

Lecture 29-30

B-Trees

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Today:

- B-Trees

Data Structures on Disk: Databases

Properties of Disk:

- **access time**
- varies by position(0-30ms)
- **size** 1 TB \longleftrightarrow 1 PB
- **grain size** 2 KB \longleftrightarrow 16 KB
- persistent

Properties of RAM:

- **access time**
- random access
- **size** 4-32GB
- **grain size** 1–16 bytes
- volatile

Consequences:

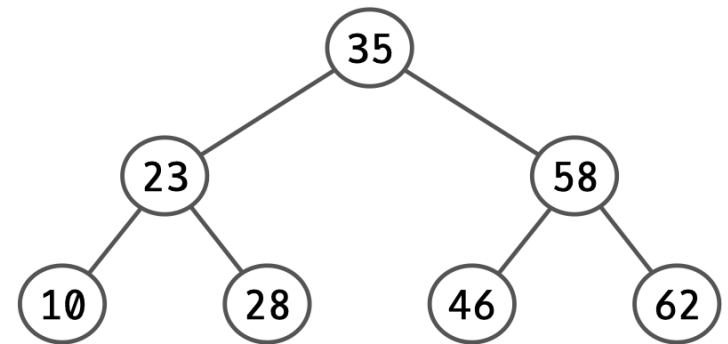
- 4 KB grain size \Rightarrow store > 1 key per node
- running time largely depends on number of disk accesses
 - a case where coefficients matter

Balanced tree on disk?

Types of Balanced Trees (Review)

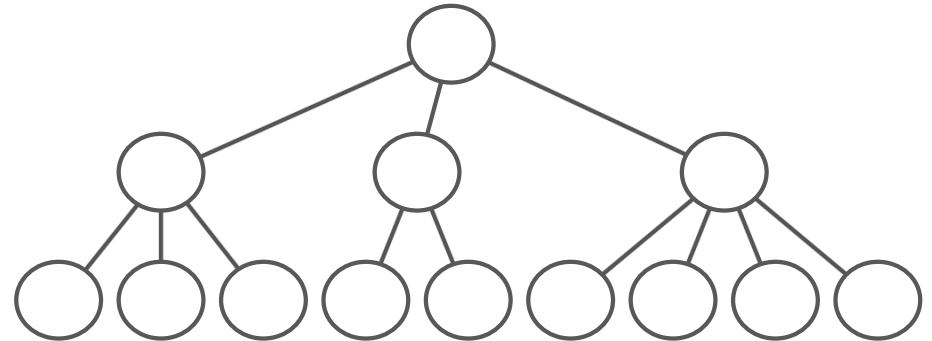
Perfect binary tree:

- a full binary tree with
- Q. How many nodes in a tree of height h ?
 $\Rightarrow N = 2^{(h+1)} - 1$
- ideal balance, but works only for $N = 0, 1, 3, 7, 15$



All leaves at same level:

- augment perfect tree with
 - branching factor between
 -
- Text



B-Tree Properties

- all leaves at same depth
- given parameter
 - branching factor is
 - each node contains
 - for an internal node with n keys
 - children satisfy

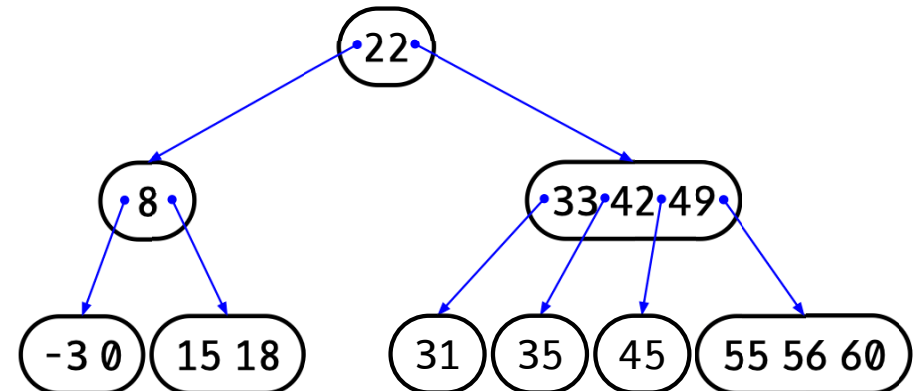
E.g., When $t = 2$, you get a

- branching factor is

Each node stored within one disk block

- higher $t \Rightarrow$

```
class BTreeNode {  
    public:  
        bool leaf;  
        int numkeys;  
        int keys[  
            BTreeNode * c[
```



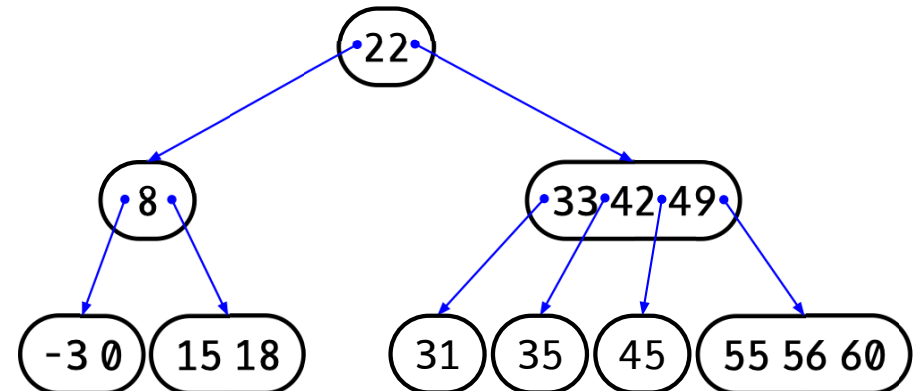
Search

Strategy: Traverse like for binary search tree except

- use to pick subrange

E.g.,

- `.search(55)`
- `.search(8)`
- `.search(-2)`



B-Tree Insert

Strategy: Perform same steps as search to locate leaf.

- new key must be

Two cases:

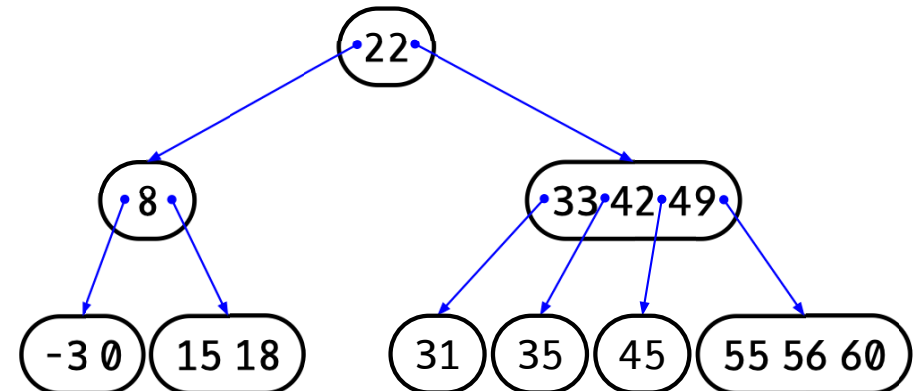
1. if leaf node has room, then
2. if leaf node is full, then

○

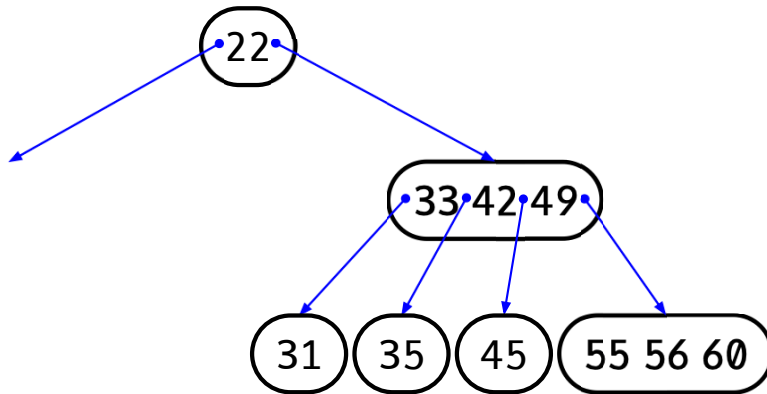
E.g.,

- `.insert(52)`

Split:



`.insert(52)`



B-Tree Growth and B-Tree Density

Q. How does tree grow in height?

- root is full \rightarrow split the root !

Q. How many keys are in a B-Tree of height h ?

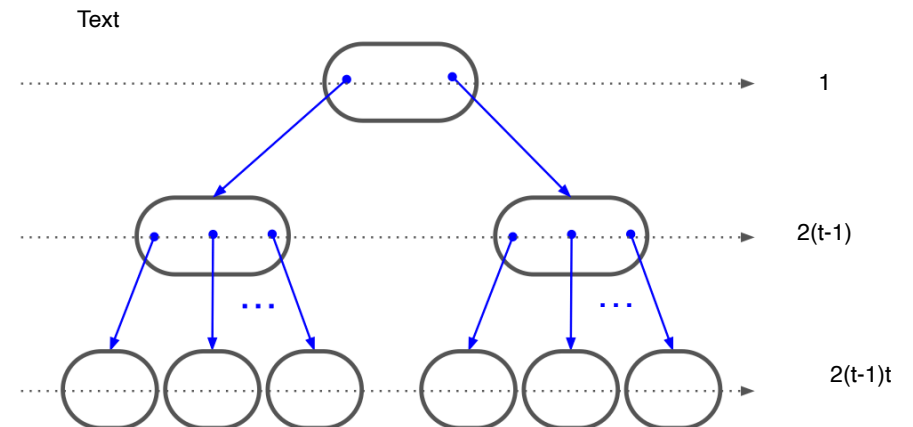
- since branching factor $\geq t$, there are at least $2 * t^h - 1$ keys in a tree of height h
- $h = o(\log t(N))$

In practice, B-Tree nodes are stored on disk

- size of each node 4 KB
- $t = 10^2 - 10^3$

Running time largely \propto # of disk access

- the higher the t , the fewer disk accesses



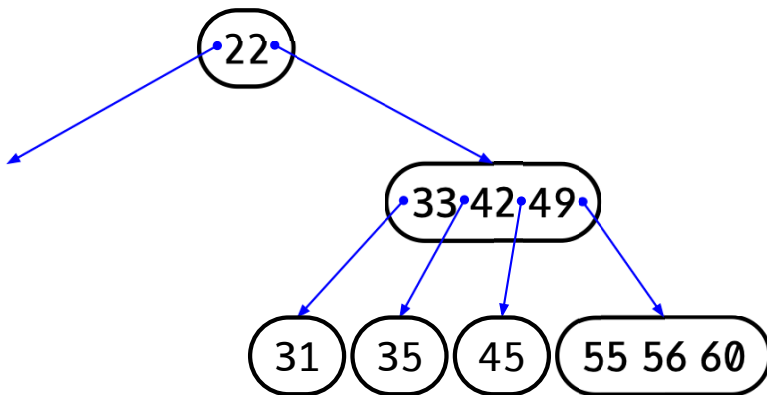
One-Pass Insert

All B-Tree operations should do at most one pass down the B-Tree

- `.insert()` did one pass down, but `.split()` may do a second pass up

New strategy: As you pass down the B-Tree, split any full node

- E.g., `.insert(52)`



Deletion

This time, worry about underfull nodes.

Two operations: merge and transfer

$x \leftarrow \text{root}$

while x not a leaf:

 if key in x : key in internal node

 delete $k = \text{seucessor of key}$

 replace key by k

 return

 find c , the child of x which might contain key

 if c has $t-1$ keys:

 transfter a key to c or merge c with sibling

 if key in x , then delete it. (leaf node)

eventually reach a node with two nodes, if key is nuked it will not be an

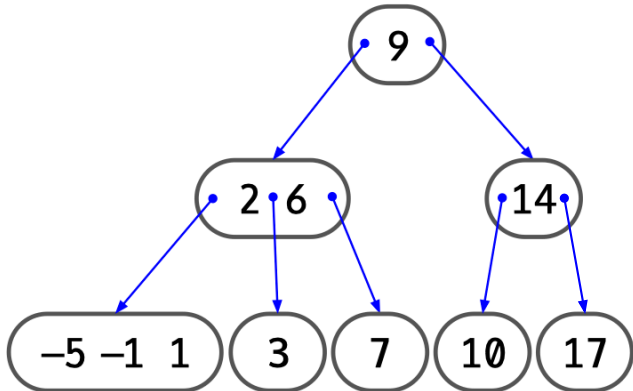
if key in x , then delete it

Merge:

Transfer:

Deletion Example

- `.delete(10)`
- OR `.delete(9)` which triggers `.delete(10)`



Text

Text
Text
Text

Extensions of B-Trees

B+-Trees

- keep all keys at the leaf level
- maximize branching factor on internal nodes
- can implement `.range()` and `.successor()` easily

B*-Trees

- keep all nodes at least 2/3 full
- fuller disk blocks \Rightarrow fuller disk blocks \rightarrow more efficient disk usage