DATABASE SYSTEMS

Practice Exercises 2

CMPT 354, Course Section of Dr. E. Ternovska

Problem 1. Consider the following set of FDs:

$$D \to AC \;, \qquad AB \to DE \;, \qquad FD \to E \;, \qquad C \to F$$

(a) Determine whether each of the following FDs is implied by the FDs above:

$$\begin{array}{cc} AC \to E & BD \to EF \\ AD \to CF & ABC \to DF \\ CD \to DE & BE \to AC \end{array}$$

(b) For each of the FDs in point (a) that are implied, give a derivation using the Armostrong's axioms.

Solution. Call Σ the given set of FDs.

Part (a)

- $\{E\} \not\subseteq C_\Sigma(AC) = ACF$, so $\Sigma \not\models AC \to E$
- $CF \subseteq C_\Sigma(AD) = ADCFE$, so $\Sigma \models AD \to CF$
- $DE \subseteq C_\Sigma(CD) = CDAFE$, so $\Sigma \models CD \to DE$
- $EF \subseteq C_\Sigma(BD) = BDACEF$, so $\Sigma \models BD \to EF$
- $DF \subseteq C_\Sigma(ABC) = ABCDEF$, so $\Sigma \models ABC \to DF$
- $AC \not\subseteq C_\Sigma(BE) = BE$, so $\Sigma \not\models BE \to AC$

Part (b)

- Derivation of $AD \to CF$

(1) $D \to AC$ [given in Σ]
(2) $AD \to AC$ [from (1) by augmentation]
(3) $C \to F$ [given in Σ]
(4) $AC \to ACF$ [from (3) by augmentation]
(5) $AD \to ACF$ [from (2) and (4) by transitivity]
(6) $AD \to CF$ [from (5) by decomposition]

- Derivation of $CD \to DE$

(1) $C \to F$ [given in Σ]
(2) $CD \to FD$ [from (1) by augmentation]
(3) $FD \to E$ [given in Σ]
(4) $CD \to E$ [from (2) and (3) by transitivity]

(5) $CD \to DE$ [from (4) by augmentation]

- Derivation of $BD \to EF$

(1) $D \to AC$ [given in Σ]
(2) $DB \to ACB$ [from (1) by augmentation]
(3) $DB \to AB$ [from (2) by decomposition]
(4) $DB \to C$ [from (2) by decomposition]
(5) $AB \to DE$ [given in Σ]
(6) $DB \to DE$ [from (3) and (5) by transitivity]
(7) $C \to F$ [given in Σ]
(8) $DB \to F$ [from (4) and (7) by transitivity]
(9) $DB \to DEF$ [from (6) and (8) by union]
(10) $DB \to EF$ [from (9) by decomposition]

- Derivation of $ABC \to DF$

(1) $AB \to DE$ [given in Σ]
(2) $ABC \to DEC$ [from (1) by augmentation]
(3) $C \to F$ [given in Σ]
(4) $DEC \to DEF$ [from (3) by augmentation]
(5) $ABC \to DEF$ [from (2) and (4) by transitivity]
(6) $ABC \to DF$ [from (5) by decomposition]

Problem 2. Consider a schema with attributes A, B, C, D, E, F and FDs

$$D \to A\ , \qquad F \to B\ , \qquad DF \to E\ , \qquad B \to C$$

(a) Find the prime attributes and candidate keys of the schema.

(b) Is the schema in BCNF? Justify your answer.

Solution. Call Σ the given set of FDs.

(a) The attributes D and F do not appear in the r.h.s. of any of the given FDs, hence every key must contain them (since they cannot be derived in any way). The closure of DF w.r.t. Σ is $ABCDEF$, so DF is the only candidate key. In turn, the prime attributes of the schema are D and F.

(b) All of the FDs in Σ are non-trivial (the l.h.s. is not a subset of the r.h.s.) so their l.h.s. would be required to be a key in order to satisfy the BCNF conditions. Let us see whether that's the case:

- $C_\Sigma(D) = DA$, so D is not a key and the FD $D \to A$ violates BCNF. At this point, we can already conclude that the given schema is not in BNCF (one violation is enough). But let us check the other FDs for instructional purposes.
- $C_\Sigma(F) = FBC$, so F is not a key and the FD $F \to B$ violates BCNF.
- $C_\Sigma(DF) = DFEABC$, so DF is a key and the FD $F \to B$ does not violate BCNF.
- $C_\Sigma(B) = BC$, so B is not a key and the FD $B \to C$ violates BCNF.

Problem 3. Let R, S and T be relations on attributes A, B, C. Given the following set of INDs:

$$R[A, B] \subseteq S[B, C] \qquad\qquad S[B, C] \subseteq T[C, A]$$

determine which of the following INDs are implied:

$$\begin{array}{ll} R[A] \subseteq T[A] & R[B] \subseteq T[B] \\ R[A] \subseteq T[B] & R[B] \subseteq T[A] \\ R[C] \subseteq T[B] & \end{array}$$

Solution. Call Σ the given set of INDs. Using the axioms for INDs we can compute the closure Σ^+ of Σ:

(1) $R[A, B] \subseteq S[B, C]$ [given in Σ]

(2) $S[B, C] \subseteq T[C, A]$ [given in Σ]

(3) $R[A] \subseteq S[B]$ [from (1) by projection]

(4) $S[B] \subseteq T[C]$ [from (2) by projection]

(5) $R[B, A] \subseteq S[C, B]$ [from (1) by permutation]

(6) $S[C, B] \subseteq T[A, C]$ [from (2) by permutation]

(7) $R[B] \subseteq S[C]$ [from (5) by projection]

(8) $S[C] \subseteq T[A]$ [from (6) by projection]

(9) $R[A, B] \subseteq T[C, A]$ [from (1) and (2) by transitivity]

(10) $R[A] \subseteq T[C]$ [from (9) by projection]

(11) $R[B, A] \subseteq T[A, C]$ [from (9) by permutation]

(12) $R[B] \subseteq T[A]$ [from (11) by projection]

(13) All trivial INDs obtained by reflexivity

$R[B] \subseteq T[A]$ is implied by Σ (because they are in Σ^+).