HW3 Part 2

Code

Arshia Singh

April 11, 2020

Question 3 - Word2Vec Embeddings

1

I think there are anumber of preprocessing steps that could improve the quality of the embedding. First I would remove stopwords and possibly also numbers. I would also stem the words so that there wouldn't be different embeddings for variants of the same word like boil and boiling. Lemmatization might also be a good option to capture words of different forms (better, good) and collate them into one canonical form.

2

Hide

```
####################################
##### Loading libraries & data #####
####################################
library(wordVectors)
library(Rtsne)
```

```
package <U+393C><U+3E31>Rtsne<U+393C><U+3E32> was built under R version 3.5.3
```

Hide

```
library(tidytext)
```

```
package <U+393C><U+3E31>tidytext<U+393C><U+3E32> was built under R version 3.5.3
```

Hide

```
library(tidyverse)
```

```
package <U+393C><U+3E31>tidyverse<U+393C><U+3E32> was built under R version 3.5.3[30
m-- [1mAttaching packages[22m --------------------------------------- tidyverse 1.3.
0 --[39m
[30m[32mv[30m [34mggplot2[30m 3.3.0     [32mv[30m [34mpurrr  [30m 0.3.3
[32mv[30m [34mtibble [30m 3.0.0     [32mv[30m [34mdplyr  [30m 0.8.5
[32mv[30m [34mtidyr  [30m 1.0.2     [32mv[30m [34mstringr[30m 1.4.0
[32mv[30m [34mreadr  [30m 1.3.1     [32mv[30m [34mforcats[30m 0.5.0[39m
package <U+393C><U+3E31>ggplot2<U+393C><U+3E32> was built under R version 3.5.3package
<U+393C><U+3E31>tibble<U+393C><U+3E32> was built under R version 3.5.3package <U+393C>
<U+3E31>tidyr<U+393C><U+3E32> was built under R version 3.5.3package <U+393C><U+3E31>r
eadr<U+393C><U+3E32> was built under R version 3.5.3package <U+393C><U+3E31>purrr<U+39
3C><U+3E32> was built under R version 3.5.3package <U+393C><U+3E31>dplyr<U+393C><U+3E3
2> was built under R version 3.5.3package <U+393C><U+3E31>stringr<U+393C><U+3E32> was
built under R version 3.5.3package <U+393C><U+3E31>forcats<U+393C><U+3E32> was built u
nder R version 3.5.3[30m-- [1mConflicts[22m ---------------------------------------
--- tidyverse_conflicts() --
[31mx[30m [34mdplyr[30m::[32mfilter()[30m masks [34mstats[30m::filter()
[31mx[30m [34mdplyr[30m::[32mlag()[30m    masks [34mstats[30m::lag()[39m
```

Hide

```
####################################
#####       Download data      #####
####################################
# -- Check to see  if file exists --
if (!file.exists("cookbooks.zip")) {
  download.file("http://archive.lib.msu.edu/dinfo/feedingamerica/cookbook_text.zip","c
ookbooks.zip")
}
unzip("cookbooks.zip",exdir="cookbooks")
if (!file.exists("cookbooks.txt")) prep_word2vec(origin="cookbooks",destination="cookb
ooks.txt",lowercase=T,bundle_ngrams=1)
# Training a Word2Vec model
if (!file.exists("cookbook_vectors.bin")) {
  model = train_word2vec("cookbooks.txt","cookbook_vectors.bin",
                         vectors=100,threads=4,window=6,
                         min_count = 10,
                         iter=5,negative_samples=15)
} else{
    model = read.vectors("cookbook_vectors.bin")
    }
```

```
Filename ends with .bin, so reading in binary format
Reading a word2vec binary file of 18952 rows and 100 columns
```

```
  |
  |
|   0%
  |
  |=
|   0%
  |
  |=
|   1%
  |
  |==
|   1%
  |
  |==
|   2%
  |
  |===
|   2%
  |
  |===
|   3%
  |
  |====
|   3%
  |
  |=====
|   3%
  |
  |=====
|   4%
  |
  |======
|   4%
  |
  |======
|   5%
  |
  |=======
|   5%
  |
  |=======
|   6%
  |
  |========
|   6%
  |
  |=========
```

```
|   6%
  |
  |=========
|   7%
  |
  |==========
|   7%
  |
  |==========
|   8%
  |
  |===========
|   8%
  |
  |===========
|   9%
  |
  |============
|   9%
  |
  |=============
|   9%
  |
  |=============
|  10%
  |
  |==============
|  10%
  |
  |==============
|  11%
  |
  |===============
|  11%
  |
  |===============
|  12%
  |
  |================
|  12%
  |
  |=================
|  12%
  |
  |=================
|  13%
  |
  |==================
|  13%
```

```
  |
  |==================
|  14%
  |
  |===================
|  14%
  |
  |===================
|  15%
  |
  |====================
|  15%
  |
  |=====================
|  15%
  |
  |=====================
|  16%
  |
  |======================
|  16%
  |
  |======================
|  17%
  |
  |=======================
|  17%
  |
  |=======================
|  18%
  |
  |========================
|  18%
  |
  |=========================
|  18%
  |
  |=========================
|  19%
  |
  |==========================
|  19%
  |
  |==========================
|  20%
  |
  |===========================
|  20%
  |
```

```
|===========================
|  21%
|
|============================
|  21%
|
|=============================
|  21%
|
|=============================
|  22%
|
|==============================
|  22%
|
|==============================
|  23%
|
|===============================
|  23%
|
|===============================
|  24%
|
|================================
|  24%
|
|=================================
|  24%
|
|=================================
|  25%
|
|==================================
|  25%
|
|==================================
|  26%
|
|===================================
|  26%
|
|====================================
|  26%
|
|====================================
|  27%
|
|=====================================
```

```
|   27%
  |
  |=====================================
|   28%
  |
  |======================================
|   28%
  |
  |======================================
|   29%
  |
  |=======================================
|   29%
  |
  |========================================
|   29%
  |
  |========================================
|   30%
  |
  |=========================================
|   30%
  |
  |=========================================
|   31%
  |
  |==========================================
|   31%
  |
  |==========================================
|   32%
  |
  |===========================================
|   32%
  |
  |============================================
|   32%
  |
  |============================================
|   33%
  |
  |=============================================
|   33%
  |
  |=============================================
|   34%
  |
  |==============================================
|   34%
```

```
  |
  |==============================================
|  35%
  |
  |===============================================
|  35%
  |
  |================================================
|  35%
  |
  |================================================
|  36%
  |
  |=================================================
|  36%
  |
  |=================================================
|  37%
  |
  |==================================================
|  37%
  |
  |==================================================
|  38%
  |
  |===================================================
|  38%
  |
  |====================================================
|  38%
  |
  |====================================================
|  39%
  |
  |=====================================================
|  39%
  |
  |=====================================================
|  40%
  |
  |======================================================
|  40%
  |
  |======================================================
|  41%
  |
  |=======================================================
|  41%
  |
```

```
|=======================================================
|  41%
|
|=======================================================
|  42%
|
|========================================================
|  42%
|
|========================================================
|  43%
|
|=========================================================
|  43%
|
|=========================================================
|  44%
|
|==========================================================
|  44%
|
|===========================================================
|  44%
|
|===========================================================
|  45%
|
|============================================================
|  45%
|
|============================================================
|  46%
|
|=============================================================
|  46%
|
|=============================================================
|  47%
|
|==============================================================
|  47%
|
|===============================================================
|  47%
|
|===============================================================
|  48%
|
|================================================================
```

```
|   48%
  |
  |=================================================================
|   49%
  |
  |==================================================================
|   49%
  |
  |==================================================================
|   50%
  |
  |===================================================================
|   50%
  |
  |====================================================================
|   50%
  |
  |====================================================================
|   51%
  |
  |=====================================================================
|   51%
  |
  |=====================================================================
|   52%
  |
  |======================================================================
|   52%
  |
  |======================================================================
|   53%
  |
  |=======================================================================
|   53%
  |
  |========================================================================
|   53%
  |
  |========================================================================
|   54%
  |
  |=========================================================================
|   54%
  |
  |=========================================================================
|   55%
  |
  |==========================================================================
|   55%
```

```
    |
    |==========================================================================
  |  56%
    |
    |===========================================================================
  |  56%
    |
    |============================================================================
  |  56%
    |
    |============================================================================
  |  57%
    |
    |=============================================================================
  |  57%
    |
    |=============================================================================
  |  58%
    |
    |==============================================================================
  |  58%
    |
    |==============================================================================
  |  59%
    |
    |===============================================================================
  |  59%
    |
    |================================================================================
  |  59%
    |
    |================================================================================
  |  60%
    |
    |=================================================================================
  |  60%
    |
    |=================================================================================
  |  61%
    |
    |==================================================================================
  |  61%
    |
    |==================================================================================
  |  62%
    |
    |===================================================================================
  |  62%
    |
```

```
  |=====================================================================================
=                                                   |   62%
  |
  |=====================================================================================
=                                                   |   63%
  |
  |=====================================================================================
==                                                  |   63%
  |
  |=====================================================================================
==                                                  |   64%
  |
  |=====================================================================================
===                                                 |   64%
  |
  |=====================================================================================
===                                                 |   65%
  |
  |=====================================================================================
====                                                |   65%
  |
  |=====================================================================================
=====                                               |   65%
  |
  |=====================================================================================
=====                                               |   66%
  |
  |=====================================================================================
======                                              |   66%
  |
  |=====================================================================================
======                                              |   67%
  |
  |=====================================================================================
=======                                             |   67%
  |
  |=====================================================================================
=======                                             |   68%
  |
  |=====================================================================================
========                                            |   68%
  |
  |=====================================================================================
=========                                           |   68%
  |
  |=====================================================================================
=========                                           |   69%
  |
  |=====================================================================================
```

```
==========                                          |  69%
  |
  |=====================================================================================
==========                                          |  70%
  |
  |=====================================================================================
===========                                         |  70%
  |
  |=====================================================================================
===========                                         |  71%
  |
  |=====================================================================================
============                                        |  71%
  |
  |=====================================================================================
=============                                       |  71%
  |
  |=====================================================================================
=============                                       |  72%
  |
  |=====================================================================================
==============                                      |  72%
  |
  |=====================================================================================
==============                                      |  73%
  |
  |=====================================================================================
===============                                     |  73%
  |
  |=====================================================================================
===============                                     |  74%
  |
  |=====================================================================================
================                                    |  74%
  |
  |=====================================================================================
=================                                   |  74%
  |
  |=====================================================================================
=================                                   |  75%
  |
  |=====================================================================================
==================                                  |  75%
  |
  |=====================================================================================
==================                                  |  76%
  |
  |=====================================================================================
===================                                 |  76%
```

```
  |
  |==================================================================================
====================                              |  76%
  |
  |==================================================================================
====================                              |  77%
  |
  |==================================================================================
=====================                             |  77%
  |
  |==================================================================================
=====================                             |  78%
  |
  |==================================================================================
======================                            |  78%
  |
  |==================================================================================
======================                            |  79%
  |
  |==================================================================================
=======================                           |  79%
  |
  |==================================================================================
========================                          |  79%
  |
  |==================================================================================
========================                          |  80%
  |
  |==================================================================================
=========================                         |  80%
  |
  |==================================================================================
=========================                         |  81%
  |
  |==================================================================================
==========================                        |  81%
  |
  |==================================================================================
==========================                        |  82%
  |
  |==================================================================================
===========================                       |  82%
  |
  |==================================================================================
============================                      |  82%
  |
  |==================================================================================
============================                      |  83%
  |
```

```
  |====================================================================================
=============================                       |  83%
  |
  |====================================================================================
=============================                       |  84%
  |
  |====================================================================================
==============================                      |  84%
  |
  |====================================================================================
==============================                      |  85%
  |
  |====================================================================================
===============================                     |  85%
  |
  |====================================================================================
================================                    |  85%
  |
  |====================================================================================
================================                    |  86%
  |
  |====================================================================================
=================================                   |  86%
  |
  |====================================================================================
=================================                   |  87%
  |
  |====================================================================================
==================================                  |  87%
  |
  |====================================================================================
==================================                  |  88%
  |
  |====================================================================================
===================================                 |  88%
  |
  |====================================================================================
====================================                |  88%
  |
  |====================================================================================
====================================                |  89%
  |
  |====================================================================================
=====================================               |  89%
  |
  |====================================================================================
=====================================               |  90%
  |
  |====================================================================================
```

```
=====================================             |  90%
  |
  |==================================================================================
=====================================             |  91%
  |
  |==================================================================================
======================================            |  91%
  |
  |==================================================================================
=======================================           |  91%
  |
  |==================================================================================
=======================================           |  92%
  |
  |==================================================================================
========================================          |  92%
  |
  |==================================================================================
========================================          |  93%
  |
  |==================================================================================
=========================================         |  93%
  |
  |==================================================================================
=========================================         |  94%
  |
  |==================================================================================
==========================================        |  94%
  |
  |==================================================================================
===========================================       |  94%
  |
  |==================================================================================
===========================================       |  95%
  |
  |==================================================================================
============================================      |  95%
  |
  |==================================================================================
============================================      |  96%
  |
  |==================================================================================
=============================================     |  96%
  |
  |==================================================================================
=============================================     |  97%
  |
  |==================================================================================
==============================================    |  97%
```

```
  |
  |=====================================================================================
=================================================   |  97%
  |
  |=====================================================================================
=================================================   |  98%
  |
  |=====================================================================================
==================================================  |  98%
  |
  |=====================================================================================
==================================================  |  99%
  |
  |=====================================================================================
=================================================== |  99%
  |
  |=====================================================================================
=================================================== | 100%
  |
  |=====================================================================================
====================================================| 100%
```

Hide

```
####################################
#####      Proximity search    #####
####################################
# -- Select ingredient and cuisine --
ingredient = 'sage'
ingredient_2 = 'thyme'
ingredient_3 = 'basil'
list_of_ingredients = c(ingredient, ingredient_2, ingredient_3)
cuisine = 'italian'
# Coordinages in 300D space of embedding for the word "sage"
model[[ingredient]]
```

```
A VectorSpaceModel object of  1  words and  100  vectors
          [,1]       [,2]       [,3]     [,4]         [,5]      [,6]
[1,] 0.2364041 -0.1724167 0.08512986 0.126502 -0.008896198 0.1683877
attr(,".cache")
<environment: 0x00000000061b9150>
```

Hide

```
# Searching closest words to sage
model %>% closest_to(model[[ingredient]]) #<- set of closest ingredients to "sage"
```

word <chr>	similarity to model[[ingredient]] <dbl>
sage	1.0000000
marjoram	0.8019883
thyme	0.7985719
savory	0.7383473
basil	0.7047918
parsley	0.6912978
knotted	0.6798792
pennyroyal	0.6741924
herbs	0.6696337
mint	0.6355805

1-10 of 10 rows

Hide

```
model %>% closest_to(model[[cuisine]], 20) #<- set of closest cuisines to "italian"
```

word <chr>	similarity to model[[cuisine]] <dbl>
italian	1.0000000
genoa	0.6986518
australian	0.6974486
hungarian	0.6897454
portuguese	0.6847645
spumante	0.6783787
tuscany	0.6758480
illyrian	0.6749846
austria	0.6713014
french	0.6660833

1-10 of 20 rows Previous **1** 2 Next

Hide

```
# Set of closest words to "sage", "thyme","basil"
model %>% closest_to(model[[list_of_ingredients]],10)
```

word <chr>	similarity to model[[list_of_ingredients]] <dbl>
thyme	0.9627577
basil	0.9343336
marjoram	0.9199597
sage	0.8821822
bayleaf	0.8100456
knotted	0.8073121
bay	0.7883564
savory	0.7838670
herbs	0.7805929
laurel	0.7707843

1-10 of 10 rows

Hide

```
####################################
#####      Proximity search    #####
####################################
# -- Select ingredient and cuisine --
ingredient = 'turmeric'
ingredient_2 = 'cumin'
ingredient_3 = 'ginger'
list_of_ingredients = c(ingredient, ingredient_2, ingredient_3)
cuisine = 'indian'
# Coordinages in 300D space of embedding for the word "turmeric"
model[[ingredient]]
```

```
A VectorSpaceModel object of  1  words and  100  vectors
             [,1]      [,2]      [,3]      [,4]       [,5]      [,6]
[1,] -0.009492924 0.5757813 0.5833898 0.6113898 0.07088909 0.4945517
attr(,".cache")
<environment: 0x000000001c25e718>
```

Hide

```
# Searching closest words to sage
model %>% closest_to(model[[ingredient]]) #<- set of closest ingredients to "turmeric"
```

word <chr>	similarity to model[[ingredient]] <dbl>
turmeric	1.0000000
tumeric	0.7466400
mustard	0.7348814
allspice	0.6762297
bruised	0.6761345
saltpeter	0.6759863
ginger	0.6758935
cummin	0.6569852
cardamoms	0.6536759
vinegar	0.6444942

1-10 of 10 rows

Hide

```
model %>% closest_to(model[[cuisine]], 20) #<- set of closest cuisines to "indian"
```

word <chr>	similarity to model[[cuisine]] <dbl>
indian	1.0000000
meal	0.6906715
oat	0.6775426
rye	0.6745011
corn	0.6699416
mush	0.6603985
mealindian	0.6603419
buckwheat	0.6407531
cornmeal	0.6294710
rice	0.5996728

1-10 of 20 rows Previous **1** 2 Next

Hide

```
# Set of closest words to "turmeric", "cumin","ginger"
model %>% closest_to(model[[list_of_ingredients]],10)
```

word <chr>	**similarity to model[[list_of_ingredients]]** <dbl>
turmeric	0.8829268
ginger	0.8482772
cumin	0.8318314
tumeric	0.8179254
coriander	0.7827304
cardamon	0.7737848
cardamom	0.7642598
mustard	0.7604513
cardamoms	0.7560645
caraway	0.7537103

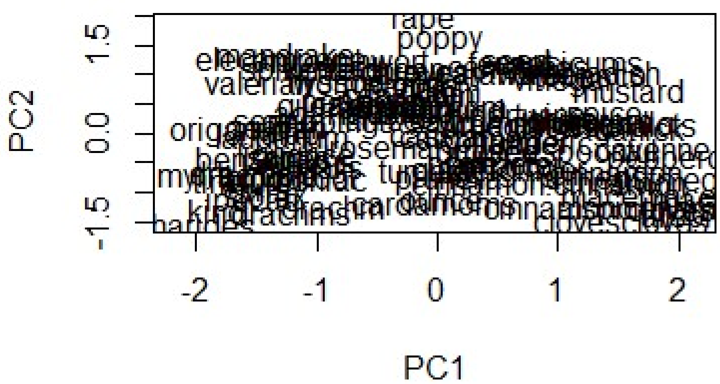
1-10 of 10 rows

My ingredients were turmeric, cumin, and ginger. The top ten ingredients closest to this set of ingredients were turmeric, ginger, cumin, tumeric, coriander, cardamoms, cardamom, alspice, mustard, and cardamon. This is somewhat interesting because there are a number of misspellings that made the list, and also because allspice and cardamom are slightly sweeter ingredients than the ones I listed. Coriander and cardamom are very common ingredients used in indian cooking, but allspice is not as common, so I thought that was a bit odd.

3

Hide

```
##############################################
#####   Using TSNE to see similarity    #####
##############################################
# We have a list of potential herb-related words from old cookbooks.
n_words = 100
closest_ingredients = closest_to(model,model[[list_of_ingredients]], n_words)$word
surrounding_ingredients = model[[closest_ingredients,average=F]]
plot(surrounding_ingredients,method="pca")
```



Hide

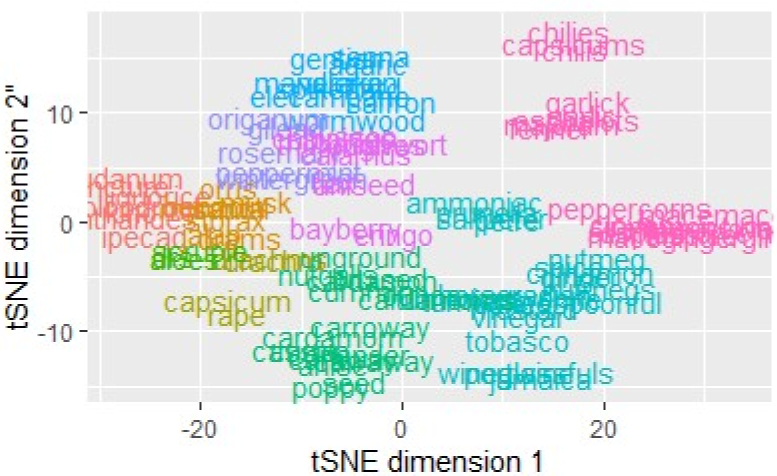
```
embedding = Rtsne(X = surrounding_ingredients, dims = 2,
                  perplexity = 4,
                  theta = 0.5,
                  eta = 10,
                  pca = TRUE, verbose = TRUE,
                  max_iter = 2000)
```

```
Performing PCA
Read the 100 x 50 data matrix successfully!
OpenMP is working. 1 threads.
Using no_dims = 2, perplexity = 4.000000, and theta = 0.500000
Computing input similarities...
Building tree...
Done in 0.05 seconds (sparsity = 0.166800)!
Learning embedding...
Iteration 50: error is 64.995886 (50 iterations in 0.39 seconds)
Iteration 100: error is 63.076374 (50 iterations in 0.19 seconds)
Iteration 150: error is 62.501965 (50 iterations in 0.08 seconds)
Iteration 200: error is 62.497400 (50 iterations in 0.24 seconds)
Iteration 250: error is 62.496773 (50 iterations in 0.16 seconds)
Iteration 300: error is 1.067029 (50 iterations in 0.77 seconds)
Iteration 350: error is 0.895766 (50 iterations in 0.30 seconds)
Iteration 400: error is 0.830265 (50 iterations in 0.17 seconds)
Iteration 450: error is 0.811456 (50 iterations in 0.68 seconds)
Iteration 500: error is 0.801824 (50 iterations in 0.19 seconds)
Iteration 550: error is 0.793186 (50 iterations in 0.16 seconds)
Iteration 600: error is 0.786645 (50 iterations in 0.20 seconds)
Iteration 650: error is 0.784075 (50 iterations in 0.16 seconds)
Iteration 700: error is 0.779695 (50 iterations in 0.14 seconds)
Iteration 750: error is 0.778561 (50 iterations in 0.45 seconds)
Iteration 800: error is 0.776323 (50 iterations in 0.08 seconds)
Iteration 850: error is 0.775540 (50 iterations in 0.12 seconds)
Iteration 900: error is 0.774019 (50 iterations in 0.30 seconds)
Iteration 950: error is 0.772533 (50 iterations in 0.10 seconds)
Iteration 1000: error is 0.774107 (50 iterations in 0.19 seconds)
Iteration 1050: error is 0.773898 (50 iterations in 0.08 seconds)
Iteration 1100: error is 0.773117 (50 iterations in 0.05 seconds)
Iteration 1150: error is 0.773097 (50 iterations in 0.10 seconds)
Iteration 1200: error is 0.771227 (50 iterations in 0.07 seconds)
Iteration 1250: error is 0.771918 (50 iterations in 0.21 seconds)
Iteration 1300: error is 0.771641 (50 iterations in 0.10 seconds)
Iteration 1350: error is 0.770813 (50 iterations in 0.05 seconds)
Iteration 1400: error is 0.770408 (50 iterations in 0.06 seconds)
Iteration 1450: error is 0.770380 (50 iterations in 0.04 seconds)
Iteration 1500: error is 0.770521 (50 iterations in 0.08 seconds)
Iteration 1550: error is 0.770443 (50 iterations in 0.11 seconds)
Iteration 1600: error is 0.769481 (50 iterations in 0.07 seconds)
Iteration 1650: error is 0.769482 (50 iterations in 0.06 seconds)
Iteration 1700: error is 0.768556 (50 iterations in 0.07 seconds)
Iteration 1750: error is 0.768749 (50 iterations in 0.07 seconds)
Iteration 1800: error is 0.769138 (50 iterations in 0.05 seconds)
Iteration 1850: error is 0.769795 (50 iterations in 0.05 seconds)
Iteration 1900: error is 0.768134 (50 iterations in 0.06 seconds)
Iteration 1950: error is 0.769592 (50 iterations in 0.08 seconds)
```

```
Iteration 2000: error is 0.769148 (50 iterations in 0.09 seconds)
Fitting performed in 6.62 seconds.
```

Hide

```
embedding_vals = embedding$Y
rownames(embedding_vals) = rownames(surrounding_ingredients)
# Looking for clusters for embedding
set.seed(10)
n_centers = 10
clustering = kmeans(embedding_vals,centers=n_centers,
                    iter.max = 5)
# Setting up data for plotting
embedding_plot = tibble(x = embedding$Y[,1],
                        y = embedding$Y[,2],
                        labels = rownames(surrounding_ingredients)) %>%
  bind_cols(cluster = as.character(clustering$cluster))
# Visualizing TSNE output
ggplot(aes(x = x, y=y,label = labels, color = cluster), data = embedding_plot) +
  geom_text() +xlab('tSNE dimension 1') +ylab('tSNE dimension 2"')+theme(legend.positi
on = 'none')
```



Hide

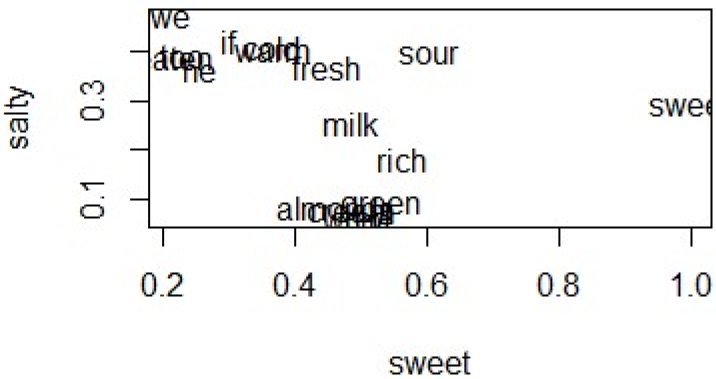
```
# Topics produced by the top 3 words
sapply(sample(1:n_centers,n_centers),function(n) {
  names(clustering$cluster[clustering$cluster==n][1:10])
})
```

```
      [,1]          [,2]         [,3]          [,4]          [,5]      [,6]
[,7]           [,8]        [,9]      [,10]
 [1,] "peppermint"  "saffron"    "tincture"    "vinegar"     "drs"     "cloves"
"flax"         "ounce"     "musk"    "drachm"
 [2,] "rosemary"    "wormwood"   "laudanum"    "nutmeg"      "aloes"   "mace"
"aniseed"      "seed"      "orris"   "drachms"
 [3,] "wintergreen" "senna"      "liquorice"   "cinnamon"    "scruple" "allspice"
"hyssop"       "bruised"   "benzoin" "capsicum"
 [4,] "origanum"    "mandrake"   "cantharides" "mustard"     NA        "garlic"
"calamus"      "caraway"   "myrrh"   "rape"
 [5,] "gilead"      "elecampane" "ipecac"      "ginger"      NA        "peppercorns"
"bayberry"     "coriander" "drams"   NA
 [6,] NA            "gentian"    "kino"        "cayenne"     NA        "fennel"
"eringo"       "anise"     "jalap"   NA
 [7,] NA            "valerian"   "bloodroot"   "spice"       NA        "eschalots"
"thoroughwort" "poppy"     "storax"  NA
 [8,] NA            "agaric"     NA            "horseradish" NA        "garlick"
"chippings"    "cassia"    "santal"  NA
 [9,] NA            "spikenard"  NA            "nutmegs"     NA        "clovescloves"
"thieves"      "carraway"  NA        NA
[10,] NA            NA           NA            "turmeric"    NA        "alspice"
NA             "cardamom"  NA        NA
```

4

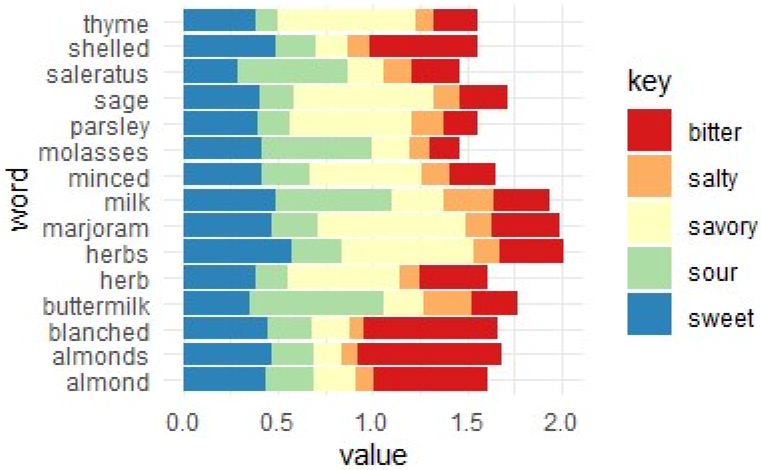
Hide

```
##################################################
#####   Plotting Sweet and Salty Dimensions  #####
##################################################
# -- Plotting across the sweet-salty plane --
tastes = model[[c("sweet","salty"),average=F]]
sweet_and_saltiness = model[1:500,] %>% cosineSimilarity(tastes)
# Filter to the top n words for sweet or salty.
top_n_words = 10
sweet_and_saltiness = sweet_and_saltiness[
  rank(-sweet_and_saltiness[,1])<top_n_words |
    rank(-sweet_and_saltiness[,2])<top_n_words,
  ]
plot(sweet_and_saltiness,type='n')
text(sweet_and_saltiness,labels=rownames(sweet_and_saltiness))
```



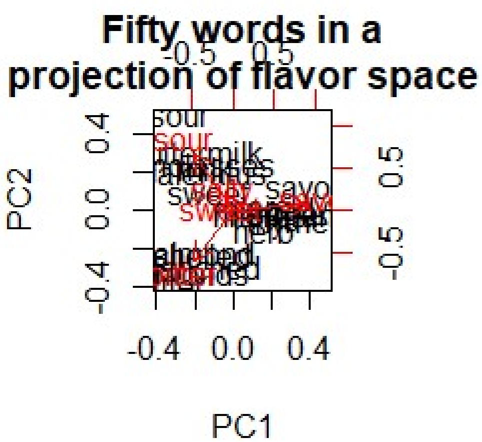
Hide

```
###########################################
#####   Plotting 5 Taste  Dimensions  #####
###########################################
# We can plot along mltiple dimensions:
tastes = c("salty","sweet","savory","bitter","sour")
common_similarities_tastes = model[1:3000,]%>% cosineSimilarity( model[[tastes,average
=F]])
high_similarities_to_tastes = common_similarities_tastes[rank(-apply(common_similariti
es_tastes,1,max)) < 20,]
# - Plotting
high_similarities_to_tastes %>%
  as_tibble(rownames='word') %>%
  filter( ! (is.element(word,tastes))) %>%
  #mutate(total = salty+sweet+savory+bitter+sour) %>%
  #mutate( sweet=sweet/total,salty=salty/total,savory=savory/total,bitter=bitter/tota
l, sour = sour/total) %>%
  #select(-total) %>%
  gather(key = 'key', value = 'value',-word) %>%
  ggplot(aes(x = word,
             y = value,
             fill = key)) + geom_bar(stat='identity') +
  coord_flip() + theme_minimal() + scale_fill_brewer(palette='Spectral')
```



Hide

```
# --- Most similar terms  ---
high_similarities_to_tastes %>%
  prcomp %>%
  biplot(main="Fifty words in a\nprojection of flavor space")
```



Hide

```
############################################
#####   Plotting 5 Temperature Dimensions   #####
############################################
rownames(model[1000:2000,])
```

```
  [1] "months"        "30"            "essence"       "mortar"        "colored"      
"larger"        "aux"           "t"            
  [9] "burning"       "tins"          "usual"         "seen"          "unless"       
"201"           "whip"          "weather"      
 [17] "raised"        "important"     "grain"         "hole"          "peeled"       
"temperature"   "forms"         "loin"         
 [25] "obtained"      "lump"          "fancy"         "remainder"     "wholesome"    
"fingers"       "plates"        "dipped"       
 [33] "consomm"       "level"         "brine"         "granulated"    "heavy"        
"remaining"     "walnut"        "beating"      
 [41] "crisp"         "teaspoons"     "lbs"           "animal"        "stalks"       
"blanched"      "bitter"        "divide"       
 [49] "against"       "scum"          "removing"      "tops"          "chapter"      
"forty"         "thought"       "griddle"      
 [57] "poor"          "week"          "perhaps"       "weeks"         "catsup"       
"preferred"     "punch"         "freeze"       
 [65] "although"      "preserves"     "wanted"        "ought"         "diet"         
"plum"          "h"             "sure"         
 [73] "beets"         "select"        "across"        "mush"          "squash"       
"jam"           "themselves"    "garnished"    
 [81] "alone"         "saltspoonful"  "allowing"      "degrees"       "perfect"      
"stones"        "bright"        "sage"         
 [89] "clams"         "tart"          "steaks"        "spanish"       "did"          
"fact"          "cauliflower"   "tail"         
 [97] "gum"           "greased"       "pack"          "trim"          "formed"       
"knead"         "gives"         "grape"        
[105] "amp"           "cleaned"       "stuff"         "sort"          "nature"       
"oatmeal"       "certain"       "bed"          
[113] "scalded"       "tenderloin"    "fowls"         "within"        "grapes"       
"along"         "olives"        "shoulder"     
[121] "w"             "molds"         "cocoa"         "flowers"       "fall"         
"measure"       "f"             "thickened"    
[129] "candy"         "plums"         "child"         "follows"       "foods"        
"238"           "25"            "9"            
[137] "en"            "calf's"        "mode"          "market"        "am"           
"value"         "shallow"       "lastly"       
[145] "digestion"     "miss"          "pretty"        "cherry"        "want"         
"lined"         "sick"          "proportions"  
[153] "diameter"      "came"          "raspberries"   "indeed"        "crab"         
"true"          "rule"          "smoked"       
[161] "apart"         "prefer"        "nutritious"    "domestic"      "burn"         
"clove"         "lime"          "rum"          
[169] "preparing"     "choose"        "pulverized"    "world"         "frozen"       
"lower"         "spoons"        "smaller"      
[177] "equally"       "bad"           "eyes"          "dr"            "useful"       
"receipt"       "blanch"        "candied"      
[185] "wafers"        "degree"        "ribs"          "tureen"        "boat"         
"sweetened"     "halves"        "repeat"       
```

```
 [193] "natural"        "change"         "bird"           "already"        "sandwiche
s"     "sticks"         "loaves"         "frosting"
 [201] "garnishing"     "moderately"     "mind"           "j"              "linen"
"puffs"          "towel"          "drained"
 [209] "fourths"        "contains"       "192"            "similar"        "226"
"proceed"        "england"        "anchovy"
 [217] "gallons"        "goods"          "convenient"     "save"           "hang"
"roots"          "honey"          "present"
 [225] "claret"         "contain"        "cork"           "human"          "turtle"
"waters"         "fast"           "ducks"
 [233] "applied"        "system"         "chestnuts"      "strength"       "somewhat"
"try"            "meringue"       "bouillon"
 [241] "hash"           "pigeons"        "covering"       "rapidly"        "says"
"finger"         "clarified"      "daily"
 [249] "solid"          "basting"        "176"            "york"           "sold"
"german"         "cause"          "itself"
 [257] "tough"          "ways"           "figs"           "mass"           "jellies"
"states"         "sand"           "flower"
 [265] "snow"           "core"           "account"        "whether"        "rises"
"nine"           "cod"            "apply"
 [273] "flannel"        "simply"         "mouth"          "lukewarm"       "muffins"
"gills"          "poached"        "pared"
 [281] "ball"           "supply"         "alcohol"        "placing"        "partly"
"g"              "harden"         "basket"
 [289] "age"            "drippings"      "substance"      "greater"        "border"
"employed"       "na"             "neatly"
 [297] "grown"          "lid"            "50"             "et"             "charlotte"
"rolling"        "steamed"        "below"
 [305] "pressed"        "joint"          "luncheon"       "original"       "otherwise"
"tied"           "takes"          "evenly"
 [313] "lengthwise"     "giving"         "former"         "alum"           "picked"
"suitable"       "16"             "st"
 [321] "cornstarch"     "pumpkin"        "cr"             "seal"           "force"
"juices"         "thickly"        "wire"
 [329] "warmed"         "cutter"         "ears"           "folded"         "subject"
"walnuts"        "lie"            "toward"
 [337] "condition"      "beautiful"      "savory"         "weigh"          "sausage"
"pouring"        "afterwards"     "something"
 [345] "sent"           "251"            "cookies"        "substances"     "chief"
"per"            "liked"          "mint"
 [353] "attention"      "strew"          "live"           "cotton"         "earth"
"teacup"         "ii"             "contents"
 [361] "sausages"       "places"         "weak"           "blanc"          "material"
"pin"            "purposes"       "scant"
 [369] "40"             "palatable"      "rabbit"         "fermentation"   "horse"
"handle"         "spirits"        "plants"
 [377] "muslin"         "described"      "cents"          "broad"          "saddle"
"strainer"       "fifty"          "cooks"
 [385] "valuable"       "entire"         "beer"           "apricot"        "deer"
```

```
"mackerel"       "anything"       "shad"
 [393] "rinse"          "coals"          "difficult"      "macaroons"      "raise"
"wrap"           "bear"           "decorate"
 [401] "easy"           "reason"         "sirloin"        "smoke"          "trout"
"cheap"          "rings"          "low"
 [409] "lift"           "everything"     "space"          "tell"           "labor"
"divided"        "golden"         "lumps"
 [417] "whose"          "moulds"         "thou"           "marrow"         "sago"
"gruel"          "entr"           "effect"
 [425] "empty"          "father"         "sprinkled"      "barrel"         "cans"
"character"      "circle"         "result"
 [433] "pink"           "seems"          "whatever"       "china"          "cucumber"
"door"           "finished"       "cooled"
 [441] "regard"         "234"            "bass"           "crabs"          "curry"
"since"          "brains"         "porcelain"
 [449] "gathered"       "alternately"    "going"          "himself"        "creamed"
"meals"          "quinces"        "horseradish"
 [457] "tarts"          "plan"           "champagne"      "show"           "trouble"
"materials"      "floor"          "kidney"
 [465] "24"             "opening"        "doing"          "port"           "supper"
"italian"        "ancient"        "sea"
 [473] "saw"            "halibut"        "hominy"         "standing"       "patties"
"thickens"       "service"        "particular"
 [481] "follow"         "souffl"         "cure"           "prunes"         "18"
"particularly"   "families"       "pea"
 [489] "14"             "aunt"           "appears"        "produce"        "agreeable"
"tub"            "rubbing"        "kernels"
 [497] "crusts"         "anchovies"      "face"           "eighth"         "help"
"sew"            "single"         "salads"
 [505] "beginning"      "silk"           "mange"          "codfish"        "juicy"
"united"         "tarragon"       "household"
 [513] "weighing"       "turnip"         "ashes"          "apricots"       "ha"
"hare"           "parsnips"       "composed"
 [521] "joints"         "boston"         "breaking"       "south"          "creams"
"mr"             "tripe"          "remains"
 [529] "avoid"          "broiling"       "blade"          "corned"         "seem"
"scalding"       "gridiron"       "individual"
 [537] "special"        "keeper"         "artichokes"     "sound"          "class"
"superior"       "brisk"          "passed"
 [545] "animals"        "exactly"        "pressing"       "neither"        "stems"
"disease"        "rump"           "beet"
 [553] "outer"          "tree"           "girl"           "moments"        "thrown"
"moisture"       "pig"            "seldom"
 [561] "whom"           "11"             "oval"           "knowledge"      "patient"
"city"           "pain"           "creamy"
 [569] "remedy"         "kidneys"        "sirup"          "spoonsful"      "told"
"big"            "pine"           "worth"
 [577] "letting"        "wines"          "economy"        "improvement"    "butterbutt
er"   "moist"          "greatly"        "knuckle"
```

```
 [585] "covers"          "evening"         "appear"          "front"           "public"         
"boy"             "foot"            "wings"          
 [593] "folktale"        "holes"           "action"          "shapes"          "finest"         
"coming"          "arranged"        "damp"           
 [601] "june"            "fasten"          "lunch"           "tablespoonsful" "rules"          
"gas"             "ma"              "forth"          
 [609] "looks"           "importance"      "rising"          "killed"          "parmesan"       
"mentioned"       "farina"          "priest"         
 [617] "suit"            "75"              "art"             "o"               "radish"         
"markets"         "rhubarb"         "touch"          
 [625] "month"           "street"          "hollow"          "depends"         "practice"       
"call"            "capers"          "sheets"         
 [633] "finally"         "went"            "sprigs"          "france"          "substitut
e"     "readily"         "cellar"          "east"           
 [641] "healthy"         "246"             "answer"          "drying"          "slip"           
"cranberry"       "livers"          "medicine"       
 [649] "alternate"       "scraped"         "july"            "youth"           "believe"        
"relish"          "apt"             "principal"      
 [657] "glassful"        "pearl"           "gras"            "skimmer"         "spirit"         
"vessels"         "clothes"         "march"          
 [665] "cuts"            "pleasant"        "none"            "considerable"    "bark"           
"object"          "improved"        "looking"        
 [673] "tails"           "shred"           "caramel"         "cracked"         "calf"           
"skewer"          "231"             "holding"        
 [681] "coloring"        "rock"            "later"           "height"          "scalloped"      
"breasts"         "stalk"           "price"          
 [689] "casserole"       "clam"            "isinglass"       "preferable"      "late"           
"april"           "probably"        "spiced"         
 [697] "saleratus"       "favorite"        "russe"           "13"              "potage"         
"pull"            "60"              "followed"       
 [705] "composition"     "eye"             "glazed"          "bananas"         "field"          
"hence"           "peck"            "escape"         
 [713] "dash"            "coat"            "hunter"          "quenelles"       "stop"           
"causes"          "coyote"          "largest"        
 [721] "boxes"           "hams"            "pancakes"        "experience"      "living"         
"company"         "danger"          "southern"       
 [729] "economical"      "remarks"         "parboil"         "saltpetre"       "dropped"        
"lima"            "merely"          "wife"           
 [737] "houses"          "rib"             "carried"         "packed"          "crushed"        
"risen"           "held"            "17"             
 [745] "straw"           "preceding"       "marble"          "timbale"         "essential"      
"gentle"          "grow"            "besides"        
 [753] "fricassee"       "pots"            "corner"          "mouse"           "skimming"       
"produced"        "working"         "america"        
 [761] "menu"            "velout"          "waste"           "word"            "sticking"       
"414"             "terrapin"        "circumstances"  
 [769] "desirable"       "laying"          "whisk"           "freely"          "marjoram"       
"receipts"        "duties"          "pear"           
 [777] "45"              "wax"             "don't"           "rabbits"         "sole"
```

```
"reach"           "fully"           "herself"
 [785] "415"             "copper"          "greens"          "heard"           "spit"
"portions"        "hollandaise"     "blackberry"
 [793] "freezer"         "surround"        "couple"          "herring"         "ring"
"shrimps"         "colors"          "heating"
 [801] "stem"            "buttermilk"      "pail"            "points"          "deal"
"stoned"          "thread"          "gems"
 [809] "fore"            "moistened"       "cleaning"        "obtain"          "mean"
"narrow"          "gather"          "servants"
 [817] "soil"            "lobsters"        "provided"        "feathers"        "pitcher"
"crack"           "22"              "species"
 [825] "chamel"          "drinks"          "maidens"         "smoothly"        "tray"
"19"              "dutch"           "rare"
 [833] "spots"           "west"            "buy"             "waffles"         "thickenin
g"      "hung"            "aid"             "regular"
 [841] "exercise"        "example"         "lost"            "tables"          "waterwate
r"      "mock"            "21"              "took"
 [849] "bouquet"         "pile"            "choice"          "ou"              "yes"
"shown"           "power"           "stewing"
 [857] "noodles"         "chiefly"         "35"              "inferior"        "increase"
"braised"         "floured"         "named"
 [865] "gingerbread"     "cultivated"      "truffle"         "n"               "intended"
"forming"         "friends"         "frances"
 [873] "render"          "100"             "larded"          "seeded"          "curd"
"hearts"          "setting"         "results"
 [881] "teeth"           "bottoms"         "turpentine"      "fare"            "native"
"paint"           "throughout"      "potatoe"
 [889] "running"         "hind"            "iced"            "gooseberries"    "extra"
"average"         "appetite"        "buckwheat"
 [897] "sense"           "ammonia"         "chowder"         "school"          "throat"
"unmold"          "cross"           "quince"
 [905] "methods"         "lemonade"        "looked"          "stains"          "prayer"
"sherbet"         "foundation"      "solution"
 [913] "europe"          "ox"              "certainly"       "principally"     "doubt"
"popular"         "freshly"         "continued"
 [921] "gooseberry"      "began"           "lead"            "tongues"         "scotch"
"tried"           "melon"           "receive"
 [929] "vent"            "firmly"          "afterward"       "boned"           "ware"
"changing"        "sister"          "skimmed"
 [937] "growing"         "196"             "smoking"         "worked"          "bind"
"myth"            "soften"          "polish"
 [945] "slack"           "27"              "principles"      "cro"             "guests"
"ago"             "sarah"           "le"
 [953] "thousand"        "store"           "please"          "branches"        "really"
"operation"       "lose"            "entremets"
 [961] "complete"        "spoil"           "26"              "favor"           "fashion"
"28"              "closed"          "difference"
 [969] "further"         "plunge"          "cent"            "x153"            "extent"
"burnt"           "sprinkling"      "crumb"
```

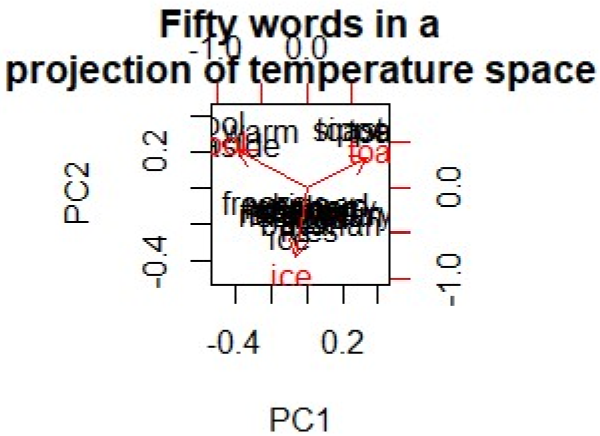
```
 [977] "invalids"        "mothers"         "strip"           "eels"            "shallots"
"note"            "cal"             "brother"
 [985] "dilute"          "zuÃ±is"          "irish"           "goes"            "distance"
"inner"           "advantage"       "wind"
 [993] "23"              "sit"             "rooms"           "business"        "fashioned"
"consists"        "brain"           "carving"
[1001] "quartered"
```

Hide

```
# We can plot along mltiple dimensions:
tastes = c("toast", "cool", "ice")
common_similarities_tastes = model[1:5000,]%>% cosineSimilarity( model[[tastes,average
=F]])
high_similarities_to_tastes = common_similarities_tastes[rank(-apply(common_similariti
es_tastes,1,max)) < 20,]
# - Plotting
high_similarities_to_tastes %>%
  as_tibble(rownames='word') %>%
  filter( ! (is.element(word,tastes))) %>%
  #mutate(total = salty+sweet+savory+bitter+sour) %>%
  #mutate( sweet=sweet/total,salty=salty/total,savory=savory/total,bitter=bitter/tota
l, sour = sour/total) %>%
  #select(-total) %>%
  gather(key = 'key', value = 'value',-word) %>%
  ggplot(aes(x = word,
             y = value,
             fill = key)) + geom_bar(stat='identity') +
  coord_flip() + theme_minimal() + scale_fill_brewer(palette='Spectral')
```

Hide

```
# --- Most similar terms  ---
high_similarities_to_tastes %>%
  prcomp %>%
  biplot(main="Fifty words in a\nprojection of temperature space")
```



I think these make sense - I would expect words associated with cool to be closer to ice than to toast, and that seems to be the case, at least somewhat. The words also make sense generally, usually you “set aside” things to cool or let dry. All the words that had the highest “ice” values were logical. “Crisp”, “graham”, and “wafer” all make sense for toast as well.

5

Hide

```
####################################
#####   Vector calculations  #####
####################################
model %>% closest_to("health") # words associated with haelthy living (if not a bit ou
tdated)
```

word <chr>	similarity to "health" <dbl>
health	1.0000000
constitution	0.7497024
piety	0.7412316
enjoyment	0.7405042
maintenance	0.7376751
interests	0.7319397

word <chr>	similarity to "health" <dbl>
intellect	0.7177833
comfort	0.7090357
thrift	0.7042710
happiness	0.7030533

1-10 of 10 rows

Hide

```
model %>% closest_to(~("health" - "cream" ),15) # number 7 is cravings
```

word <chr>	similarity to ("health" - "cream") <dbl>
health	0.7817636
indulgences	0.5683702
interests	0.5622593
physical	0.5604732
lives	0.5563459
dispensing	0.5547542
degradation	0.5542852
promoted	0.5508144
correctness	0.5505568
profess	0.5468486

1-10 of 15 rows Previous **1** 2 Next

Hide

```
model %>% closest_to(~"orange" + ("pretzel"- "salty"),15)
```

word <chr>	similarity to "orange" + ("pretzel" - "salty") <dbl>
orange	0.5575795
pretzel	0.5352269

word <chr>	similarity to "orange" + ("pretzel" - "salty") <dbl>
bergamot	0.4654249
ratafias	0.4578950
flora	0.4577522
owings	0.4516814
kisses	0.4509562
cheesecake	0.4497312
rhubarb	0.4495824
331lemon	0.4447752

1-10 of 15 rows Previous **1** 2 Next

Hide

```
model %>% closest_to(~"french" + ("florentine" - "kebab"),15)
```

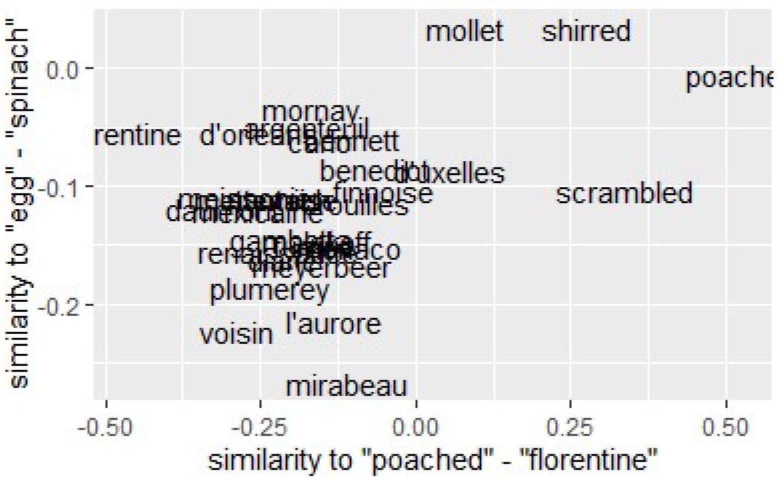
word <chr>	similarity to "french" + ("florentine" - "kebab") <dbl>
florentine	0.6645936
douglas	0.5914435
rudini	0.5773749
choux	0.5658704
doria	0.5634538
ratrice	0.5568442
smolenska	0.5550327
chamberlain	0.5540282
d'orleans	0.5534945
chaud	0.5532485

1-10 of 15 rows Previous **1** 2 Next

Hide

```
top_evaluative_words = model %>%
  closest_to(~ "poached"+"florentine",n=30)
goodness = model %>%
  closest_to(~ "poached"-"florentine",n=Inf)
taste = model %>%
  closest_to(~ "egg" - "spinach", n=Inf)
top_evaluative_words %>%
  inner_join(goodness) %>%
  inner_join(taste) %>%
  ggplot() +
  geom_text(aes(x=`similarity to "poached" - "florentine"`,
                y=`similarity to "egg" - "spinach"`,
                label=word))
```

```
Joining, by = "word"
Joining, by = "word"
```



Hide

```
####################################
#####   Vector calculations   #####
####################################
model %>% closest_to("bake") # words associated with haelthy living (if not a bit outd
ated)
```

word <chr>	similarity to "bake" <dbl>
bake	1.0000000
oven	0.7032936
moderate	0.6270468

word <chr>	similarity to "bake" <dbl>
quick	0.6030007
tins	0.5961357
pans	0.5832742
brickloaf	0.5696879
moderatemoderate	0.5641077
buttered	0.5596496
gem	0.5585893

1-10 of 10 rows

Hide

```
model %>% closest_to(~("bake" - "sweet" ),15) # number 7 is cravings
```

word <chr>	similarity to ("bake" - "sweet") <dbl>
bake	0.7132357
oven	0.5026668
tins	0.4846343
moderate	0.4816047
pans	0.4454488
buttered	0.4437071
slack	0.4059095
gem	0.3987069
patty	0.3937922
quick	0.3928549

1-10 of 15 rows Previous 1 2 Next

Hide

```
model %>% closest_to(~"chicken" + ("bake"- "marinate"),15)
```

word <chr>	similarity to "chicken" + ("bake" - "marinate") <dbl>
bake	0.5299452
pie	0.4877393
chicken	0.4743963
pigeon	0.4690289
baked	0.4533274
patties	0.4409676
oven	0.4331568
ramekins	0.4274505
rice	0.4225753
oven.egg	0.4119723

1-10 of 15 rows Previous **1** 2 Next

Hide

```
model %>% closest_to(~"southern" + ("spicy" - "meat"),15)
```

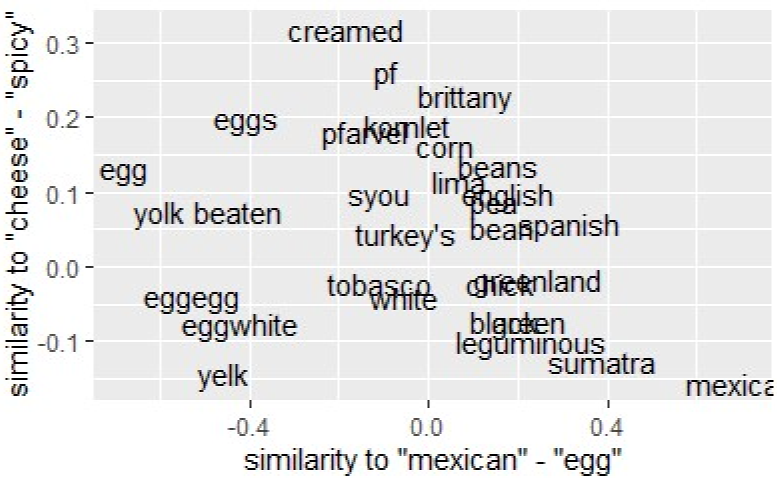
word <chr>	similarity to "southern" + ("spicy" - "meat") <dbl>
spicy	0.6874666
southern	0.5910659
mexico	0.5470400
siberia	0.5393642
countries	0.5352862
sumatra	0.5277036
connoisseurs	0.5223007
coffees	0.5177040
european	0.5103582
eastern	0.5020361

1-10 of 15 rows Previous **1** 2 Next

Hide

```
top_evaluative_words = model %>%
  closest_to(~ "mexican"+"egg",n=30)
goodness = model %>%
  closest_to(~ "mexican"-"egg",n=Inf)
taste = model %>%
  closest_to(~ "cheese"-"spicy", n=Inf)
top_evaluative_words %>%
  inner_join(goodness) %>%
  inner_join(taste) %>%
  ggplot() +
  geom_text(aes(x=`similarity to "mexican" - "egg"`,
                y=`similarity to "cheese" - "spicy"`,
                label=word))
```

```
Joining, by = "word"
Joining, by = "word"
```



6

I thought it was interesting that looking at “bake - sweet” got rid of the term muffin which made sense, but didn’t produce any savory items specifically. It was mostly technical terms or tools associated with baking. This might be because most of the recipes were for sweet baked good, so most of the “non-sweet” items were just neutral in flavor.

It was odd that “southern+(spicy-meat)” mostly lead to a list of a lot of other cuisines, I didn’t expect that and I’m not really sure why that was the case. Logically, I thought it might lead to some vegetables like okra or a list of beans, but that wasn’t the case at all.

My graph of “mexican-egg”vs“cheese-spicy” was very interesting! The top right area shows enchiladas which indeed are mexican, do not contain eggs, and often contain cheese. However, they can be spicy so it made total sense for it to be somewhat low on the “cheese-spicy” similarity scale. It was weird that the term “greenland” showed up so high on the “mexican-egg” scale. I’m not sure why.

7

Hide

```
if (!file.exists("cookbooks.zip")) {
  download.file("http://archive.lib.msu.edu/dinfo/feedingamerica/cookbook_text.zip","c
ookbooks.zip")
}
unzip("cookbooks.zip",exdir="cookbooks")
if (!file.exists("cookbooks2.txt")) prep_word2vec(origin="cookbooks",destination="cook
books2.txt",lowercase=T,bundle_ngrams=2)
# Training a Word2Vec model
if (!file.exists("cookbook_vectors2.bin")) {
  model2 = train_word2vec("cookbooks2.txt","cookbook_vectors2.bin",
                         vectors=100,threads=4,window=6,
                         min_count = 10,
                         iter=5,negative_samples=15)
} else{
    model2 = read.vectors("cookbook_vectors2.bin")
}
```

```
Starting training using file C:/Users/Arshia/Documents/Georgetown/ANLY601 Advanced Mac
hine Learning/Advanced-Machine-Learning/Assignment 3/cookbooks2.txt
100K
200K
300K
400K
500K
600K
700K
800K
900K
1000K
1100K
1200K
1300K
1400K
1500K
1600K
1700K
1800K
1900K
2000K
2100K
2200K
2300K
2400K
2500K
2600K
2700K
2800K
2900K
3000K
3100K
3200K
3300K
3400K
3500K
3600K
3700K
3800K
3900K
4000K
4100K
4200K
4300K
4400K
4500K
4600K
```

```
4700K
4800K
4900K
5000K
5100K
5200K
5300K
5400K
5500K
5600K
5700K
5800K
5900K
6000K
6100K
6200K
6300K
6400K
6500K
6600K
6700K
6800K
6900K
7000K
7100K
7200K
7300K
7400K
7500K
7600K
7700K
7800K
7900K
8000K
8100K
8200K
8300K
8400K
8500K
8600K
8700K
8800K
8900K
9000K
9100K
9200K
9300K
9400K
9500K
```

```
9600K
9700K
9800K
Vocab size: 25375
Words in train file: 9660178
```

```
Filename ends with .bin, so reading in binary format
Reading a word2vec binary file of 25375 rows and 100 columns
```

Hide

```
terms <- rownames(model2)
bigr <- "_"
bigrams <- terms[grepl(bigr,terms, fixed=TRUE)]
bigrams[1:50]
```

```
 [1] "1_2"              "an_hour"          "224_la"           "do_not"           "has_bee
n"        "160_160"         "an_inch"
 [8] "bread_crumbs"     "have_been"        "melted_butter"    "lemon_juice"      "twenty_m
inutes"  "as_soon"         "ten_minutes"
[15] "32_32"            "three_quarters"   "five_minutes"     "baking_powder"    "powdered
_sugar"  "as_possible"     "at_once"
[22] "an_ounce"         "an_illustration" "fifteen_minutes" "small_pieces"     "more_tha
n"       "few_minutes"     "moderate_oven"
[29] "four_hours"       "chopped_parsley" "white_wine"       "ice_cream"        "thin_sli
ces"     "frying_pan"      "over_night"
[36] "at_least"         "does_not"         "have_ready"       "four_ounces"      "table_sp
oonful"  "hard_boiled"     "any_other"
[43] "just_before"      "cool_place"       "table_spoonfuls" "lemon_peel"       "3_4"
"sifted_flour"    "stew_pan"
[50] "those_who"
```

These all make sense and most are specific to cooking and timing/measurement: “melted butter”, “bread crumbs”, “lemon juice”, “powdered sugar”, “frying pan”, “hard boiled”, “chopped parsley”, “baking powder”, “white wine”, “lemon peel”

Question 4 - Gaussian Processes

Hide

```
library(MASS)
```

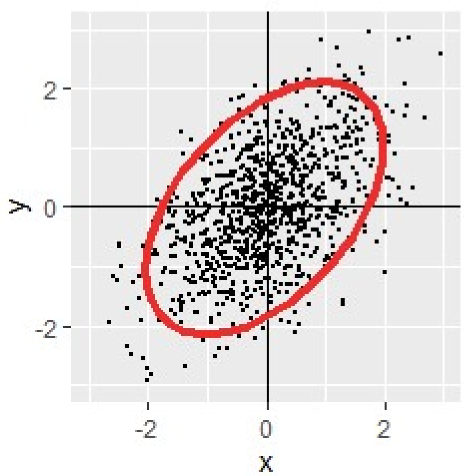
```
Attaching package: <U+393C><U+3E31>MASS<U+393C><U+3E32>

The following object is masked from <U+393C><U+3E31>package:dplyr<U+393C><U+3E32>:

    select
```

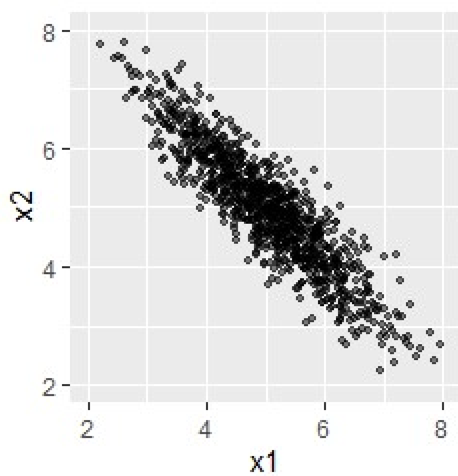
Hide

```
##########################################################
## Plotting Gaussian plots
##########################################################
# Multivariate normal 0 variance
set.seed(1234)
d2 = mvrnorm(n =1000, mu =c(0,0), Sigma = matrix(c(1,0.5,0.5,1),ncol = 2))
ggplot(aes(x = x, y= y), data = tibble(x = d2[,1], y = d2[,2])) +
  geom_point(size = 0.5)+
  geom_vline(xintercept = 0) + geom_hline(yintercept = 0) +
  stat_ellipse(size = 1.5, color = 'firebrick2')  +
  coord_fixed(ratio = 1)+ ylim(c(-3,3))+xlim(c(-3,3))
```



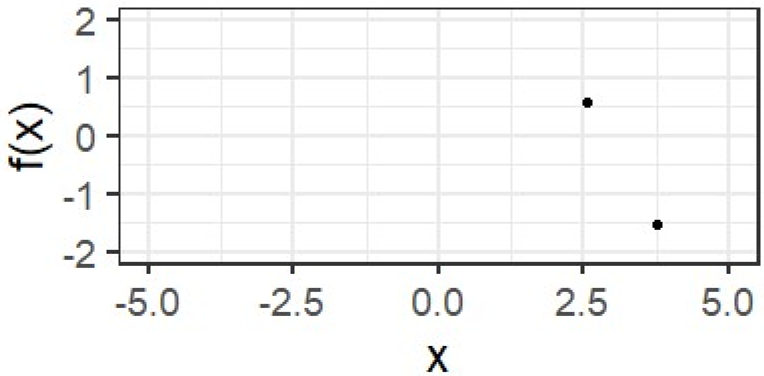
Hide

```
###################################
## Simulating a bivariate normal #
###################################
n_samples = 1000
d = 2
Z = matrix(rnorm(n_samples * d), ncol = 2)
rho = -0.9
mu = c(5,5)
Sigma = matrix(c(1,rho,rho,1), ncol = 2)
L = chol(Sigma)
X = mu + Z %*% L
colnames(X) = c('x1','x2')
ggplot(aes(x =x1, y = x2), data = as_tibble(X)) +
  geom_point(size = 1, alpha =0.5)+
  coord_fixed(ratio = 1) +xlim(c(2,8))+ylim(c(2,8))
```



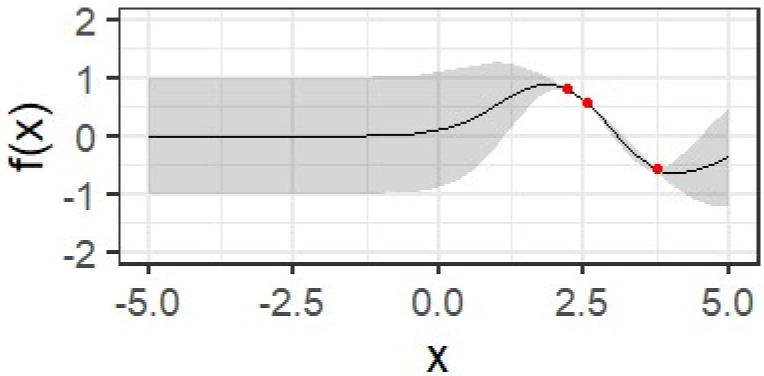
Hide

```
#########################################################
## 3 Random points on a graph
#########################################################
n = 50
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
f = sin(x_observed) + rnorm(3)
theme_set(theme_bw(base_size = 18))
ggplot(aes(x = x, y= y), data = tibble(x = x_observed, y = f)) +
  geom_point() +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```
########################################################
## 3 Random points on a graph
########################################################
# Kernel matrix
K = function(x,x_prime,l){
  d = sapply(x, FUN = function(x_in)(x_in - x_prime)^2)
  return(t(exp(-1/(2*l^2) *d)))
}
# Generating Data
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
x_prime = seq(-5,5,length.out = n)
f = sin(x_observed)
# Setting up GP
mu = 0
mu_star = 0
l = 1
# Covariance of f
K_f = K(x_observed,x_observed,l)
# Marginal and conditional covariance of f_star|f
K_star = K(x_observed,x_prime,l)
K_starstar = K(x_prime,x_prime,l)
# Conditional distribution of  f_star|f
mu_star = mu_star + t(K_star) %*% solve(K_f) %*% (f - mu)
Sigma_star = K_starstar - t(K_star)%*% t(solve(K_f)) %*% K_star
# Re-arranging values for plotting
plot_gp = tibble(x = x_prime,
                 y = mu_star %>% as.vector(),
                 sd_prime = sqrt(diag(Sigma_star)))
# Plotting values
ggplot(aes(x = x, y = y), data = plot_gp) +
  geom_line()+
  geom_ribbon(aes(ymin = y-sd_prime,ymax = y+sd_prime), alpha = 0.2)+
  geom_point(aes(x =x , y= y), data = tibble(x = x_observed, y = f),
             color = 'red') +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```
x = c(1,2,3)
x_prime= c(1,2,3)
########################################################
## Examples of GPs
########################################################
# Kernel matrix
Wiener_Process = function(x){
  return(sapply(x, FUN = function(x_in)(pmin(x_in, x))))
}
Ornstein_Uhlenbeck= function(x){
  d = sapply(x, FUN = function(x_in)(abs(x_in- x)))
  return(exp(-d))
}
Browninan_bridge = function(x){
  # x in (0,1)
  d1 = sapply(x, FUN = function(x_in)(pmin(x_in, x)))
  d2 = sapply(x, FUN = function(x_in)(x_in * x))
  return(d1-d2)
}
kernel_rbf = function(x){
  exp(-as.matrix(dist(x, diag = T))^2/2)
}
sampling_from_a_gp = function(x_min = 0,
                              x_max=1,
                              kernel_in,
                              n = 50,
                              n_gps = 10){

  # Simulation
  x = seq(x_min, x_max,length.out = n)
  K = kernel_in(x)
  L = chol(K + 1e-6*diag(n))
  f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)

  # Reshaping
  colnames(f_prior) = paste0('Simulation ', seq(1:n_gps))
  f_prior_long_format = f_prior %>% as_tibble() %>%
    bind_cols(x = x) %>%
    pivot_longer(cols = starts_with("sim"))

  # Plot
  p = ggplot(aes(x = x, color = name, y = value),
             data = f_prior_long_format) +
    geom_line()+theme(legend.position = 'bottom')+
    guides(color=guide_legend(title=""))+
    ylab('f(x)')
  return(list('data_out' = f_prior, 'plot' = p))
```

```
}
sampling_from_a_gp(kernel_in = Browninan_bridge, n_gps = 5, n = 1000)
```

```
$`data_out`
       Simulation 1  Simulation 2  Simulation 3  Simulation 4  Simulation 5
  [1,]  0.0012061728  0.0012377338 -0.0003325691 -0.0001664464 -0.0001575622
  [2,] -0.0306451950  0.0378766928  0.0282135051  0.0478212293 -0.0442844758
  [3,] -0.0298519288  0.0279481460  0.0339105107  0.1077928408 -0.0002522062
  [4,]  0.0435072632  0.0995436300  0.0713300021  0.0871352992  0.0294462497
  [5,]  0.0109034746  0.0960214559  0.1015185464  0.1050882039  0.0482939649
  [6,] -0.0856840651  0.0631094908  0.0637869446  0.1170742734  0.0818769947
  [7,] -0.0884818185  0.0529436194  0.0810874056  0.1458084689  0.0898030193
  [8,] -0.0962057968  0.0625039807  0.0686261550  0.1472779718  0.0952506801
  [9,] -0.0435185036  0.1281080063  0.0771313242  0.1009221943  0.0975549410
 [10,] -0.0577365372  0.1555904599  0.0560355780  0.1003893586  0.1308066206
 [11,] -0.0403795026  0.2202845569  0.0207337869  0.0338217075  0.1082910872
 [12,] -0.0492116673  0.2363259307  0.0275806401  0.0271399244  0.1835490991
 [13,] -0.0456501135  0.2594346470  0.0227442144  0.0662866028  0.1852989596
 [14,] -0.0468329278  0.2760937933  0.0011778784  0.0828130748  0.1843584794
 [15,] -0.1263269531  0.2212097079 -0.0376807688  0.1017172414  0.1302569117
 [16,] -0.1111523161  0.1655524083 -0.0215373041  0.0930027720  0.0981750117
 [17,] -0.1222635284  0.1820311160  0.0154590146  0.1236959059  0.1308171223
 [18,] -0.0808006207  0.1062396354 -0.0230510979  0.1209168448  0.1392551488
 [19,] -0.1158174635  0.1618501492 -0.0402464407  0.1203460012  0.1517211135
 [20,] -0.1213119803  0.1706406817 -0.0974785158  0.1272136790  0.1660679083
 [21,] -0.0974556548  0.2210340935 -0.1179034474  0.0791859431  0.1651469884
 [22,] -0.1120759855  0.1890963727 -0.1457496165  0.0649190795  0.2400863189
 [23,] -0.1661355617  0.2132550978 -0.2056685552  0.0984165254  0.2379235550
 [24,] -0.1557799684  0.2103404822 -0.2004550153  0.1800763106  0.2263044515
 [25,] -0.1257630439  0.1677691006 -0.1887027882  0.1002836159  0.2492814609
 [26,] -0.1508593345  0.1923868206 -0.1933499921  0.1115117120  0.2562161456
 [27,] -0.1507395489  0.2191941163 -0.2210083430  0.0941411115  0.2312957779
 [28,] -0.1949891192  0.2154800604 -0.2860868996  0.1321510571  0.2156476230
 [29,] -0.2174510115  0.2124038141 -0.3337096405  0.1278353125  0.2134053564
 [30,] -0.2376973560  0.2130297739 -0.2928134905  0.0894366034  0.2569950027
 [31,] -0.2153918852  0.2101607492 -0.2505741882  0.1235751586  0.2512356305
 [32,] -0.2397470844  0.2412667759 -0.2449710711  0.1437688428  0.3006689523
 [33,] -0.2723556769  0.2635929467 -0.2507576426  0.1193062082  0.2434700818
 [34,] -0.2210409905  0.2471791965 -0.2268810083  0.1149877537  0.2637992122
 [35,] -0.2204239343  0.2376097226 -0.2364784418  0.1006636722  0.2245182817
 [36,] -0.2336152274  0.2723318203 -0.2336096516  0.1084582154  0.1965367841
 [37,] -0.2237773562  0.2490695846 -0.2662953950  0.1779631166  0.2373307362
 [38,] -0.2109326775  0.2649428108 -0.2479525356  0.2125638346  0.2145079530
 [39,] -0.2435196290  0.2417161536 -0.2305147495  0.2436397750  0.2282747407
 [40,] -0.2420874198  0.2227763712 -0.2609572994  0.2838030668  0.2541064159
 [41,] -0.3073807479  0.1804926717 -0.2055008285  0.3228735881  0.2634252724
 [42,] -0.3231376987  0.1373503177 -0.2289386754  0.2953225507  0.1850140647
 [43,] -0.3225982654  0.1207448944 -0.2576100478  0.3456088467  0.1471102917
 [44,] -0.3713062244  0.1480481128 -0.2468126690  0.3500046861  0.1590800880
 [45,] -0.3432921893  0.1059886593 -0.2274344674  0.3269015123  0.1477590295
 [46,] -0.3348647913  0.0442744917 -0.1872432742  0.3248949232  0.1469418575
```

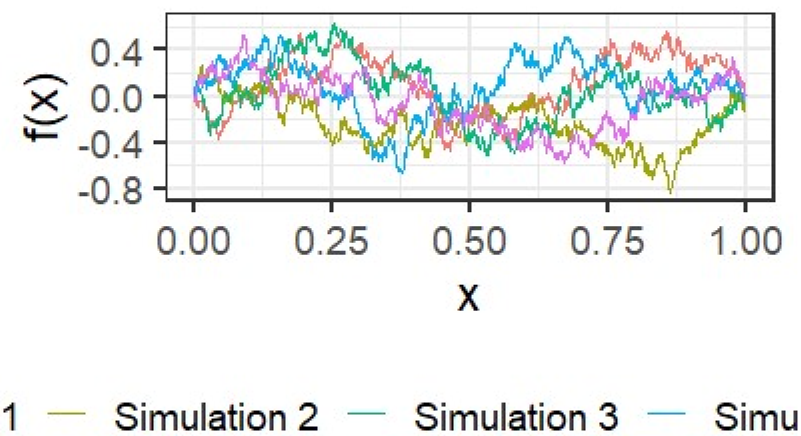
```
[47,] -0.3334230643  0.0228566843 -0.1915932749  0.3579976664  0.1294688011
[48,] -0.3117296467  0.0527176352 -0.1735198834  0.3052790095  0.1372699176
[49,] -0.3196210704  0.0307475890 -0.1705961074  0.3103233175  0.1087852534
[50,] -0.3098353284  0.0131575289 -0.1619172493  0.2950429401  0.2349246710
[51,] -0.2999975955 -0.0019246327 -0.1345171103  0.2883800719  0.2182927897
[52,] -0.2586696059 -0.0476884475 -0.1249002391  0.2890473527  0.2167971728
[53,] -0.2236235987 -0.0389279632 -0.0248998460  0.3237025871  0.1656367287
[54,] -0.2483953247 -0.0507348327 -0.0048190756  0.3250403646  0.1947509420
[55,] -0.2574913948 -0.0501905202 -0.0139773754  0.2723779496  0.1872941624
[56,] -0.2669010008 -0.0642423223 -0.0050457561  0.2264816060  0.1850707100
[57,] -0.2249945406  0.0418707733  0.0133657255  0.2406380658  0.1410404569
[58,] -0.2177618536  0.0211335867 -0.0056156370  0.3045644148  0.1636160512
[59,] -0.2322718648 -0.0393208208 -0.0062365060  0.2826651370  0.1543167163
[60,] -0.2308264572 -0.0500875152  0.0262861259  0.2409333650  0.1714247872
[61,] -0.2154895190 -0.0430076249 -0.0374972647  0.2179197271  0.1936557653
[62,] -0.1886087927 -0.0138224887 -0.0307802082  0.1731501355  0.2195475245
[63,] -0.2166159670  0.0123967186 -0.0158362403  0.1492674655  0.2251328398
[64,] -0.1925230149  0.0056791106 -0.0094530075  0.1595472907  0.2610919657
[65,] -0.1782485576  0.0069940376 -0.0420992175  0.2177340363  0.2631108019
[66,] -0.1647425937 -0.0199411468 -0.0436779389  0.1971418765  0.3129623278
[67,] -0.1281912069 -0.0587342981 -0.0264770268  0.1376142672  0.2771570636
[68,] -0.1564614902 -0.0560552668  0.0183135851  0.1427842644  0.2496803714
[69,] -0.1287374618 -0.0836780524  0.0474791524  0.1299243683  0.2416533420
[70,] -0.1126129244 -0.0474276527  0.0322702540  0.1083737833  0.2481374170
[71,] -0.1020083884  0.0047280661  0.0503104271  0.0785394757  0.2488977063
[72,] -0.0468326225  0.0284746205  0.0414628039  0.1197882399  0.3123237053
[73,] -0.0471195857  0.0617119950  0.0992589878  0.1426047669  0.3013596712
[74,] -0.0573942447  0.0335175204  0.1432018727  0.1363535471  0.3442852481
[75,] -0.0526201207  0.0536150901  0.1603620890  0.1468385572  0.3427201179
[76,] -0.0645919000  0.1003617154  0.1155573263  0.1745118749  0.3285582991
[77,] -0.0395534555  0.1228125765  0.0479166868  0.1819778874  0.3620583891
[78,] -0.0449562709  0.1341002446  0.0401476087  0.2111428670  0.3333791533
[79,] -0.0367235510  0.0908490641  0.0225403091  0.1674500343  0.3029550835
[80,] -0.0290896352  0.0625639554  0.0435514338  0.2080927457  0.2891835377
[81,] -0.0008196417  0.0471791370  0.0461353362  0.2454109370  0.3188064792
[82,] -0.0014572326  0.0627683622  0.0170540501  0.2511069028  0.3631354530
[83,] -0.0311495115  0.0910175111  0.0092183047  0.2527260114  0.4026557866
[84,] -0.0394484304  0.0498530337 -0.0678406795  0.2359198028  0.4197700938
[85,] -0.0673947898  0.0645060234 -0.0875134092  0.2133640139  0.4025852863
[86,] -0.0831257599  0.0875016072 -0.0609407140  0.2493553066  0.4289716773
[87,] -0.0615931620  0.0315935744 -0.0150113255  0.2412133394  0.4225478697
[88,] -0.0169459548 -0.0099030980  0.0183595366  0.2665479673  0.4844532192
[89,]  0.0131149112  0.0103207189  0.0206548600  0.2976543195  0.5341375624
[90,]  0.0058648104 -0.0134189489  0.0093420063  0.3237312907  0.5195989273
[91,] -0.0145566378  0.0031886115  0.0091558469  0.2666441519  0.5037973470
[92,]  0.0134597489  0.0398247562  0.0225027718  0.2729777630  0.5226080952
[93,]  0.0621949062  0.0340001670  0.0195628588  0.2555908556  0.4959927508
[94,]  0.0301825711  0.0295205467  0.0237250162  0.2885953334  0.4964011314
[95,]  0.0683474967  0.0283889143  0.0260065070  0.2722433890  0.4573330071
```

```
 [96,]  0.0526359148  0.0093669906  0.0859852397  0.3177460079  0.4184074615
 [97,] -0.0253011376  0.0016556322  0.0438341434  0.3296131209  0.3826788933
 [98,]  0.0012235954  0.0085958948  0.0501113468  0.2935077296  0.3506928791
 [99,] -0.0021208001 -0.0004985869 -0.0003090532  0.2704039531  0.3293283428
[100,]  0.0371476844  0.0061743485 -0.0289446782  0.2334227334  0.3455915250
[101,]  0.0381234135 -0.0186740839  0.0167579462  0.2329176414  0.2941549053
[102,]  0.0610041248 -0.0316308952  0.0016605264  0.2782689643  0.3016941355
[103,]  0.0136381803 -0.0556997948  0.0267573497  0.2586806040  0.2835236280
[104,]  0.0259792143 -0.0373710654 -0.0094348989  0.2962126291  0.2439587386
[105,]  0.0421286862 -0.0174287525 -0.0078690042  0.2906941011  0.2516434752
[106,]  0.0217957764  0.0348679001 -0.0301691983  0.3114098159  0.2732333449
[107,]  0.0162806312  0.0617520223 -0.0739752771  0.3574339563  0.2938482871
[108,]  0.0190410814  0.1058413609 -0.0855621344  0.3020154820  0.2659425242
[109,] -0.0111319271  0.1138786637 -0.0909207885  0.2698828270  0.2746795299
[110,] -0.0503741300  0.0739898001 -0.1173946243  0.3171439710  0.3038957480
[111,] -0.0475316705  0.0593813055 -0.1085130920  0.3040648947  0.2698236655
[112,] -0.0299626047  0.0523414696 -0.0845994115  0.3033686932  0.2512999126
[113,] -0.0349907686  0.0708062757 -0.0700707050  0.3247892906  0.2734716385
[114,] -0.0270812381  0.0747779448  0.0017612598  0.3406255174  0.2774777971
[115,] -0.0213145561  0.0942167811 -0.0354787547  0.4059352962  0.2280928473
[116,] -0.0050049236  0.0826998694 -0.0946893993  0.3994176874  0.2428926706
[117,] -0.0037009596  0.0908374953 -0.1127021201  0.4029896553  0.2388768210
[118,]  0.0403493910  0.1075064575 -0.0633133960  0.3676272334  0.2424468580
[119,]  0.0306426645  0.1053637350 -0.0577671662  0.3790102461  0.2366907715
[120,]  0.0810555501  0.0917351313 -0.0765861873  0.4176535814  0.2191350532
[121,]  0.1058254316  0.0474528912 -0.0639767093  0.4485503364  0.1516558357
[122,]  0.1200993862  0.0687579080 -0.0701522080  0.4380973325  0.1321424416
[123,]  0.1226910401  0.0909536092 -0.0665987839  0.4010105335  0.1507010579
[124,]  0.1481470817  0.0952003646 -0.0356062476  0.4013419561  0.1367337410
[125,]  0.2030479453  0.0526606495  0.0128077404  0.4585508709  0.1844242532
[126,]  0.1823674054  0.0234089539  0.0796769709  0.4883550784  0.2188041006
[127,]  0.2143960000  0.0474568579  0.0915332156  0.4827772420  0.2455855018
[128,]  0.1855815579  0.0243546462  0.1198846566  0.5284084809  0.2243730380
[129,]  0.1836837700  0.0723970315  0.1360449422  0.4910206448  0.2093167796
[130,]  0.2131469025  0.0322408911  0.1229597091  0.5071902284  0.1880175818
[131,]  0.2564556622  0.0336461808  0.1510069573  0.5190773722  0.1340425311
[132,]  0.2122816993  0.0227641920  0.2008663729  0.4896448072  0.1171458798
[133,]  0.2380085239  0.0685447744  0.2142619168  0.4795389655  0.1116182914
[134,]  0.2534957470  0.1025272079  0.2090879895  0.4899180607  0.1638347818
[135,]  0.2381929796  0.0638800424  0.2503451562  0.4567977338  0.1036826654
[136,]  0.3200303178  0.0466150019  0.2645167916  0.4384060087  0.1153132654
[137,]  0.3109875561  0.0711831666  0.2633118604  0.3571687238  0.0930833943
[138,]  0.2508285232  0.1440283774  0.2212317184  0.3542581348  0.0204782498
[139,]  0.2845616266  0.0900101092  0.2197259275  0.3299502655  0.0254412880
[140,]  0.3098864050  0.1399894068  0.1762802432  0.2382164132  0.0171702147
[141,]  0.2439320924  0.0889335090  0.1859891986  0.2656747990  0.0252587133
[142,]  0.2366050552  0.0790044243  0.1801304839  0.2540864433  0.0530778234
[143,]  0.2625850220  0.0735427157  0.1717098190  0.2951670756  0.0613337518
[144,]  0.2637305845  0.0292780871  0.1551132228  0.2819845394  0.0394579055
```

```
[145,]  0.2938250735  0.0560405139  0.1807859429  0.3110662514  0.0326064402
[146,]  0.2801221404  0.0060865204  0.2122030333  0.3738437321  0.0177846721
[147,]  0.2616878793  0.0414034279  0.2022794881  0.3552589883  0.0047708642
[148,]  0.2364289782  0.0450587209  0.1522128927  0.3727445811 -0.0277344395
[149,]  0.1679240945 -0.0040964250  0.1432377193  0.4369173099 -0.0521743171
[150,]  0.1757036932 -0.0521174306  0.1738265042  0.4141387167 -0.0926940565
[151,]  0.1879796288 -0.0541520963  0.2046364132  0.4311120407 -0.1065722465
[152,]  0.2023975646 -0.0611605439  0.2489888879  0.4856016975 -0.1089737210
[153,]  0.1794802129 -0.0962969767  0.2741442287  0.4788626483 -0.1284755486
[154,]  0.1662489129 -0.1436329547  0.2724929323  0.4894822084 -0.0869579154
[155,]  0.1606192169 -0.1763466207  0.3439636161  0.4973463867 -0.0308696722
[156,]  0.1362364522 -0.1501225780  0.3261657211  0.5298303360  0.0102537123
[157,]  0.1736836966 -0.1509109436  0.3024233079  0.5061792289  0.0253661924
[158,]  0.1802936718 -0.1776907839  0.2983426136  0.5336905616  0.1068633922
[159,]  0.2155558138 -0.1586255021  0.3000668799  0.5039646243  0.0983402029
[160,]  0.2294383815 -0.2106152989  0.2942767890  0.4491781872  0.1176407920
[161,]  0.2214207275 -0.2007546332  0.2973049548  0.4626215642  0.0990797280
[162,]  0.2774551468 -0.2203345877  0.3533533619  0.4481000841  0.0377407705
[163,]  0.2090920838 -0.2452406766  0.3822349443  0.4785344490  0.0443623797
[164,]  0.1854470266 -0.2329729592  0.4104294873  0.4674707375  0.0893254773
[165,]  0.2179301274 -0.1569505804  0.3970467217  0.4447696791  0.0997335984
[166,]  0.2319852391 -0.1209604412  0.3888127622  0.4034784547  0.0951247160
[167,]  0.1823246494 -0.1258210250  0.4746538001  0.3880915153  0.1533601367
[168,]  0.2015224290 -0.0738999659  0.4437380794  0.4202050197  0.1496011699
[169,]  0.2386026966 -0.0755962401  0.4370223989  0.4274491706  0.1261546331
[170,]  0.2543628282 -0.0545654484  0.4642001208  0.4120613031  0.1038395297
[171,]  0.2852383436 -0.0451230521  0.4296882967  0.4003864477  0.0862217013
[172,]  0.2950805995  0.0377548001  0.4441517368  0.3580194863  0.0560826610
[173,]  0.3449637921 -0.0005276789  0.4965654104  0.3275618538  0.0653575930
[174,]  0.3661930646 -0.0212474497  0.4964145753  0.2748851342  0.0347386681
[175,]  0.3755946756 -0.0223238518  0.4865829981  0.2432772038  0.0367263955
[176,]  0.3861081124  0.0167377703  0.4673282152  0.2575883054  0.0201837449
[177,]  0.4047809158  0.0093449119  0.4369694610  0.2057342038  0.0050168206
[178,]  0.4102777799 -0.0610863500  0.4415282649  0.2348315438  0.0038835650
[179,]  0.4349709905 -0.0363670303  0.4117758535  0.2523912541  0.0027435674
[180,]  0.4040059865 -0.0083464280  0.4158284233  0.2687198507  0.0025072410
[181,]  0.3814468416 -0.0003215802  0.3862252555  0.2903119134  0.0688211139
[182,]  0.3991986618 -0.0446543206  0.4328521449  0.3193175280  0.0609683426
[183,]  0.3777417868 -0.0003049977  0.4183604698  0.2800564304  0.0964588408
[184,]  0.3830023231 -0.0133226162  0.4020747122  0.2939782702  0.0508507083
[185,]  0.4074566583  0.0539980594  0.4278065937  0.2965446731  0.0505179146
[186,]  0.4167407418  0.0086683447  0.4323681918  0.2559720769  0.0419445953
[187,]  0.4109903520  0.0302223903  0.4483924691  0.2581418100  0.0457473476
[188,]  0.4725849086 -0.0069602260  0.4121184573  0.2192347470  0.0090914735
[189,]  0.4685363005  0.0185617246  0.4154027002  0.2355248206  0.0377341173
[190,]  0.4958683850  0.0072953433  0.4516871663  0.1758230759  0.0100812920
[191,]  0.5098594587  0.0135002657  0.4834422879  0.1682458914  0.0272551952
[192,]  0.5434303594 -0.0115505746  0.4617606059  0.1646051276  0.1047404233
[193,]  0.5147656823 -0.0320654860  0.4785052160  0.1632787870  0.0910359498
```

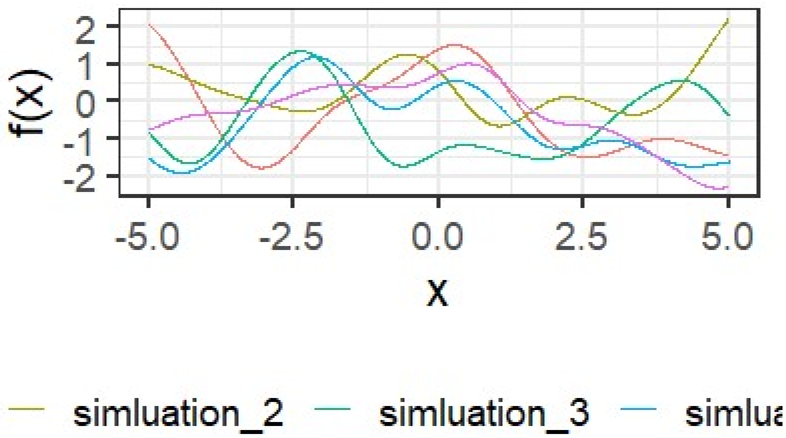
```
 [194,]  0.4555757988 -0.0415166620  0.4355135751  0.2350448346  0.0588891079
 [195,]  0.4884758717 -0.0917460703  0.4635036568  0.2676994815  0.0537047105
 [196,]  0.4573843467 -0.1178910155  0.4553431850  0.2404230440  0.0439760410
 [197,]  0.4301362319 -0.1431644059  0.4382305553  0.2811430846  0.0654636129
 [198,]  0.4200092367 -0.0958198260  0.3712328424  0.2382062210  0.0284005021
 [199,]  0.3832402009 -0.0963795711  0.4087276805  0.2500621761 -0.0007251281
 [200,]  0.3807642504 -0.0434841863  0.4204936589  0.2395247812 -0.0045828791
 [ reached getOption("max.print") -- omitted 800 rows ]

$plot
```



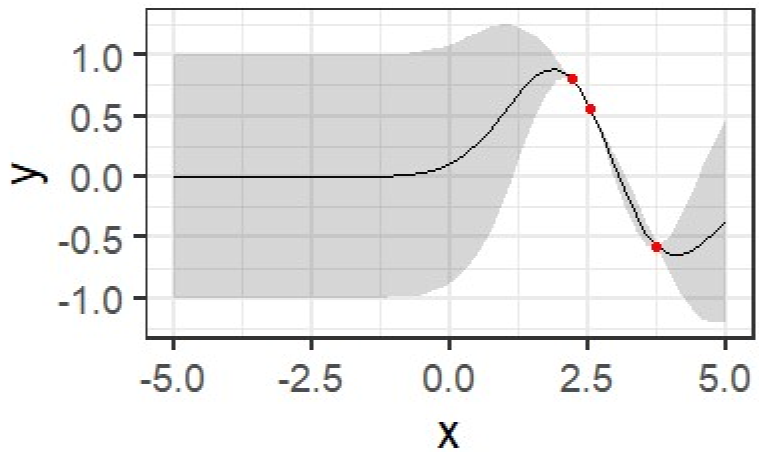
Hide

```
###########################################################
## Generating a sample from a GP
###########################################################
# Sampling from the prior GP
kernel_rbf = function(x){
  exp(-as.matrix(dist(x, diag = T))^2/2)
}
n = 1000
n_gps = 5
x = seq(-5,5,length.out = n)
K = kernel_rbf(x)
L = chol(K + 1e-6*diag(n))
f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)
colnames(f_prior) = paste0('simluation_', seq(1:n_gps))
f_prior_long_format = f_prior %>% as_tibble() %>% bind_cols(x = x) %>% pivot_longer(co
ls = starts_with("sim"))
ggplot(aes(x = x, color = name, y = value), data = f_prior_long_format) + geom_line
()+theme(legend.position = 'bottom')+
  guides(color=guide_legend(title=""))+
  ylab('f(x)')
```



Hide

```
########################################################
## Learning values from a GP
########################################################
n = 50
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
x_prime = seq(-5,5,length.out = n)
f = sin(x_observed)
mu = 0
mu_star = 0
l = 1
K = function(x,x_prime,l){
  d = sapply(x, FUN = function(x_in)(x_in - x_prime)^2)
  return(t(exp(-1/(2*l^2) *d)))
}
K_f = K(x_observed,x_observed,l)
K_star = K(x_observed,x_prime,l)
K_starstar = K(x_prime,x_prime,l)
mu_star = mu_star + t(K_star) %*% solve(K_f) %*% (f - mu)
Sigma_star = K_starstar - t(K_star)%*% t(solve(K_f)) %*% K_star
# Re-arranging values for plotting
plot_gp = tibble(x = x_prime,
                 y = mu_star %>% as.vector(),
                 sd_prime = sqrt(diag(Sigma_star)))
# Simulating values from posterior
simulated_gp_posterior = t(chol(Sigma_star + 1e-6*diag(ncol(Sigma_star)))) %*% matrix
(rnorm(n*n_gps), ncol = n_gps) +
  matrix(rep(mu_star, n_gps), ncol= n_gps)
colnames(simulated_gp_posterior) = paste0('simluation_', seq(1:n_gps))
f_posterior_long_format = simulated_gp_posterior %>% as_tibble() %>% bind_cols(x = x_p
rime) %>% pivot_longer(cols = starts_with("sim"))
# Plotting values
ggplot(aes(x = x, y = y), data = plot_gp) +
  geom_line()+
  geom_ribbon(aes(ymin = y-sd_prime,ymax = y+sd_prime), alpha = 0.2)+
  geom_point(aes(x =x , y= y), data = tibble(x = x_observed, y = f), color = 'red') #+
```



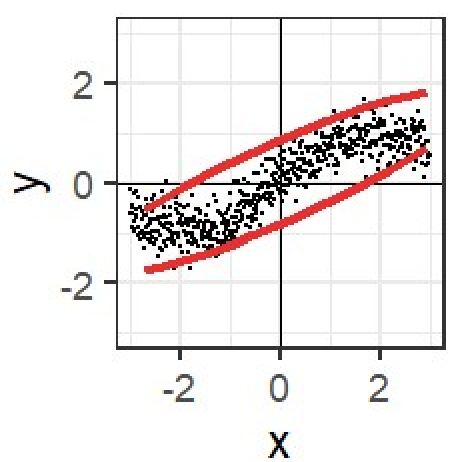
Hide

```
#geom_line(aes(x = x, color = name, y = value), data = f_posterior_long_format) + geom
_line()
```

Part 1

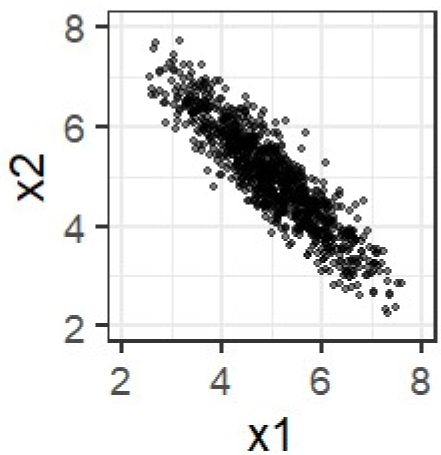
Hide

```
d2 = read.csv("C:/Users/Arshia/Documents/Georgetown/ANLY601 Advanced Machine Learning/
Assignment 3/Part 2/kernel_regression_1.csv")
ggplot(aes(x = x, y= y), data = tibble(x = d2[,1], y = d2[,2])) +
  geom_point(size = 0.5)+
  geom_vline(xintercept = 0) + geom_hline(yintercept = 0) +
  stat_ellipse(size = 1.5, color = 'firebrick2')  +
  coord_fixed(ratio = 1)+ ylim(c(-3,3))+xlim(c(-3,3))
```



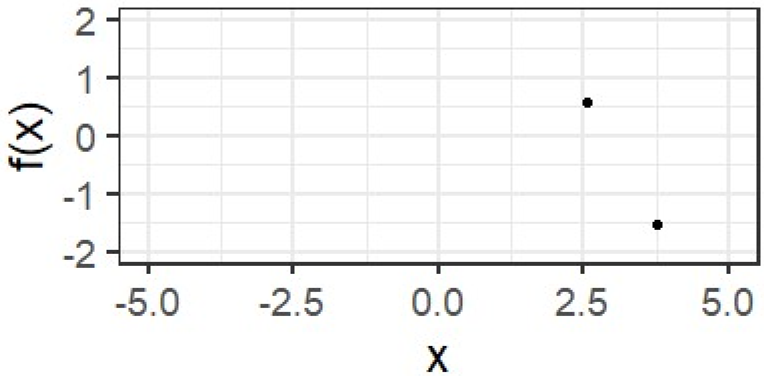
Hide

```
##################################
## Simulating a bivariate normal #
##################################
n_samples = 1000
d = 2
Z = matrix(rnorm(n_samples * d), ncol = 2)
rho = -0.9
mu = c(5,5)
Sigma = matrix(c(1,rho,rho,1), ncol = 2)
L = chol(Sigma)
X = mu + Z %*% L
colnames(X) = c('x1','x2')
ggplot(aes(x =x1, y = x2), data = as_tibble(X)) +
  geom_point(size = 1, alpha =0.5)+
  coord_fixed(ratio = 1) +xlim(c(2,8))+ylim(c(2,8))
```



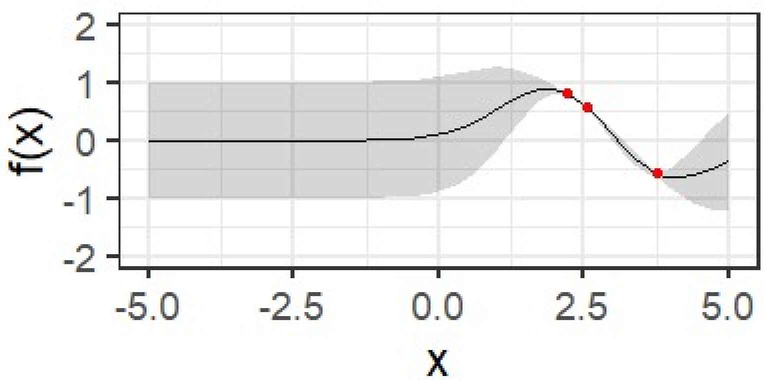
Hide

```
########################################################
## 3 Random points on a graph
########################################################
n = 50
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
f = sin(x_observed) + rnorm(3)
theme_set(theme_bw(base_size = 18))
ggplot(aes(x = x, y= y), data = tibble(x = x_observed, y = f)) +
  geom_point() +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```
########################################################
## 3 Random points on a graph
########################################################
# Kernel matrix
K = function(x,x_prime,l){
  d = sapply(x, FUN = function(x_in)(x_in - x_prime)^2)
  return(t(exp(-1/(2*l^2) *d)))
}
# Generating Data
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
x_prime = seq(-5,5,length.out = n)
f = sin(x_observed)
# Setting up GP
mu = 0
mu_star = 0
l = 1
# Covariance of f
K_f = K(x_observed,x_observed,l)
# Marginal and conditional covariance of f_star|f
K_star = K(x_observed,x_prime,l)
K_starstar = K(x_prime,x_prime,l)
# Conditional distribution of  f_star|f
mu_star = mu_star + t(K_star) %*% solve(K_f) %*% (f - mu)
Sigma_star = K_starstar - t(K_star)%*% t(solve(K_f)) %*% K_star
# Re-arranging values for plotting
plot_gp = tibble(x = x_prime,
                 y = mu_star %>% as.vector(),
                 sd_prime = sqrt(diag(Sigma_star)))
# Plotting values
ggplot(aes(x = x, y = y), data = plot_gp) +
  geom_line()+
  geom_ribbon(aes(ymin = y-sd_prime,ymax = y+sd_prime), alpha = 0.2)+
  geom_point(aes(x =x , y= y), data = tibble(x = x_observed, y = f),
             color = 'red') +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```r
x = c(1,2,3)
x_prime= c(1,2,3)
#########################################################
## Examples of GPs
#########################################################
# Kernel matrix
Wiener_Process = function(x){
  return(sapply(x, FUN = function(x_in)(pmin(x_in, x))))
}
Ornstein_Uhlenbeck= function(x){
  d = sapply(x, FUN = function(x_in)(abs(x_in- x)))
  return(exp(-d))
}
Browninan_bridge = function(x){
  # x in (0,1)
  d1 = sapply(x, FUN = function(x_in)(pmin(x_in, x)))
  d2 = sapply(x, FUN = function(x_in)(x_in * x))
  return(d1-d2)
}
kernel_rbf = function(x){
  exp(-as.matrix(dist(x, diag = T))^2/2)
}
sampling_from_a_gp = function(x_min = 0,
                              x_max=1,
                              kernel_in,
                              n = 50,
                              n_gps = 10){

  # Simulation
  x = seq(x_min, x_max,length.out = n)
  K = kernel_in(x)
  L = chol(K + 1e-6*diag(n))
  f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)

  # Reshaping
  colnames(f_prior) = paste0('Simulation ', seq(1:n_gps))
  f_prior_long_format = f_prior %>% as_tibble() %>%
    bind_cols(x = x) %>%
    pivot_longer(cols = starts_with("sim"))

  # Plot
  p = ggplot(aes(x = x, color = name, y = value),
             data = f_prior_long_format) +
    geom_line()+theme(legend.position = 'bottom')+
    guides(color=guide_legend(title=""))+
    ylab('f(x)')
  return(list('data_out' = f_prior, 'plot' = p))
```

```
}
sampling_from_a_gp(kernel_in = Browninan_bridge, n_gps = 5, n = 1000)
```

```
$`data_out`
        Simulation 1  Simulation 2  Simulation 3  Simulation 4  Simulation 5
   [1,]  0.0012061728  0.0012377338 -0.0003325691 -0.0001664464 -0.0001575622
   [2,] -0.0306451950  0.0378766928  0.0282135051  0.0478212293 -0.0442844758
   [3,] -0.0298519288  0.0279481460  0.0339105107  0.1077928408 -0.0002522062
   [4,]  0.0435072632  0.0995436300  0.0713300021  0.0871352992  0.0294462497
   [5,]  0.0109034746  0.0960214559  0.1015185464  0.1050882039  0.0482939649
   [6,] -0.0856840651  0.0631094908  0.0637869446  0.1170742734  0.0818769947
   [7,] -0.0884818185  0.0529436194  0.0810874056  0.1458084689  0.0898030193
   [8,] -0.0962057968  0.0625039807  0.0686261550  0.1472779718  0.0952506801
   [9,] -0.0435185036  0.1281080063  0.0771313242  0.1009221943  0.0975549410
  [10,] -0.0577365372  0.1555904599  0.0560355780  0.1003893586  0.1308066206
  [11,] -0.0403795026  0.2202845569  0.0207337869  0.0338217075  0.1082910872
  [12,] -0.0492116673  0.2363259307  0.0275806401  0.0271399244  0.1835490991
  [13,] -0.0456501135  0.2594346470  0.0227442144  0.0662866028  0.1852989596
  [14,] -0.0468329278  0.2760937933  0.0011778784  0.0828130748  0.1843584794
  [15,] -0.1263269531  0.2212097079 -0.0376807688  0.1017172414  0.1302569117
  [16,] -0.1111523161  0.1655524083 -0.0215373041  0.0930027720  0.0981750117
  [17,] -0.1222635284  0.1820311160  0.0154590146  0.1236959059  0.1308171223
  [18,] -0.0808006207  0.1062396354 -0.0230510979  0.1209168448  0.1392551488
  [19,] -0.1158174635  0.1618501492 -0.0402464407  0.1203460012  0.1517211135
  [20,] -0.1213119803  0.1706406817 -0.0974785158  0.1272136790  0.1660679083
  [21,] -0.0974556548  0.2210340935 -0.1179034474  0.0791859431  0.1651469884
  [22,] -0.1120759855  0.1890963727 -0.1457496165  0.0649190795  0.2400863189
  [23,] -0.1661355617  0.2132550978 -0.2056685552  0.0984165254  0.2379235550
  [24,] -0.1557799684  0.2103404822 -0.2004550153  0.1800763106  0.2263044515
  [25,] -0.1257630439  0.1677691006 -0.1887027882  0.1002836159  0.2492814609
  [26,] -0.1508593345  0.1923868206 -0.1933499921  0.1115117120  0.2562161456
  [27,] -0.1507395489  0.2191941163 -0.2210083430  0.0941411115  0.2312957779
  [28,] -0.1949891192  0.2154800604 -0.2860868996  0.1321510571  0.2156476230
  [29,] -0.2174510115  0.2124038141 -0.3337096405  0.1278353125  0.2134053564
  [30,] -0.2376973560  0.2130297739 -0.2928134905  0.0894366034  0.2569950027
  [31,] -0.2153918852  0.2101607492 -0.2505741882  0.1235751586  0.2512356305
  [32,] -0.2397470844  0.2412667759 -0.2449710711  0.1437688428  0.3006689523
  [33,] -0.2723556769  0.2635929467 -0.2507576426  0.1193062082  0.2434700818
  [34,] -0.2210409905  0.2471791965 -0.2268810083  0.1149877537  0.2637992122
  [35,] -0.2204239343  0.2376097226 -0.2364784418  0.1006636722  0.2245182817
  [36,] -0.2336152274  0.2723318203 -0.2336096516  0.1084582154  0.1965367841
  [37,] -0.2237773562  0.2490695846 -0.2662953950  0.1779631166  0.2373307362
  [38,] -0.2109326775  0.2649428108 -0.2479525356  0.2125638346  0.2145079530
  [39,] -0.2435196290  0.2417161536 -0.2305147495  0.2436397750  0.2282747407
  [40,] -0.2420874198  0.2227763712 -0.2609572994  0.2838030668  0.2541064159
  [41,] -0.3073807479  0.1804926717 -0.2055008285  0.3228735881  0.2634252724
  [42,] -0.3231376987  0.1373503177 -0.2289386754  0.2953225507  0.1850140647
  [43,] -0.3225982654  0.1207448944 -0.2576100478  0.3456088467  0.1471102917
  [44,] -0.3713062244  0.1480481128 -0.2468126690  0.3500046861  0.1590800880
  [45,] -0.3432921893  0.1059886593 -0.2274344674  0.3269015123  0.1477590295
  [46,] -0.3348647913  0.0442744917 -0.1872432742  0.3248949232  0.1469418575
```

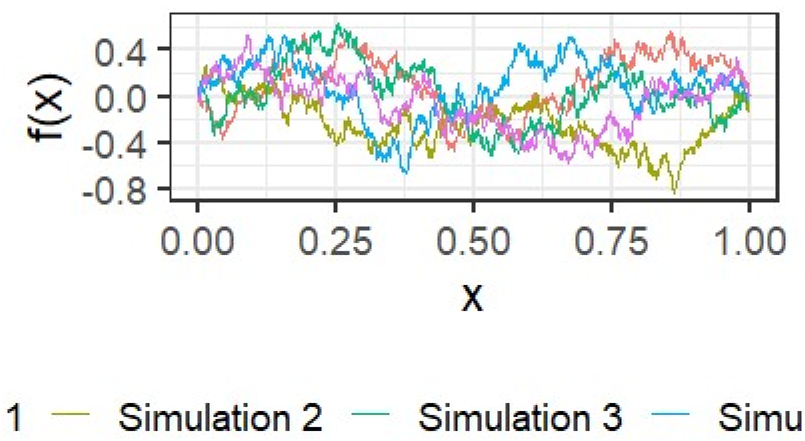
```
 [47,] -0.3334230643  0.0228566843 -0.1915932749  0.3579976664  0.1294688011
 [48,] -0.3117296467  0.0527176352 -0.1735198834  0.3052790095  0.1372699176
 [49,] -0.3196210704  0.0307475890 -0.1705961074  0.3103233175  0.1087852534
 [50,] -0.3098353284  0.0131575289 -0.1619172493  0.2950429401  0.2349246710
 [51,] -0.2999975955 -0.0019246327 -0.1345171103  0.2883800719  0.2182927897
 [52,] -0.2586696059 -0.0476884475 -0.1249002391  0.2890473527  0.2167971728
 [53,] -0.2236235987 -0.0389279632 -0.0248998460  0.3237025871  0.1656367287
 [54,] -0.2483953247 -0.0507348327 -0.0048190756  0.3250403646  0.1947509420
 [55,] -0.2574913948 -0.0501905202 -0.0139773754  0.2723779496  0.1872941624
 [56,] -0.2669010008 -0.0642423223 -0.0050457561  0.2264816060  0.1850707100
 [57,] -0.2249945406  0.0418707733  0.0133657255  0.2406380658  0.1410404569
 [58,] -0.2177618536  0.0211335867 -0.0056156370  0.3045644148  0.1636160512
 [59,] -0.2322718648 -0.0393208208 -0.0062365060  0.2826651370  0.1543167163
 [60,] -0.2308264572 -0.0500875152  0.0262861259  0.2409333650  0.1714247872
 [61,] -0.2154895190 -0.0430076249 -0.0374972647  0.2179197271  0.1936557653
 [62,] -0.1886087927 -0.0138224887 -0.0307802082  0.1731501355  0.2195475245
 [63,] -0.2166159670  0.0123967186 -0.0158362403  0.1492674655  0.2251328398
 [64,] -0.1925230149  0.0056791106 -0.0094530075  0.1595472907  0.2610919657
 [65,] -0.1782485576  0.0069940376 -0.0420992175  0.2177340363  0.2631108019
 [66,] -0.1647425937 -0.0199411468 -0.0436779389  0.1971418765  0.3129623278
 [67,] -0.1281912069 -0.0587342981 -0.0264770268  0.1376142672  0.2771570636
 [68,] -0.1564614902 -0.0560552668  0.0183135851  0.1427842644  0.2496803714
 [69,] -0.1287374618 -0.0836780524  0.0474791524  0.1299243683  0.2416533420
 [70,] -0.1126129244 -0.0474276527  0.0322702540  0.1083737833  0.2481374170
 [71,] -0.1020083884  0.0047280661  0.0503104271  0.0785394757  0.2488977063
 [72,] -0.0468326225  0.0284746205  0.0414628039  0.1197882399  0.3123237053
 [73,] -0.0471195857  0.0617119950  0.0992589878  0.1426047669  0.3013596712
 [74,] -0.0573942447  0.0335175204  0.1432018727  0.1363535471  0.3442852481
 [75,] -0.0526201207  0.0536150901  0.1603620890  0.1468385572  0.3427201179
 [76,] -0.0645919000  0.1003617154  0.1155573263  0.1745118749  0.3285582991
 [77,] -0.0395534555  0.1228125765  0.0479166868  0.1819778874  0.3620583891
 [78,] -0.0449562709  0.1341002446  0.0401476087  0.2111428670  0.3333791533
 [79,] -0.0367235510  0.0908490641  0.0225403091  0.1674500343  0.3029550835
 [80,] -0.0290896352  0.0625639554  0.0435514338  0.2080927457  0.2891835377
 [81,] -0.0008196417  0.0471791370  0.0461353362  0.2454109370  0.3188064792
 [82,] -0.0014572326  0.0627683622  0.0170540501  0.2511069028  0.3631354530
 [83,] -0.0311495115  0.0910175111  0.0092183047  0.2527260114  0.4026557866
 [84,] -0.0394484304  0.0498530337 -0.0678406795  0.2359198028  0.4197700938
 [85,] -0.0673947898  0.0645060234 -0.0875134092  0.2133640139  0.4025852863
 [86,] -0.0831257599  0.0875016072 -0.0609407140  0.2493553066  0.4289716773
 [87,] -0.0615931620  0.0315935744 -0.0150113255  0.2412133394  0.4225478697
 [88,] -0.0169459548 -0.0099030980  0.0183595366  0.2665479673  0.4844532192
 [89,]  0.0131149112  0.0103207189  0.0206548600  0.2976543195  0.5341375624
 [90,]  0.0058648104 -0.0134189489  0.0093420063  0.3237312907  0.5195989273
 [91,] -0.0145566378  0.0031886115  0.0091558469  0.2666441519  0.5037973470
 [92,]  0.0134597489  0.0398247562  0.0225027718  0.2729777630  0.5226080952
 [93,]  0.0621949062  0.0340001670  0.0195628588  0.2555908556  0.4959927508
 [94,]  0.0301825711  0.0295205467  0.0237250162  0.2885953334  0.4964011314
 [95,]  0.0683474967  0.0283889143  0.0260065070  0.2722433890  0.4573330071
```

```
 [96,]  0.0526359148  0.0093669906  0.0859852397  0.3177460079  0.4184074615
 [97,] -0.0253011376  0.0016556322  0.0438341434  0.3296131209  0.3826788933
 [98,]  0.0012235954  0.0085958948  0.0501113468  0.2935077296  0.3506928791
 [99,] -0.0021208001 -0.0004985869 -0.0003090532  0.2704039531  0.3293283428
[100,]  0.0371476844  0.0061743485 -0.0289446782  0.2334227334  0.3455915250
[101,]  0.0381234135 -0.0186740839  0.0167579462  0.2329176414  0.2941549053
[102,]  0.0610041248 -0.0316308952  0.0016605264  0.2782689643  0.3016941355
[103,]  0.0136381803 -0.0556997948  0.0267573497  0.2586806040  0.2835236280
[104,]  0.0259792143 -0.0373710654 -0.0094348989  0.2962126291  0.2439587386
[105,]  0.0421286862 -0.0174287525 -0.0078690042  0.2906941011  0.2516434752
[106,]  0.0217957764  0.0348679001 -0.0301691983  0.3114098159  0.2732333449
[107,]  0.0162806312  0.0617520223 -0.0739752771  0.3574339563  0.2938482871
[108,]  0.0190410814  0.1058413609 -0.0855621344  0.3020154820  0.2659425242
[109,] -0.0111319271  0.1138786637 -0.0909207885  0.2698828270  0.2746795299
[110,] -0.0503741300  0.0739898001 -0.1173946243  0.3171439710  0.3038957480
[111,] -0.0475316705  0.0593813055 -0.1085130920  0.3040648947  0.2698236655
[112,] -0.0299626047  0.0523414696 -0.0845994115  0.3033686932  0.2512999126
[113,] -0.0349907686  0.0708062757 -0.0700707050  0.3247892906  0.2734716385
[114,] -0.0270812381  0.0747779448  0.0017612598  0.3406255174  0.2774777971
[115,] -0.0213145561  0.0942167811 -0.0354787547  0.4059352962  0.2280928473
[116,] -0.0050049236  0.0826998694 -0.0946893993  0.3994176874  0.2428926706
[117,] -0.0037009596  0.0908374953 -0.1127021201  0.4029896553  0.2388768210
[118,]  0.0403493910  0.1075064575 -0.0633133960  0.3676272334  0.2424468580
[119,]  0.0306426645  0.1053637350 -0.0577671662  0.3790102461  0.2366907715
[120,]  0.0810555501  0.0917351313 -0.0765861873  0.4176535814  0.2191350532
[121,]  0.1058254316  0.0474528912 -0.0639767093  0.4485503364  0.1516558357
[122,]  0.1200993862  0.0687579080 -0.0701522080  0.4380973325  0.1321424416
[123,]  0.1226910401  0.0909536092 -0.0665987839  0.4010105335  0.1507010579
[124,]  0.1481470817  0.0952003646 -0.0356062476  0.4013419561  0.1367337410
[125,]  0.2030479453  0.0526606495  0.0128077404  0.4585508709  0.1844242532
[126,]  0.1823674054  0.0234089539  0.0796769709  0.4883550784  0.2188041006
[127,]  0.2143960000  0.0474568579  0.0915332156  0.4827772420  0.2455855018
[128,]  0.1855815579  0.0243546462  0.1198846566  0.5284084809  0.2243730380
[129,]  0.1836837700  0.0723970315  0.1360449422  0.4910206448  0.2093167796
[130,]  0.2131469025  0.0322408911  0.1229597091  0.5071902284  0.1880175818
[131,]  0.2564556622  0.0336461808  0.1510069573  0.5190773722  0.1340425311
[132,]  0.2122816993  0.0227641920  0.2008663729  0.4896448072  0.1171458798
[133,]  0.2380085239  0.0685447744  0.2142619168  0.4795389655  0.1116182914
[134,]  0.2534957470  0.1025272079  0.2090879895  0.4899180607  0.1638347818
[135,]  0.2381929796  0.0638800424  0.2503451562  0.4567977338  0.1036826654
[136,]  0.3200303178  0.0466150019  0.2645167916  0.4384060087  0.1153132654
[137,]  0.3109875561  0.0711831666  0.2633118604  0.3571687238  0.0930833943
[138,]  0.2508285232  0.1440283774  0.2212317184  0.3542581348  0.0204782498
[139,]  0.2845616266  0.0900101092  0.2197259275  0.3299502655  0.0254412880
[140,]  0.3098864050  0.1399894068  0.1762802432  0.2382164132  0.0171702147
[141,]  0.2439320924  0.0889335090  0.1859891986  0.2656747990  0.0252587133
[142,]  0.2366050552  0.0790044243  0.1801304839  0.2540864433  0.0530778234
[143,]  0.2625850220  0.0735427157  0.1717098190  0.2951670756  0.0613337518
[144,]  0.2637305845  0.0292780871  0.1551132228  0.2819845394  0.0394579055
```

```
[145,]  0.2938250735  0.0560405139  0.1807859429  0.3110662514  0.0326064402
[146,]  0.2801221404  0.0060865204  0.2122030333  0.3738437321  0.0177846721
[147,]  0.2616878793  0.0414034279  0.2022794881  0.3552589883  0.0047708642
[148,]  0.2364289782  0.0450587209  0.1522128927  0.3727445811 -0.0277344395
[149,]  0.1679240945 -0.0040964250  0.1432377193  0.4369173099 -0.0521743171
[150,]  0.1757036932 -0.0521174306  0.1738265042  0.4141387167 -0.0926940565
[151,]  0.1879796288 -0.0541520963  0.2046364132  0.4311120407 -0.1065722465
[152,]  0.2023975646 -0.0611605439  0.2489888879  0.4856016975 -0.1089737210
[153,]  0.1794802129 -0.0962969767  0.2741442287  0.4788626483 -0.1284755486
[154,]  0.1662489129 -0.1436329547  0.2724929323  0.4894822084 -0.0869579154
[155,]  0.1606192169 -0.1763466207  0.3439636161  0.4973463867 -0.0308696722
[156,]  0.1362364522 -0.1501225780  0.3261657211  0.5298303360  0.0102537123
[157,]  0.1736836966 -0.1509109436  0.3024233079  0.5061792289  0.0253661924
[158,]  0.1802936718 -0.1776907839  0.2983426136  0.5336905616  0.1068633922
[159,]  0.2155558138 -0.1586255021  0.3000668799  0.5039646243  0.0983402029
[160,]  0.2294383815 -0.2106152989  0.2942767890  0.4491781872  0.1176407920
[161,]  0.2214207275 -0.2007546332  0.2973049548  0.4626215642  0.0990797280
[162,]  0.2774551468 -0.2203345877  0.3533533619  0.4481000841  0.0377407705
[163,]  0.2090920838 -0.2452406766  0.3822349443  0.4785344490  0.0443623797
[164,]  0.1854470266 -0.2329729592  0.4104294873  0.4674707375  0.0893254773
[165,]  0.2179301274 -0.1569505804  0.3970467217  0.4447696791  0.0997335984
[166,]  0.2319852391 -0.1209604412  0.3888127622  0.4034784547  0.0951247160
[167,]  0.1823246494 -0.1258210250  0.4746538001  0.3880915153  0.1533601367
[168,]  0.2015224290 -0.0738999659  0.4437380794  0.4202050197  0.1496011699
[169,]  0.2386026966 -0.0755962401  0.4370223989  0.4274491706  0.1261546331
[170,]  0.2543628282 -0.0545654484  0.4642001208  0.4120613031  0.1038395297
[171,]  0.2852383436 -0.0451230521  0.4296882967  0.4003864477  0.0862217013
[172,]  0.2950805995  0.0377548001  0.4441517368  0.3580194863  0.0560826610
[173,]  0.3449637921 -0.0005276789  0.4965654104  0.3275618538  0.0653575930
[174,]  0.3661930646 -0.0212474497  0.4964145753  0.2748851342  0.0347386681
[175,]  0.3755946756 -0.0223238518  0.4865829981  0.2432772038  0.0367263955
[176,]  0.3861081124  0.0167377703  0.4673282152  0.2575883054  0.0201837449
[177,]  0.4047809158  0.0093449119  0.4369694610  0.2057342038  0.0050168206
[178,]  0.4102777799 -0.0610863500  0.4415282649  0.2348315438  0.0038835650
[179,]  0.4349709905 -0.0363670303  0.4117758535  0.2523912541  0.0027435674
[180,]  0.4040059865 -0.0083464280  0.4158284233  0.2687198507  0.0025072410
[181,]  0.3814468416 -0.0003215802  0.3862252555  0.2903119134  0.0688211139
[182,]  0.3991986618 -0.0446543206  0.4328521449  0.3193175280  0.0609683426
[183,]  0.3777417868 -0.0003049977  0.4183604698  0.2800564304  0.0964588408
[184,]  0.3830023231 -0.0133226162  0.4020747122  0.2939782702  0.0508507083
[185,]  0.4074566583  0.0539980594  0.4278065937  0.2965446731  0.0505179146
[186,]  0.4167407418  0.0086683447  0.4323681918  0.2559720769  0.0419445953
[187,]  0.4109903520  0.0302223903  0.4483924691  0.2581418100  0.0457473476
[188,]  0.4725849086 -0.0069602260  0.4121184573  0.2192347470  0.0090914735
[189,]  0.4685363005  0.0185617246  0.4154027002  0.2355248206  0.0377341173
[190,]  0.4958683850  0.0072953433  0.4516871663  0.1758230759  0.0100812920
[191,]  0.5098594587  0.0135002657  0.4834422879  0.1682458914  0.0272551952
[192,]  0.5434303594 -0.0115505746  0.4617606059  0.1646051276  0.1047404233
[193,]  0.5147656823 -0.0320654860  0.4785052160  0.1632787870  0.0910359498
```

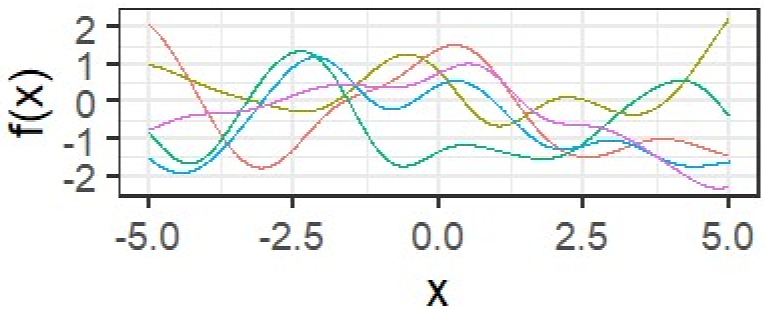
```
 [194,]  0.4555757988 -0.0415166620  0.4355135751  0.2350448346  0.0588891079
 [195,]  0.4884758717 -0.0917460703  0.4635036568  0.2676994815  0.0537047105
 [196,]  0.4573843467 -0.1178910155  0.4553431850  0.2404230440  0.0439760410
 [197,]  0.4301362319 -0.1431644059  0.4382305553  0.2811430846  0.0654636129
 [198,]  0.4200092367 -0.0958198260  0.3712328424  0.2382062210  0.0284005021
 [199,]  0.3832402009 -0.0963795711  0.4087276805  0.2500621761 -0.0007251281
 [200,]  0.3807642504 -0.0434841863  0.4204936589  0.2395247812 -0.0045828791
 [ reached getOption("max.print") -- omitted 800 rows ]

$plot
```



Hide

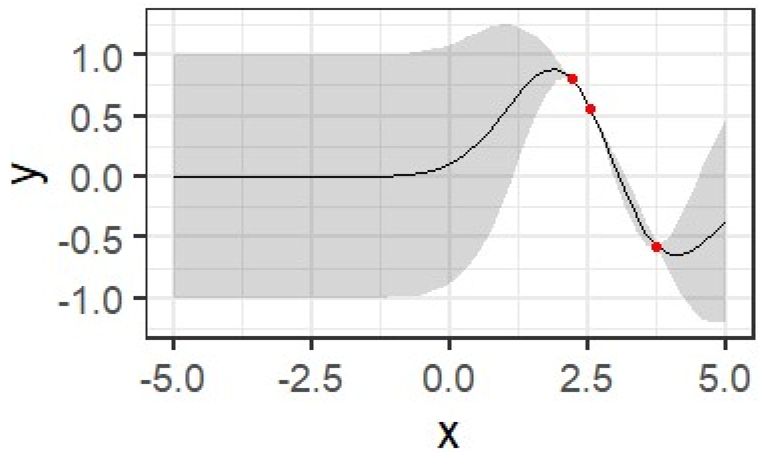
```
###########################################################
## Generating a sample from a GP
###########################################################
# Sampling from the prior GP
kernel_rbf = function(x){
  exp(-as.matrix(dist(x, diag = T))^2/2)
}
n = 1000
n_gps = 5
x = seq(-5,5,length.out = n)
K = kernel_rbf(x)
L = chol(K + 1e-6*diag(n))
f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)
colnames(f_prior) = paste0('simluation_', seq(1:n_gps))
f_prior_long_format = f_prior %>% as_tibble() %>% bind_cols(x = x) %>% pivot_longer(co
ls = starts_with("sim"))
ggplot(aes(x = x, color = name, y = value), data = f_prior_long_format) + geom_line
()+theme(legend.position = 'bottom')+
  guides(color=guide_legend(title=""))+
  ylab('f(x)')
```



— simluation_2 — simluation_3 — simlua

Hide

```
########################################################
## Learning values from a GP
########################################################
n = 50
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
x_prime = seq(-5,5,length.out = n)
f = sin(x_observed)
mu = 0
mu_star = 0
l = 1
K = function(x,x_prime,l){
  d = sapply(x, FUN = function(x_in)(x_in - x_prime)^2)
  return(t(exp(-1/(2*l^2) *d)))
}
K_f = K(x_observed,x_observed,l)
K_star = K(x_observed,x_prime,l)
K_starstar = K(x_prime,x_prime,l)
mu_star = mu_star + t(K_star) %*% solve(K_f) %*% (f - mu)
Sigma_star = K_starstar - t(K_star)%*% t(solve(K_f)) %*% K_star
# Re-arranging values for plotting
plot_gp = tibble(x = x_prime,
                 y = mu_star %>% as.vector(),
                 sd_prime = sqrt(diag(Sigma_star)))
# Simulating values from posterior
simulated_gp_posterior = t(chol(Sigma_star + 1e-6*diag(ncol(Sigma_star)))) %*% matrix
(rnorm(n*n_gps), ncol = n_gps) +
  matrix(rep(mu_star, n_gps), ncol= n_gps)
colnames(simulated_gp_posterior) = paste0('simluation_', seq(1:n_gps))
f_posterior_long_format = simulated_gp_posterior %>% as_tibble() %>% bind_cols(x = x_p
rime) %>% pivot_longer(cols = starts_with("sim"))
# Plotting values
ggplot(aes(x = x, y = y), data = plot_gp) +
  geom_line()+
  geom_ribbon(aes(ymin = y-sd_prime,ymax = y+sd_prime), alpha = 0.2)+
  geom_point(aes(x =x , y= y), data = tibble(x = x_observed, y = f), color = 'red') #+
```

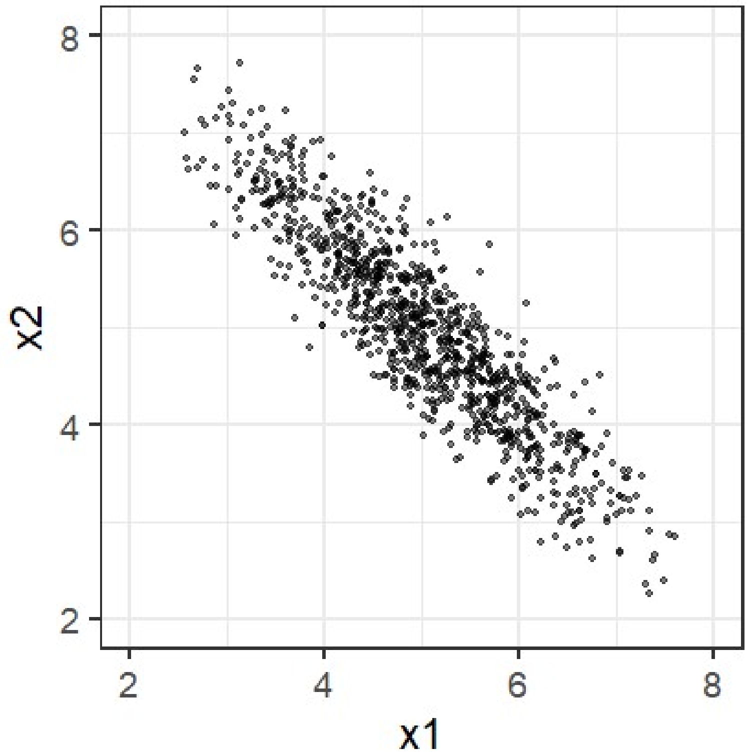


Hide

```
#geom_line(aes(x = x, color = name, y = value), data = f_posterior_long_format) + geom
_line()
```

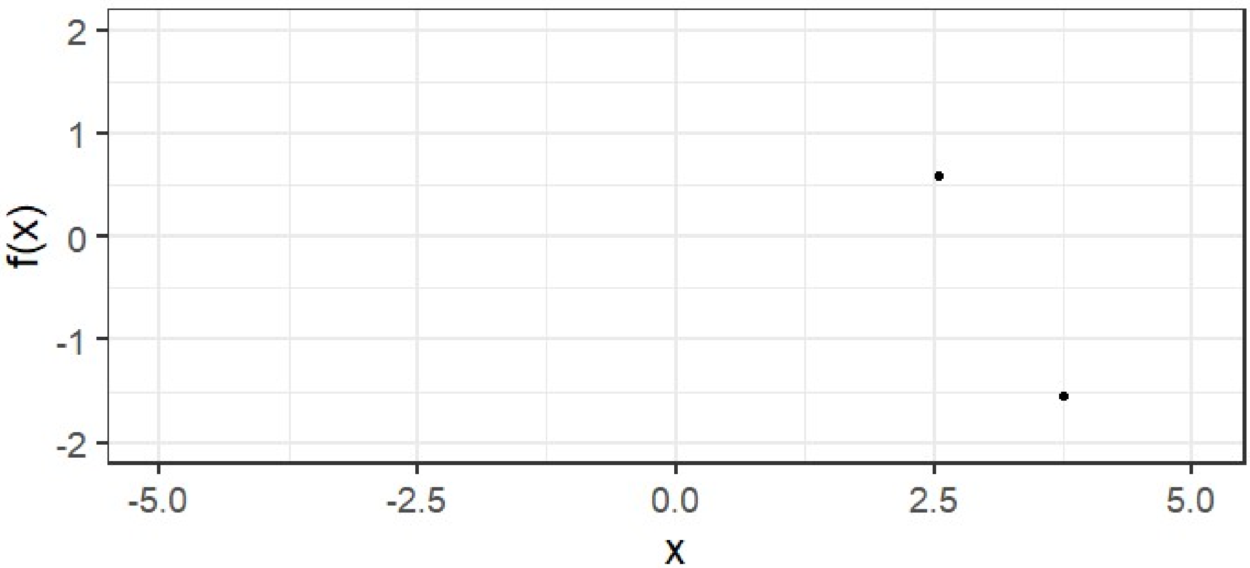
Hide

```
##################################
## Simulating a bivariate normal #
##################################
n_samples = 1000
d = 2
Z = matrix(rnorm(n_samples * d), ncol = 2)
rho = -0.9
mu = c(5,5)
Sigma = matrix(c(1,rho,rho,1), ncol = 2)
L = chol(Sigma)
X = mu + Z %*% L
colnames(X) = c('x1','x2')
ggplot(aes(x =x1, y = x2), data = as_tibble(X)) +
  geom_point(size = 1, alpha =0.5)+
  coord_fixed(ratio = 1) +xlim(c(2,8))+ylim(c(2,8))
```



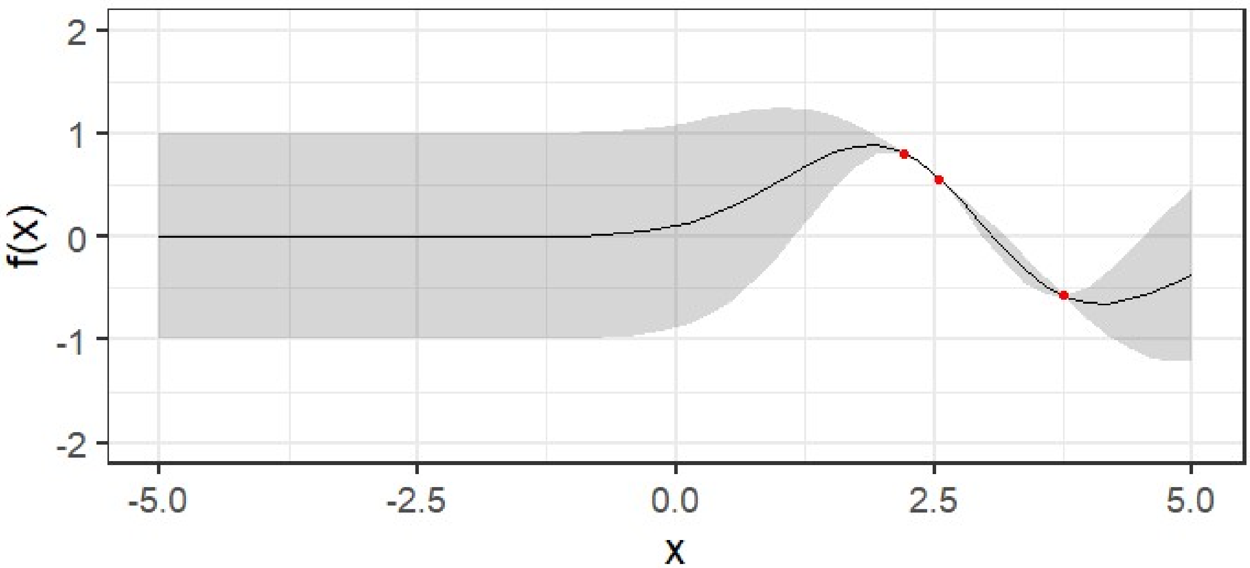
Hide

```
#########################################################
## 3 Random points on a graph
#########################################################
n = 50
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
f = sin(x_observed) + rnorm(3)
theme_set(theme_bw(base_size = 18))
ggplot(aes(x = x, y= y), data = tibble(x = x_observed, y = f)) +
  geom_point() +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```
########################################################
## 3 Random points on a graph
########################################################
# Kernel matrix
K = function(x,x_prime,l){
  d = sapply(x, FUN = function(x_in)(x_in - x_prime)^2)
  return(t(exp(-1/(2*l^2) *d)))
}
# Generating Data
set.seed(12345)
x_observed = sample(seq(-5,5,0.05), size = 3)
x_prime = seq(-5,5,length.out = n)
f = sin(x_observed)
# Setting up GP
mu = 0
mu_star = 0
l = 1
# Covariance of f
K_f = K(x_observed,x_observed,l)
# Marginal and conditional covariance of f_star|f
K_star = K(x_observed,x_prime,l)
K_starstar = K(x_prime,x_prime,l)
# Conditional distribution of  f_star|f
mu_star = mu_star + t(K_star) %*% solve(K_f) %*% (f - mu)
Sigma_star = K_starstar - t(K_star)%*% t(solve(K_f)) %*% K_star
# Re-arranging values for plotting
plot_gp = tibble(x = x_prime,
                 y = mu_star %>% as.vector(),
                 sd_prime = sqrt(diag(Sigma_star)))
# Plotting values
ggplot(aes(x = x, y = y), data = plot_gp) +
  geom_line()+
  geom_ribbon(aes(ymin = y-sd_prime,ymax = y+sd_prime), alpha = 0.2)+
  geom_point(aes(x =x , y= y), data = tibble(x = x_observed, y = f),
             color = 'red') +
  xlim(c(-5,5))+ylim(c(-2,2))+
  coord_fixed(ratio = 1) +ylab('f(x)')
```



Hide

```
x = c(1,2,3)
x_prime= c(1,2,3)
########################################################
## Examples of GPs
########################################################
# Kernel matrix
Wiener_Process = function(x){
  return(sapply(x, FUN = function(x_in)(pmin(x_in, x))))
}
Ornstein_Uhlenbeck= function(x){
  d = sapply(x, FUN = function(x_in)(abs(x_in- x)))
  return(exp(-d))
}
Browninan_bridge = function(x){
  # x in (0,1)
  d1 = sapply(x, FUN = function(x_in)(pmin(x_in, x)))
  d2 = sapply(x, FUN = function(x_in)(x_in * x))
  return(d1-d2)
}
kernel_rbf = function(x, theta){
  exp(-as.matrix(dist(x, diag = T))^2/(2*theta))
}
sampling_from_a_gp = function(x_min = 0,
                              x_max=1,
                              kernel_in,
                              n = 50,
                              n_gps = 10){

  # Simulation
  x = seq(x_min, x_max,length.out = n)
  K = kernel_in(x)
  L = chol(K + 1e-6*diag(n))
  f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)

  # Reshaping
  colnames(f_prior) = paste0('Simulation ', seq(1:n_gps))
  f_prior_long_format = f_prior %>% as_tibble() %>%
    bind_cols(x = x) %>%
    pivot_longer(cols = starts_with("sim"))

  # Plot
  p = ggplot(aes(x = x, color = name, y = value),
             data = f_prior_long_format) +
    geom_line()+theme(legend.position = 'bottom')+
    guides(color=guide_legend(title=""))+
    ylab('f(x)')
  return(list('data_out' = f_prior, 'plot' = p))
```

```
}
sampling_from_a_gp(kernel_in = Browninan_bridge, n_gps = 5, n = 1000)
```

```
$`data_out`
        Simulation 1  Simulation 2  Simulation 3  Simulation 4  Simulation 5
   [1,]  0.0012061728  0.0012377338 -0.0003325691 -0.0001664464 -0.0001575622
   [2,] -0.0306451950  0.0378766928  0.0282135051  0.0478212293 -0.0442844758
   [3,] -0.0298519288  0.0279481460  0.0339105107  0.1077928408 -0.0002522062
   [4,]  0.0435072632  0.0995436300  0.0713300021  0.0871352992  0.0294462497
   [5,]  0.0109034746  0.0960214559  0.1015185464  0.1050882039  0.0482939649
   [6,] -0.0856840651  0.0631094908  0.0637869446  0.1170742734  0.0818769947
   [7,] -0.0884818185  0.0529436194  0.0810874056  0.1458084689  0.0898030193
   [8,] -0.0962057968  0.0625039807  0.0686261550  0.1472779718  0.0952506801
   [9,] -0.0435185036  0.1281080063  0.0771313242  0.1009221943  0.0975549410
  [10,] -0.0577365372  0.1555904599  0.0560355780  0.1003893586  0.1308066206
  [11,] -0.0403795026  0.2202845569  0.0207337869  0.0338217075  0.1082910872
  [12,] -0.0492116673  0.2363259307  0.0275806401  0.0271399244  0.1835490991
  [13,] -0.0456501135  0.2594346470  0.0227442144  0.0662866028  0.1852989596
  [14,] -0.0468329278  0.2760937933  0.0011778784  0.0828130748  0.1843584794
  [15,] -0.1263269531  0.2212097079 -0.0376807688  0.1017172414  0.1302569117
  [16,] -0.1111523161  0.1655524083 -0.0215373041  0.0930027720  0.0981750117
  [17,] -0.1222635284  0.1820311160  0.0154590146  0.1236959059  0.1308171223
  [18,] -0.0808006207  0.1062396354 -0.0230510979  0.1209168448  0.1392551488
  [19,] -0.1158174635  0.1618501492 -0.0402464407  0.1203460012  0.1517211135
  [20,] -0.1213119803  0.1706406817 -0.0974785158  0.1272136790  0.1660679083
  [21,] -0.0974556548  0.2210340935 -0.1179034474  0.0791859431  0.1651469884
  [22,] -0.1120759855  0.1890963727 -0.1457496165  0.0649190795  0.2400863189
  [23,] -0.1661355617  0.2132550978 -0.2056685552  0.0984165254  0.2379235550
  [24,] -0.1557799684  0.2103404822 -0.2004550153  0.1800763106  0.2263044515
  [25,] -0.1257630439  0.1677691006 -0.1887027882  0.1002836159  0.2492814609
  [26,] -0.1508593345  0.1923868206 -0.1933499921  0.1115117120  0.2562161456
  [27,] -0.1507395489  0.2191941163 -0.2210083430  0.0941411115  0.2312957779
  [28,] -0.1949891192  0.2154800604 -0.2860868996  0.1321510571  0.2156476230
  [29,] -0.2174510115  0.2124038141 -0.3337096405  0.1278353125  0.2134053564
  [30,] -0.2376973560  0.2130297739 -0.2928134905  0.0894366034  0.2569950027
  [31,] -0.2153918852  0.2101607492 -0.2505741882  0.1235751586  0.2512356305
  [32,] -0.2397470844  0.2412667759 -0.2449710711  0.1437688428  0.3006689523
  [33,] -0.2723556769  0.2635929467 -0.2507576426  0.1193062082  0.2434700818
  [34,] -0.2210409905  0.2471791965 -0.2268810083  0.1149877537  0.2637992122
  [35,] -0.2204239343  0.2376097226 -0.2364784418  0.1006636722  0.2245182817
  [36,] -0.2336152274  0.2723318203 -0.2336096516  0.1084582154  0.1965367841
  [37,] -0.2237773562  0.2490695846 -0.2662953950  0.1779631166  0.2373307362
  [38,] -0.2109326775  0.2649428108 -0.2479525356  0.2125638346  0.2145079530
  [39,] -0.2435196290  0.2417161536 -0.2305147495  0.2436397750  0.2282747407
  [40,] -0.2420874198  0.2227763712 -0.2609572994  0.2838030668  0.2541064159
  [41,] -0.3073807479  0.1804926717 -0.2055008285  0.3228735881  0.2634252724
  [42,] -0.3231376987  0.1373503177 -0.2289386754  0.2953225507  0.1850140647
  [43,] -0.3225982654  0.1207448944 -0.2576100478  0.3456088467  0.1471102917
  [44,] -0.3713062244  0.1480481128 -0.2468126690  0.3500046861  0.1590800880
  [45,] -0.3432921893  0.1059886593 -0.2274344674  0.3269015123  0.1477590295
  [46,] -0.3348647913  0.0442744917 -0.1872432742  0.3248949232  0.1469418575
```

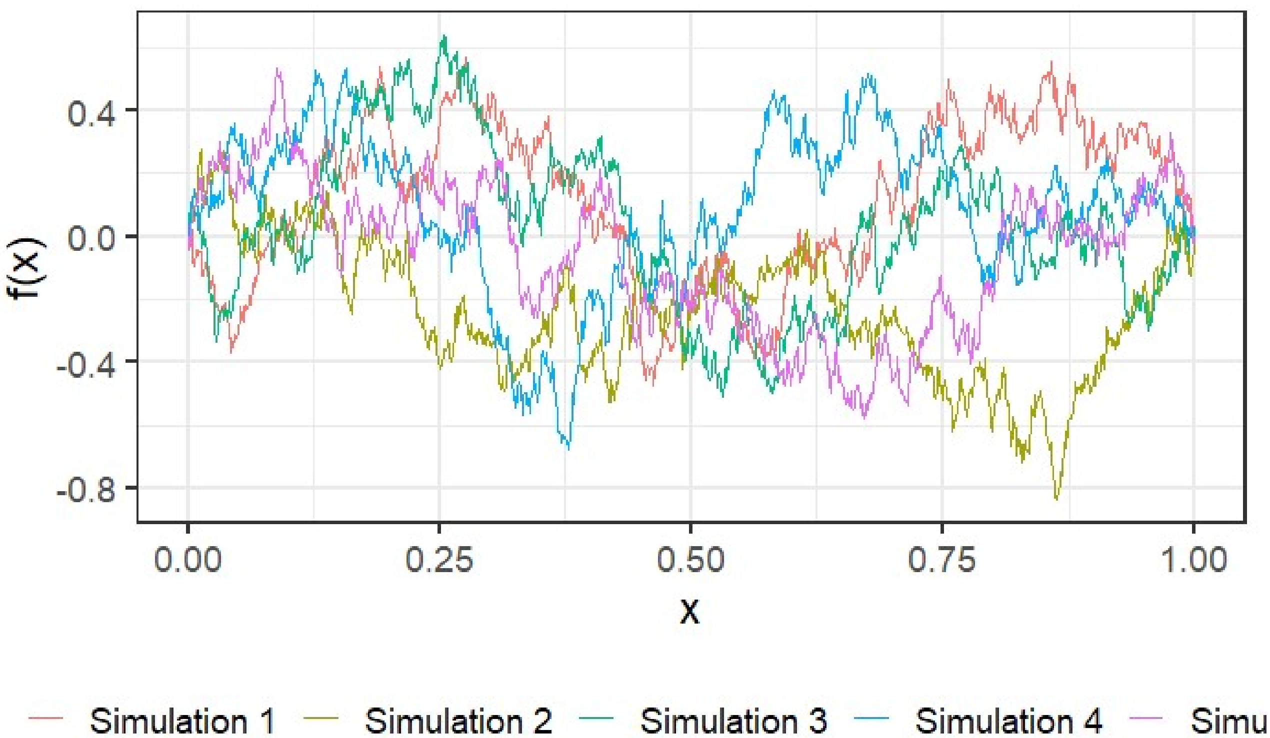
```
 [47,] -0.3334230643  0.0228566843 -0.1915932749  0.3579976664  0.1294688011
 [48,] -0.3117296467  0.0527176352 -0.1735198834  0.3052790095  0.1372699176
 [49,] -0.3196210704  0.0307475890 -0.1705961074  0.3103233175  0.1087852534
 [50,] -0.3098353284  0.0131575289 -0.1619172493  0.2950429401  0.2349246710
 [51,] -0.2999975955 -0.0019246327 -0.1345171103  0.2883800719  0.2182927897
 [52,] -0.2586696059 -0.0476884475 -0.1249002391  0.2890473527  0.2167971728
 [53,] -0.2236235987 -0.0389279632 -0.0248998460  0.3237025871  0.1656367287
 [54,] -0.2483953247 -0.0507348327 -0.0048190756  0.3250403646  0.1947509420
 [55,] -0.2574913948 -0.0501905202 -0.0139773754  0.2723779496  0.1872941624
 [56,] -0.2669010008 -0.0642423223 -0.0050457561  0.2264816060  0.1850707100
 [57,] -0.2249945406  0.0418707733  0.0133657255  0.2406380658  0.1410404569
 [58,] -0.2177618536  0.0211335867 -0.0056156370  0.3045644148  0.1636160512
 [59,] -0.2322718648 -0.0393208208 -0.0062365060  0.2826651370  0.1543167163
 [60,] -0.2308264572 -0.0500875152  0.0262861259  0.2409333650  0.1714247872
 [61,] -0.2154895190 -0.0430076249 -0.0374972647  0.2179197271  0.1936557653
 [62,] -0.1886087927 -0.0138224887 -0.0307802082  0.1731501355  0.2195475245
 [63,] -0.2166159670  0.0123967186 -0.0158362403  0.1492674655  0.2251328398
 [64,] -0.1925230149  0.0056791106 -0.0094530075  0.1595472907  0.2610919657
 [65,] -0.1782485576  0.0069940376 -0.0420992175  0.2177340363  0.2631108019
 [66,] -0.1647425937 -0.0199411468 -0.0436779389  0.1971418765  0.3129623278
 [67,] -0.1281912069 -0.0587342981 -0.0264770268  0.1376142672  0.2771570636
 [68,] -0.1564614902 -0.0560552668  0.0183135851  0.1427842644  0.2496803714
 [69,] -0.1287374618 -0.0836780524  0.0474791524  0.1299243683  0.2416533420
 [70,] -0.1126129244 -0.0474276527  0.0322702540  0.1083737833  0.2481374170
 [71,] -0.1020083884  0.0047280661  0.0503104271  0.0785394757  0.2488977063
 [72,] -0.0468326225  0.0284746205  0.0414628039  0.1197882399  0.3123237053
 [73,] -0.0471195857  0.0617119950  0.0992589878  0.1426047669  0.3013596712
 [74,] -0.0573942447  0.0335175204  0.1432018727  0.1363535471  0.3442852481
 [75,] -0.0526201207  0.0536150901  0.1603620890  0.1468385572  0.3427201179
 [76,] -0.0645919000  0.1003617154  0.1155573263  0.1745118749  0.3285582991
 [77,] -0.0395534555  0.1228125765  0.0479166868  0.1819778874  0.3620583891
 [78,] -0.0449562709  0.1341002446  0.0401476087  0.2111428670  0.3333791533
 [79,] -0.0367235510  0.0908490641  0.0225403091  0.1674500343  0.3029550835
 [80,] -0.0290896352  0.0625639554  0.0435514338  0.2080927457  0.2891835377
 [81,] -0.0008196417  0.0471791370  0.0461353362  0.2454109370  0.3188064792
 [82,] -0.0014572326  0.0627683622  0.0170540501  0.2511069028  0.3631354530
 [83,] -0.0311495115  0.0910175111  0.0092183047  0.2527260114  0.4026557866
 [84,] -0.0394484304  0.0498530337 -0.0678406795  0.2359198028  0.4197700938
 [85,] -0.0673947898  0.0645060234 -0.0875134092  0.2133640139  0.4025852863
 [86,] -0.0831257599  0.0875016072 -0.0609407140  0.2493553066  0.4289716773
 [87,] -0.0615931620  0.0315935744 -0.0150113255  0.2412133394  0.4225478697
 [88,] -0.0169459548 -0.0099030980  0.0183595366  0.2665479673  0.4844532192
 [89,]  0.0131149112  0.0103207189  0.0206548600  0.2976543195  0.5341375624
 [90,]  0.0058648104 -0.0134189489  0.0093420063  0.3237312907  0.5195989273
 [91,] -0.0145566378  0.0031886115  0.0091558469  0.2666441519  0.5037973470
 [92,]  0.0134597489  0.0398247562  0.0225027718  0.2729777630  0.5226080952
 [93,]  0.0621949062  0.0340001670  0.0195628588  0.2555908556  0.4959927508
 [94,]  0.0301825711  0.0295205467  0.0237250162  0.2885953334  0.4964011314
 [95,]  0.0683474967  0.0283889143  0.0260065070  0.2722433890  0.4573330071
```

```
 [96,]  0.0526359148  0.0093669906  0.0859852397  0.3177460079  0.4184074615
 [97,] -0.0253011376  0.0016556322  0.0438341434  0.3296131209  0.3826788933
 [98,]  0.0012235954  0.0085958948  0.0501113468  0.2935077296  0.3506928791
 [99,] -0.0021208001 -0.0004985869 -0.0003090532  0.2704039531  0.3293283428
[100,]  0.0371476844  0.0061743485 -0.0289446782  0.2334227334  0.3455915250
[101,]  0.0381234135 -0.0186740839  0.0167579462  0.2329176414  0.2941549053
[102,]  0.0610041248 -0.0316308952  0.0016605264  0.2782689643  0.3016941355
[103,]  0.0136381803 -0.0556997948  0.0267573497  0.2586806040  0.2835236280
[104,]  0.0259792143 -0.0373710654 -0.0094348989  0.2962126291  0.2439587386
[105,]  0.0421286862 -0.0174287525 -0.0078690042  0.2906941011  0.2516434752
[106,]  0.0217957764  0.0348679001 -0.0301691983  0.3114098159  0.2732333449
[107,]  0.0162806312  0.0617520223 -0.0739752771  0.3574339563  0.2938482871
[108,]  0.0190410814  0.1058413609 -0.0855621344  0.3020154820  0.2659425242
[109,] -0.0111319271  0.1138786637 -0.0909207885  0.2698828270  0.2746795299
[110,] -0.0503741300  0.0739898001 -0.1173946243  0.3171439710  0.3038957480
[111,] -0.0475316705  0.0593813055 -0.1085130920  0.3040648947  0.2698236655
[112,] -0.0299626047  0.0523414696 -0.0845994115  0.3033686932  0.2512999126
[113,] -0.0349907686  0.0708062757 -0.0700707050  0.3247892906  0.2734716385
[114,] -0.0270812381  0.0747779448  0.0017612598  0.3406255174  0.2774777971
[115,] -0.0213145561  0.0942167811 -0.0354787547  0.4059352962  0.2280928473
[116,] -0.0050049236  0.0826998694 -0.0946893993  0.3994176874  0.2428926706
[117,] -0.0037009596  0.0908374953 -0.1127021201  0.4029896553  0.2388768210
[118,]  0.0403493910  0.1075064575 -0.0633133960  0.3676272334  0.2424468580
[119,]  0.0306426645  0.1053637350 -0.0577671662  0.3790102461  0.2366907715
[120,]  0.0810555501  0.0917351313 -0.0765861873  0.4176535814  0.2191350532
[121,]  0.1058254316  0.0474528912 -0.0639767093  0.4485503364  0.1516558357
[122,]  0.1200993862  0.0687579080 -0.0701522080  0.4380973325  0.1321424416
[123,]  0.1226910401  0.0909536092 -0.0665987839  0.4010105335  0.1507010579
[124,]  0.1481470817  0.0952003646 -0.0356062476  0.4013419561  0.1367337410
[125,]  0.2030479453  0.0526606495  0.0128077404  0.4585508709  0.1844242532
[126,]  0.1823674054  0.0234089539  0.0796769709  0.4883550784  0.2188041006
[127,]  0.2143960000  0.0474568579  0.0915332156  0.4827772420  0.2455855018
[128,]  0.1855815579  0.0243546462  0.1198846566  0.5284084809  0.2243730380
[129,]  0.1836837700  0.0723970315  0.1360449422  0.4910206448  0.2093167796
[130,]  0.2131469025  0.0322408911  0.1229597091  0.5071902284  0.1880175818
[131,]  0.2564556622  0.0336461808  0.1510069573  0.5190773722  0.1340425311
[132,]  0.2122816993  0.0227641920  0.2008663729  0.4896448072  0.1171458798
[133,]  0.2380085239  0.0685447744  0.2142619168  0.4795389655  0.1116182914
[134,]  0.2534957470  0.1025272079  0.2090879895  0.4899180607  0.1638347818
[135,]  0.2381929796  0.0638800424  0.2503451562  0.4567977338  0.1036826654
[136,]  0.3200303178  0.0466150019  0.2645167916  0.4384060087  0.1153132654
[137,]  0.3109875561  0.0711831666  0.2633118604  0.3571687238  0.0930833943
[138,]  0.2508285232  0.1440283774  0.2212317184  0.3542581348  0.0204782498
[139,]  0.2845616266  0.0900101092  0.2197259275  0.3299502655  0.0254412880
[140,]  0.3098864050  0.1399894068  0.1762802432  0.2382164132  0.0171702147
[141,]  0.2439320924  0.0889335090  0.1859891986  0.2656747990  0.0252587133
[142,]  0.2366050552  0.0790044243  0.1801304839  0.2540864433  0.0530778234
[143,]  0.2625850220  0.0735427157  0.1717098190  0.2951670756  0.0613337518
[144,]  0.2637305845  0.0292780871  0.1551132228  0.2819845394  0.0394579055
```

```
[145,]  0.2938250735  0.0560405139  0.1807859429  0.3110662514  0.0326064402
[146,]  0.2801221404  0.0060865204  0.2122030333  0.3738437321  0.0177846721
[147,]  0.2616878793  0.0414034279  0.2022794881  0.3552589883  0.0047708642
[148,]  0.2364289782  0.0450587209  0.1522128927  0.3727445811 -0.0277344395
[149,]  0.1679240945 -0.0040964250  0.1432377193  0.4369173099 -0.0521743171
[150,]  0.1757036932 -0.0521174306  0.1738265042  0.4141387167 -0.0926940565
[151,]  0.1879796288 -0.0541520963  0.2046364132  0.4311120407 -0.1065722465
[152,]  0.2023975646 -0.0611605439  0.2489888879  0.4856016975 -0.1089737210
[153,]  0.1794802129 -0.0962969767  0.2741442287  0.4788626483 -0.1284755486
[154,]  0.1662489129 -0.1436329547  0.2724929323  0.4894822084 -0.0869579154
[155,]  0.1606192169 -0.1763466207  0.3439636161  0.4973463867 -0.0308696722
[156,]  0.1362364522 -0.1501225780  0.3261657211  0.5298303360  0.0102537123
[157,]  0.1736836966 -0.1509109436  0.3024233079  0.5061792289  0.0253661924
[158,]  0.1802936718 -0.1776907839  0.2983426136  0.5336905616  0.1068633922
[159,]  0.2155558138 -0.1586255021  0.3000668799  0.5039646243  0.0983402029
[160,]  0.2294383815 -0.2106152989  0.2942767890  0.4491781872  0.1176407920
[161,]  0.2214207275 -0.2007546332  0.2973049548  0.4626215642  0.0990797280
[162,]  0.2774551468 -0.2203345877  0.3533533619  0.4481000841  0.0377407705
[163,]  0.2090920838 -0.2452406766  0.3822349443  0.4785344490  0.0443623797
[164,]  0.1854470266 -0.2329729592  0.4104294873  0.4674707375  0.0893254773
[165,]  0.2179301274 -0.1569505804  0.3970467217  0.4447696791  0.0997335984
[166,]  0.2319852391 -0.1209604412  0.3888127622  0.4034784547  0.0951247160
[167,]  0.1823246494 -0.1258210250  0.4746538001  0.3880915153  0.1533601367
[168,]  0.2015224290 -0.0738999659  0.4437380794  0.4202050197  0.1496011699
[169,]  0.2386026966 -0.0755962401  0.4370223989  0.4274491706  0.1261546331
[170,]  0.2543628282 -0.0545654484  0.4642001208  0.4120613031  0.1038395297
[171,]  0.2852383436 -0.0451230521  0.4296882967  0.4003864477  0.0862217013
[172,]  0.2950805995  0.0377548001  0.4441517368  0.3580194863  0.0560826610
[173,]  0.3449637921 -0.0005276789  0.4965654104  0.3275618538  0.0653575930
[174,]  0.3661930646 -0.0212474497  0.4964145753  0.2748851342  0.0347386681
[175,]  0.3755946756 -0.0223238518  0.4865829981  0.2432772038  0.0367263955
[176,]  0.3861081124  0.0167377703  0.4673282152  0.2575883054  0.0201837449
[177,]  0.4047809158  0.0093449119  0.4369694610  0.2057342038  0.0050168206
[178,]  0.4102777799 -0.0610863500  0.4415282649  0.2348315438  0.0038835650
[179,]  0.4349709905 -0.0363670303  0.4117758535  0.2523912541  0.0027435674
[180,]  0.4040059865 -0.0083464280  0.4158284233  0.2687198507  0.0025072410
[181,]  0.3814468416 -0.0003215802  0.3862252555  0.2903119134  0.0688211139
[182,]  0.3991986618 -0.0446543206  0.4328521449  0.3193175280  0.0609683426
[183,]  0.3777417868 -0.0003049977  0.4183604698  0.2800564304  0.0964588408
[184,]  0.3830023231 -0.0133226162  0.4020747122  0.2939782702  0.0508507083
[185,]  0.4074566583  0.0539980594  0.4278065937  0.2965446731  0.0505179146
[186,]  0.4167407418  0.0086683447  0.4323681918  0.2559720769  0.0419445953
[187,]  0.4109903520  0.0302223903  0.4483924691  0.2581418100  0.0457473476
[188,]  0.4725849086 -0.0069602260  0.4121184573  0.2192347470  0.0090914735
[189,]  0.4685363005  0.0185617246  0.4154027002  0.2355248206  0.0377341173
[190,]  0.4958683850  0.0072953433  0.4516871663  0.1758230759  0.0100812920
[191,]  0.5098594587  0.0135002657  0.4834422879  0.1682458914  0.0272551952
[192,]  0.5434303594 -0.0115505746  0.4617606059  0.1646051276  0.1047404233
[193,]  0.5147656823 -0.0320654860  0.4785052160  0.1632787870  0.0910359498
```

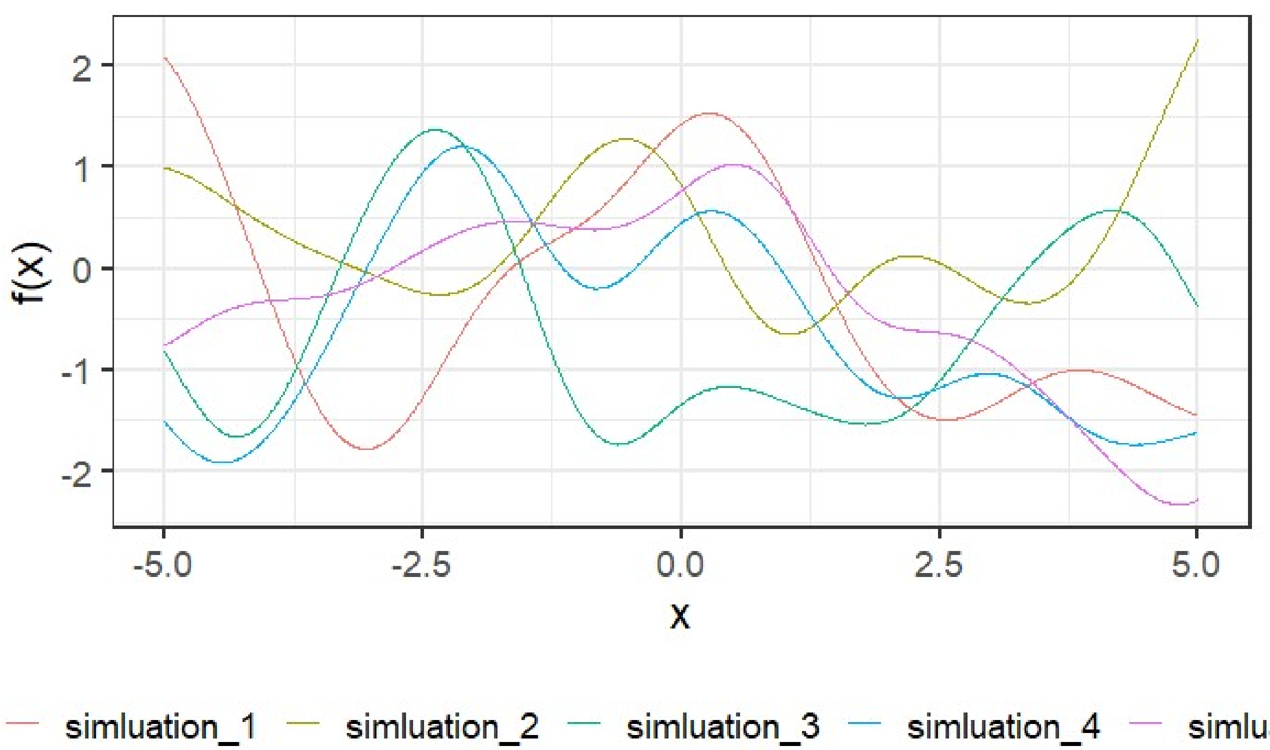
```
 [194,]  0.4555757988 -0.0415166620  0.4355135751  0.2350448346  0.0588891079
 [195,]  0.4884758717 -0.0917460703  0.4635036568  0.2676994815  0.0537047105
 [196,]  0.4573843467 -0.1178910155  0.4553431850  0.2404230440  0.0439760410
 [197,]  0.4301362319 -0.1431644059  0.4382305553  0.2811430846  0.0654636129
 [198,]  0.4200092367 -0.0958198260  0.3712328424  0.2382062210  0.0284005021
 [199,]  0.3832402009 -0.0963795711  0.4087276805  0.2500621761 -0.0007251281
 [200,]  0.3807642504 -0.0434841863  0.4204936589  0.2395247812 -0.0045828791
 [ reached getOption("max.print") -- omitted 800 rows ]

$plot
```



Hide

```
##########################################################
## Generating a sample from a GP
##########################################################
# Sampling from the prior GP
kernel_rbf = function(x){
  exp(-as.matrix(dist(x, diag = T))^2/2)
}
n = 1000
n_gps = 5
x = seq(-5,5,length.out = n)
K = kernel_rbf(x)
L = chol(K + 1e-6*diag(n))
f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)
colnames(f_prior) = paste0('simluation_', seq(1:n_gps))
f_prior_long_format = f_prior %>% as_tibble() %>% bind_cols(x = x) %>% pivot_longer(co
ls = starts_with("sim"))
ggplot(aes(x = x, color = name, y = value), data = f_prior_long_format) + geom_line
()+theme(legend.position = 'bottom')+
  guides(color=guide_legend(title=""))+
  ylab('f(x)')
```



Hide

```
##########################################################
## Learning values from a GP
##########################################################
```

Part 2 (Question 2: Time Series)

Hide

```
tims <- function(x, c=2, p=3, l=1.5, sig=2, sigv=1.25, sigb=2){
   d=sapply(x, FUN = function(x_in)(x_in- x))
   xc=sapply(x, FUN = function(x_in)((x_in- c)*(x-c)))
   per=(sig^2)*exp(-2*(sin(((pi*abs(d))/p)/(l^2))^2))
   return(per+(per)*exp(-d^2/(2*l^2))+sigb^2+(sigv^2)*xc)
}
sampling_from_a_gp = function(x_min = 0,
                              x_max=10,
                              kernel_in,
                              n = 50,
                              n_gps = 10){

  # Simulation
  x = seq(x_min, x_max,length.out = n)
  K = kernel_in(x)
  L = chol(K + 1e-6*diag(n))
  f_prior = t(L) %*% matrix(rnorm(n*n_gps), ncol = n_gps)

  # Reshaping
  colnames(f_prior) = paste0('Simulation ', seq(1:n_gps))
  f_prior_long_format = f_prior %>% as_tibble() %>%
    bind_cols(x = x) %>%
    pivot_longer(cols = starts_with("sim"))

  # Plot
  p = ggplot(aes(x = x, color = name, y = value),
             data = f_prior_long_format) +
    geom_line()+theme(legend.position = 'bottom')+
    guides(color=guide_legend(title=""))+
    ylab('f(x)')
  return(list('data_out' = f_prior, 'plot' = p))
}
sampling_from_a_gp(kernel_in = tims, n_gps = 10, n = 1000)
```

```
$`data_out`
        Simulation 1 Simulation 2 Simulation 3 Simulation 4 Simulation 5  Simulation
6 Simulation 7 Simulation 8 Simulation 9 Simulation 10
   [1,]  7.211798e+00  -4.63161003 12.110558178 -3.856438043   4.38908584 -3.145233871
2 -2.088166625   4.38087494 -4.337385966 -1.015762e+01
   [2,]  7.219677e+00  -4.65319936 12.095592302 -3.862607912   4.38098330 -3.089414593
3 -2.027317279   4.34089111 -4.319830838 -1.013903e+01
   [3,]  7.228144e+00  -4.67666608 12.075179397 -3.871679535   4.37191456 -3.032395071
3 -1.965534424   4.30418733 -4.308661378 -1.012308e+01
   [4,]  7.235934e+00  -4.69738539 12.057434425 -3.878503250   4.36533979 -2.975368037
5 -1.904037875   4.26816848 -4.290467303 -1.010379e+01
   [5,]  7.247034e+00  -4.71794606 12.036979218 -3.884407345   4.35681431 -2.920686131
6 -1.844012333   4.23145378 -4.272258702 -1.008379e+01
   [6,]  7.254995e+00  -4.73882255 12.019012100 -3.893430144   4.35231760 -2.863149276
3 -1.781769809   4.19515865 -4.257580991 -1.006580e+01
   [7,]  7.264025e+00  -4.75943402 11.999710059 -3.900382655   4.34644896 -2.805630911
8 -1.723719345   4.16333131 -4.233956907 -1.004862e+01
   [8,]  7.273295e+00  -4.77792358 11.975536137 -3.906644889   4.34179875 -2.749812768
1 -1.666331080   4.13101763 -4.217496210 -1.002855e+01
   [9,]  7.284353e+00  -4.79729594 11.953596367 -3.913123606   4.33628627 -2.689588609
4 -1.608263687   4.09734471 -4.195042644 -1.000963e+01
  [10,]  7.293394e+00  -4.81566876 11.931753307 -3.919291069   4.33239311 -2.634011492
5 -1.547874841   4.06561845 -4.173344939 -9.988642e+00
  [11,]  7.304631e+00  -4.83367742 11.908549413 -3.927701894   4.32946782 -2.575410313
8 -1.489120149   4.03439846 -4.149350443 -9.966044e+00
  [12,]  7.313853e+00  -4.84977422 11.882628375 -3.931143317   4.32726660 -2.518626479
7 -1.429668883   4.00378037 -4.124169679 -9.945958e+00
  [13,]  7.323288e+00  -4.86804636 11.861269337 -3.935166513   4.32517587 -2.459885870
9 -1.374321207   3.97739471 -4.099994350 -9.925743e+00
  [14,]  7.334488e+00  -4.88562092 11.836316880 -3.941781895   4.32309843 -2.402692662
5 -1.317193695   3.94617819 -4.073354785 -9.902937e+00
  [15,]  7.343653e+00  -4.90044077 11.808173459 -3.945327843   4.32219658 -2.344739225
4 -1.261794540   3.91633712 -4.046394387 -9.881350e+00
  [16,]  7.351664e+00  -4.91698288 11.784836484 -3.948312298   4.32117057 -2.286474225
1 -1.206515205   3.88972274 -4.016990729 -9.860967e+00
  [17,]  7.364385e+00  -4.93179598 11.758066495 -3.953772149   4.32389243 -2.231465175
1 -1.146271432   3.86276466 -3.987238189 -9.841233e+00
  [18,]  7.374002e+00  -4.94579571 11.729190017 -3.957208542   4.32207115 -2.173627968
9 -1.095079936   3.83748546 -3.960136089 -9.818937e+00
  [19,]  7.385480e+00  -4.96097796 11.702761179 -3.960202438   4.32396060 -2.118019253
5 -1.039278181   3.81272827 -3.926652187 -9.795918e+00
  [20,]  7.394947e+00  -4.97217529 11.670493278 -3.962882520   4.32442454 -2.061493501
6 -0.986395462   3.78759416 -3.896128820 -9.774006e+00
  [21,]  7.406481e+00  -4.98638113 11.644623758 -3.964052331   4.32563616 -2.005072770
4 -0.933009011   3.76499431 -3.865196202 -9.752172e+00
  [22,]  7.414039e+00  -4.99755737 11.612856549 -3.965846134   4.32796327 -1.950529239
3 -0.878236102   3.74154073 -3.831100650 -9.728633e+00
  [23,]  7.425827e+00  -5.01108380 11.581202549 -3.968814332   4.32909897 -1.895936272
```

```
4 -0.825469636   3.72058228 -3.795003546 -9.707247e+00
  [24,]  7.434039e+00  -5.01903450 11.549258096 -3.969151389   4.33286107 -1.841335635
5 -0.773853692   3.69875731 -3.761851731 -9.687040e+00
  [25,]  7.443981e+00  -5.03148937 11.519048254 -3.969300113   4.33753351 -1.789850812
6 -0.723755550   3.67755563 -3.726391569 -9.664242e+00
  [26,]  7.452975e+00  -5.04156317 11.488838635 -3.965682698   4.34110629 -1.736689268
0 -0.672881710   3.65632110 -3.691264425 -9.642476e+00
  [27,]  7.463600e+00  -5.05002531 11.451533204 -3.964724131   4.34221852 -1.681746615
1 -0.621841920   3.63733485 -3.654852389 -9.620359e+00
  [28,]  7.472333e+00  -5.06103473 11.419039741 -3.959649078   4.34499783 -1.631787212
3 -0.569456765   3.61748697 -3.619186804 -9.596568e+00
  [29,]  7.480576e+00  -5.06900905 11.386194932 -3.962738278   4.35147263 -1.581214282
2 -0.519772465   3.60004076 -3.579413841 -9.578274e+00
  [30,]  7.489974e+00  -5.07675775 11.350897144 -3.956662054   4.35540322 -1.532027993
3 -0.473426794   3.58053091 -3.543792537 -9.557750e+00
  [31,]  7.497157e+00  -5.08240539 11.316032551 -3.952453818   4.35826719 -1.483453512
9 -0.424410972   3.56524389 -3.502192356 -9.534756e+00
  [32,]  7.506812e+00  -5.08935500 11.280965073 -3.948947451   4.36382482 -1.434076877
1 -0.376530856   3.54705133 -3.465074853 -9.513809e+00
  [33,]  7.513198e+00  -5.09506305 11.241838124 -3.942657885   4.36788456 -1.384783086
2 -0.328485255   3.53364615 -3.425780154 -9.490942e+00
  [34,]  7.518948e+00  -5.10041916 11.208936594 -3.936016867   4.37247634 -1.339378518
8 -0.283153284   3.51838202 -3.385846141 -9.473003e+00
  [35,]  7.525100e+00  -5.10609793 11.170533653 -3.928204437   4.37840620 -1.295740366
9 -0.236508644   3.50340326 -3.344416270 -9.450927e+00
  [36,]  7.532703e+00  -5.11117940 11.132050000 -3.922720700   4.38359159 -1.249121744
0 -0.190518301   3.48686532 -3.304486802 -9.430845e+00
  [37,]  7.537154e+00  -5.11384393 11.093848171 -3.914700585   4.38742299 -1.208277370
4 -0.146596332   3.47471495 -3.261365430 -9.410891e+00
  [38,]  7.543922e+00  -5.11800581 11.057595949 -3.904414380   4.39136667 -1.165095999
0 -0.102710967   3.46289572 -3.220745323 -9.392405e+00
  [39,]  7.549375e+00  -5.12249205 11.017771833 -3.894847818   4.39846976 -1.122463689
0 -0.058330096   3.45071310 -3.180359816 -9.373744e+00
  [40,]  7.553545e+00  -5.12342801 10.979863488 -3.882570613   4.39922275 -1.084212434
2 -0.016740237   3.43655054 -3.138349010 -9.351804e+00
  [41,]  7.556872e+00  -5.12425057 10.938871166 -3.869984136   4.40559121 -1.044322565
6  0.027698942   3.42557144 -3.094910854 -9.332376e+00
  [42,]  7.561663e+00  -5.12487089 10.899377483 -3.859024264   4.41095406 -1.007851874
6  0.069502668   3.41363437 -3.053971641 -9.316126e+00
  [43,]  7.562582e+00  -5.12624516 10.859285276 -3.846124332   4.41422968 -0.970304723
8  0.111059035   3.40483534 -3.010825397 -9.296376e+00
  [44,]  7.565348e+00  -5.12517179 10.818404727 -3.829571168   4.41637263 -0.936349679
4  0.150110567   3.39739850 -2.971582139 -9.280210e+00
  [45,]  7.570637e+00  -5.12720656 10.775192325 -3.815311106   4.42215028 -0.901023723
0  0.191333047   3.38662458 -2.928270854 -9.260220e+00
  [46,]  7.569737e+00  -5.12643384 10.735272714 -3.799374114   4.42621251 -0.868460486
4  0.231473743   3.37851145 -2.885687072 -9.243813e+00
  [47,]  7.571301e+00  -5.12467587 10.692765239 -3.782159887   4.43340622 -0.836936053
1  0.269758215   3.37127250 -2.842512614 -9.227500e+00
```

```
  [48,]  7.569783e+00  -5.12395579 10.652461722 -3.766630770   4.43519568 -0.807606517
6  0.307483625   3.36228150 -2.800131113 -9.209861e+00
  [49,]  7.570731e+00  -5.11947596 10.607327122 -3.746728668   4.43731422 -0.779134075
5  0.345116862   3.35654063 -2.758809610 -9.194443e+00
  [50,]  7.569340e+00  -5.11844575 10.565987994 -3.728137412   4.44193207 -0.751928352
7  0.380466482   3.34918279 -2.718177056 -9.176766e+00
  [51,]  7.566721e+00  -5.11436274 10.523058987 -3.706569537   4.44399372 -0.727918308
6  0.418628224   3.34542907 -2.676178635 -9.163102e+00
  [52,]  7.564321e+00  -5.11160293 10.480824946 -3.683728451   4.44645359 -0.703902255
6  0.453087311   3.34168030 -2.633086761 -9.147524e+00
  [53,]  7.560186e+00  -5.10734759 10.436523253 -3.661195397   4.44879021 -0.680140958
6  0.488932082   3.33750238 -2.594861819 -9.133592e+00
  [54,]  7.554951e+00  -5.10566995 10.393461827 -3.640535981   4.45219435 -0.658513723
6  0.522775025   3.33213678 -2.552982098 -9.119820e+00
  [55,]  7.550936e+00  -5.10038658 10.349449858 -3.614685565   4.45286779 -0.638071008
6  0.556606620   3.32845707 -2.509497226 -9.106983e+00
  [56,]  7.546910e+00  -5.09626864 10.306472039 -3.591380675   4.45573958 -0.619244365
9  0.588738610   3.32446482 -2.471767165 -9.092702e+00
  [57,]  7.540235e+00  -5.09087149 10.262727921 -3.565197000   4.45650213 -0.603218057
9  0.623755293   3.32276366 -2.431937070 -9.077573e+00
  [58,]  7.532690e+00  -5.08520702 10.217265912 -3.538081218   4.45768117 -0.586049089
3  0.654144065   3.32112946 -2.392302144 -9.065566e+00
  [59,]  7.525967e+00  -5.07899455 10.170510234 -3.510908002   4.45998124 -0.572826569
9  0.687445776   3.32172510 -2.354703694 -9.055917e+00
  [60,]  7.518354e+00  -5.07397027 10.129748021 -3.482208649   4.45917871 -0.559391529
5  0.717530807   3.32087042 -2.315243054 -9.041939e+00
  [61,]  7.507129e+00  -5.06644480 10.083421554 -3.454296686   4.45949693 -0.548972006
2  0.746633094   3.31952660 -2.277238144 -9.031543e+00
  [62,]  7.499188e+00  -5.06098593 10.039868424 -3.423951059   4.45840118 -0.538073248
6  0.773972220   3.32281105 -2.240426367 -9.019525e+00
  [63,]  7.487705e+00  -5.05371052  9.994842427 -3.393766301   4.46085495 -0.530133483
8  0.806271961   3.32292206 -2.201505335 -9.010429e+00
  [64,]  7.476379e+00  -5.04484198  9.949042823 -3.363853251   4.45759904 -0.524216386
6  0.833868920   3.32406121 -2.167032953 -9.002674e+00
  [65,]  7.463769e+00  -5.03939870  9.904393165 -3.330496371   4.45783720 -0.517686648
2  0.860427743   3.32597839 -2.130758036 -8.993426e+00
  [66,]  7.451164e+00  -5.03186854  9.860074204 -3.294162314   4.45468198 -0.515134324
3  0.888198683   3.32959905 -2.097538370 -8.986742e+00
  [67,]  7.435520e+00  -5.02397475  9.814608216 -3.263822652   4.45526186 -0.513404214
1  0.914780710   3.33346704 -2.061984306 -8.973117e+00
  [68,]  7.421470e+00  -5.01781479  9.769546050 -3.229977468   4.45113260 -0.513204896
8  0.941940666   3.33607404 -2.027974476 -8.964973e+00
  [69,]  7.406698e+00  -5.00818224  9.724387872 -3.193240400   4.44986993 -0.511498268
5  0.965084989   3.33876801 -1.995405652 -8.960524e+00
  [70,]  7.387992e+00  -5.00248736  9.679731442 -3.158815891   4.44785520 -0.514216240
0  0.990031946   3.34665366 -1.964000759 -8.949885e+00
  [71,]  7.373519e+00  -4.99178596  9.635134610 -3.119859927   4.44390037 -0.519244572
3  1.013380780   3.35005423 -1.932950075 -8.945490e+00
  [72,]  7.354824e+00  -4.98377994  9.589482983 -3.082903109   4.43782630 -0.520988657
```

```
2  1.037061024   3.35744518 -1.902047990 -8.936316e+00
  [73,]  7.335269e+00  -4.97481134  9.546084534 -3.045066822   4.43542032 -0.530167449
5  1.062192762   3.36140258 -1.875422896 -8.931590e+00
  [74,]  7.316156e+00  -4.96651444  9.499799906 -3.008049687   4.43274200 -0.535928878
9  1.084057765   3.37024956 -1.844412666 -8.926311e+00
  [75,]  7.293432e+00  -4.96106949  9.457138550 -2.968421803   4.42784362 -0.545887519
5  1.105322336   3.37819198 -1.815994462 -8.919598e+00
  [76,]  7.271755e+00  -4.95326502  9.411518160 -2.926766950   4.42279183 -0.558186652
3  1.126007262   3.38793294 -1.790390232 -8.916639e+00
  [77,]  7.250180e+00  -4.94243263  9.368428412 -2.886437894   4.41584973 -0.566851652
6  1.147719953   3.39628895 -1.763669755 -8.912716e+00
  [78,]  7.226886e+00  -4.93373570  9.324889423 -2.848332839   4.41153805 -0.581258593
0  1.168562112   3.40322210 -1.740490457 -8.904113e+00
  [79,]  7.203003e+00  -4.92614948  9.277413902 -2.804908204   4.40556383 -0.594195713
9  1.187343434   3.41243282 -1.715544288 -8.903130e+00
  [80,]  7.174920e+00  -4.91800300  9.235053779 -2.763649240   4.40038073 -0.608194047
2  1.207386980   3.42362810 -1.692568889 -8.898099e+00
  [81,]  7.151673e+00  -4.90858431  9.189576431 -2.722916050   4.39287529 -0.625695147
2  1.225513298   3.43366341 -1.672567195 -8.896272e+00
  [82,]  7.123071e+00  -4.89999881  9.145812336 -2.680056474   4.38548962 -0.645768429
1  1.244667688   3.44482586 -1.651989629 -8.894245e+00
  [83,]  7.093381e+00  -4.89205817  9.102937684 -2.635421441   4.37832540 -0.662642563
4  1.260601395   3.45419225 -1.631181709 -8.892592e+00
  [84,]  7.066736e+00  -4.88138166  9.058869867 -2.592772569   4.37050010 -0.683163365
4  1.280170509   3.46507438 -1.613949264 -8.889568e+00
  [85,]  7.037316e+00  -4.87514841  9.015881797 -2.549608891   4.36381837 -0.705609869
4  1.296169262   3.48028689 -1.595475427 -8.888884e+00
  [86,]  7.006649e+00  -4.86964498  8.971101082 -2.503832222   4.35422864 -0.728096703
7  1.311009267   3.49078884 -1.581267759 -8.886364e+00
  [87,]  6.976161e+00  -4.86161814  8.929367859 -2.461906570   4.34681771 -0.752901619
5  1.329155466   3.50346595 -1.563432024 -8.884225e+00
  [88,]  6.943326e+00  -4.85238804  8.886659708 -2.416401926   4.33872536 -0.778410614
9  1.342933250   3.51962718 -1.550158316 -8.884773e+00
  [89,]  6.908836e+00  -4.84441635  8.843532618 -2.371502109   4.32989488 -0.803613665
9  1.357421296   3.52978146 -1.537143194 -8.884293e+00
  [90,]  6.874106e+00  -4.83805135  8.800190579 -2.323469877   4.32039080 -0.828391690
0  1.372529210   3.54564370 -1.524530351 -8.884419e+00
  [91,]  6.840623e+00  -4.83029303  8.759674422 -2.278336800   4.30930023 -0.857191733
1  1.386335955   3.55871996 -1.511955857 -8.883956e+00
  [92,]  6.804740e+00  -4.82374443  8.716267229 -2.234193749   4.30087506 -0.883537665
6  1.402412884   3.57201945 -1.503098072 -8.885794e+00
  [93,]  6.767250e+00  -4.81988983  8.676715080 -2.189216854   4.29309331 -0.915905093
8  1.416150526   3.58758258 -1.494028720 -8.885707e+00
  [94,]  6.730049e+00  -4.81145307  8.633370221 -2.142782253   4.28228257 -0.945666389
6  1.430231022   3.60222005 -1.486382998 -8.883231e+00
  [95,]  6.690387e+00  -4.80501940  8.590817089 -2.097486983   4.26799316 -0.975555416
9  1.442918120   3.61863523 -1.478551506 -8.887600e+00
  [96,]  6.652661e+00  -4.79885301  8.551145778 -2.051086002   4.25898433 -1.007157744
8  1.457285025   3.63541722 -1.473412741 -8.888577e+00
```

```
  [97,]  6.610563e+00  -4.79456139  8.510580201 -2.004970887   4.24893529 -1.039758999
4  1.467013378   3.65111171 -1.467098620 -8.889320e+00
  [98,]  6.569611e+00  -4.78951768  8.467968102 -1.960405346   4.23991487 -1.073142960
3  1.478025561   3.66280286 -1.463915849 -8.894536e+00
  [99,]  6.528086e+00  -4.78501464  8.426583102 -1.913018249   4.22699132 -1.106139899
1  1.491140056   3.68172558 -1.461152555 -8.896348e+00
 [100,]  6.484971e+00  -4.77894433  8.386127415 -1.868243861   4.21467547 -1.143449277
6  1.501646728   3.69948389 -1.459989849 -8.895957e+00
 [ reached getOption("max.print") -- omitted 900 rows ]

$plot
```