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DYNAMIC EFFECTS OF PERMANENT AND TEMPORARY TAX POLICIES IN A q MODEL OF INVESTMENT

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This paper incorporates the tax policy analysis of Hall and Jorgenson into a dynamic optimizing model with adjustment costs to develop a q model of investment. This framework is particularly useful for analyzing the dynamic effects on investment of permanent and temporary changes in tax policy. It is shown that, contrary to the conventional intertemporal substitution argument, a temporary investment tax credit need not be more expansionary than a permanent investment tax credit. The role of depreciation allowances in determining the dynamic response of investment to temporary changes in the tax rate is also investigated.

1. Introduction

In this paper, we analyze the dynamic effects of various policies in a q model of investment. Tobin (1969) has defined q as the ratio of the market value of existing capital to the replacement cost of capital. Subsequent empirical studies of investment [Ciccolo (1975), von Furstenberg (1977), Abel (1980a)] have examined the role of q as a determinant of investment behavior. In this paper we provide a rigorous foundation for a version of the q theory of investment based on intertemporal optimization by firms in the presence of convex costs of adjustment. By integrating the tax policy analysis of Hall and Jorgenson [1967, 1971] with the adjustment cost literature [Eisner and Strotz (1963), Lucas (1967a, b), Gould (1968), Treadway (1969), Mussa (1977)], we can analyze the dynamic effects of permanent and temporary policies in an optimizing framework. In this model, a variable similar to Tobin's q is the key determinant of investment, and focusing on q greatly facilitates the economic interpretation of the analysis.

The analysis of permanent changes in the tax code leads to essentially the same results in the q model as in the Hall–Jorgenson analysis. However, we point out an interpretation of tax neutrality which differs somewhat from the Hall–Jorgenson definition. The major advantage of the framework in this paper is that it permits a rigorous dynamic analysis of temporary fiscal policies.[1] It is shown that the increase in the rate of investment due to a

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[1]Hall (1971) analyzes the dynamic effects of fiscal policy in a model without adjustment costs. The inclusion of adjustment costs in this paper allows us to develop a q theory of investment.

temporary investment tax credit is at least as great as the increase due to a permanent tax credit of the same size. It would appear that the reason for this finding is that under the temporary investment tax credit, there is an intertemporal shifting of investment expenditures to take advantage of the credit while it is in effect. However, for a competitive firm with constant returns to scale facing constant output and factor prices over time, temporary and permanent investment tax credits of the same size have the same stimulus to investment. Thus an argument based on intertemporal shifting of investment does not appear to be a completely satisfactory explanation of the greater stimulus due to temporary changes mentioned above.

In the analysis of temporary tax rate changes, the age profile of depreciation allowances is an important factor. We confine the analysis to two interesting special cases: (a) depreciation allowances which are proportional to physical depreciation, and (b) immediate write-off of some fraction of capital expenditures, as suggested by Auerbach and Jorgenson (1980). For each of these depreciation allowance schedules, we show that a permanent increase in the corporate tax rate leads to a larger immediate drop in the rate of investment than does a temporary tax increase of the same magnitude. Indeed, for a short-lived temporary tax increase, the rate of investment can rise rather than fall if firms are allowed to immediately write-off the cost of capital.

Section 2 of the paper sets up the model and derives the relation between investment and a shadow price q. The effects of permanent tax policies are analyzed in section 3. Section 4 analyzes the dynamic effects of temporary changes in the investment tax credit and section 5 presents an analysis of temporary changes in the corporate tax rate.

2. The model

The model of the firm is based on the assumption that the firm acts to maximize the present value of its after-tax cash flow. Let $\pi(K_t)$ be the instantaneous real operating profit at time t for a firm with capital stock K_t. More precisely, $\pi(K_t)$ represents the maximized value of revenue minus variable costs, given the capital stock K_t. We assume that $\pi'>0$ and $\pi''\leq 0$. Note that π'' equals zero for a perfectly competitive firm with a constant returns to scale production function. Let I_t be the rate of real gross investment and assume that the cost of purchasing and installing capital is an increasing function of the rate of gross investment, $c(I_t)$, with $c(0)=0$, $c'(I_t)>0$, $c''(I_t)>0$ for $I_t>0$. Finally, assume that the physical depreciation of capital is proportional to the capital stock so that net investment is $\dot{K}_t=I_t-\delta K_t$, where δ is the rate of depreciation.

We will analyze the dynamic effects of several tax policies by incorporating the tax policy analysis of Hall and Jorgenson (1967, 1971) into this adjustment cost framework. Taxable corporate income at time t, which is operating profits less depreciation deductions, is taxed at rate τ_t.[2] Depreciation deductions are based on the original cost of capital. Let $D(x)$ be the depreciation deduction per dollar of original cost for an asset of age x. Thus, the present value of the entire stream of depreciation deductions per dollar of original cost is

$$z=\int_0^\infty D(x)\,\mathrm{e}^{-rx}\,\mathrm{d}x, \tag{1}$$

where r is the after-tax discount rate. Since a depreciation deduction of $D(x)$ reduces the firm's tax bill by $\tau_t D(x)$ at time t, the present value of the tax bill saving due to depreciation deductions on a dollar of capital installed at time t is

$$D_t^*=\int_t^\infty \tau_s D(s-t)\,\mathrm{e}^{-r(s-t)}\,\mathrm{d}s. \tag{2}$$

Note that if the corporate tax rate has a constant value τ, then $D^*=\tau z$.

We model the investment tax credit (ITC) as an immediate rebate on the cost of installed capital. Let k_t be the rate of the ITC at time t. If the gross cost of investment at time t is $c(I_t)$, the firm receives an immediate rebate of $k_t c(I_t)$. Taking account of the ITC and the allowable depreciation deductions, the net cost of capital at time t is $(1-k_t-D_t^*)c(I_t)$.[3] We can combine this cost with the after-tax operating profit to derive an expression for V_{t_0}, the present value of the after-tax cash flow, calculated at time t_0,

$$\begin{aligned} V_{t_0}=&\int_{t_0}^\infty \{(1-\tau_t)\pi(K_t)-(1-k_t-D_t^*)c(I_t)\}\,\mathrm{e}^{-r(t-t_0)}\,\mathrm{d}t \\ &+\int_{t_0}^\infty \tau_t \int_0^\infty D(t-t_0+v)c(I_{t_0-v})\,\mathrm{d}v\,\mathrm{e}^{-r(t-t_0)}\,\mathrm{d}t. \end{aligned} \tag{3}$$

The second line in the expression above represents the tax bill saving due to depreciation deductions on existing capital installed before time t_0. This term

[2] We assume that the firm is financed entirely by equity so that no interest payments enter into the calculation of taxable income.

[3] This expression for the net cost of investment is based on the fact that the depreciation base is equal to the original gross cost of investment $c(I_t)$. Under the Long Amendment, which was repealed in 1964, the depreciation base was reduced by the amount of the ITC so that the net cost of investment was $(1-k)(1-D^*)c(I_t)$.

will be irrelevant for investment decisions from time t_0 onward and hence will be ignored.

We assume that the firm chooses the time path of gross investment to maximize V_{t_0} subject to the conditions that $\dot{K}_t = I_t - \delta K_t$ and K_{t_0} is given. The Hamiltonian for this problem is

$$H_t = e^{-rt}\{(1-\tau_t)\pi(K_t) - (1-k_t-D_t^*)c(I_t) + q_t^*(I_t - \delta K_t)\}, \tag{4}$$

where q_t^* is the shadow price of installed capital. Solving the first-order condition $\partial H_t/\partial I_t = 0$, we obtain

$$(1-k_t-D_t^*)c'(I_t) = q_t^*. \tag{5}$$

Eq. (5) demonstrates that the rate of gross investment is chosen so as to equate the net marginal cost of investment with the shadow price of capital. This equation may be rewritten as

$$I_t = c'^{-1}(q_t) \quad \text{where} \quad q_t = \frac{q_t^*}{1-k_t-D_t^*}. \tag{6}$$

Since $c(\cdot)$ is an increasing convex function, $c'^{-1}(\cdot)$ is an increasing function. Therefore, gross investment is an increasing function of q_t. Observe that q_t is the ratio of the shadow price of installed capital, q_t^*, to the net marginal cost of uninstalled capital, $(1-k_t-D_t^*)$.[4] Hence investment is determined by a mechanism similar to Tobin's (1969) q theory of investment. Note that eq. (6) can be combined with the definition of net investment to obtain the equation of motion for the capital stock

$$\dot{K}_t = I(q_t) - \delta K_t, \qquad I'(q_t) > 0. \tag{7}$$

To determine the dynamic behavior of the shadow price q_t^*, we solve the necessary condition $(\mathrm{d}/\mathrm{d}t)(q_t^* e^{-rt}) = -\partial H_t/\partial K_t$ to obtain

$$\dot{q}_t^* = (r+\delta)q_t^* - (1-\tau_t)\pi'(K_t). \tag{8}$$

One interpretation of eq. (8) is that the shadow return from holding and

[4]It is convenient to think of the function $c(I_t)$ as being the sum of the purchase price of uninstalled capital and the cost of installation. In a one-sector model, with a homogeneous good, the purchase price of I_t units of uninstalled capital (output) is simply I_t, if output is numeraire. If the installation cost function is an increasing convex function $c^*(I_t)$, then $c(I_t) = I_t + c^*(I_t)$ is an increasing convex function. Note that the net marginal cost which appears on the left of eq. (5) is $(1-k_t-D_t^*)c'(I_t) = (1-k_t-D_t^*) + (1-k_t-D_t^*)c^{*\prime}(I_t)$. The term $1-k_t-D_t^*$ thus represents the net marginal cost of uninstalled capital, and $(1-k_t-D_t^*)c^{*\prime}(I_t)$ is the net marginal installation cost.

using capital, which is the sum of the shadow capital gain, $\dot{q}_t^*$, and the after-tax rental $(1-\tau_t)\pi'(K_t)$, is equal to the required return on capital $(r+\delta)q_t^*$. If q_t^* is interpreted as a market price of installed capital, then eq. (8) can be viewed as an arbitrage condition. If investors can earn a rate of return r on other assets, then in the absence of uncertainty, arbitrage would force the price of capital q^* to be such that the rate of return on capital, net of depreciation, $[(1-\tau_t)\pi'(K_t)/q_t^* + \dot{q}_t^*/q_t^* - \delta]$, is equal to r as in eq. (8).

The solution to the firm's optimization problem must also satisfy the transversality condition

$$\lim_{t\to\infty} q_t^* \mathrm{e}^{-rt} = 0. \tag{9}$$

Solving the equation of motion for q_t^* (8) and using the transversality condition we obtain

$$q_t^* = \int_t^\infty (1-\tau_s)\pi'(K_s)\,\mathrm{e}^{-(r+\delta)(s-t)}\,\mathrm{d}s. \tag{10}$$

Thus, q_t^* is the present value of after-tax marginal products accruing to the undepreciated portion of capital installed at time t.

For the purposes of tax policy analysis, it will be convenient to analyze the dynamic behavior of q_t rather than q_t^*. Differentiating q_t, as defined in eq. (6), with respect to time, and using eq. (8), we obtain

$$\dot{q}_t = (r+\delta)q_t - \frac{1-\tau_t}{1-k_t-D_t^*}\pi'(K_t) + \frac{\dot{k}_t+\dot{D}_t^*}{1-k_t-D_t^*}q_t. \tag{11}$$

Solving eq. (11) and applying the transversality condition we obtain[5]

$$q_t = \int_t^\infty (1-\tau_s)\pi'(K_s)\,\mathrm{e}^{-(r+\delta)(s-t)}\,\mathrm{d}s/(1-k_t-D_t^*). \tag{12}$$

Thus q_t is the present value of after-tax marginal products accruing to one dollar of capital installed at time t. Note that, in general, q_t depends on the future path of the capital stock.[6]

To simplify the dynamic analysis, we assume, for the moment, that the tax

[5]If we assume that the net marginal cost of uninstalled capital is bounded away from zero so that $1-k_t-D_t^* \geq c$ for some positive c, then using the transversality condition eq. (9), we find that $\lim_{t\to\infty} q_t \mathrm{e}^{-rt} \leq \lim_{t\to\infty} c^{-1} q_t^* \mathrm{e}^{-rt} = c^{-1}\lim_{t\to\infty} q_t^* \mathrm{e}^{-rt} = 0$. Also note that as long as k_t and D_t^* are non-negative, $q_t \geq q_t^*$ so that $\lim_{t\to\infty} q_t \mathrm{e}^{-rt} \geq \lim_{t\to\infty} q_t^* \mathrm{e}^{-rt} = 0$. Thus $\lim_{t\to\infty} q_t \mathrm{e}^{-rt} = 0$ which may be combined with eq. (11) to yield eq. (12).

[6]In the special case in which $\pi''=0$, the rental $\pi'(K)$ is independent of the stock of capital and q_t can be calculated without knowledge of the future path of the capital stock.

parameters remain constant over time, so that eq. (11) simplifies to

$$\dot{q}_t = (r+\delta)q_t - T\pi'(K_t) \quad \text{where} \quad T = \frac{1-\tau}{1-k-D^*}.^7 \tag{13}$$

Eqs. (7) and (13) comprise a system of two differential equations in the two variables K_t and q_t. These two equations describe the dynamic behavior of K_t and q_t and, when combined with the transversality condition, these equations allow us to find the value of q_t, given the value of K_t. The dynamic analysis of this system is conducted using the phase diagram in fig. 1.

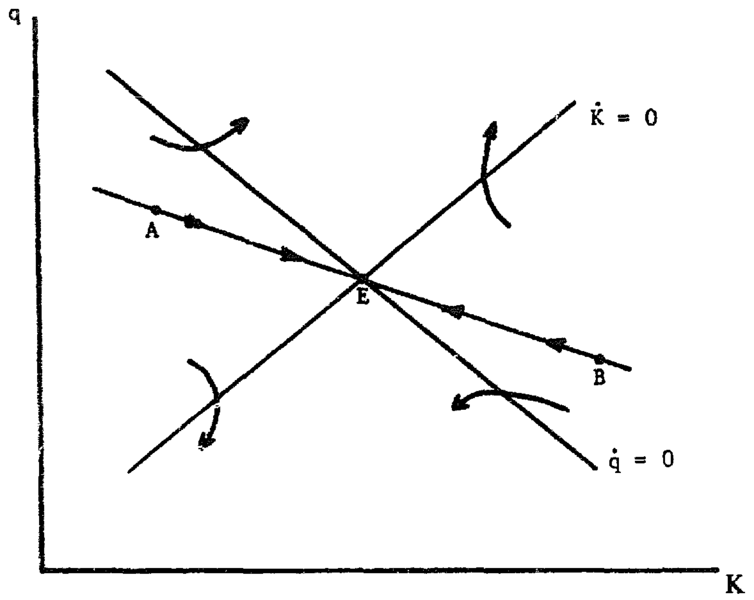

Fig. 1

The phase diagram in fig. 1 is constructed by finding the locus of points for which $\dot{K}=0$ and the locus for which $\dot{q}=0$. The $\dot{K}=0$ locus is the set of points in the (K, q) plane for which gross investment $I(q)$ is equal to depreciation, δK. This locus slopes upward because an increase in K leads to a higher level of depreciation which must be offset by a higher rate of gross investment. The higher rate of gross investment is called forth by an increase in q. For points above the $\dot{K}=0$ locus, gross investment exceeds depreciation so that net investment is positive. For points below the $\dot{K}=0$ locus, net investment is negative.

[7] Note that the Hall–Jorgenson cost of capital can be written as $(1/T)((r+\delta)q_t - \dot{q}_t)$ so that T also captures the effect of permanent tax policy in the Hall–Jorgenson analysis.

The $\dot{q}=0$ locus may be determined by setting $\dot{q}$ equal to zero in eq. (13) to obtain

$$q_t = \frac{T\pi'(K_t)}{r+\delta}, \qquad \dot{q}=0 \text{ locus.} \tag{14}$$

One interpretation of eq. (14) is that $T\pi'(K_t)$ is the tax-adjusted rental to capital and that $T\pi'(K_t)/(r+\delta)$ is the present value of these rentals, if $\pi'(K_t)$ is expected to remain constant. Thus the $\dot{q}=0$ locus represents, for any value of capital stock, the present value of tax-adjusted rentals to capital, calculated under the assumption that the capital stock remains constant. In general, this assumption will not be true except in the steady state, so that along the path to the steady state, the system will not be on the $\dot{q}=0$ locus (unless $\pi'' \equiv 0$).

If $\pi''<0$, the $\dot{q}=0$ locus slopes downward since an increase in K reduces the marginal product $\pi'(K)$ and hence reduces q according to eq. (14). For points above the $\dot{q}=0$ locus, the required return, $(r+\delta)q$, exceeds the tax-adjusted marginal product, $T\pi'(K)$. Therefore, the capital gain, $\dot{q}$, must be positive to equate the realized return with the required return. Thus, for points above the $\dot{q}=0$ locus, q is rising; for points below the $\dot{q}=0$ locus, q is falling.

Fig. 1 presents the $\dot{q}=0$ and $\dot{K}=0$ loci and the vectors of motion corresponding to eqs. (7) and (13). The steady-state equilibrium of the system (7) and (13) is a saddle point equilibrium. Given any level of the capital stock, there is a unique value of q which lies on the path to the steady state. Imposing the transversality condition requires that the system not blow up asymptotically, i.e., that it lies on the stable path to the steady state. Thus, the system must lie on AE or BE.

3. Unanticipated permanent changes in tax policy

In this section we assume that initially the tax parameters τ, k, and $D(\cdot)$ are expected to remain constant over time, and we examine the response to an unanticipated permanent change in one or more of these parameters. In terms of the differential equation system (7) and (13), we consider the effects of a once-and-for-all change in $T=(1-\tau)/(1-k-D^*)=(1-\tau)/(1-k-\tau z)$. Clearly, the $\dot{K}=0$ locus in the (K,q) plane is unaffected by the change in tax policy. Therefore, we merely need to determine how the $\dot{q}=0$ locus shifts when T changes.

It is clear that an increase in the investment tax credit, k, increases the tax parameter T. Also any change in the schedule of depreciation allowances which increases z will lead to an increase in T. As for the effect of changes in the corporate tax rate τ, note that τ appears with a negative coefficient in both the numerator and denominator of T. Thus, in order to determine the effect of τ on T, we partially differentiate T with respect to τ to obtain

$$\frac{\partial T}{\partial \tau} = -\frac{(1-k-z)}{(1-k-\tau z)^2}. \tag{15}$$

If $k+z$ equals one, then changes in the corporate tax rate τ have no effect on T. That is, as long as the present value of the depreciation allowance z is equal to the net purchase price of capital, $1-k$, changes in the tax rate are neutral in the sense that they have no effect on T. Note that this neutrality condition

$$k+z=1 \tag{16}$$

differs from the Hall–Jorgenson condition for neutrality of taxes[8]

$$k+\tau z=\tau. \tag{17}$$

Before commenting on the difference between the neutrality conditions (16) and (17), we note that in the special case in which $k=0$, both conditions simplify to $z=1$ and $T=1$.[9]

The neutrality condition (16) is the condition under which changes in τ, holding constant the tax parameters k and z, do not affect the tax parameter T. When condition (16) holds, it is easily shown that $T=1/z$ regardless of the value of τ. Therefore ceteris paribus changes in τ do not affect T. The Hall–Jorgenson neutrality condition (17) does not apply to changes in τ holding k and z constant.[10] Rewriting eq. (17) as $\tau=k/(1-z)$, it is clear that any change in τ requires a change in $k/(1-z)$ in order to satisfy condition (17) (assuming $k\neq 0$ and $z\neq 1$). When condition (17) holds, the tax parameter T is equal to 1. Thus condition (17) specifies the combinations of tax parameters τ, k, and z for which the tax system is neutral in the sense that investment behavior is unaffected by the existence of taxes. It is as much a condition for neutrality of the investment tax credit, or neutrality of the depreciation allowance, as it is a condition for neutrality of the corporate tax rate. For neutrality of ceteris paribus changes in the corporate tax rate, eq. (16) is the appropriate condition. Note that if eq. (16) is satisfied, however, investment may be distorted by the existence of taxes.

We will maintain the assumption that $k+z$ is less than unity so that, from (15), a cut in the corporate tax rate increases T. To summarize briefly, we have shown that the tax parameter T increases in response to an increase in

[8] See Hall and Jorgenson (1971, p. 18).

[9] Also in the special case in which $\tau=1$, conditions (16) and (17) are identical and $T=0$.

[10] In the special case in which $k=0$, the Hall–Jorgenson neutrality condition is $z=1$. In this case, the tax parameter T will be invariant to ceteris paribus changes in τ.

the investment tax credit k, an increase in the present value of depreciation deductions z and, if $k+z<1$, to a decrease in the corporate tax rate. In the analysis below we examine the effect of a permanent increase in T, recognizing that it could arise from any of these changes in the tax code.

Suppose that there is a permanent unanticipated increase in the tax parameter T from T_0 to T_1. This increase in T causes the $\dot{q}=0$ locus to shift proportionately upward by the factor T_1/T_0 as shown in fig. 2. The steady state of the system shifts from E_0 to E_1 with a higher capital stock and a higher value of q in order to call forth the higher level of replacement investment. Note that, in general, the steady-state value of q is $T(\pi'(K_{ss})/(r+\delta))$, where K_{ss} is the steady-state capital stock. Since the marginal product of capital $\pi'(K)$ is lower at the new steady-state capital stock K_1 than at the original steady-state capital stock K_0, the steady-state value of q rises less than in proportion to T_1/T_0. The effect on the steady-state value of the shadow price q^* is easily determined by recalling that $q^*=(1-k-\tau z)q$. If the change in $T=(1-\tau)/(1-k-\tau z)$ is due to either an increase k or an increase in z, then $(1-k-\tau z)$ decreases to a fraction T_0/T_1 of its initial value. Since q increases by a factor less than T_1/T_0, the product $(1-k-\tau z)q=q^*$ is lower in the new steady state than in the original steady state. As for a decrease in the corporate tax rate τ, it is clear that $1-k-\tau z$ increases so that q^* is higher in

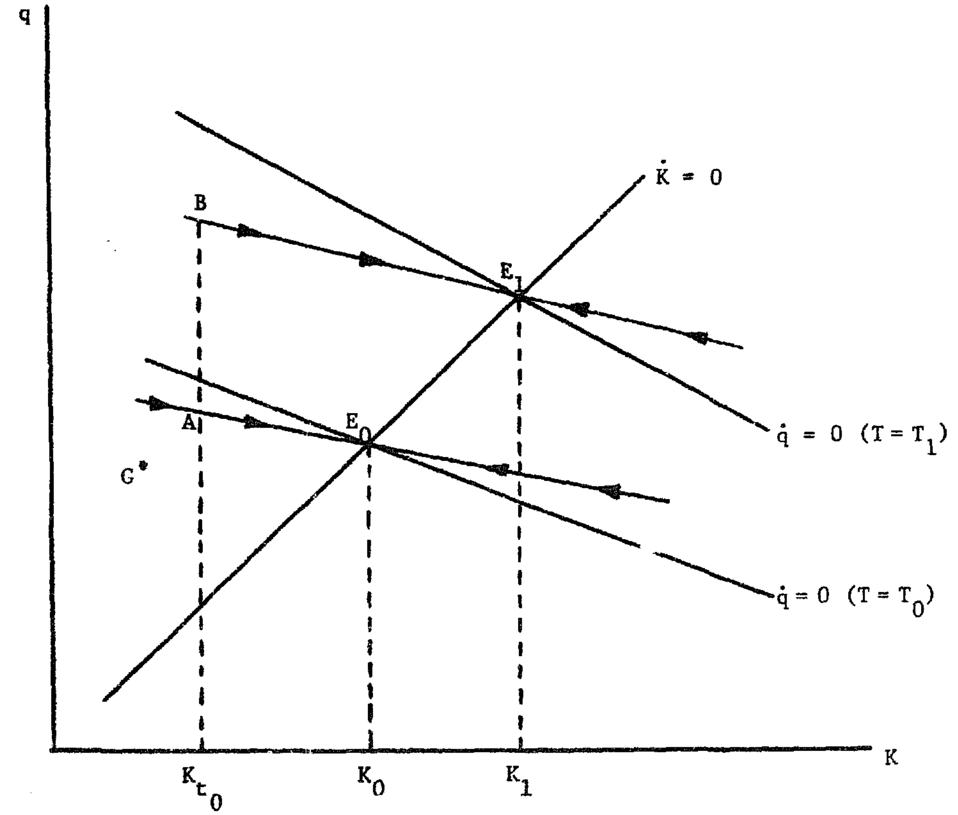

Fig. 2

the new steady state. Although the steady-state shadow price of installed capital q^* falls when k or z is increased, the value of the firm as defined in eq. (3) increases in response to an increase in k or z, as well as in response to a decrease in τ.[11]

Suppose that the capital is K_{t_0} and the system is initially at point A in fig. 2. In response to the unanticipated permanent increase in T, the system jumps immediately to B on the path to the new steady state. The capital stock cannot jump at a point in time but the shadow price q jumps upward in response to the increase in T. The upward jump in q causes an upward jump in the rate of investment.

Up to this point we have shown that the increase in T causes the $\dot{q}=0$ locus to shift upward, but we have merely asserted that the stable path BE_1 lies above AE_0. To demonstrate that the stable path does indeed shift upward recall that the equation of motion for q is

$$\dot{q}_t=(r+\delta)q_t-T\pi'(K_t). \tag{18}$$

Therefore, for any point in the (K,q) plane, the increase in T causes a decrease in $\dot{q}_t$; specifically, if $\dot{q}$ is negative under the original tax regime, it will be more negative under the new tax regime. Now consider any point below the path AE_0, such as G. Under the initial tax regime, q was falling too quickly to reach E_0 from such a point. Since q falls even more rapidly at G under the new regime, the stable path to E_1 could not pass through this point. Furthermore, the stable path to E_1 under the new tax regime cannot contain any points on AE_0, since if it did, the system would then pass below AE_0, contradicting the argument above. Hence the stable path to E_1 must lie above AE_0. Therefore, the unanticipated permanent increase in T causes an upward jump in q and in the rate of investment.

4. Temporary changes in the investment tax credit

A temporary ITC is generally believed to have a more stimulative effect on current investment than a permanent ITC because it induces an intertemporal shifting of investment toward the present and near-future in order to take advantage of the ITC while it is available. In this section, we demonstrate that unless $\pi''=0$, a temporary ITC does have a more stimulative effect than a permanent ITC. However, if $\pi''=0$, then permanent and temporary ITC's of the same magnitude have the same effect on current

[11] Let $\{I_t^*\}$ be the time path of gross investment under the initial tax regime. Consider an increase in k or z. If the firm maintained the original path of investment $\{I_t^*\}$, then V_{t_0} in eq. (3) would increase. If the firm chooses to change the time path of investment, the value V_{t_0} must increase even further. A similar argument applies to decreases in the tax rate τ if we assume that the firm's taxable income is non-negative at every point in time.

investment. This fact leads us to re-examine the conventional intertemporal shifting argument.

Suppose that at time t_0 it is unexpectedly announced that the ITC will be raised immediately from k_0 to k_1 and then will be reduced back to its original level at some future date t_1. In terms of the tax parameter T, there is a temporary increase from T_0 to T_1 followed by a return to T_0 at time t_1. In fig. 3, the stable path corresponding to a permanent value of T equal to T_0 passes through point E_0. The stable path corresponding to a permanent value of T_1 passes through point E_1. The dashed line in fig. 3 is T_1/T_0 times as high as the stable path through E_0.

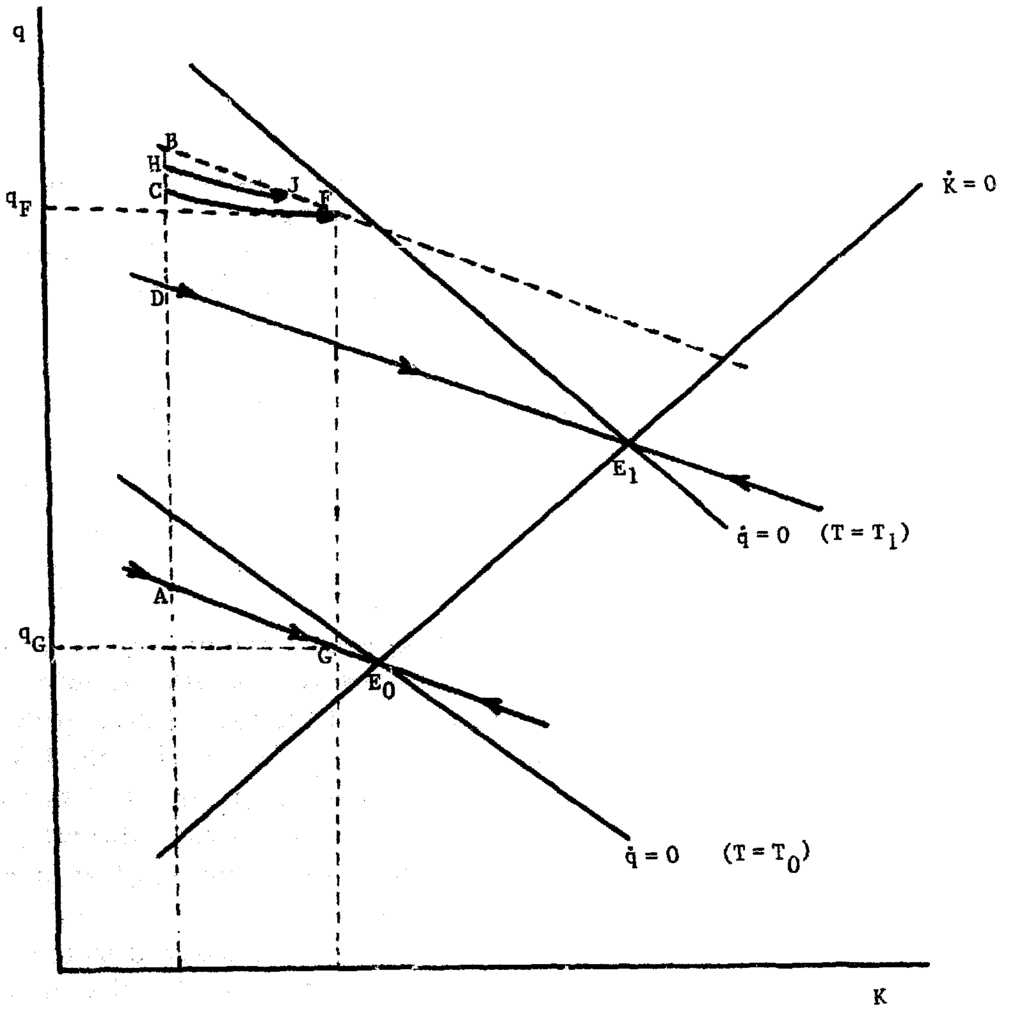

Fig. 3

Suppose that at time t_0 the system is at point A when the increased ITC is unexpectedly announced. If the increase in the ITC lasts only instantaneously, there is an immediate upward jump to B and then an immediate return to A. The effect of this policy is to cause an instantaneous jump in the rate of investment to the higher level corresponding to point B. Since the higher ITC is in effect for only an infinitesimal period of time, the path of the capital stock is unaffected relative to what it would have been in the absence of this policy. Thus, the shadow price q_t^* never changes from its value along the stable path through E_0. Since the numerator of q is unchanged by the policy, and since the denominator is decreased to T_0/T_1 of its initial value, the value of q jumps by a factor T_1/T_0.

Now consider the polar opposite case in which the ITC lasts forever. As discussed in section 2, there is an immediate jump to point D and the system proceeds along the stable path to E_1. Observe that the shadow price q_t^* falls immediately because the path of the capital stock will be higher, thereby reducing future rentals $\pi'(K)$.

Now consider the case of the temporary increase in the ITC. The system jumps immediately to point C and proceeds, according to the new laws of motion, to point F. At time t_1, when the system reaches point F, the ITC returns to its initial value, and there is an instantaneous downward jump to G. Then the system proceeds smoothly to the steady state E_0.

The dynamic behavior of q_t merits further comment. As in the analysis of permanent policies, there is an immediate (unanticipated) jump in q_t. In addition, there is a second jump in q_t at time t_1, when the ITC returns to its initial value. At first glance, this might appear to violate the requirement that there be no anticipated windfall capital gains or losses. However, it is not the ratio q_t which is required to move continuously throughout the future. Rather, it is the numerator of q_t, i.e., the shadow price q_t^*, which cannot have any anticipated discontinuous movements.[12] Although q_t is anticipated to fall discontinuously from q_F to q_G, q_t^* does not jump discontinuously since $q_F^* =((1-\tau)/T_1)$, $q_F=((1-\tau)/T_0)$, $q_G=q_G^*$, where q_F^* and q_G^* are the values of q^* at points F and G, respectively. Thus, the anticipated discontinuity in q does not reflect an anticipated discontinuity in q^*. However, the discontinuity in q does imply that there will be a downward jump in the rate of investment at time t_1. This jump in the rate of investment is due to the fact that the ITC is a reward to the act of investment, not to the holding of capital. When the reward to investment stops abruptly, the rate of investment falls abruptly.

The size of the upward jump in q_t and the downward jump in q_t^* at time t_0 is related to the length of time for which the increased ITC is expected to be in effect. In fig. 3, the path HJ corresponds to a more short-lived increase in the ITC than that represented by CF.[13] The more short-lived policy induces a larger jump in q_t and the rate of investment and a smaller downward jump

[12]However, q_t^* has an unanticipated drop when the ITC is announced.

[13]Since the rate of investment is higher along HJ than along CF, and since the capital stock at J is lower than the capital stock at F, the system traverses HJ in less time than it traverses CF. This line of argument is based on Krugman (1977).

in q_t^*, since it leads to a smaller increase in the future time path of the capital stock and a smaller decrease in future rentals $\pi'(K)$.

We have shown that for the case in which $\pi''<0$ a temporary increase in the ITC has a greater stimulus to investment than a permanent increase in the ITC. However, this result does not hold in the case in which $\pi''=0$.[14] If $\pi''=0$, then the rental $\pi'(K)$ is independent of the capital stock. If the production function, output prices, and factor prices are fixed over time then $\pi'(K)$ is constant over time. Let ϕ be this constant pre-tax rental. In this case, the equation of motion for q is

$$\dot{q}_t=(r+\delta)q_t-T\phi \tag{19}$$

which is independent of the capital stock. Note that the $\dot{q}=0$ locus is given by

$$q_t=T\frac{\phi}{r+\delta} \tag{20}$$

which is a horizontal line in the (K,q) plane as shown in fig. 4. Above the $\dot{q}=0$ locus, q is rising and below the $\dot{q}=0$ locus, q is falling. Therefore, the steady state E is a saddle point equilibrium and the stable path to the steady state coincides with the horizontal $\dot{q}=0$ locus. The economic interpretation is that q_t is the present value of tax-adjusted rentals $T\phi$, discounted by $r+\delta$. Since the tax-adjusted rental, $T\phi$, is constant, the value of q is simply

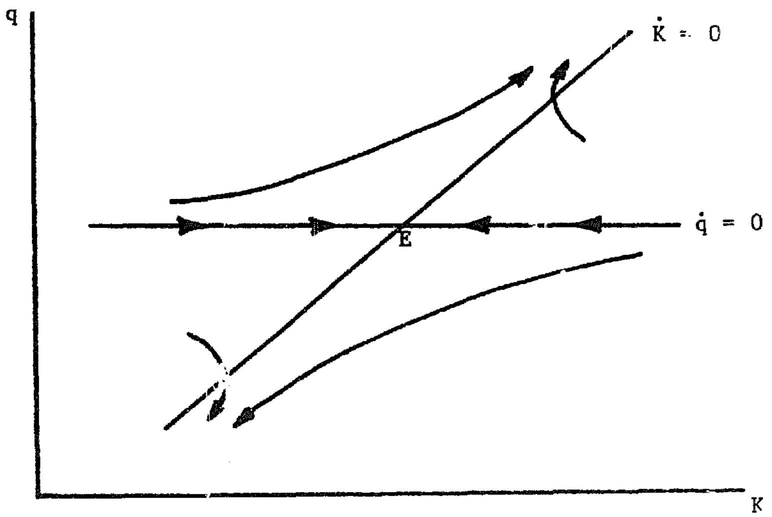

Fig. 4

[14]Recall that if the production function is linearly homogeneous and if the firm is a price-taker in its output and factor markets, then $\pi''=0$.

$T\phi/(r+\delta)$, which is also constant. Since q_t remains constant over time, investment is also constant over time.[15]

Now consider an increase in the investment tax credit which increases T from T_0 to T_1. From eq. (20), it is clear that the $\dot{q}=0$ locus shifts upward by the factor T_1/T_0, as shown in fig. 5. Suppose that the system is initially at point A in fig. 5 when the unanticipated increase in the ITC occurs. If the increase is permanent, the system jumps to point B and proceeds along the stable path to E_1. Note that the value of q at B is T_1/T_0 times as large as the value of q at A. The present value of the pre-tax rentals $\phi/(r+\delta)$ is not affected by the increase in T so the increase in q is proportional to the increase in T.

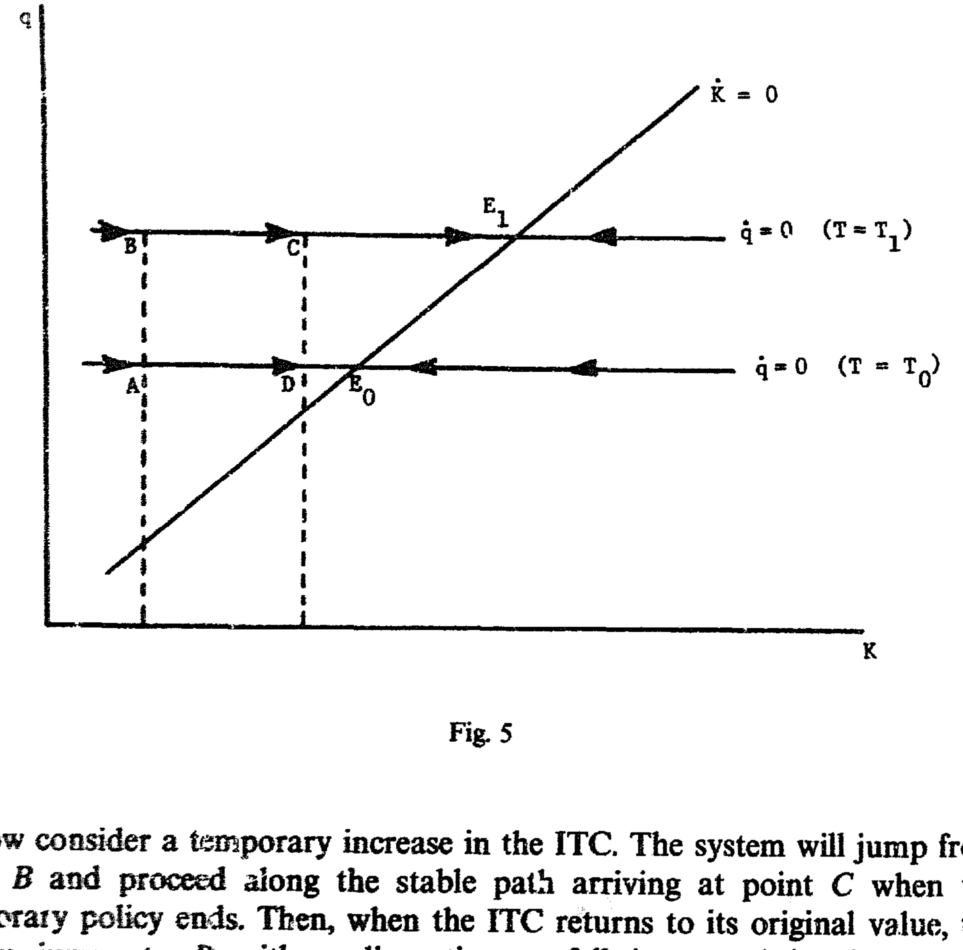

Fig. 5

Now consider a temporary increase in the ITC. The system will jump from A to B and proceed along the stable path arriving at point C when the temporary policy ends. Then, when the ITC returns to its original value, the system jumps to D with a discontinuous fall in q and in the rate of investment. The important point to note is that the size of the immediate increase in the rate of investment is independent of the duration of the ITC

[15]Gould (1968) has shown that for a fixed linearly homogeneous production function and constant prices, the rate of investment is constant.

increase. That is, a temporary ITC policy is no more stimulative than a permanent ITC policy, in contrast to the case in which $\pi''<0$. The lesson to be learned from this example is that a simple intertemporal substitution story about buying capital when it is cheap will not suffice to explain the greater stimulus of the temporary ITC increase. This type of story would seem to imply a greater stimulus for a temporary ITC even when $\pi''=0$. The reason that the temporary ITC has a greater stimulus than the permanent ITC, when $\pi''<0$, is that the permanent ITC leads to a higher time path of the capital stock and hence a lower time path of rentals $\pi'(K)$, when appropriately discounted. The more long-lived is the ITC policy, the more will the present value of future rentals be depressed and hence the smaller will be the stimulating effect of the policy. However, in this case with $\pi''=0$, the time path of rentals $\pi'(K)$ is independent of the path of the capital stock and hence does not depend on the duration of the ITC policy. Although the greater stimulus of the temporary ITC policy in the case in which $\pi''<0$ may be described as some type of intertemporal substitution, the analysis of the $\pi''=0$ case makes it clear that an intertemporal substitution explanation must be more sophisticated than simply buying capital when it is cheap before the higher ITC ends.

5. Temporary changes in the corporate tax rate

In this section we analyze the dynamic effect of temporary changes in the tax rate. Since the present value of the tax savings due to depreciation deductions on a given unit of capital depends on the tax rate throughout the depreciable life of capital, our analysis must take account of anticipated tax rate changes during the depreciable life of capital. Indeed, the analysis of temporary tax rate changes depends on the age profile of the depreciation deduction schedule. In this section, we will consider the following two depreciation allowance schemes: (a) depreciation deductions which are proportional to physical depreciation so that

$$D(x)=\lambda e^{-\delta x} \quad \text{and} \quad z=\lambda/(r+\delta), \tag{21}$$

(b) immediate expensing of some fraction z of the original gross cost of investment, as suggested by Auerbach and Jorgenson (1980).[16]

Suppose that at time t_0 it is unexpectedly announced that there will be an immediate, but temporary, decrease in the tax rate from τ_0 to τ_1. Then at some future time t_1, the tax rate will return permanently to the value τ_0.

[16]See Abel (1980b) for a discussion of the effects of temporary tax rate changes under more general depreciation allowance schemes. Although the depreciation allowance schemes are more general in Abel (1980b), the analysis of that paper is confined to the special case in which $\pi''=0$.

Since the tax rate is expected to change at time t_1, the value of D_t^* is not constant, in general, for $t<t_1$. Therefore the appropriate equation of motion for q_t is (11) rather than (13). If we assume that the ITC is expected to remain constant over time, the equation of motion for q_t is

$$\dot{q}_t=\frac{1}{1-k-D_t^*}[A_t q_t-(1-\tau_t)\pi'(K_t)], \tag{22}$$

where $A_t=(1-k-D_t^*)(r+\delta)+\dot{D}_t^*$.

Hence the $\dot{q}_t=0$ locus is the set of (K_t, q_t) for which

$$A_t q_t=(1-\tau_t)\pi'(K_t), \qquad \dot{q}_t=0. \tag{23}$$

Note that at a point in time, A_t and $(1-\tau_t)$ are fixed so that the $\dot{q}=0$ locus slopes downward in the (K_t, q_t) plane if $\pi''<0$, and is a horizontal line if $\pi''=0$, as discussed in section 4. However, A_t and $(1-\tau_t)$ can change so that $\dot{q}_t=0$ locus can shift over time.

It can be shown that under either of the two depreciation deduction schemes considered here, A_t is constant until time t_1 and then is constant at a lower value from time t_1 onward.[17] Specifically,

$$\begin{aligned} A_t &= (r+\delta)(1-k-\tau_1 z) \quad \text{for } t<t_1, \\ &= (r+\delta)(1-k-\tau_0 z) \quad \text{for } t\geqq t_1. \end{aligned} \tag{24}$$

Thus, it is clear from eq. (23) that the $\dot{q}=0$ locus remains fixed between t_0 and t_1, and then remains fixed at a lower level from time t_1 onward. Note that for any other depreciation allowance schemes, A_t is continually changing[18] and the $\dot{q}=0$ locus does not remain fixed during the low tax regime prevailing until time t_1.

Now we are prepared to analyze the dynamic response to a temporary decrease in the corporate tax rate. Suppose that initially the tax rate is τ_0, and is expected to remain equal to τ_0, so that the system is at point A in fig. 6. Assume that the depreciation deduction scheme is given by eq. (21), so that $z=\lambda/(r+\delta)$. Consider an unanticipated decrease in the tax rate from τ_0 to τ_1

[17] Observe that $\dot{A}_t=-(r+\delta)\dot{D}_t^*+\ddot{D}_t^*$ and that $D_t^*=\tau_1\int_t^{t_1} D(s-t)e^{-r(s-t)}ds+\tau_0\int_{t_1}^{\infty} D(s-t)e^{-r(s-t)}ds$. Differentiating D_t^* twice with respect to time, we obtain $\dot{D}_t^*=-(\tau_1-\tau_0)D(t_1-t)e^{-r(t_1-t)}$ and $\ddot{D}_t^*=(\tau_1-\tau_0)D'(t_1-t)e^{-r(t_1-t)}+r\dot{D}_t^*$. Therefore $\dot{A}_t=(\tau_1-\tau_0)e^{-r(t_1-t)}\{\delta D(t_1-t)+D'(t_1-t)\}$, for $t<t_1$. Under the proportional depreciation allowance scheme in eq. (21), $D(t_1-t)=\lambda e^{-\delta(t_1-t)}$ and $D'(t_1-t)=-\lambda\delta e^{-\delta(t_1-t)}$, so that $\dot{A}_t$ is equal to zero for $t<t_1$. Under the Auerbach–Jorgenson immediate write-off scheme, $D_t^*=\tau_1 z$ and $\dot{D}_t^*\equiv 0$, so that A_t takes on the constant value $(r+\delta)(1-k-\tau_1 z)$ for $t<t_1$.

[18] From the expression for $\dot{A}_t$ in footnote 17, it is clear that, in general, $\dot{A}$ is not identically zero.

occurs at time t_0. If the tax cut is permanent, the system jumps upward to point B and proceeds along the stable path to the steady state E_1. However, if the tax cut is temporary, the system jumps upward to point C and proceeds along the path CD. At time t_1, when the temporary tax cut ends, the system arrives at point D and then proceeds along the stable path to the steady state E_0. For a more long-lived tax cut, the system would jump upward to point F and then proceed along FG, arriving at G when the temporary policy ends. Thus the more long-lasting is the tax cut, the greater will be the upward jump in q_t and the upward jump in the rate of investment.

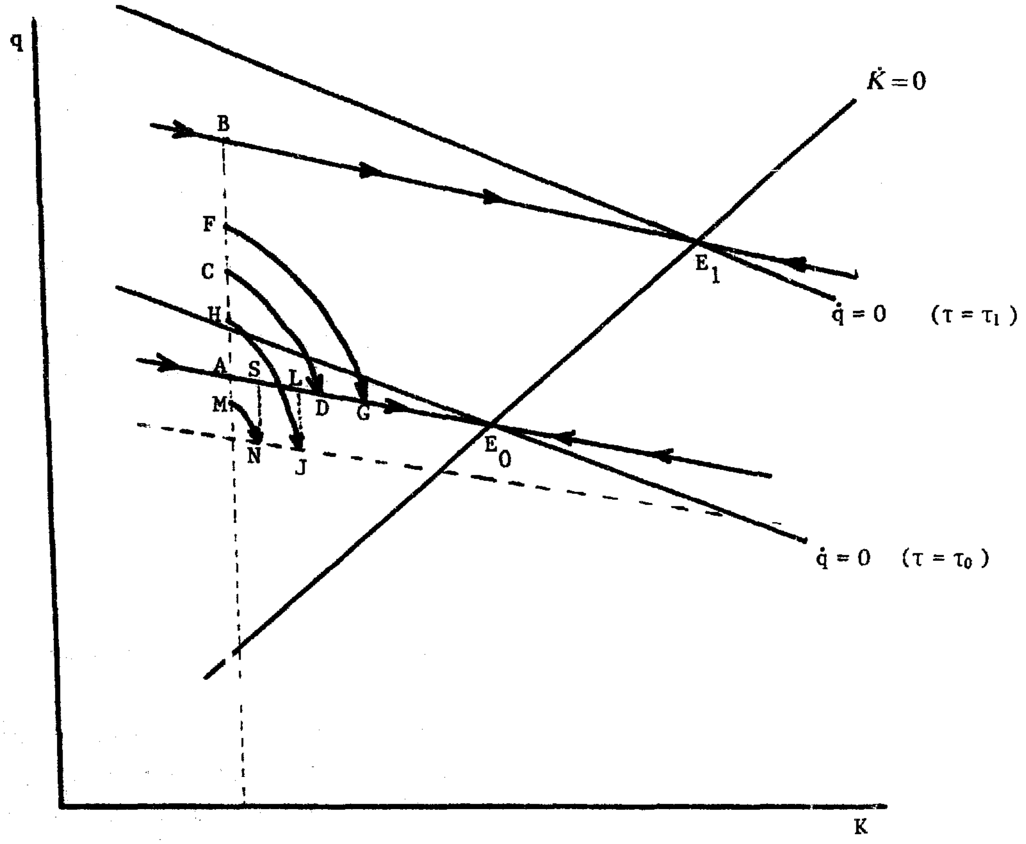

Fig. 6

As discussed above, with the proportional depreciation deduction scheme in eq. (21), the $\dot{q}_t=0$ locus is fixed for $t_0<t<t_1$. With a fixed $\dot{q}=0$ locus during the temporary tax cut, we know that the point D (or G) must be approached from above in fig. 6, and that $\dot{q}$ is negative along CD (or FG). These facts allow us to illustrate the dynamic response to a temporary tax decrease using the phase diagram in fig. 6. If the $\dot{q}_t=0$ locus were continually shifting during the temporary tax cut, we would not be able to use the simple phase diagram in fig. 6.

Now consider the effects of a temporary tax decrease under the alternative depreciation deduction scheme in which firms are allowed to write off immediately a fraction z of the cost of investment. If the system is at point A in fig. 6 when the temporary tax cut suddenly goes into effect, there is an immediate upward jump to point H. The system then proceeds along HJ, arriving at J when the tax cut ends; at this time, there is an upward jump to L and the system proceeds to the steady state E_0.

The anticipated upward jump in q when the temporary policy ends is due to the requirement that the shadow price q_t^* have no anticipated jumps. When the policy ends at time t_1, q_t^*, which is the numerator of q, cannot jump. The denominator of q, $1-k-D^*$, jumps downward from $1-k-\tau_1 z$ to $1-k-\tau_0 z$ (recall that $\tau_1<\tau_0$) at time t_1, so there is an upward jump in q and investment at time t_1. The reason for the upward jump in investment is that tax savings per dollar of investment jumps upward from $\tau_1 z$ to $\tau_0 z$, thereby causing investment to jump upward.

Under the immediate write-off scheme, it is not necessary that investment immediately jump upward in response to a temporary tax cut. If the tax cut is of short duration, the system may jump downward to a point such as M and then proceed along MN, arriving at point N when the tax cut ends. Upon removal of the tax cut, the system jumps upward from N to S and then proceeds to the steady state E_0. The reason that a short-lived tax cut will cause investment to fall is that q_t^*, the present value of after-tax rentals $(1-\tau_t)\pi'(K_t)$, will not be greatly affected by a short-lived tax decrease. However, under the immediate write-off scheme, the temporary tax cut has as large an effect on the denominator of q as a permanent tax cut. The reduced tax rate reduces the tax savings due to the depreciation deduction and hence increases the denominator of q. If the duration of the tax cut is short enough, the increase in the denominator of q will outweigh the increase in q^*; hence q and investment will fall.[19]

To compare the dynamic responses to temporary tax rate changes under the two alternative depreciation schemes, suppose that the fraction z under the Auerbach–Jorgenson immediate write-off scheme is equal to $\lambda/(r+\delta)$ under the proportional scheme in eq. (21). Suppose also that the temporary tax cut corresponding to path HJ under the Auerbach–Jorgenson scheme is of the same magnitude and duration as the temporary tax cut corresponding to path CD under the scheme in eq. (21). Fig. 6 is drawn to indicate the fact that the capital stock at time t_1 is smaller under the Auerbach–Jorgenson scheme (point L) than under the proportional depreciation allowance scheme in eq. (21) (point D). Although this proposition is true for the two cases in which $\pi''=0$ and $\pi''<0$, it is more easily and intuitively established for the

[19]Eisner (1969) discusses this phenomenon for the case of accelerated depreciation.

case in which $\pi''=0$. Therefore, we first prove the result for the $\pi''=0$ case and then proceed to the $\pi''<0$ case in which we use proof by contradiction.

Let the superscript 1 denote that the depreciation deduction scheme is given by eq. (21) and let the superscript 2 denote that the depreciation scheme is the Auerbach–Jorgenson immediate write-off scheme. For example, $K_{t_1}^{(1)}$ is the level of the capital stock under eq. (21) when the temporary tax cut ends and $K_{t_1}^{(2)}$ is the level of the capital stock under the immediate write-off scheme when the temporary tax cut ends. We first observe that

$$D_t^{*(1)}=\tau_1\frac{\lambda}{r+\delta}+(\tau_0-\tau_1)\frac{\lambda}{r+\delta}e^{-(r+\delta)(t_1-t)} \quad \text{for} \quad t<t_1, \tag{25a}$$

$$D_t^{*(2)}=\tau_1\frac{\lambda}{r+\delta} \quad \text{for} \quad t<t_1. \tag{25b}$$

Therefore, since $\tau_0>\tau_1$, the present value of tax savings due to the depreciation deduction is greater under the proportional scheme in eq. (21) than under the Auerbach–Jorgenson scheme, i.e., $D_t^{*(1)}>D_t^{*(2)}$. This finding is clear because tax saving due to depreciation deductions is proportional to the tax rate, and all depreciation deductions under the Auerbach–Jorgenson scheme are taken during the low tax regime. From eq. (25) it follows that

$$1-k-D_t^{*(1)}<1-k-D_t^{*(2)} \quad \text{for} \quad t<t_1. \tag{26}$$

That is, during the low tax regime, the denominator of q_t is lower for the proportional depreciation deduction scheme than for the Auberbach–Jorgenson scheme.

Now suppose $\pi''=0$ so that $\pi'(K_s)$ takes on the constant value of ϕ for all s. From eq. (10), it is obvious that q_t^* (the numerator of q_t) is identical under the two alternative depreciation deduction schemes. Since the denominator of q_t is lower under the proportional depreciation deduction scheme, we obtain

$$q_t^{(1)}>q_t^{(2)} \quad \text{for} \quad t<t_1. \tag{27}$$

Since investment is an increasing function of q_t, $I_t^{(1)}>I_t^{(2)}$ for all $t<t_1$ and hence $K_{t_1}^{(1)}>K_{t_1}^{(2)}$.[20]

For the case in which $\pi''<0$, it is no longer true that the numerator of q_t, i.e. q_t^*, is identical under the two alternative depreciation schemes. The difficulty arises because future rentals to capital depend on the future path of the capital stock. Suppose, contrary to the desired result, that $K_{t_1}^{(1)}\leqq K_{t_1}^{(2)}$. Since at time t_1, $q_{t_1}^{(1)}$ and $q_{t_1}^{(2)}$ are both on the stable path to E_0, it is clear

[20] Note that $K_t^{(i)}=K_{t_0}e^{-\delta(t-t_0)}+\int_{t_0}^{t} I_s^{(i)}e^{-\delta(t-s)}ds$. Since $I_t^{(1)}>I_t^{(2)}$ for all $t<t_1$, it is clear that $K_t^{(1)}>K_t^{(2)}$ for $t\leqq t_1$.

that $q_{t_1}^{(1)} \geqq q_{t_1}^{(2)}$. Furthermore, since the denominators of $q_t^{(1)}$ and $q_t^{(2)}$ are equal (since the tax rate is constant from time t_1 onward), $q_t^{*(1)} \geqq q_t^{*(2)}$. Consider some time t' just before the tax cut ends, i.e., $t' = t_1 - \varepsilon$ for small $\varepsilon > 0$. It follows that $q_{t'}^{*(1)} \geqq q_{t'}^{*(2)}$, and since, from eq. (26), $1-k-D_{t'}^{*(1)} < 1-k-D_{t'}^{*(2)}$, we obtain $q_{t'}^{(1)} > q_{t'}^{(2)}$. Therefore $\dot{K}_{t'}^{(1)} > \dot{K}_{t'}^{(2)}$ so that the positive gap $K_t^{(2)} - K_t^{(1)}$ becomes wider as we go backward in time from t_1. This widening gap will prevent $\pi'(K_t^{(1)})$ from ever falling below $\pi'(K_t^{(2)})$ for $t < t_1$ and thus prevents $q_t^{*(1)}$ from ever falling below $q_t^{*(2)}$ as t moves backward from t_1. Hence $q_t^{*(1)} \geqq q_t^{*(2)}$ and therefore $q_t^{(1)} > q_t^{(2)}$ for all $t < t_1$. But since investment is an increasing function of q_t, if $q_t^{(1)}$ always exceeds $q_t^{(2)}$ for $t < t_1$, then $K_{t_1}^{(1)} > K_{t_1}^{(2)}$, which is a contradiction. Therefore, $K_{t_1}^{(1)} > K_{t_1}^{(2)}$. Thus, this result holds both when $\pi'' = 0$ and when $\pi'' < 0$.

As a final comparison of the response to a temporary tax cut under the alternative depreciation schemes, we examine the immediate effect on investment. As fig. 6 is drawn there is a larger impact effect on investment under the proportional depreciation scheme than under the Auerbach–Jorgenson scheme (point C lies above point H). We are not able to prove that this result must be true for the general case with $\pi'' \leqq 0$. However, this result must hold in the case in which $\pi'' = 0$. Indeed, as demonstrated above, $I_t^{(1)} > I_t^{(2)}$ for all $t < t_1$ when $\pi'' = 0$. Thus, in this case, the impact effect of the temporary tax cut is larger under the proportional depreciation scheme than under the Auerbach–Jorgenson scheme.

6. Conclusion

By integrating the Hall–Jorgenson tax policy analysis with the q model of investment, we have developed a useful framework to analyze the dynamic effects of tax policy on investment. In particular, this integrated framework provides insights into the analysis of temporary fiscal policies. We have shown that a temporary investment tax credit provides a greater stimulus to investment than a permanent investment tax credit, except for a competitive firm with constant returns to scale. This important exception led us to re-examine the simple intertemporal substitution argument which would appear to explain the greater stimulus of a temporary investment tax credit than a permanent credit. The analysis of temporary tax rate changes emphasized the importance of the age profile of depreciation deductions. The dynamic responses to temporary tax rate changes were then analyzed and compared under two alternative depreciation allowance schedules. If firms are allowed to write off some fraction of capital expenditures immediately, as suggested by Auerbach and Jorgenson (1980), the dynamic response to a temporary tax rate change can differ substantially from the response under a regime in which depreciation deductions are proportional to physical depreciation.

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