

$$\bar{\Sigma} = \left\{ \begin{array}{l} D \rightarrow A \\ F \rightarrow B \\ DF \rightarrow E \\ B \rightarrow C \end{array} \right\}$$

$U = ABCDEF$

Find a lossless BCNF decomposition
Is it dependency-preserving?

A relation with FDs F is in **BCNF** if for every $X \rightarrow Y$ in F

- ▶ $Y \subseteq X$ (the FD is trivial), or
- ▶ X is a key

$$S = (U, \bar{\Sigma})$$

$$\begin{array}{cc} \swarrow & \searrow \\ (U_1, \bar{\Sigma}_1) & (U_2, \bar{\Sigma}_2) \end{array}$$

Given a set of attributes U and a set of FDs F ,
a **decomposition** of (U, F) is a set

$$(U_1, F_1), \dots, (U_n, F_n)$$

such that $U = \bigcup_{i=1}^n U_i$ and F_i is a set of FDs over U_i

BCNF decomposition if each (U_i, F_i) is in BCNF

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$$F \models Y \rightarrow Z \text{ if and only if } Z \subseteq C_F(Y)$$

$$\pi_V(\Sigma)$$

The **projection** of F on $V \subseteq U$

$$\pi_V(F) = \{X \rightarrow Y \mid X, Y \subseteq V, \underline{Y \subseteq C_F(X)}\}$$

is the set of all FDs over V that are implied by F

Can be often represented compactly as a set of FDs F' such that

$$\forall X, Y \subseteq V \quad F' \models X \rightarrow Y \iff F \models X \rightarrow Y$$

$$\pi_{\{FBC\}}(\bar{\Sigma}) = \{B \rightarrow C, F \rightarrow B\}$$

all trivial
dept.

$$\text{e.g. } FB \rightarrow B$$

$$F \rightarrow C$$

$FC \rightarrow C$... by aug

Essential axioms

Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

Other axioms

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

$$\bar{Z} = \left\{ \begin{array}{l} D \rightarrow A \\ F \rightarrow B \\ DF \rightarrow E \\ B \rightarrow C \end{array} \right\}$$

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Find a lossless BCNF decomposition
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1. $S := \{(U, F)\}$
2. While there is $(U_i, F_i) \in S$ not in BCNF:
Replace (U_i, F_i) by **decompose** (U_i, F_i)
3. Remove any (U_i, F_i) for which there is (U_j, F_j) with $U_i \subseteq U_j$
4. Return S *remove smaller tables*

Subprocedure **decompose** (U, F) :

1. Choose $(X \rightarrow Y) \in F$ that violates BCNF
2. Set $V := C_F(X)$ and $Z := U - V$
3. Return $(V, \pi_V(F))$ and $(XZ, \pi_{XZ}(F))$

$F \rightarrow B$

attr. closure of LHS
FBC

What is "left"
 $F \underline{ADE}$

$A \not\rightarrow B \not\rightarrow C \not\rightarrow D \not\rightarrow E \not\rightarrow F - FBC = ADE$

$$\Sigma = \begin{cases} D \rightarrow A \\ F \rightarrow B \\ DF \rightarrow E \\ B \rightarrow C \end{cases}$$

$U = ABCDEF$

Find a lossless BCNF decomposition

Is it dependency-preserving?

$$S = (ABCDEF, \Sigma)$$

1. $R \rightarrow (X, Y)$
 2. Split into $R_1(X)$ and $R_2(Y)$
 3. Repeat (1, 2) for R_1 and R_2
 4. Return any R_1, R_2 for which there is $R_1(X), R_2(Y)$
 5. Return R
- Algorithm: Decompose(R, F)
1. Choose $R \rightarrow (X, Y)$ that violates BCNF
 2. Set $R_1 = R(X)$ and $R_2 = R(Y)$
 3. Return R_1, R_2 and $Decompose(R_1, F)$

$$C_{\Sigma}(F) = FBC$$

$F \rightarrow B$

$x \rightarrow y$

not in BCNF \Rightarrow decompose

$$S_1 = (FBC, \{F \rightarrow B, B \rightarrow C\})$$

$$S_2 = (FADE, \{D \rightarrow A, DF \rightarrow E\})$$

$$\Sigma_1 = \Pi_V(\Sigma)$$

$$X \quad U-V$$

$$\Sigma_2 = \Pi(\Sigma)_{X, (U-V)}$$

$B \rightarrow C$

$B \rightarrow C$

violates BCNF

$$C_{\Sigma_1}(B) = BC$$

$$S_{11} = (BC, \{B \rightarrow C\})$$

$$S_{12} = (BF, \{F \rightarrow B\})$$

$D \rightarrow A$

$$S_{21} = (DA, \{D \rightarrow A\})$$

$$S_{22} = (DEF, \{DF \rightarrow E\})$$