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(ass)ignment Solution

sample $(x_1, x_2 \dots x_n)$

mean $= \theta_1$ var $= \theta_2$

$L = \prod f(x_i \mid \theta)$

$$L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_2)^2}{2\theta_2}}$$

Take log on both sides

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i \theta_i)^2$$

Differentiate w.r.t θ_1 & θ_2 & then equate to 0

for θ_1

$$\frac{\partial \ln L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

$$\frac{\partial \ln L}{\partial \theta_1} = 0$$

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta_i}) = 0$$

$$\sum_{i=1}^{n} (x_i - \hat{\theta_i}) = 0$$

$$\hat{\theta_i} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So MLE for θ_1 is sample mean

for θ_2:

$$\frac{\partial \ln L(\theta_1, \theta_2 \mid x_1, x_2 \dots x_n)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_i)^2$$

$$\frac{\partial \ln L}{\partial \theta_2} = 0$$

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\hat{\theta}_2^2} \sum (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

So MLE for θ_2 is sample variance

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Q2. Bernoulli distⁿ:

parameter → $\theta \in \Theta = (0,1)$ unknown

→ m (known +'ve Z)

The likelihood function is →

$$L(\theta|x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

Since Xi follows bernoulli distⁿ

$$P(X_i = x_i|\theta) = \theta^{x_i}(1-\theta)^{m-x_i} \quad \text{for each } i$$

Taking log on both sides

$$\ln L(\theta|x_1, x_2 \ldots x_n) = \sum_{x_i=1}^{n} \ln\left(\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n} (x_i \ln\theta (m-x_i) \ln(1-\theta))$$

Differentiate w.r.t θ

$$\frac{\partial}{\partial\theta} \ln L(\theta|x_1, x_2 \ldots x_n) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = mn - \sum_{i=1}^{n}\frac{x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n}\frac{x_i}{m \cdot n}$$

So. max likelihood estimating θ is

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n \cdot m}$$