CS251 - Data Structures and Algorithms
Fall 2024

PSO 1, Week 1

Question 1

Let c be the cost of calling the function WORK. That is, the cost of the function is constant, regardless of the input value. Determine the respective closed-form $T(n)$ for the cost of calling WORK.

```
1: function A1(n : ℤ⁺)
2:     val ← 0
3:     for i from 1 to n by multiplying by 3 do
4:         for j from i to i² do
5:             val ← val + WORK(n)
6:         end for
7:     end for
8:     return val
9: end function
```

Question 2

Derive the closed-form $T(n)$ for the value returned by the following algorithm:

```
1: function A2(n : ℤ⁺)
2:     sum ← 0
3:     for i from 0 to n^4 − 1 do
4:         for j from i to n^3 − 1 do
5:             sum ← sum + 1
6:         end for
7:     end for
8:     return sum
9: end function
```

Question 3

(a) The following statements are true or false?

1. $n^2 = O(5^{\log n})$
2. $\frac{\log n}{\log \log n} = O(\sqrt{\log n})$
3. $n^{\log n} = \Omega(n!)$

(b) Sort the following functions in increasing order of asymptotic (big-O) complexity:

$$f_1(n) = n^{\sqrt{n}}, \quad f_2(n) = 2^n, \quad f_3(n) = n^{10} \cdot 2^{n/2}, \quad f_4(n) = \binom{n}{2}$$

Question 4

(a) Show that $\max\{f(n), g(n)\} = \Theta(f(n)+g(n))$ for any $f(n)$ and $g(n)$ that eventually become and stay positive.

(b) Give an example of f and g such that f is not $O(g)$ and g is also not $O(f)$.

(c) Prove that $n(2+\sin(n\pi/2))$ is $\Theta(n)$. (Note that the limit $\lim_{n\to\infty} 2+\sin(n\pi/2)$ does not exist.)