ME598/494 Homework 4

1. (10 points) Sketch graphically the problem

$$\begin{aligned}
&\min\ f(\mathbf{x}) = (x_1+1)^2 + (x_2-2)^2 \\
&\text{subject to } g_1 = x_1 - 2 \le 0, \quad g_3 = -x_1 \le 0, \\
&g_2 = x_2 - 1 \le 0, \quad g_4 = -x_2 \le 0.
\end{aligned}$$

   Find the optimum graphically. Determine directions of feasible descent at the corner points of the feasible domain. Show the gradient directions of f and g_is at these points. Verify graphical results analytically using the KKT conditions.

2. (10 points) Graph the problem

$$\begin{aligned}
&\min f = -x_1, \text{subject to} \\
&g_1 = x_2 - (1-x_1)^3 \le 0 \quad \text{and} \quad x_2 \ge 0.
\end{aligned}$$

   Find the solution graphically. Then apply the optimality conditions. Can you find a solution based on the optimality conditions? Why? (From Kuhn and Tucker, 1951.)

3. (20 points) Find a local solution to the problem

$$\begin{aligned}
&\max\ f = x_1x_2 + x_2x_3 + x_1x_3 \\
&\text{subject to } h = x_1 + x_2 + x_3 - 3 = 0.
\end{aligned}$$

   You can use either the reduced gradient or the Lagrangian method.

4. (40 points) Find the solution to

$$\begin{aligned}
&\min\ f = x_1^2 + x_2^2 + x_3^2 \\
&\text{subject to } h_1 = x_1^2/4 + x_2^2/5 + x_3^2/25 - 1 = 0 \\
&\text{and } h_2 = x_1 + x_2 - x_3 = 0,
\end{aligned}$$

   by implementing the generalized reduced gradient algorithm.

   (Bonus 40 points) Implement a differentiable program using pytorch to solve this problem (maximum 140 points). Here, you will consider the update of state variables as an iterative program with a fixed number of iterations. At the kth step of GRG, this can be written as $s(k+1) = solve(s(k), d(k+1))$.

5. (20 points) Consider the following garbage truck routing problem. Let there be N sites to be visited and consider them as nodes of a graph. The cost of moving from node i to j is c_{ij} if there is an edge between the nodes, or ∞ if there is none. Site 0 is the truck station where the truck starts and returns.

   Formulate the problem to minimize the total cost while the truck visits all sites and returns to the station.