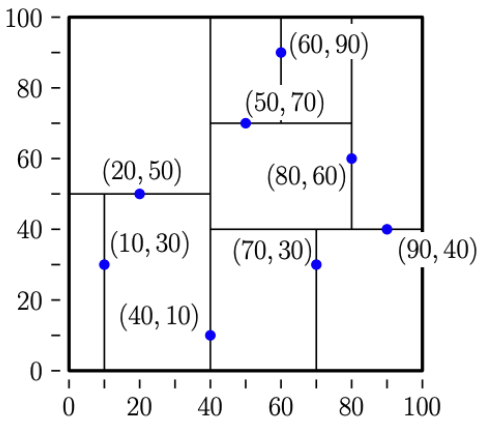
CS251 - Data Structures and Algorithms
Fall 2024

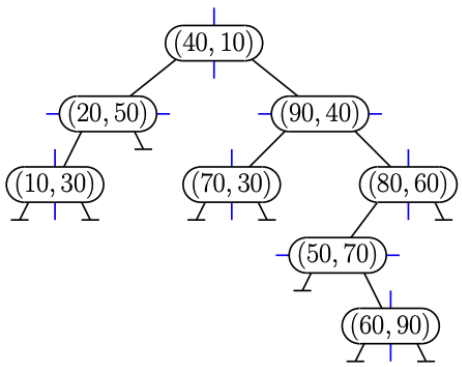
PSO 13, Week 16

Question 1

(1) **(Quadtrees)** Insert the following points (in order) into an empty Point-region Quadtree: (35,40), (50,10), (60,75), (80,65), (85,15), (5,45), (25,35), (90,5).

(2) **(kd-trees)** Consider the kd-tree shown below. We assume it's a standard kd-tree where the cutting dimensions alternates between x and y with each level. Show the final tree after the operation **insert((70,50))**.





Question 2

(Red-Black tree)

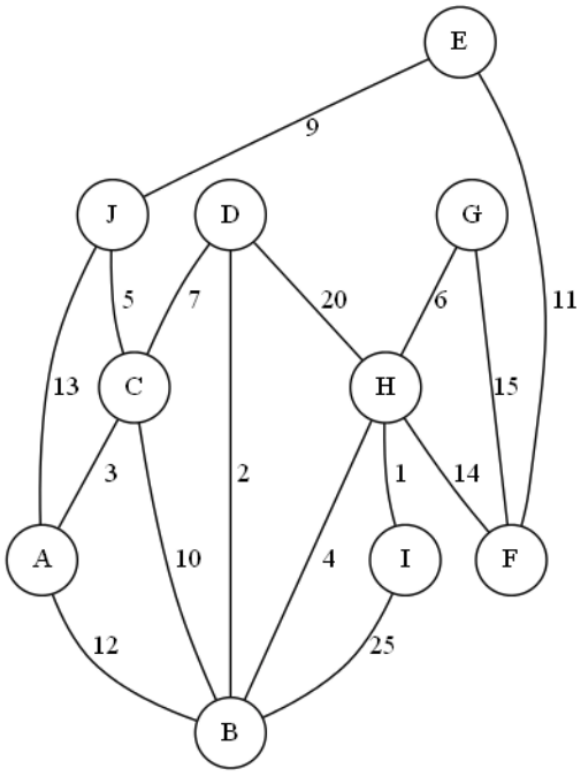
(1) Create a Red-Black tree by inserting the following sequence of nodes: 8, 18, 5, 15, 17, 25, 40, 80.

(2) Delete 18 in the resulting Red-Black tree from (1).

Question 3

(Prim's algorithm & MST)

(1) Run the Prim's algorithm on the following graph with starting vertex A.



(2) Let G be a connected undirected graph of 100 vertices and 300 edges. The total weight of an MST of G is 500. When the weight of each edge of G is increased by 5, what is the total weight of the MST of the updated graph?

Question 4

(Bloom Filter)

(1) **(What is a false positive?)** Suppose you have a Bloom Filter with three hash functions and a bit array of length 13. The three hash functions h_1, h_2, and h_3 produce the following values for the three objects to be inserted into the Bloom Filter.

$$\begin{array}{lll} h_1(object_1) = 3 & h_2(object_1) = 12 & h_3(object_1) = 11 \\ h_1(object_2) = 11 & h_2(object_2) = 1 & h_3(object_2) = 9 \\ h_1(object_3) = 8 & h_2(object_3) = 3 & h_3(object_3) = 12 \end{array}$$

(1.1) What is the status of the Bloom Filter after inserting the three objects?

(1.2) Suppose another (fourth) object produces the hash values of 3, 12, and 9 for the hash functions h_1, h_2, and h_3 respectively. What will the exist() function return if the other three objects have already been inserted? Does the object look as if it exists in the Bloom filter?

(2) Suppose you have a Bloom Filter consisting of an m-bit array and k hash functions with values ranging from 0 to $m - 1$. What is the false positive rate after n objects have been inserted into the Bloom Filter?

(3) What is the ratio of m to n if you want to achieve a false positive rate of 1% with three hash functions?