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Assignment Solution

sample (x_1, x_2, \dots, x_n)

mean $= \theta_1$ var $= \theta_2$

$$L = \prod f(x_i | \theta)$$

$$L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Take log on both sides

$$\ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^n (x_i - \theta_1)^2$$

Differentiate w.r.t θ_1 & θ_2 & then equate to 0

for θ_1

$$\frac{\partial \ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n)}{\partial \theta_1} = \frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1)$$

$$\frac{\partial \ln L}{\partial \theta_1} = 0$$

$$\frac{1}{\theta_2} \sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\sum_{i=1}^n (x_i - \theta_1) = 0$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^n x_i$$

So MLE for θ_1 is sample mean

for θ_2 :

$$\frac{\partial \ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \theta_1)^2$$

$$\frac{\partial \ln L}{\partial \theta_2} = 0$$

$$\frac{\partial \ln L}{\partial \theta_2}$$

$$-\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \theta_1)^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^n (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \theta_1)^2$$

So MLE for θ_2 is sample variance

$$0 = \ln L$$

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$$\ln \sum_{i=1}^n (x_i - \theta_1)^2 = \ln \theta_2^n$$

So MLE for θ_2 is sample variance

$$1 + \theta_2 = \ln L = \ln \left(\frac{1}{\theta_2^n} \sum_{i=1}^n (x_i - \theta_1)^2 \right)$$

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Q2. Bernoulli distⁿ.
parameter $\rightarrow \theta \in \Theta = (0, 1)$ unknown
 $\rightarrow m$ (known $\forall i \in \mathbb{Z}$)

The likelihood function is \rightarrow

$$L(\theta | x_1, x_2, \dots, x_n) = \prod_{i=1}^n p(x_i = x_i | \theta)$$

Since X_i follows Bernoulli distⁿ

$$P(X_i = x_i | \theta) = \theta^{x_i} (1-\theta)^{m-x_i} \text{ for each } i$$

Taking log on both sides

$$\begin{aligned} \ln L(\theta | x_1, x_2, \dots, x_n) &= \sum_{i=1}^n \ln (\theta^{x_i} (1-\theta)^{m-x_i}) \\ &= \sum_{i=1}^n (x_i \ln \theta + (m-x_i) \ln(1-\theta)) \end{aligned}$$

Differentiate w.r.t θ

$$\frac{\partial}{\partial \theta} \ln L(\theta | x_1, x_2, \dots, x_n) = 0$$

$$\sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^n \frac{x_i}{\theta} = mn - \sum_{i=1}^n \frac{x_i}{1-\theta}$$

$$\therefore \theta = \frac{\sum_{i=1}^n x_i}{m \cdot n}$$

So. max likelihood estimating θ is

$$\hat{\theta}_{MLE} = \sum_{i=1}^n \frac{n_i}{n-m}$$

the likelihood function is

$$L(\theta) = \prod_{i=1}^n \binom{n}{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

the log likelihood function is

$$\ln L(\theta) = \sum_{i=1}^n \left[\ln \binom{n}{x_i} + x_i \ln \theta + (n-x_i) \ln (1-\theta) \right]$$

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$$\ln L(\theta) = \sum_{i=1}^n \left[\ln \binom{n}{x_i} + x_i \ln \theta + (n-x_i) \ln (1-\theta) \right]$$

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^n \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right]$$

the log likelihood function is

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^n \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right]$$