FDs
$$\Sigma = \begin{Bmatrix} D \to AC \\ AB \to DE \\ FD \to E \\ C \to F \end{Bmatrix}$$

Implication problem: $\Sigma \models \varphi$?

use attribute closure to solve

$$\Sigma \models Y \to Z \text{ iff } Z \subseteq C_{\Sigma}(Y)$$

1) $\Sigma \overset{?}{\models} AC \to E$ no

$C_{\Sigma}(AC) = ACF \quad E \not\subseteq ACF$

2) $\Sigma \overset{?}{\models} \boxed{AD \to \underline{CF}}$ yes

$C_{\Sigma}(AD) = ADCFE \quad CF \subseteq ADCFE$

3) $\Sigma \overset{?}{\models} CD \to DE$ yes

$C_{\Sigma}(CD) = CDFAE \quad DE \subseteq CDFAE$

FDs

$$\Sigma = \begin{Bmatrix} D \to AC \\ AB \to DE \\ FD \to E \\ C \to F \end{Bmatrix}$$

For each implied FD, give a derivation using the Amstrong axioms

Show $\Sigma \vdash AD \to CF$

(1) $D \to AC$ given in Σ

(2) $AD \to AC$ from (1) by Augm.

(3) $C \to F$ given in Σ

(4) $AC \to ACF$ from (3) by Aug.

(5) $AD \to ACF$ from (2) (4) by trans

(6) $AD \to CF$ from (5) by decomp.

Essential axioms

Reflexivity: If $Y \subseteq X$, then $X \to Y$

Augmentation: If $X \to Y$, then $XZ \to YZ$ for any Z

Transitivity: If $X \to Y$ and $Y \to Z$, then $X \to Z$

Other axioms

Union: If $X \to Y$ and $X \to Z$, then $X \to YZ$

Decomposition: If $X \to YZ$, then $X \to Y$ and $X \to Z$

FDs

$$\Sigma = \begin{Bmatrix} D \to A \\ F \to B \\ DF \to E \\ B \to C \end{Bmatrix}$$

Find prime attributes & candidate keys of the schema ABCDEF.

Note: cannot derive D, F

X is a key of R with attributes ABCDEF if for any two tuples t_1, t_2 of R $\pi_X(t_1) = \pi_X(t_2) \Rightarrow t_1 = t_2$

Let R be a relation with set of attributes U and FDs F

$X \subseteq U$ is a key for R if $F \models X \to U$

Equivalently, X is a key if $C_F(X) = U$ (why?)

Candidate keys

Keys X such that, for each $Y \subset X$, Y is not a key

Intuitively, keys with a minimal set of attributes

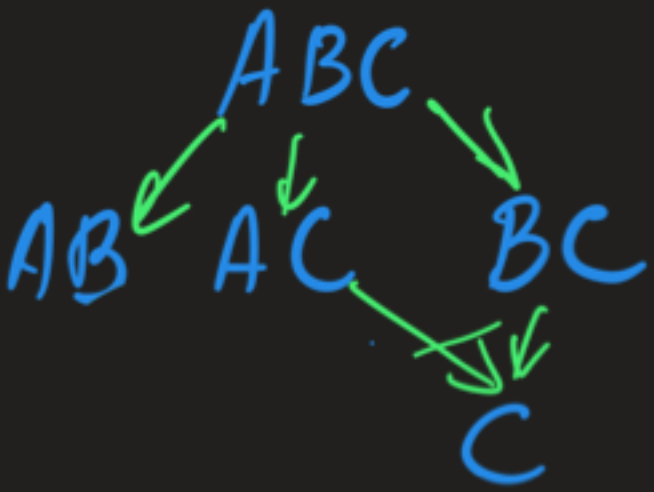
Prime attribute: an attribute of a candidate key

FDs

$$\Sigma = \begin{Bmatrix} D \to A \\ F \to B \\ DF \to E \\ B \to C \end{Bmatrix}$$

Find prime attributes & candidate keys of the schema $ABC\underline{D}E\underline{F}$.

Note: cannot derive D, F



1. ck := ∅
2. G := DAG of the powerset 2^U of U
   - Nodes are elements of 2^U (sets of attributes)
   - There is an edge from X to Y if $Y \subset X$
3. Repeat until G is empty:

   Find a node X without children

   if $C_F(X) = U$:

   ck := ck $\cup \{X\}$

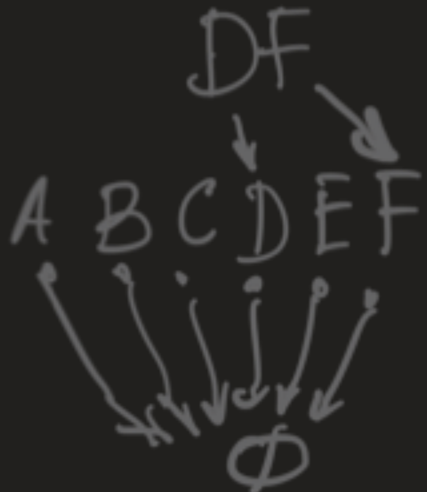
   Delete X and all its ancestors from G

   else:

   Delete X from G

Add one attribute at a time

Node X: Compute $C_F(X)$



FDs $\Sigma = \begin{cases} D \to A \\ F \to B \\ DF \to E \\ B \to C \end{cases}$

Find prime attributes & candidate keys of the schema ABCDEF.

DF

Note: cannot derive D, F

$\Rightarrow$ every key must contain D and F

$C_\Sigma(DF) = DFABEC$

$\Rightarrow$ DF is a key & candidate key

1. ck := $\varnothing$
2. G := DAG of the powerset 2^U of U
   - Nodes are elements of 2^U (sets of attributes)
   - There is an edge from X to Y if $Y \subset X$
3. Repeat until G is empty:

   Find a node X without children

   if $C_F(X) = U$:

   ck := ck $\cup \{X\}$

   Delete X and all its ancestors from G

   else:

   Delete X from G

Add one attribute at a time

Node X: Compute $C_F(X)$

FDs

$U = ABCDEF$

$\Sigma = \begin{Bmatrix} * \; D \to A \\ * \; F \to B \\ DF \to E \\ * \; B \to C \end{Bmatrix}$

$\boxed{\text{Is } (U, \Sigma) \text{ in BCNF?}}$

no

$C_\Sigma(DF) = DFABEC$

$A\underline{BC} \to \underline{BC}$ - trivial

Let R be a relation with set of attributes U and FDs F

$X \subseteq U$ is a key for R if $F \models X \to U$

Equivalently, X is a key if $C_F(X) = U$ (why?)

Candidate keys

Keys X such that, for each $Y \subset X$, Y is not a key

Intuitively, keys with a minimal set of attributes

Prime attribute: an attribute of a candidate key

A relation with FDs F is in BCNF if for every $X \to Y$ in F

- $Y \subseteq X$ (the FD is trivial), or
- X is a key

INDs

Let R, S, T be relations on attributes ABC
Given the following INDs,

$$\Sigma = \left\{ \begin{array}{l} R[AB] \subseteq S[BC] \\ S[BC] \subseteq T[CA] \end{array} \right\}$$

determine which of the following INDs are implied.

Compute the closure Σ^+ of Σ :

$\Sigma^+ = \Sigma \cup$ all derived INDs
including trivial INDs $R[X] \subseteq R[X]$
reflexivity

(1) $R[AB] \subseteq S[BC]$ in Σ
(2) $S[BC] \subseteq T[CA]$ in Σ
(3) $R[A] \subseteq S[B]$ pr. (1)
(4) $R[B] \subseteq \underline{S[C]}$ pr. (1)
(5) $\underline{S[C]} \subseteq T[A]$ pr. (2)
(6) $R[B] \subseteq T[A]$

Axiomatization

Reflexivity: $R[X] \subseteq R[X]$

Transitivity: If $R[X] \subseteq S[Y]$ and $S[Y] \subseteq T[Z]$, then $R[X] \subseteq T[Z]$

Projection: If $R[X, Y] \subseteq S[W, Z]$ with $|X| = |W|$,
then $R[X] \subseteq S[W]$

Permutation: If $R[A_1, \ldots, A_n] \subseteq S[B_1, \ldots, B_n]$,
then $R[A_{i_1}, \ldots, A_{i_n}] \subseteq S[B_{i_1}, \ldots, B_{i_n}]$,
where $i_1, \ldots, i_n$ is a permutation of $1, \ldots, n$

Sound and complete derivation procedure for INDs