Comparison of Two Quantum Error-Correcting code

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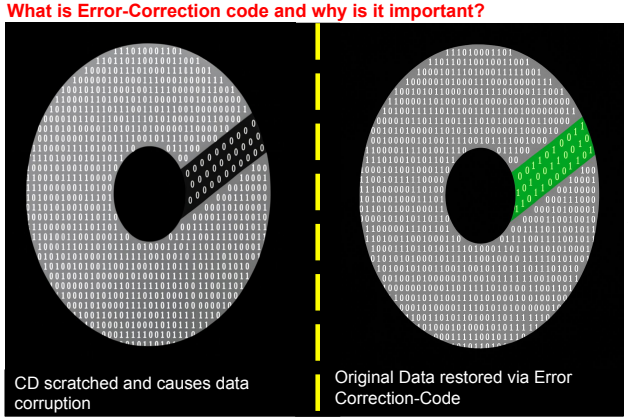
INTRODUCTION

What is Error-Correction code and why is it important?



Shor 9

[[9, 1, 3]] Shor code

9 Physical Qubits

Distance of 3 corrects single bit errors and can ONLY detect up to 2 errors, but not correct them

1 Logical Qubit

Logical Codewords.

$$|\bar{0}\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle + |111\rangle\right)^{\otimes 3}$$

$$|\bar{1}\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - |111\rangle\right)^{\otimes 3}.$$

Works for any arbitrary superpositions of the form $\alpha\,|0\rangle_L + \beta\,|1\rangle_L$

Steane 7

[[7, 1, 3]] Steane code

7 Physical Qubits

1 Logical Qubit

Distance of 3 corrects single bit errors and can ONLY detect up to 2 errors, but not correct them

Logical Codewords.

$$|\bar{0}\rangle = \frac{1}{\sqrt{8}}\Big(|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle\Big)$$

$$|\bar{1}\rangle = \frac{1}{\sqrt{8}}\Big(|1111111\rangle + |0101010\rangle + |1001100\rangle + |0011001\rangle + |1110000\rangle + |0100101\rangle + |1000011\rangle + |0010110\rangle\Big).$$

Works for any arbitrary superpositions of the form $\alpha\,|0\rangle_L + \beta\,|1\rangle_L$

FINDINGS

Figure 1. Steane 7 Qubit code

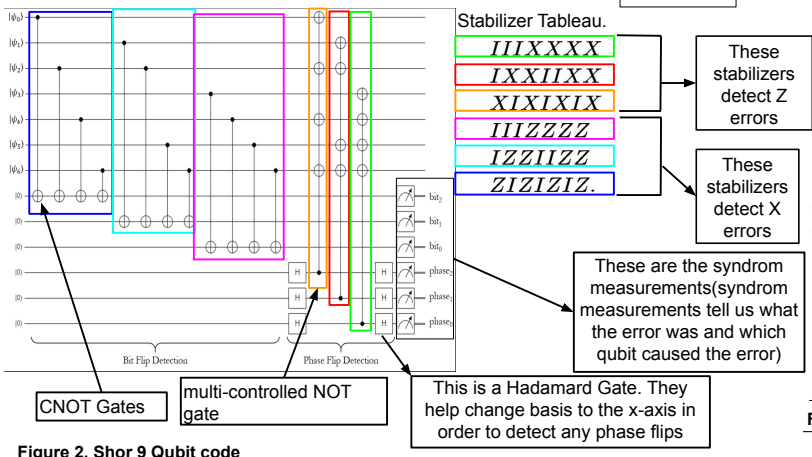


Figure 2. Shor 9 Qubit code

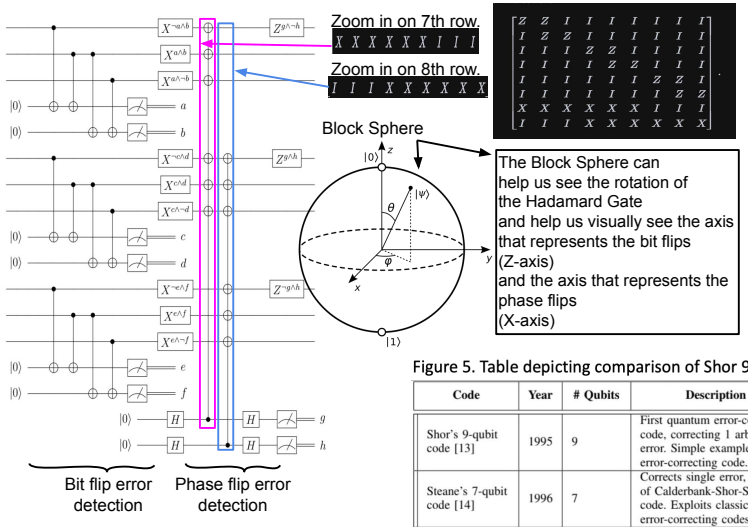


Figure 3. Monte Carlo simulation of Steane 7 with 100,000 samples

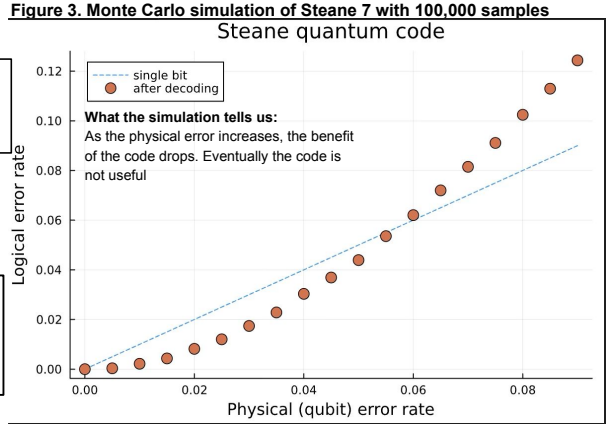


Figure 4. Monte Carlo simulation of Shor 9 with 100,000 samples

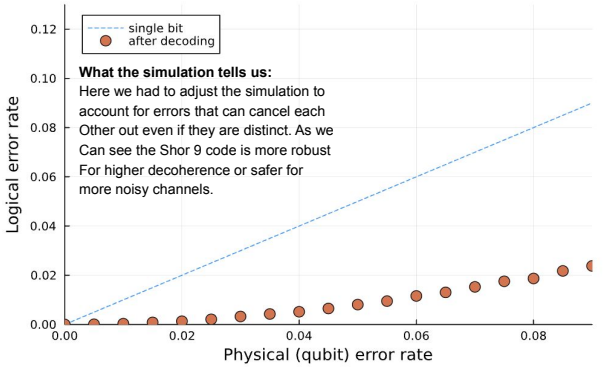
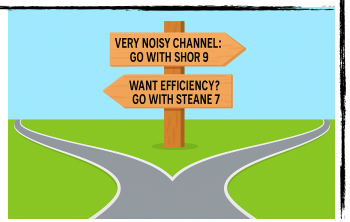


Figure 5. Table depicting comparison of Shor 9 and Steane's 7 (Paper on comparison of Steane and Shor codes)

Code	Year	# Qubits	Description	Complexity	Decoding Algorithm	Advantage	Disadvantage
Shor's 9-qubit code [13]	1995	9	First quantum error-correcting code, correcting 1 arbitrary error. Simple example of an error-correcting code.	Moderate (9 physical qubits for 1 logical qubit)	Syndrome measurement, lookup table	Good for understanding basic error correction concepts.	Requires 9 qubits, not very resource-efficient.
Steane's 7-qubit code [14]	1996	7	Corrects single error, example of Calderbank-Shor-Steane code. Exploits classical error-correcting codes.	Moderate (7 physical qubits for 1 logical qubit)	Syndrome measurement, lookup table	More resource-efficient than Shor's code, easy to implement.	Only corrects single error, not suitable for larger systems.

CONCLUSION

- The **Steane code** [[7,1,3]] is more qubit-efficient, using fewer physical qubits while still correcting arbitrary single-qubit errors.
- The **Shor code** [[9,1,3]] offers stronger protection against noise by separating bit-flip and phase-flip error correction.
- Steane is better suited for **low to moderate noise** environments due to its efficiency.
- Shor is more effective in **highly noisy** or decoherence-heavy systems where resilience is critical.



References