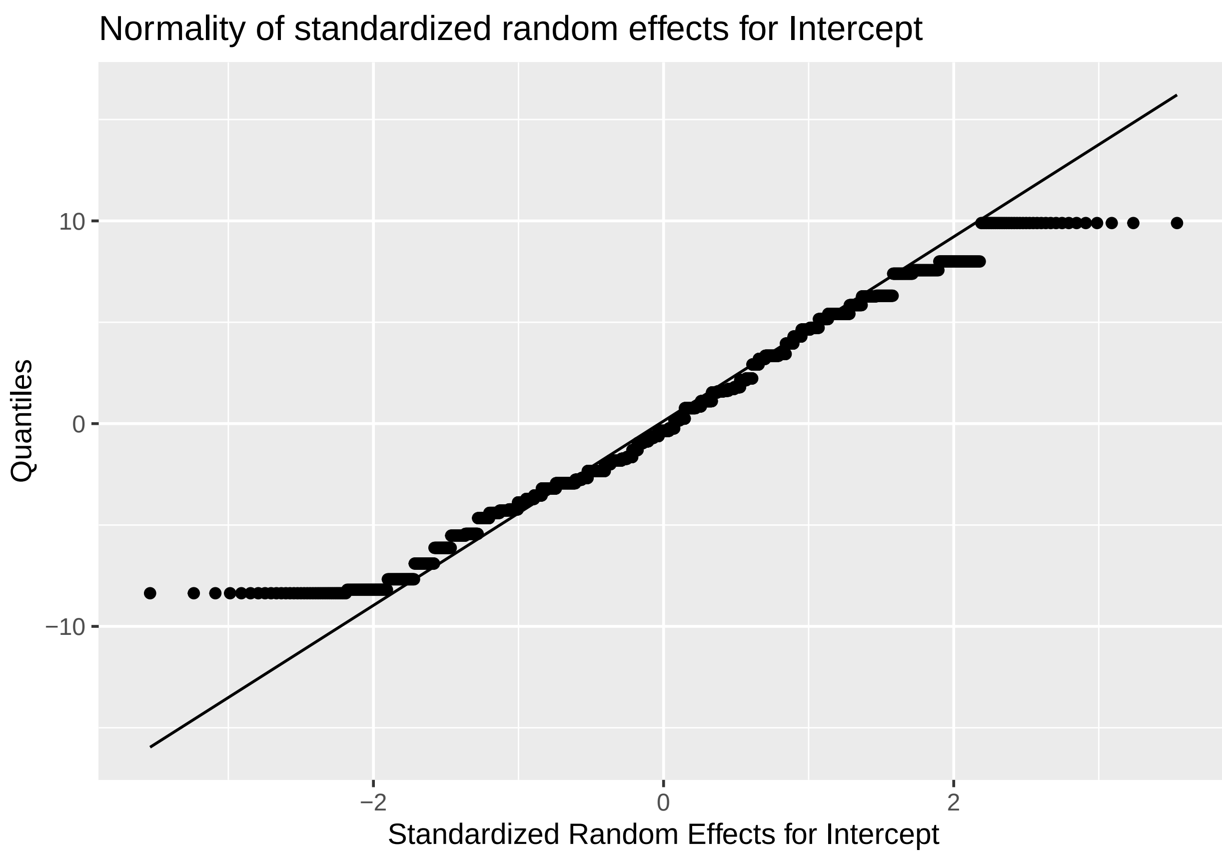

```
ggsave('conditional_plot_1.png')

marginal_plot_1 <- ggplot(data = as.data.frame(residuals_11),
                          mapping = aes(y = residuals_11$.mar.resid,
                                        x = residuals_11$.mar.fitted)) +
  xlab("Marginal Fitted Values") +
  ylab("Marginal Residual Values") +
  ggtitle("Marginal residuals as a function of marginal fitted values") +
  geom_point(pch = 20) +
  geom_smooth()

marginal_plot_1
```




```
ggsave('marginal_plot_1.png')

params <- data.frame(cbind(std_resid, std_ranef_intercept))

plot_std_resid_1 <- params %>%
  ggplot(aes(sample = std_resid)) +
  stat_qq() +
  xlab("Standardized Residuals") +
  ylab("Quantiles") +
  ggtitle("Normality of standardized residuals") +
  stat_qq_line()

plot_std_resid_1
```




```
ggsave('plot_std_resid_1.png')

plot_std_ranef_intercept_1 <- params %>%
  ggplot(aes(sample = std_ranef_intercept)) +
  stat_qq() +
  xlab("Standardized Random Effects for Intercept") +
  ylab("Quantiles") +
  ggtitle("Normality of standardized random effects for Intercept") +
  stat_qq_line()

plot_std_ranef_intercept_1
```




```
ggsave('plot_std_ranef_intercept_1.png')
```

```
AIC_BIC_DIC <- cbind(AIC = sapply(list(model_2, multilevel_model_1),
                                  AIC),

                     BIC = sapply(list(model_2, multilevel_model_1),
                                  BIC),

                     DIC = sapply(list(multilevel_model_1, multilevel_model_1),
                                  extractDIC))

AIC_BIC_DIC <- t(as.tibble(AIC_BIC_DIC))
AIC_BIC_DIC[3, 1] = AIC_BIC_DIC[1, 1]
colnames(AIC_BIC_DIC) <- c("model_2", "multilevel_model_1")
kable(t(AIC_BIC_DIC), digits = 2)
```

	AIC	BIC	DIC
model_2	11226.94	11314.26	11226.94
multilevel_model_1	10458.10	10551.24	10426.10

1.30 Evaluating random effects for `Instrument`, `Harmony`, and `Voice`

As a next step, we test if our model could be improved by adding random slops for the three design variables. This is done by creating seven new `lmer()` models as follows in which we have included varying combinations of random effects for person/instrument, person/harmony, and person/voice. Upon computing the AIC, BIC, and DIC values for them all, we see that the multi-level model containing the random effect estimates for `(1 | Subject) + (0 + Instrument | Subject) + (0 + Harmony | Subject)` is the one yielding minimum values for all three of those. Thus, we will choose this as the best one so far.

```
multilevel_model_2 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              (1 | Subject) +
                              (0 + Instrument | Subject) +
                              (0 + Harmony | Subject) +
                              (0 + Voice | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_3 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              (1 | Subject) +
                              (0 + Instrument | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_4 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              (1 | Subject) +
                              (0 + Harmony | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_5 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              (1 | Subject) +
                              (0 + Voice | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_6 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              (1 | Subject) +
                              (0 + Instrument | Subject) +
```

```r
                             (0 + Harmony | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_7 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             (1 | Subject) +
                             (0 + Instrument | Subject) +
                             (0 + Voice | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_8 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             (1 | Subject) +
                             (0 + Harmony | Subject) +
                             (0 + Voice | Subject),
                           data = ratings,
                           REML = FALSE)
```

```r
AIC_BIC_DIC <- cbind(AIC = sapply(list(multilevel_model_2,
                                       multilevel_model_3,
                                       multilevel_model_4,
                                       multilevel_model_5,
                                       multilevel_model_6,
                                       multilevel_model_7,
                                       multilevel_model_8),
                                  AIC),

                     BIC = sapply(list(multilevel_model_2,
                                     multilevel_model_3,
                                     multilevel_model_4,
                                     multilevel_model_5,
                                     multilevel_model_6,
                                     multilevel_model_7,
                                     multilevel_model_8),
                                  BIC),

                     DIC = sapply(list(multilevel_model_2,
                                     multilevel_model_3,
                                     multilevel_model_4,
                                     multilevel_model_5,
                                     multilevel_model_6,
                                     multilevel_model_7,
                                     multilevel_model_8),
                                  extractDIC))
```

```
AIC_BIC_DIC <- t(as.tibble(AIC_BIC_DIC))
colnames(AIC_BIC_DIC) <- c("multilevel_model_2",
                          "multilevel_model_3",
                          "multilevel_model_4",
                          "multilevel_model_5",
                          "multilevel_model_6",
                          "multilevel_model_7",
                          "multilevel_model_8")
kable(t(AIC_BIC_DIC), digits = 2)
```

	AIC	BIC	DIC
multilevel_model_2	9950.18	10171.39	9874.18
multilevel_model_3	10086.96	10215.03	10042.96
multilevel_model_4	10377.57	10528.92	10325.57
multilevel_model_5	10470.10	10598.17	10426.10
multilevel_model_6	9940.24	10126.52	9876.24
multilevel_model_7	10098.45	10261.44	10042.45
multilevel_model_8	10389.57	10575.85	10325.57

1.31 Examining the performance of MLM

We examine the influence of the three main experimental factors in our updated MLM using couple of approaches:

- **ANOVA:** we run the `anova()` function which confirms that the effect of these three experimental factors is significant.
- **Plotting residuals:** we plot the conditional and marginal residuals as a function of conditional and fitted values respectively. The smooth fitted line for both is almost a horizontal line centered at zero suggesting that the data fits our current model well.
- **Checking normality:** The standardized residuals show a normal fit with some outliers on either tails but they do not seem to pose a big issue.
- **Fixed and random effect variances:** From the `summary()` output, we see that $\hat{\sigma}_0^2 = 2.539209$, $\hat{\tau}_1^2 = 1.63176$, $\hat{\tau}_3^2 = 3.509338$, $\hat{\tau}_4^2 = 3.1353 \times 10^{-2}$, $\hat{\tau}_5^2 = 1.534336$, and $\hat{\tau}_6^2 = 4.644 \times 10^{-3}$.

These do seem to be an improvement from the previous multi-level model that we had.

```
multilevel_model_6 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             (1 + Instrument + Harmony | Subject),
                           data = ratings,
                           REML = FALSE)

anova(multilevel_model_6)
```

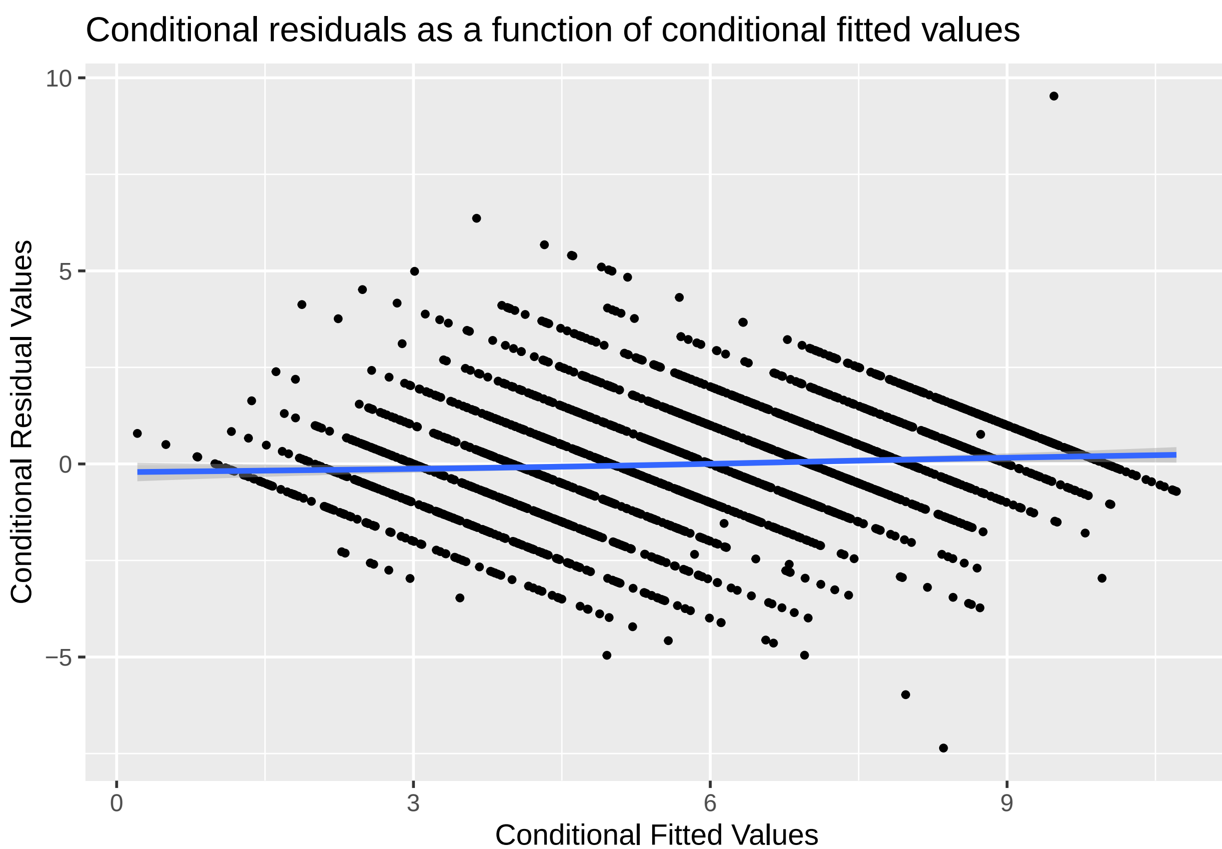
```
## Analysis of Variance Table
##                npar Sum Sq Mean Sq  F value
## Instrument        2 610.79 305.396 126.1343
## Harmony           3  53.92  17.973   7.4231
## Voice             2  84.96  42.482  17.5458
## Harmony:Voice     6  80.38  13.397   5.5331
```

```
summary(multilevel_model_6)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: Classical ~ Instrument + Harmony + Voice + Harmony:Voice + (1 +
##     Instrument + Harmony | Subject)
##    Data: ratings
##
##      AIC      BIC   logLik deviance df.resid
##   9937.3  10146.8  -4932.6   9865.3     2457
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7284 -0.5798  0.0192  0.5701  6.1221
##
## Random effects:
##  Groups   Name             Variance Std.Dev. Corr
##  Subject  (Intercept)      2.539209 1.59349
##           Instrumentpiano  1.631760 1.27740  -0.39
##           Instrumentstring 3.509338 1.87332  -0.57  0.66
##           HarmonyI-V-IV    0.031353 0.17707   0.83 -0.77 -0.52
##           HarmonyI-V-VI    1.534336 1.23868  -0.03 -0.27 -0.43  0.00
##           HarmonyIV-I-V    0.004644 0.06815   0.27 -0.53  0.18  0.68 -0.12
##  Residual                  2.421195 1.55602
## Number of obs: 2493, groups:  Subject, 70
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                   4.2524     0.2232  19.055
## Instrumentpiano               1.3702     0.1710   8.012
## Instrumentstring              3.1274     0.2365  13.223
## HarmonyI-V-IV                 0.1553     0.1543   1.007
## HarmonyI-V-VI                 1.1387     0.2127   5.353
## HarmonyIV-I-V                -0.1335     0.1524  -0.876
## Voicepar3rd                  -0.2707     0.1523  -1.778
## Voicepar5th                  -0.2364     0.1526  -1.549
## HarmonyI-V-IV:Voicepar3rd  -0.3651     0.2158  -1.692
## HarmonyI-V-VI:Voicepar3rd  -0.6799     0.2160  -3.147
## HarmonyIV-I-V:Voicepar3rd   0.4854     0.2157   2.250
## HarmonyI-V-IV:Voicepar5th  -0.1891     0.2162  -0.874
## HarmonyI-V-VI:Voicepar5th  -0.4259     0.2160  -1.972
## HarmonyIV-I-V:Voicepar5th   0.0752     0.2156   0.349
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

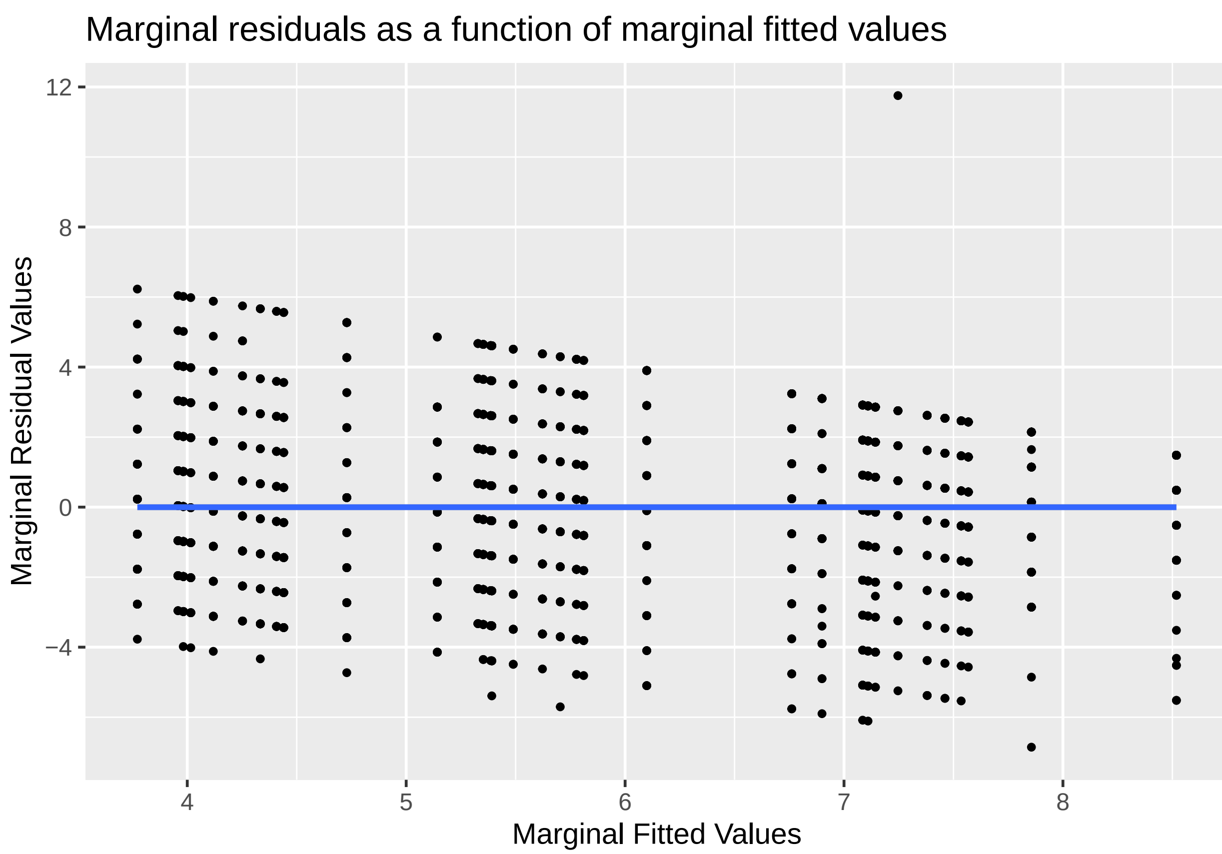
```
residuals_21 <- hlm_resid(multilevel_model_6,
                          level = 1,
```

```
                            include.ls = F)

residuals_21_std <- hlm_resid(multilevel_model_6,
                                level = 1,
                                include.ls = F,
                                standardize = T)

residuals_22 <- hlm_resid(multilevel_model_6,
                            level = "Subject",
                            include.ls = F)

conditional_plot_2 <- ggplot(data = as.data.frame(residuals_21),
                               mapping = aes(y = residuals_21$.resid,
                                             residuals_21$.fitted)) +
  xlab("Conditional Fitted Values") +
  ylab("Conditional Residual Values") +
  ggtitle("Conditional residuals as a function of conditional fitted values") +
  geom_point(pch = 20) +
  geom_smooth()

conditional_plot_2
```

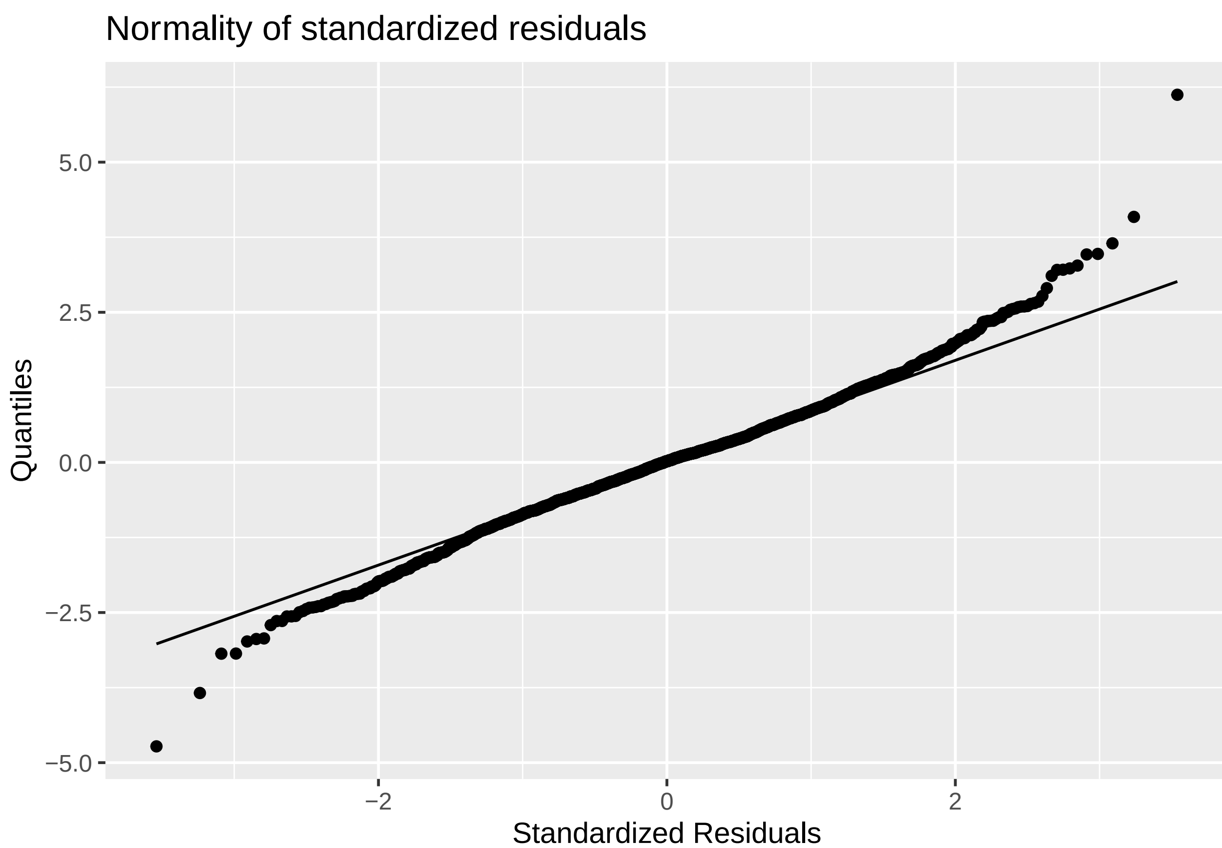



```
ggsave('conditional_plot_2.png')

marginal_plot_2 <- ggplot(data = as.data.frame(residuals_21),
```

```
                          mapping = aes(y = residuals_21$.mar.resid,
                                        x = residuals_21$.mar.fitted)) +
  xlab("Marginal Fitted Values") +
  ylab("Marginal Residual Values") +
  ggtitle("Marginal residuals as a function of marginal fitted values") +
  geom_point(pch = 20) +
  geom_smooth()

marginal_plot_2
```




```
ggsave('marginal_plot_2.png')

std_resid <- residuals_21_std$.std.resid
params <- data.frame(cbind(std_resid))

plot_std_resid_1 <- params %>%
  ggplot(aes(sample = std_resid)) +
  stat_qq() +
  xlab("Standardized Residuals") +
  ylab("Quantiles") +
  ggtitle("Normality of standardized residuals") +
  stat_qq_line()

plot_std_resid_1
```




```
ggsave('plot_std_resid_1.png')
```

1.32 Selecting covariates for fixed effects in model

In order to determine which person covariates should be added to the model as fixed effects, we will first only consider the fixed effects part of our multi-level model as described above and treat is as an OLS `lm()` function to perform analyses. we will perform `regsubsets()` on it to determine the optimal subset of person covariates that should be included in the model.

The model with the lowest BIC has the following covariates: `Harmony`, `Instrument`, `Voice`, `KnowAxis`.

The model with the lowest AIC has the following covariates: `Harmony`, `Instrument`, `Voice`, `SelfDeclare`, `X1990s2000s.minus.1960s1970s`, `GuitarPlay`, `X2ndInstr`

We will include all the covariates as selected by the lowest AIC and BIC models, and add them as fixed level predictors within my chosen multi-level model.

```
regsubsets <- regsubsets(Classical ~
                           Harmony +
                           Instrument +
                           Voice +
                           Selfdeclare +
                           OMSI +
                           X16.minus.17 +
                           ConsInstr +
                           ConsNotes +
```

```
                           Instr.minus.Notes +
                           PachListen +
                           ClsListen +
                           KnowRob +
                           KnowAxis +
                           X1990s2000s +
                           X1990s2000s.minus.1960s1970s +
                           CollegeMusic +
                           NoClass +
                           APTheory +
                           Composing +
                           PianoPlay +
                           GuitarPlay +
                           X1stInstr +
                           X2ndInstr,
                         data = ratings,
                         method = "exhaustive",
                         really.big = T,
                         nvmax = 99)

p <- dim(summary(regsubsets)$which)[2]
n <- dim(ratings)[1]

aic <- n * log(summary(regsubsets)$rss) + 2 * (p + 2)

results <- data.frame(summary(regsubsets)$which,
                      BIC = summary(regsubsets)$bic,
                      AIC = aic)

results[which(results$AIC == min(results$AIC) | results$BIC == min(results$BIC)), ]
```

```
##    X.Intercept. HarmonyI.V.IV HarmonyI.V.VI HarmonyIV.I.V Instrumentpiano
## 4          TRUE         FALSE          TRUE         FALSE            TRUE
## 11         TRUE          TRUE          TRUE          TRUE            TRUE
##    Instrumentstring Voicepar3rd Voicepar5th Selfdeclare  OMSI X16.minus.17
## 4              TRUE       FALSE       FALSE       FALSE FALSE        FALSE
## 11             TRUE        TRUE        TRUE        TRUE FALSE        FALSE
##    ConsInstr ConsNotes Instr.minus.Notes PachListen ClsListen KnowRob KnowAxis
## 4      FALSE     FALSE             FALSE      FALSE     FALSE   FALSE     TRUE
## 11     FALSE     FALSE             FALSE      FALSE     FALSE   FALSE    FALSE
##    X1990s2000s X1990s2000s.minus.1960s1970s CollegeMusic NoClass APTheory
## 4        FALSE                        FALSE        FALSE   FALSE    FALSE
## 11       FALSE                         TRUE        FALSE   FALSE    FALSE
##    Composing PianoPlay GuitarPlay X1stInstr X2ndInstr       BIC      AIC
## 4      FALSE     FALSE      FALSE     FALSE     FALSE -71.40665 16509.98
## 11     FALSE     FALSE       TRUE     FALSE      TRUE -41.25725 16423.16
```

1.33 Analyzing for changes in random effects on the model

In order to see if there should be any changes in the random effects, we implement an idea similar to **forward selection** but using DIC. The idea here is that we first add create six new models each containing one of the new predictors added under the fixed effects part of the model. we then compare the DIC for all these models and if the lowest DIC of these models containing the new predictor in the random effects part is lower than the DIC of the base model without any of the new predictors in its random effects part, then we add that predictor into the random effects. Taking that model then as the base, we repeat the process where we create five new models each containing one of the new predictors added and compare their DICs with that of the model selected in previous step.

Our findings are as follows:

Step 1: Model with random effects part `(1 + Instrument + Harmony + KnowAxis | Subject)` has lowest DIC among all models, lower than the previous `(1 + Instrument + Harmony | Subject)`, so we select that.

Step 2: Model with random effects part `(1 + Instrument + Harmony + KnowAxis + X2ndInstr | Subject)` has the lowest DIC among all models, but not lower than the best model selected at previous step with fixed effects part `(1 + Instrument + Harmony + KnowAxis | Subject)`. Thus, our optimal model does not change.

1.34 Step 1: Add `KnowAxis` as a random effect covariate

```
multilevel_model_7 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             as.factor(Selfdeclare) +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             (1 + Instrument + Harmony | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_8 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             as.factor(Selfdeclare) +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             (1 + Instrument + Harmony + as.factor(Selfdeclare) | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_9 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             as.factor(Selfdeclare) +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
```

```r
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_10 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + X1990s2000s.minus.1960s1970s | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_11 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(GuitarPlay) | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_12 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(X2ndInstr) | Subject),
                            data = ratings,
                            REML = FALSE)

DIC <- cbind(DIC = sapply(list(multilevel_model_7,
                               multilevel_model_8,
                               multilevel_model_9,
                               multilevel_model_10,
                               multilevel_model_11,
```

```
                                multilevel_model_12),
                           extractDIC))

DIC <- t(as.tibble(DIC))
colnames(DIC) <- c("multilevel_model_7",
                   "multilevel_model_8",
                   "multilevel_model_9",
                   "multilevel_model_10",
                   "multilevel_model_11",
                   "multilevel_model_12")
kable(t(DIC), digits = 3)
```

	DIC
multilevel_model_7	1078.344
multilevel_model_8	1078.736
multilevel_model_9	1078.299
multilevel_model_10	1078.305
multilevel_model_11	1078.301
multilevel_model_12	1078.541

1.35 Step 2: Add `X2ndInstr` as a random effect covariate

```
multilevel_model_13 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) + as.factor(Selfdeclare) |
                            data = ratings,
                            REML = FALSE)

multilevel_model_14 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) + X1990s2000s.minus.1960s19
                            data = ratings,
                            REML = FALSE)
```

```
multilevel_model_15 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) + as.factor(GuitarPlay) | S
                            data = ratings,
                            REML = FALSE)

multilevel_model_16 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              as.factor(Selfdeclare) +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) + as.factor(X2ndInstr) | Su
                            data = ratings,
                            REML = FALSE)

DIC <- cbind(DIC = sapply(list(multilevel_model_9,
                                multilevel_model_13,
                                multilevel_model_14,
                                multilevel_model_15,
                                multilevel_model_16),
                           extractDIC))

DIC <- t(as.tibble(DIC))
colnames(DIC) <- c("multilevel_model_9",
                   "multilevel_model_13",
                   "multilevel_model_14",
                   "multilevel_model_15",
                   "multilevel_model_16")
kable(t(DIC), digits = 3)
```

	DIC
multilevel_model_9	1078.299
multilevel_model_13	1078.739
multilevel_model_14	1078.336
multilevel_model_15	1079.325
multilevel_model_16	1078.309

1.36 Testing second hypothesis

Upon running the function `count()` and doing some preliminary analysis on the data set we see that about 60% of the survey participants have selected a value of 1 or 2 for `Selfdeclare`, and the remaining 40% have selected $3, 4$, or 5. Thus, we dichotomize that variable by creating a new variable called `self_declared_musician` where:

- `self_declared_musician = yes`, if `Selfdeclare` $\in \{1, 2\}$
- `self_declared_musician = no`, if `Selfdeclare` $\in \{3, 4, 5\}$

We then replace the variable `Selfdeclare` with as the fixed level covariate in our model for the purpose of this exercise.

Then, in order to see which interactions with `self_declared_musician` on the fixed level are useful, we ucreate new models where each of them contains an interaction with `self_declared_musician` and one of the other fixed level covariates, and compare the AIC and BIC with the base level model which has no interactions with `self_declared_musician`. If any model containing interaction with `self_declared_musician` has a lower AIC/BIC than the base-level model, then it would suggest that the dichotomized musician variable is sensitive to interaction with that other fixed level covariate.

We find that the models containing interaction terms `self_declared_musician:Instrument` and `self_declared_musician:Harmony` have a lower AIC than the model not containing any interaction terms with `self_declared_musician` and the other fixed level covariates, suggesting that `Instrument` and `Harmony` are sensitive to the dichotomization of `self_declared_musiciant`. Out of these two, the model containing the interaction term `self_declared_musician:Instrument` has a lower BIC suggesting that `Instrument` is the most sensitive.

Contextual interpretation: What this means is that the type of instrument that an individual plays could be influential towards whether they consider themselves as a musician or not.

```
ratings$self_declared_musician <- ifelse(ratings$Selfdeclare <= 2, "no", "yes")

multilevel_model_17 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              self_declared_musician +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_18 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              self_declared_musician +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
```

```
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             self_declared_musician:Instrument +
                             (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_19 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             self_declared_musician +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             self_declared_musician:Harmony +
                             (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_20 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             self_declared_musician +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             self_declared_musician:Voice +
                             (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_21 <- lmer(Classical ~
                             Instrument +
                             Harmony +
                             Voice +
                             Harmony:Voice +
                             self_declared_musician +
                             as.factor(KnowAxis) +
                             X1990s2000s.minus.1960s1970s +
                             as.factor(GuitarPlay) +
                             as.factor(X2ndInstr) +
                             self_declared_musician:as.factor(KnowAxis) +
                             (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                           data = ratings,
                           REML = FALSE)

multilevel_model_22 <- lmer(Classical ~
```

```
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              self_declared_musician +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              self_declared_musician:X1990s2000s.minus.1960s1970s +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_23 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              self_declared_musician +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              self_declared_musician:as.factor(GuitarPlay) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                            data = ratings,
                            REML = FALSE)

multilevel_model_24 <- lmer(Classical ~
                              Instrument +
                              Harmony +
                              Voice +
                              Harmony:Voice +
                              self_declared_musician +
                              as.factor(KnowAxis) +
                              X1990s2000s.minus.1960s1970s +
                              as.factor(GuitarPlay) +
                              as.factor(X2ndInstr) +
                              self_declared_musician:as.factor(X2ndInstr) +
                              (1 + Instrument + Harmony + as.factor(KnowAxis) | Subject),
                            data = ratings,
                            REML = FALSE)

AIC_BIC <- cbind(AIC = sapply(list(multilevel_model_17,
                                   multilevel_model_18,
                                   multilevel_model_19,
                                   multilevel_model_20,
                                   multilevel_model_21,
                                   multilevel_model_22,
                                   multilevel_model_23,
                                   multilevel_model_24),
                              AIC),
```

```r
                 BIC = sapply(list(multilevel_model_17,
                                   multilevel_model_18,
                                   multilevel_model_19,
                                   multilevel_model_20,
                                   multilevel_model_21,
                                   multilevel_model_22,
                                   multilevel_model_23,
                                   multilevel_model_24),
                                  BIC))

AIC_BIC <- t(as.tibble(AIC_BIC))
colnames(AIC_BIC) <- c("multilevel_model_17",
                       "multilevel_model_18",
                       "multilevel_model_19",
                       "multilevel_model_20",
                       "multilevel_model_21",
                       "multilevel_model_22",
                       "multilevel_model_23",
                       "multilevel_model_24")
kable(t(AIC_BIC))
```

	AIC	BIC
multilevel_model_17	1179.325	1361.065
multilevel_model_18	1176.912	1365.921
multilevel_model_19	1176.076	1368.720
multilevel_model_20	1180.091	1369.101
multilevel_model_21	1179.325	1361.065
multilevel_model_22	1179.325	1361.065
multilevel_model_23	1179.325	1361.065
multilevel_model_24	1179.325	1361.065

1.37 Summary output of final model

```r
summary(multilevel_model_9)
```

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula:
## Classical ~ Instrument + Harmony + Voice + Harmony:Voice + as.factor(Selfdeclare) +
##     as.factor(KnowAxis) + X1990s2000s.minus.1960s1970s + as.factor(GuitarPlay) +
##     as.factor(X2ndInstr) + (1 + Instrument + Harmony + as.factor(KnowAxis) |
##     Subject)
##    Data: ratings
##
##      AIC      BIC   logLik deviance df.resid
##   1178.3   1360.0   -539.1   1078.3      230
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.0010 -0.6266 -0.0263  0.6008  5.6514
```

```
##
## Random effects:
##  Groups   Name                 Variance  Std.Dev. Corr
##  Subject  (Intercept)          0.2091199 0.45730
##           Instrumentpiano      0.0027190 0.05214   0.24
##           Instrumentstring     2.2144858 1.48811  -1.00 -0.23
##           HarmonyI-V-IV        0.1639360 0.40489  -0.61  0.62  0.61
##           HarmonyI-V-VI        1.8800964 1.37117   0.70 -0.53 -0.70 -0.99
##           HarmonyIV-I-V        0.5523051 0.74317  -0.63  0.60  0.64  1.00 -1.00
##           as.factor(KnowAxis)5 0.0006984 0.02643  -1.00 -0.28  1.00  0.57 -0.67
##  Residual                      2.4583357 1.56791
##
##
##
##
##
##
##
##   0.60
##
## Number of obs: 280, groups:  Subject, 8
##
## Fixed effects:
##                             Estimate Std. Error t value
## (Intercept)                  3.52670    0.57383   6.146
## Instrumentpiano              1.71037    0.23450   7.294
## Instrumentstring             3.38542    0.57274   5.911
## HarmonyI-V-IV                0.53014    0.48404   1.095
## HarmonyI-V-VI                2.47032    0.66680   3.705
## HarmonyIV-I-V                0.58163    0.53190   1.093
## Voicepar3rd                 -0.15468    0.45783  -0.338
## Voicepar5th                  0.47826    0.46235   1.034
## as.factor(Selfdeclare)2     -1.00963    0.52279  -1.931
## as.factor(Selfdeclare)3      0.97850    0.40007   2.446
## as.factor(Selfdeclare)4     -0.64329    0.56881  -1.131
## as.factor(KnowAxis)5         0.99503    0.48702   2.043
## X1990s2000s.minus.1960s1970s  0.04429    0.05283   0.838
## as.factor(X2ndInstr)1       -0.06253    0.40211  -0.156
## as.factor(X2ndInstr)2       -2.30108    0.63015  -3.652
## HarmonyI-V-IV:Voicepar3rd   -0.62793    0.65067  -0.965
## HarmonyI-V-VI:Voicepar3rd   -1.00402    0.64820  -1.549
## HarmonyIV-I-V:Voicepar3rd   -0.01923    0.65067  -0.030
## HarmonyI-V-IV:Voicepar5th   -0.74642    0.65071  -1.147
## HarmonyI-V-VI:Voicepar5th   -0.98479    0.65094  -1.513
## HarmonyIV-I-V:Voicepar5th   -1.50624    0.65076  -2.315
## fit warnings:
## fixed-effect model matrix is rank deficient so dropping 3 columns / coefficients
## optimizer (nloptwrap) convergence code: 0 (OK)
## boundary (singular) fit: see help('isSingular')
```

```
AIC_BIC_DIC <- cbind(AIC = sapply(list(model_2,
                                       multilevel_model_1,
                                       multilevel_model_6,
                                       multilevel_model_9), AIC),
```

```r
                    BIC = sapply(list(model_2,
                                      multilevel_model_1,
                                      multilevel_model_6,
                                      multilevel_model_9), BIC),

                    DIC = sapply(list(multilevel_model_1,
                                      multilevel_model_1,
                                      multilevel_model_6,
                                      multilevel_model_9),
                    extractDIC))

AIC_BIC_DIC <- t(as.tibble(AIC_BIC_DIC))
AIC_BIC_DIC[3, 1] = AIC_BIC_DIC[1, 1]

colnames(AIC_BIC_DIC) <- c("Best Linear Model",
                           "MLM (random intercept)",
                           "MLM (random intercept and slopes)",
                           "Best Multilevel Model")

kable(t(AIC_BIC_DIC), digits = 2)
```

	AIC	BIC	DIC
Best Linear Model	11226.94	11314.26	11226.94
MLM (random intercept)	10458.10	10551.24	10426.10
MLM (random intercept and slopes)	9937.27	10146.84	9865.27
Best Multilevel Model	1178.30	1360.04	1078.30