## **APPENDIX**

## term\_project.py

import pandas as pd
import helper as helpme
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller
import seaborn as sns
from sklearn.model\_selection import train\_test\_split
from statsmodels.tsa.seasonal import seasonal\_decompose
import numpy as np
from statsmodels.tsa.holtwinters import ExponentialSmoothing
import arma\_estimator as my\_arma
from scipy import signal
import statsmodels.api as sm
from scipy.stats import chi2

```
coffee data=pd.read csv("data/COFFEE.csv")
coffee_data.head()
corn data=pd.read csv("data/corn.csv")
corn_data.head()
cotton_data=pd.read_csv("data/COTTON.csv")
cotton data.head()
gold_data=pd.read_csv("data/GOLD.csv")
gold_data.head()
lumber_data=pd.read_csv("data/LUMBER.csv")
lumber_data.head()
oil_data=pd.read_csv("data/OIL.csv")
oil_data.head()
wheat data=pd.read csv("data/WHEAT.csv")
wheat_data.head()
snp_data=pd.read_csv("data/S&P500.csv")
snp data.head()
```

```
oil_data=helpme.datetime_transformer(oil_data,['DATE'])
oil_data=oil_data.rename(columns={"CLOSING PRICE": "oil"})
oil_data.drop(oil_data.columns.difference(['DATE','oil','DATE_year','DATE_month','DATE
_day']), 1, inplace=True)
gold_data=helpme.datetime_transformer(gold_data,['DATE'])
gold_data=gold_data.rename(columns={"CLOSING PRICE": "gold"})
gold_data.drop(gold_data.columns.difference(['DATE','gold','DATE_year','DATE_month','
DATE_day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(oil_data, gold_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['DATE_year','DATE_month','DATE_day'])
all_data.drop(all_data.columns.difference(['DATE','oil','gold','DATE_year','DATE_month','
DATE_day']), 1, inplace=True)
coffee_data=helpme.datetime_transformer(coffee_data,['Date'])
coffee_data=coffee_data.rename(columns={"Price": "coffee"})
coffee data.drop(coffee data.columns.difference(['Date','coffee','Date year','Date month'
,'Date_day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(all_data, coffee_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date_year','Date_month','Date_day'])
all_data.drop(all_data.columns.difference(['DATE','oil','gold','coffee','DATE_year','DATE_
month', 'DATE_day']), 1, inplace=True)
corn_data=helpme.datetime_transformer(corn_data,['Date'])
corn_data=corn_data.rename(columns={"Price": "corn"})
corn_data.drop(corn_data.columns.difference(['Date','corn','Date_year','Date_month','Dat
e_day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(all_data, corn_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date_year','Date_month','Date_day'])
all_data.drop(all_data.columns.difference(['DATE','oil','gold','coffee','corn','DATE_year','D
ATE_month','DATE_day']), 1, inplace=True)
cotton_data=helpme.datetime_transformer(cotton_data,['Date'])
cotton_data=cotton_data.rename(columns={"Price": "cotton"})
```

```
cotton_data.drop(cotton_data.columns.difference(['Date','cotton','Date_year','Date_month
','Date day']), 1, inplace=True)
#join and drop unnecessary
all data = pd.merge(all data, cotton data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date_year','Date_month','Date_day'])
all data.drop(all data.columns.difference(['DATE','oil','gold','coffee','corn','cotton','DATE
_year','DATE_month','DATE_day']), 1, inplace=True)
lumber data=helpme.datetime transformer(lumber data,['Date'])
lumber_data=lumber_data.rename(columns={"Price": "lumber"})
lumber_data.drop(lumber_data.columns.difference(['Date','lumber','Date_year','Date_mon
th','Date_day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(all_data, lumber_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date_year','Date_month','Date_day'])
all data.drop(all data.columns.difference(['DATE','oil','gold','coffee','corn','cotton','lumbe
r','DATE_year','DATE_month','DATE_day']), 1, inplace=True)
wheat data=helpme.datetime transformer(wheat data,['Date'])
wheat_data=wheat_data.rename(columns={"Price": "wheat"})
wheat_data.drop(wheat_data.columns.difference(['Date','wheat','Date_year','Date_month',
'Date day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(all_data, wheat_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date year','Date month','Date day'])
all_data.drop(all_data.columns.difference(['DATE','oil','gold','coffee','corn','cotton','lumbe
r','wheat','DATE_year','DATE_month','DATE_day']), 1, inplace=True)
snp_data=helpme.datetime_transformer(snp_data,['Date'])
snp_data=snp_data.rename(columns={"Close": "snp"})
snp_data.drop(snp_data.columns.difference(['Date','snp','Date_year','Date_month','Date_
day']), 1, inplace=True)
#join and drop unnecessary
all_data = pd.merge(all_data, snp_data, how='inner',
left_on=['DATE_year','DATE_month','DATE_day'], right_on =
['Date_year','Date_month','Date_day'])
```

```
all_data.drop(all_data.columns.difference(['DATE','oil','gold','coffee','corn','cotton','lumbe
r','wheat','snp','DATE_year','DATE_month','DATE_day']), 1, inplace=True)
all data
#prep
#all_data.to_csv(r'data/all_data.csv', index = False)
#plottiong dependent variable against time
plt.plot(all_data["snp"])
plt.title("5 Years data of SNP 500")
plt.xlabel("Days")
plt.ylabel("SNP500 Closing Index")
plt.show()
#adf of the dependent variable
dependent_variable=all_data["snp"]
result = adfuller(dependent_variable)
print('ADF for dependent variable:')
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
       print('\t%s: %.3f' % (key, value))
# p-value is > 0.05:
# Fail to reject the null hypothesis (H0),
# the data does have a unit root and is definetely not stationary.
#plotting acf of dependent variable
helpme.acf plotter(dependent variable,len(dependent variable))
corr_d={'coffee': all_data['coffee'],
       'corn': all_data['corn'],
       'cotton': all_data['cotton'],
              'gold': all_data['gold'],
              'lumber': all_data['lumber'],
              'oil': all_data['oil'],
              'wheat': all_data['wheat'],
              'snp': all_data['snp']}
df=pd.DataFrame(data=corr_d)
sns.heatmap(df.corr(),annot=True)
```

```
#checking for nans
helpme.nan_checker(all_data)
#splitting training and testing
Xd={'coffee': all_data['coffee'],
  'corn': all_data['corn'],
  'cotton': all_data['cotton'],
       'gold': all_data['gold'],
       'lumber': all_data['lumber'],
       'oil': all_data['oil'],
       'wheat': all_data['wheat']}
X_df=pd.DataFrame(data=Xd)
#X df
Yd={'snp': all_data['snp']}
Y df=pd.DataFrame(data=Yd)
#setting index for holt
#Y_df
# 80% train, 20% test
x_train, x_test, y_train, y_test = train_test_split(X_df, Y_df, train_size = 0.8, test_size =
0.2, shuffle = False)
#x train
#x test
#y_train['snp']
#y_test
# making dependent variable stationary using first differencing method
Y = y_train.values
diff = []
for i in range(1, len(Y)):
       value = Y[i] - Y[i - 1]
       diff.append(value)
#before detrending training portion of dependent variable
plt.plot(Y)
```

```
plt.title("Before detrending")
plt.xlabel("Days")
plt.ylabel("SNP500 Closing Index 80% Train")
plt.show()
#after detrending training portion of dependent variable
plt.plot(diff)
plt.title("After detrending using first differencing")
plt.xlabel("Days")
plt.ylabel("SNP500 Closing Index 80% Train")
plt.show()
#adf of the differenced traing dependent variable
diff train dependent variable=diff
result = adfuller(diff train dependent variable)
print('ADF for differenced traing dependent variable:')
print('ADF Statistic: %f' % result[0])
print('p-value: %f' % result[1])
print('Critical Values:')
for key, value in result[4].items():
       print('\t%s: %.3f' % (key, value))
# p-value is less than 0.05:
# Reject the null hypothesis (H0),
# the data does not have a unit root and is definetely stationary.
# multiplicative decomposition
y_arr=np.array(Y_df)
result=seasonal_decompose(y_arr,model='multiplicative', freq=1)
result.plot()
plt.show()
# additive decomposition
result2=seasonal_decompose(y_arr,model='additive', freq=1)
result2.plot()
plt.show()
#hotwinter model: <start>
days=range(1,len(all_data)+1)
holt_Y={'day': days,
```

```
'snp': all data['snp']}
holt_df=pd.DataFrame(data=holt_Y)
#holt_df['day']=pd.to_datetime(holt_df['day'])
holt_df.set_index('day',inplace=True)
holt_df.index.freq='M'
holt_train, holt_test = holt_df.iloc[:996,0], holt_df.iloc[995:,0]
holt_model = ExponentialSmoothing(holt_train, trend='add', damped=False,
seasonal='add', seasonal periods=2).fit()
holt_predictions = holt_model.predict(start=holt_test.index[0], end=holt_test.index[-1])
plt.plot(holt_train.index, holt_train, label="train")
plt.plot(holt_test.index, holt_test, label="test")
plt.plot(holt_predictions.index, holt_predictions, label="predictions")
plt.show()
#holtwinter model: <end>
#holts accuracy <start>
forecast_error=holt_test-holt_predictions
#forecast error
holt_forecast_error_d = {'Y': holt_test,
   'Y': holt predictions,
  'e=Y-Y`': forecast error}
holt_forecast_error_df = pd.DataFrame(data=holt_forecast_error_d)
holt_forecast_error_df
RMSE = np.sqrt(np.mean(forecast error**2))
print("The Holt Root Mean Square of Forecast Error is "+str(RMSE))
mean_forecast_error=np.mean(forecast_error)
print("The Holts Mean of Forecast Error is "+str(mean_forecast_error))
def standard_error(forecast_error,num_of_predictors):
  return np.sqrt(np.sum(forecast_error**2)/(len(forecast_error)-num_of_predictors-1))
se=standard_error(forecast_error,1)
print("The standard error using holt is "+str(se))
R_Squared=((np.corrcoef(holt_test, holt_predictions)[0, 1])**2)
```

```
print("The R^2 using holt is "+str(R_Squared))
T=len(holt_test) #size of sample
k=1 #number of predictors
Adjusted_R_Squared=(1-((1-R_Squared)*((T-1)/(T-k-1))))
print("The Adj R^2 using holt is "+str(Adjusted_R_Squared))
#holst accuracy end
# multiple linear regression start
X_M = x_train[["coffee", "corn", "cotton", "gold", "lumber", "oil", "wheat"]]
Y_M = y_train["snp"]
X M=np.mat(X M)
Y_M=np.mat(Y_M)
lin_reg_model = sm.OLS(Y_M,X_M)
results = lin_reg_model.fit()
print (results.params)
test pred ols=results.predict(x test)
test_pred_ols
print(results.summary())
# multiple linear regression end
ry=my_arma.acf_values(diff_train_dependent_variable,100)
ax=my_arma.cal_GPAC(ry,25,25)
x_axis_labels=range(1,25)
plt.figure(figsize = (25,25))
hmap = sns.heatmap(ax, xticklabels=x_axis_labels, annot=True, linewidths=.5, vmin=-0.5,
vmax=0.5, cmap="YIGnBu")
hmap
print("Estimated parameters for ARMA(15,16) :")
```

```
my arma.levenburgMarquardt(diff train dependent variable,15,16,120)
def arma_yhat(na,theta,sampleSize):
      T=sampleSize
      mu, sigma =0, 1
      e = np.random.normal(mu, sigma, T)
      num=[1]
      den=[1]
      den=np.concatenate((den,theta[0:na]))
      num=np.concatenate((num,theta[na:]))
      if len(num)<len(den):
             z=np.zeros(len(den)-len(num))
             num=np.concatenate((num,z),axis=None)
      elif len(num)>len(den):
             z=np.zeros(len(num)-len(den))
             den=np.concatenate((den,z),axis=None)
      system = (num,den,1)
      t in=np.arange(0,T)
      t out, y = signal.dlsim(system,e,t=t in)
      return y
theta=[-0.0451566 . -0.00802712 . 0.18580267 . 0.39536419 . -0.18220256 .
    0.16160088, -0.02275384, 0.19785412, 0.0784035, 0.00378659,
    0.22446751, 0.10849447, 0.23731303, 0.03900169, 0.06361364,
    -0.04673583, -0.05854274, 0.23820085, 0.34755728, -0.22201763,
    0.15678795, 0.04174483, 0.06039233, 0.08552093, -0.01700574,
    0.25083316, 0.05473999, 0.25375103, -0.0786032, 0.033727,
    0.045867581
my arma yhat=arma yhat(15,theta,len(diff train dependent variable))
print("Using parameters from my ARMA:")
plt.figure()
plt.plot(diff train dependent variable, 'r', label="Actual")
plt.plot(my_arma_yhat,'b',label="implemented_prediction")
plt.xlabel("Samples")
plt.ylabel("Magnitude")
plt.legend()
plt.title("ACTUAL vs Implemented ARMA prediction")
plt.show()
#using stats model for ARMA
```

```
import statsmodels.api as sm
na=15
nb=16
arma model=sm.tsa.ARMA(diff train dependent variable,(na,nb)).fit(trend='nc',disp=0)
for i in range (na):
       print("The AR coeff a{}".format(i), "is:",arma_model.params[i])
for i in range (nb):
       print("The MA coeff b{}".format(i), "is:",arma_model.params[i+na])
print(arma_model.summary())
# prediction
arma_model_y_hat=arma_model.predict(start=1, end=len(diff_train_dependent_variable))
# residual testing and chisquare test
plt.figure()
plt.plot(diff_train_dependent_variable,'r',label="Actual")
plt.plot(arma model y hat, 'b', label="prediction")
plt.xlabel("Samples")
plt.ylabel("Magnitude")
plt.legend()
plt.title("ACTUAL vs ARMA prediction")
plt.show()
#residual testing and chi square test of ARMA(15,16)
"""lags=365
e=diff_train_dependent_variable-arma_model_y_hat
re=helpme.autocorrelation cal(e,lags)
Q=len(diff_train_dependent_variable)*np.sum(np.square(re[lags:]))
DOF=lags-na-nb
alfa=0.01
chi_critical=chi2.ppf(1-alfa,DOF)
if Q<chi_critical:
      print("the residual is white")
else:
       print("the residual is not white")
lbvalue,pvalue=sm.stats.acorr_ljungbox(e,lags=[lags])
print(lbvalue)
print(pvalue)"""
```

```
na2=16
nb2=23
arma_model2=sm.tsa.ARMA(diff_train_dependent_variable,(na2,nb2)).fit(trend='nc',disp=
for i in range (na2):
      print("The AR coeff a{}".format(i), "is:",arma_model2.params[i])
for i in range (nb2):
      print("The MA coeff b{}".format(i), "is:",arma_model2.params[i+na2])
print(arma_model2.summary())
# prediction
arma_model_y_hat2=arma_model2.predict(start=1,
end=len(diff_train_dependent_variable))
# residual testing and chisquare test
plt.figure()
plt.plot(diff_train_dependent_variable,'r',label="Actual")
plt.plot(arma_model_y_hat2,'b',label="prediction")
plt.xlabel("Samples")
plt.ylabel("Magnitude")
plt.legend()
plt.title("ACTUAL vs ARMA prediction")
plt.show()
arma_model_forecast=arma_model.forecast(steps=len(y_test), exog=None, alpha=0.05)
arma_model2_forecast=arma_model2.forecast(steps=len(y_test), exog=None,
alpha=0.05)
plt.plot(arma_model_forecast[0])
plt.show()
plt.plot(arma_model2_forecast[0])
```

```
plt.show()
```

```
def reverse_transform_for_differencing(original_input_list,
differenced df list with predicted values):
  """ returns transformed values for predicted values only"""
  last_index = len(original_input_list) - 1
  prediction_range = len(differenced_df_list_with_predicted_values) -
len(original input list) + 1
  back transformed = []
  predicted_sum = 0
  for i in range(prediction_range):
    predicted_sum += differenced_df_list_with_predicted_values[last_index + i]
    predicted_value = original_input_list[last_index] + predicted_sum
    back_transformed.append(predicted_value)
  return back_transformed
#arma (15,16)
diff_forecast1=diff+arma_model_forecast[0].tolist()
rev_pred=reverse_transform_for_differencing(Y,diff_forecast1)
rev_pred_arr=np.array(rev_pred)
plt.plot(Y_df.values, label="actual")
plt.plot(np.concatenate((Y,rev_pred_arr)), label="prediction")
#plt.plot(Y, label="actual")
plt.show()
#arma (16,23)
diff_forecast2=diff+arma_model2_forecast[0].tolist()
rev_pred2=reverse_transform_for_differencing(Y,diff_forecast2)
rev_pred_arr2=np.array(rev_pred2)
plt.plot(Y_df.values, label="actual")
plt.plot(np.concatenate((Y,rev_pred_arr2)), label="prediction")
#plt.plot(Y, label="actual")
plt.show()
```

```
# arma (15,16) forecast errors
arma1_forecast_error=y_test.values-rev_pred_arr
#forecast error
"""holt_forecast_error_d = {'Y': holt_test,
  'Y': holt_predictions,
  'e=Y-Y`': forecast error}
holt_forecast_error_df = pd.DataFrame(data=holt_forecast_error_d)
holt forecast error df"""
RMSE_arma1 = np.sqrt(np.mean(arma1_forecast_error**2))
print("The ARMA(15,16) Root Mean Square of Forecast Error is "+str(RMSE_arma1))
arma1_mean_forecast_error=np.mean(arma1_forecast_error)
print("The ARMA(15,16) Mean of Forecast Error is "+str(arma1_mean_forecast_error))
se_arma1=standard_error(arma1_forecast_error,1)
print("The standard error using ARMA(15,16) is "+str(se_arma1))
"""arma1_R_Squared=((np.corrcoef(y_test.values, rev_pred_arr)[0, 1])**2)
print("The R^2 using ARMA(15,16) is "+str(arma1_R_Squared))
T=len(y_test) #size of sample
k=1 #number of predictors
arma1_Adjusted_R_Squared=(1-((1-R_Squared)*((T-1)/(T-k-1))))
print("The Adj R^2 using ARMA(15,16) is "+str(arma1_Adjusted_R_Squared))
# arma (16,23) forecast errors
arma2_forecast_error=y_test.values-rev_pred_arr2
RMSE_arma2 = np.sqrt(np.mean(arma2_forecast_error**2))
print("The ARMA(16,23) Root Mean Square of Forecast Error is "+str(RMSE_arma2))
arma2_mean_forecast_error=np.mean(arma2_forecast_error)
print("The ARMA(16,23) Mean of Forecast Error is "+str(arma2 mean forecast error))
```

se\_arma2=standard\_error(arma2\_forecast\_error,1)
print("The standard error using ARMA(16,23) is "+str(se\_arma2))

## ols.py

```
import pandas as pd
import helper as helpme
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller
import seaborn as sns
from sklearn.model selection import train test split
from statsmodels.tsa.seasonal import seasonal decompose
import numpy as np
from statsmodels.tsa.holtwinters import ExponentialSmoothing
import arma_estimator as my_arma
from scipy import signal
import statsmodels.api as sm
all_data=pd.read_csv("data/all_data_numeric.csv")
all_data.head()
train, test = train test split(all data, train size = 0.8, test size = 0.2, shuffle = False)
X = train[["coffee", "corn", "cotton", "gold", "lumber", "oil", "wheat"]]
X = sm.add\_constant(X)
Y= train["snp"]
x_test=test[["coffee", "corn", "cotton", "gold", "lumber", "oil", "wheat"]]
x_test = sm.add_constant(x_test)
lin_reg_model = sm.OLS(Y,X.astype(float))
results = lin_reg_model.fit()
print (results.params)
test_pred_ols=results.predict(x_test)
test_pred_ols
print(results.summary())
plt.plot(np.concatenate((Y,test["snp"])), label="actual")
plt.plot(np.concatenate((Y,test pred ols)), label="prediction")
#plt.plot(Y, label="actual")
```

```
plt.show()

forecast_error=test["snp"]-test_pred_ols

RMSE = np.sqrt(np.mean(forecast_error**2))

print("The Root Mean Square of Forecast Error using OLS is "+str(RMSE))

mean_forecast_error=np.mean(forecast_error)

print("The Mean of Forecast Error using is "+str(mean_forecast_error))

def standard_error(forecast_error,num_of_predictors):
    return np.sqrt(np.sum(forecast_error**2)/(len(forecast_error)-num_of_predictors-1))

se=standard_error(forecast_error,1)

print("The standard error using OLS is "+str(se))
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
def datetime_transformer(df, datetime_vars):
  The datetime transformer
  Parameters
  df: dataframe
  datetime_vars : the datetime variables
  Returns
  The dataframe where datetime_vars are transformed into the following 3 datetime
types:
  year, month, and day
  # The dictionary with key as datetime type and value as datetime type operator
  dict_ = {'year' : lambda x : x.dt.year,
       'month': lambda x: x.dt.month,
       'day' : lambda x : x.dt.day}
  # Make a copy of df
  df_datetime = df.copy(deep=True)
  # For each variable in datetime vars
  for var in datetime vars:
    # Cast the variable to datetime
    df_datetime[var] = pd.to_datetime(df_datetime[var])
    # For each item (datetime_type and datetime_type_operator) in dict_
    for datetime_type, datetime_type_operator in dict_.items():
       # Add a new variable to df_datetime where:
       # the variable's name is var + '_' + datetime_type
       # the variable's values are the ones obtained by datetime_type_operator
       df_datetime[var + '_' + datetime_type] = datetime_type_operator(df_datetime[var])
```

# Remove datetime\_vars from df\_datetime

helper.py

```
# df_datetime = df_datetime.drop(columns=datetime_vars)
  return df_datetime
def autocorrelation_cal(y,k):
  T=len(y)
  mean_y=np.mean(y)
  numerator=0
  denominator=0
  T k=0
  for t in range(0,T):
    denominator=denominator+(np.square(y[t]-mean_y))
  for t in range(k,T):
    numerator=numerator+((y[t]-mean_y)*(y[t-k]-mean_y))
  T_k=numerator/denominator
  return T k
def acf_plotter(y,l):
  #acf over y with 100 samples
  lags=[]
  autoCorr=[]
  max_lag=l
  for i in range(0,max_lag):
    lags.append(i)
    autoCorr.append(autocorrelation_cal(y,i))
  #making symmetrical acf plot about y axis
  autoCorr_copy=autoCorr[1:].copy()
  autoCorr_copy.reverse()
  autoCorr_copy=np.concatenate((autoCorr_copy,autoCorr),axis=None)
  lags_rev=lags[1:].copy()
  lags_rev.reverse()
  lags_rev=np.negative(lags_rev)
  lags_rev=np.concatenate((lags_rev,lags),axis=None)
  #plotting acf over y
  plt.stem(lags_rev,autoCorr_copy,use_line_collection=True)
  plt.title("ACF Plot for Sample size {} with {} lags".format(len(y),l))
```

```
arma estimator.py
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import seaborn as sns
from scipy import signal
import copy
def cal_GPAC(acf_values,j_max,k_max):
  gpac_ndarray=np.zeros((j_max,k_max-1))
  for k in range(1,k_max):
    for j in range(0,j_max):
      #form the denominator matrix (k*k)
      den_mat=np.zeros((k,k))
      for row in range(k):
        for col in range(k):
           den_mat[row][col]=acf_values[abs(j+row-col)]
  #form the numerator matrix (same as denominator matrix except for last column)
      num mat=copy.deepcopy(den mat)
      for row in range(k):
        num_mat[row][k-1]=acf_values[j+row+1]
      det_num=np.linalg.det(num_mat)
      det_den=np.linalg.det(den_mat)
      gpac_ndarray[j][k-1]=det_num/det_den
  # return the GPAC ndarray
  return gpac_ndarray
def autocorrelation_cal(y,k):
  T=len(y)
  mean_y=np.mean(y)
  numerator=0
  denominator=0
  T k=0
  for t in range(0,T):
    denominator=denominator+(np.square(y[t]-mean_y))
  for t in range(k,T):
    numerator=numerator+((y[t]-mean_y)*(y[t-k]-mean_y))
    T k=numerator/denominator
  return T_k
```

```
def acf_values(y,ml):
  #lags=[]
  autoCorr=[]
  max lag=ml
  for i in range(0,max_lag):
    #lags.append(i)
    autoCorr.append(autocorrelation_cal(y,i))
  return autoCorr
def calc_e(y,na,theta):
  num = [1]
  den = [1]
  den=np.concatenate((den,theta[0:na]))
  num=np.concatenate((num,theta[na:]))
  if len(num)<len(den):
    z=np.zeros(len(den)-len(num))
    num=np.concatenate((num,z),axis=None)
  elif len(num)>len(den):
    z=np.zeros(len(num)-len(den))
    den=np.concatenate((den,z),axis=None)
  system = (den,num,1)
  T=len(y)
  t_in=np.arange(0,T)
  t_out, e = signal.dlsim(system,y,t=t_in)
  return e
def levenburgMarquardtStepOne(y,na,nb,theta,delta,N,n):
  e=calc_e(y,na,theta)
  E=np.mat(e)
  SSE=E.T.dot(E)
  X=np.zeros((N,n))
  X=np.mat(X)
  for i in range (0,n):#1 \leq i \leq n
    theta_copy=copy.deepcopy(theta)
    theta_copy[i]=theta_copy[i]+delta
    e2=calc_e(y,na,theta_copy)
    x=e-e2
    x=x/delta
```

```
X[:,i]=x
  A=X.T.dot(X)
  g=X.T.dot(e)
  return A,q,SSE
def levenburgMarquardtStepTwo(y,na,nb,theta,A,g,mu,n):
  I=np.identity(n)
  del_theta=np.linalg.inv(A+(mu*l)).dot(g)
  del_theta_arr=np.array(del_theta).flatten()
  theta_new=theta+del_theta_arr
  e_new=calc_e(y,na,theta_new)
  E_NEW=np.mat(e_new)
  SSE_NEW=E_NEW.T.dot(E_NEW)
  return SSE_NEW,del_theta_arr,theta_new
def levenburgMarquardt(y,na,nb,numOflter):
  # returns the estimated parameter
  # input parameters are:
  # y (generated using arma process)
  # order of ar process in arma, na
  # order of ma process in ma, nb
  #step 1
  # defining maximum number of iteration
  # the very first theta
  N=len(y)
  n=na+nb
  theta=np.zeros((n))
  delta=0.001
  A,g,SSE=levenburgMarquardtStepOne(y,na,nb,theta,delta,N,n)
  mu=0.01
  SSE_NEW,del_theta,theta_new=levenburgMarquardtStepTwo(y,na,nb,theta,A,g,mu,n)
  iterator=0
  maxIterations=numOfIter
  mu_max=10000000000
  while iterator < maxiterations:
    if SSE NEW < SSE:
      mag_del_theta = np.linalg.norm(del_theta)
```

```
if mag_del_theta < 1:
         theta=theta_new
         sigma_e_sq=SSE_NEW/(N-n)
         cov=np.multiply(sigma_e_sq,np.linalg.inv(A))
         conf=np.diagonal(np.sqrt(cov))
         print("i="+str(iterator)+", SSE new less than SSE old, ||del_theta||<0.001:")
         print("theta=")
         print(theta)
         print("confidence interval = +/-"+str(conf))
         print("Estimated variance of error:")
         print(sigma_e_sq)
         print("covariance matrix:")
         print(cov)
         print("SSE=")
         print(SSE)
         break
         #return theta
      else:
         theta=theta_new
         mu=mu/10
    while SSE NEW > SSE:
      mu=mu*10
      if mu>mu_max:
         print("i="+str(iterator)+", mu>mu_max")
         print(theta)
         #return theta
SSE_NEW,del_theta,theta_new=levenburgMarquardtStepTwo(y,na,nb,theta,A,g,mu,n)
    iterator=iterator+1
    if iterator>maxIterations:
      print("i="+str(iterator)+", iter > maxIter")
      print(theta)
      #return theta
    theta=theta_new
```

A,g,SSE=levenburgMarquardtStepOne(y,na,nb,theta,delta,N,n)

 $SSE\_NEW, del\_theta, theta\_new=levenburgMarquardtStepTwo(y, na, nb, theta, A, g, mu, n)$ 

return theta