INTRO TO QUANTUM COMPUTING

Lab #3

VECTORS AND COMPLEX NUMBERS

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11/04/2020

PROGRAM FOR TODAY

- Announcements
- Canvas attendance quiz
- Pre-lab student feedback
- Popular questions from last week
- Lab content
- Post-lab student feedback

Canvas attendance quiz passcode: 8682

ANNOUNCEMENTS

Friday homework review sessions

- Review, ask questions, work through weekly homework problems with an instructor
- Friday 4-5 p.m. EST
- Zoom links posted on Discord (#course-announcements)
- Recordings will be made available if you cannot attend live

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CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number : **7**
  - Passcode: **8682**
- The decimal number 2 is represented in binary as:
  - 11
  - 10
  - 00
  - 01
- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week’s content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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QUESTIONS FROM PAST WEEK

Homework 2, problem 2, part iii)

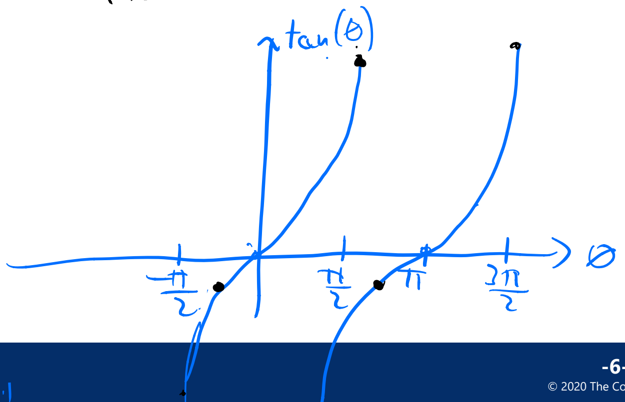
Convert the following points from Cartesian coordinates to polar coordinates. You may use a calculator for this problem:

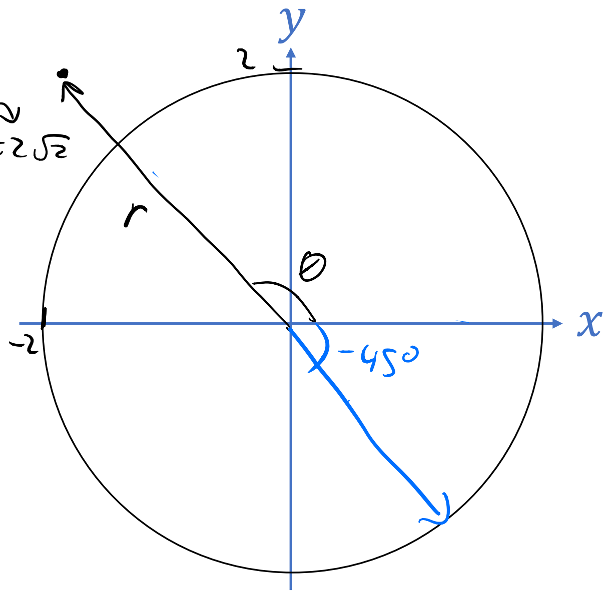
x=-2, y=2

$\sqrt{4}$

$$r = \sqrt{x^2+y^2} = \sqrt{(-2)^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan(-1) = -45^\circ$$





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QUESTIONS FROM PAST WEEK

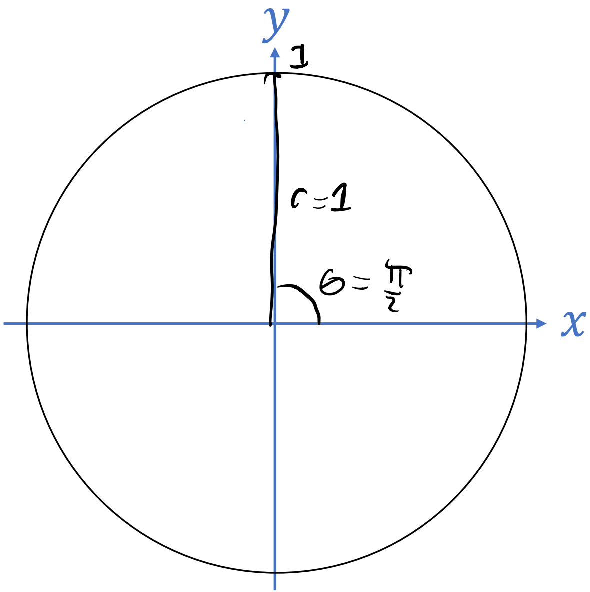
Homework 2, problem 2, part iv)

Convert the following points from Cartesian coordinates to polar coordinates. You may use a calculator for this problem:

x=0, y=1

$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{1}{0}\right)$

can't divide by 0!!



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LEARNING OBJECTIVES FOR LAB 3

- Solidify understanding of vectors
  - Representation
  - Addition
  - Multiplication
  - Decomposition

- Perform basic operations on Complex numbers
  - Representation
  - Modulus and Conjugate
  - Multiplication in cartesian form
  - Multiplication in polar form*

- Applying vectors and complex numbers to quantum computing*
  - The Bloch sphere*

*Optional content

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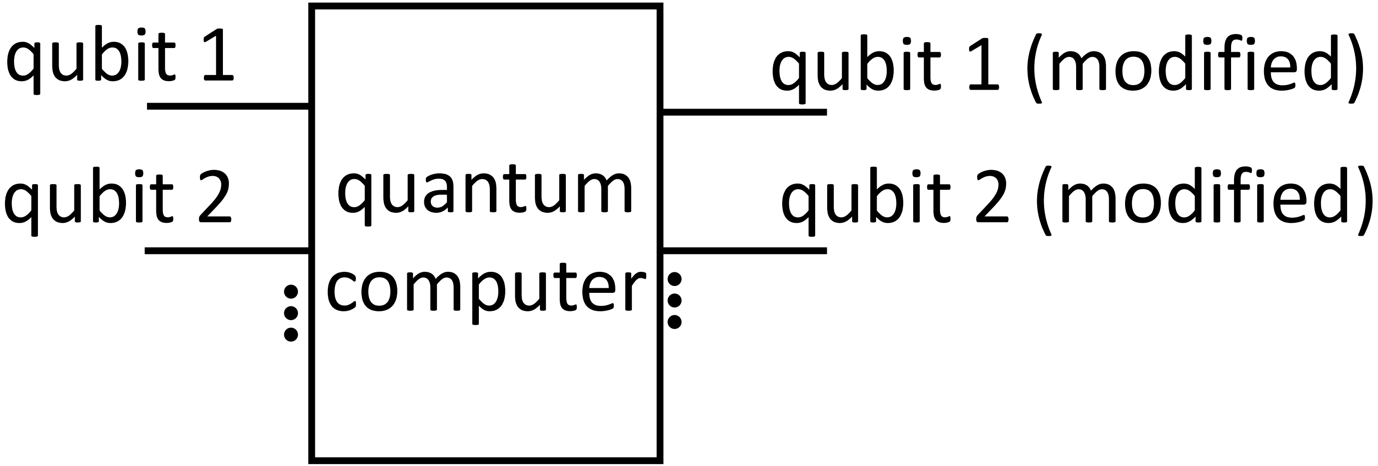
SCHEMATIC OF A QUANTUM COMPUTER



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REPRESENTING QUBITS

classical bit

either 0 or 1

REPRESENTING QUBITS

classical bit

qubit

either 0 or 1

superposition of 0 and 1

$|0\rangle, |1\rangle$

$\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$

$0.5\begin{pmatrix}0\\1\end{pmatrix} + 0.5\begin{pmatrix}1\\0\end{pmatrix}$

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VECTOR NOTATION

scalar

$$v = 2$$

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VECTOR NOTATION

scalar

$$v = 2$$

vector

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

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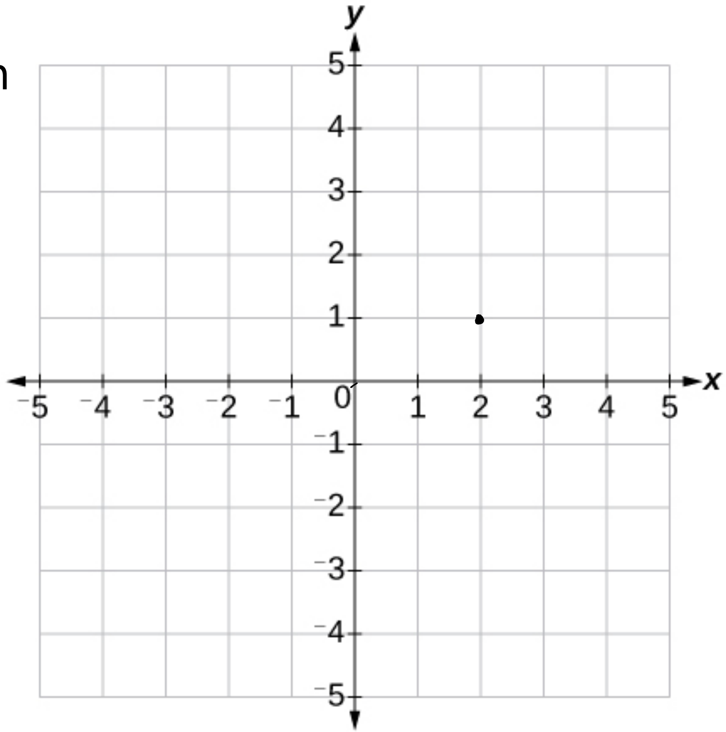
VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

$\begin{pmatrix} x \\ y \end{pmatrix}$



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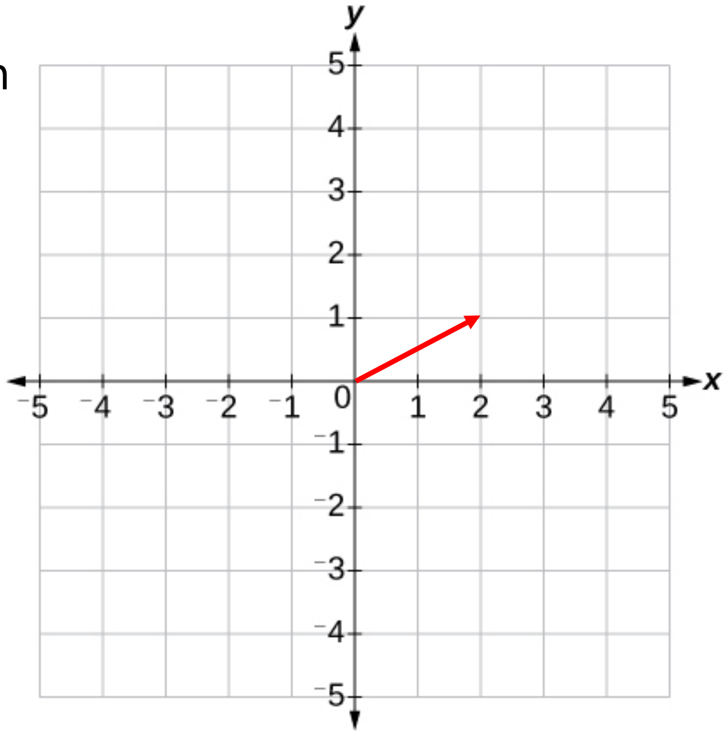
VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

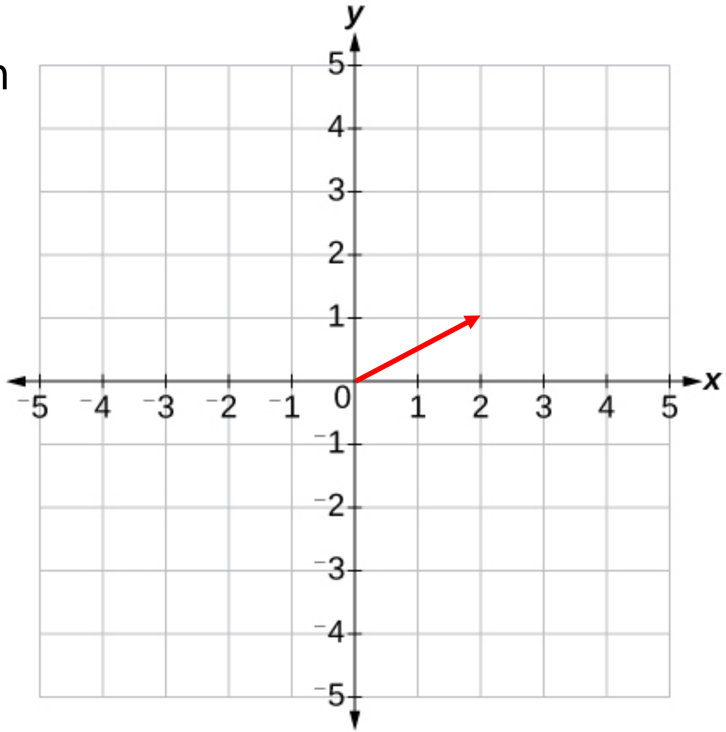


VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

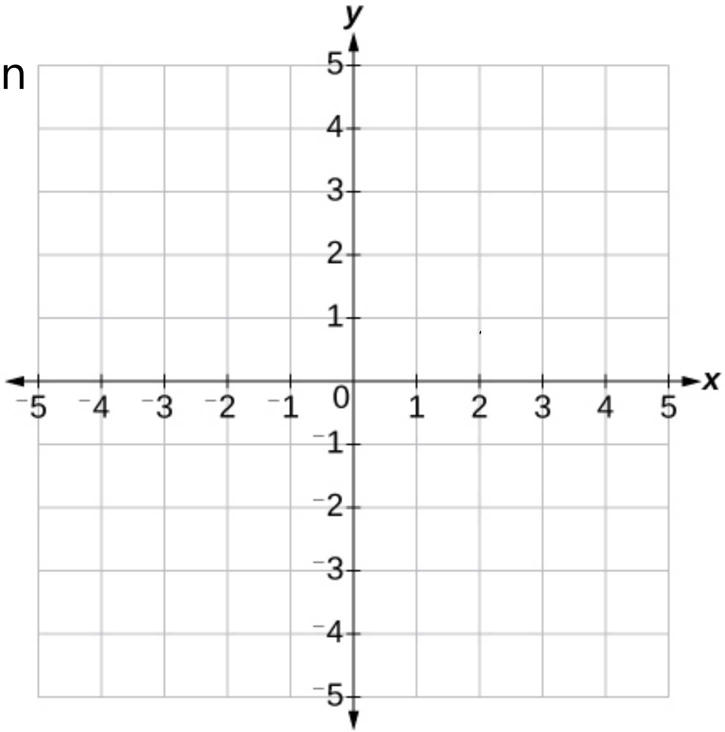
$$v_x = 2$$
$$v_y = 1$$



VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?



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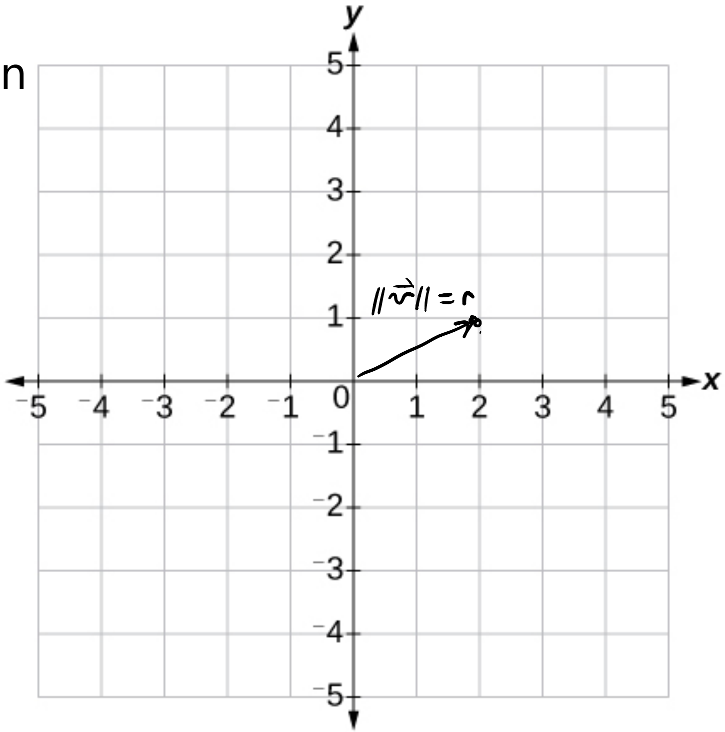
VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?

$$\boxed{\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}} = r$$



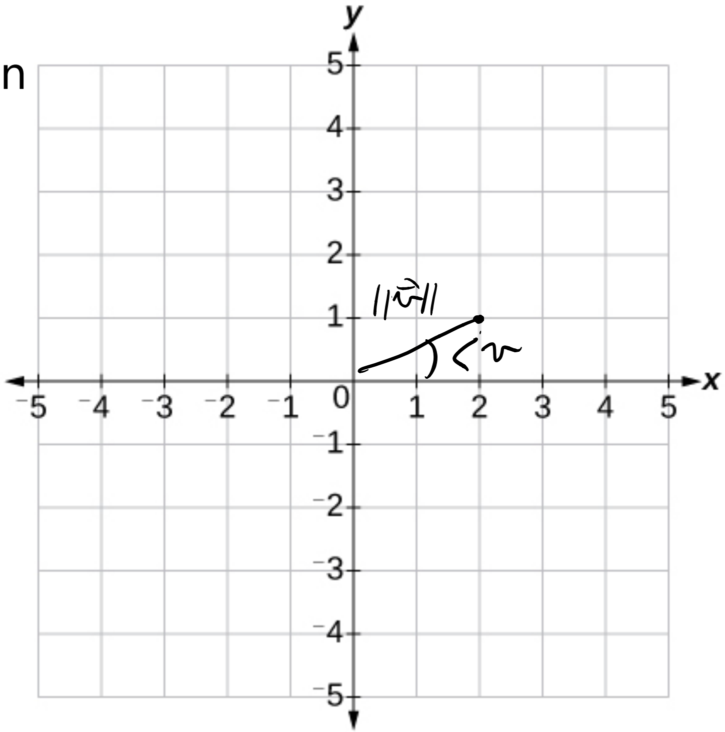
VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2} = r$$

$$\angle \vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \theta$$



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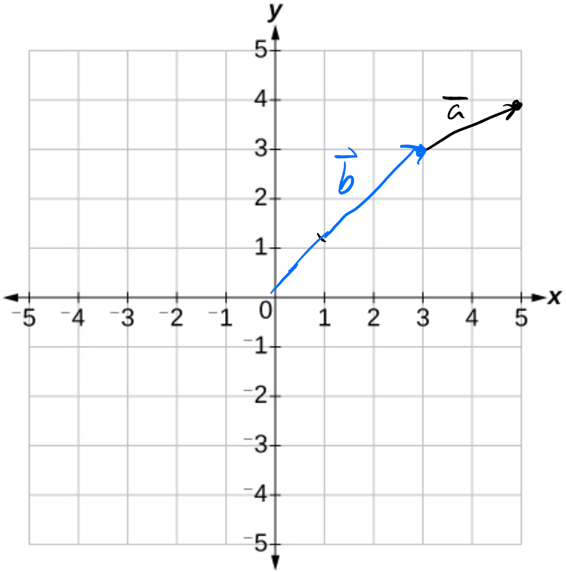
VECTOR ADDITION

Add the vectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$$\vec{a} + \vec{b} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix}$$



$$\begin{pmatrix} 2+3 \\ 1+3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

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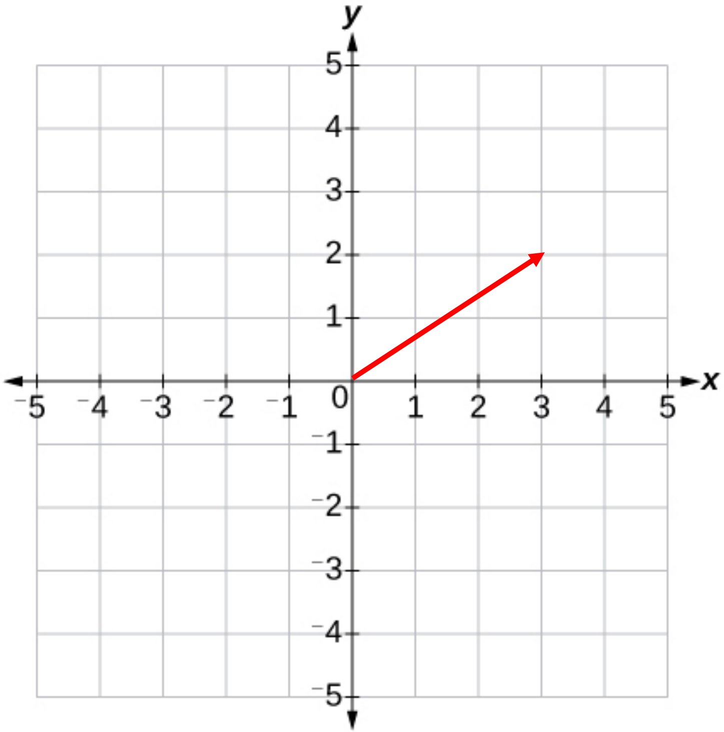
VECTOR DECOMPOSITION

$$\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix}$$

$$\vec{v} = \begin{pmatrix}v_x\\v_y\end{pmatrix} = \begin{pmatrix}v_x\\0\end{pmatrix} + \begin{pmatrix}0\\v_y\end{pmatrix}$$

$$= \begin{pmatrix}0.5v_x\\0\end{pmatrix} + \begin{pmatrix}0.5v_x\\v_y\end{pmatrix}$$



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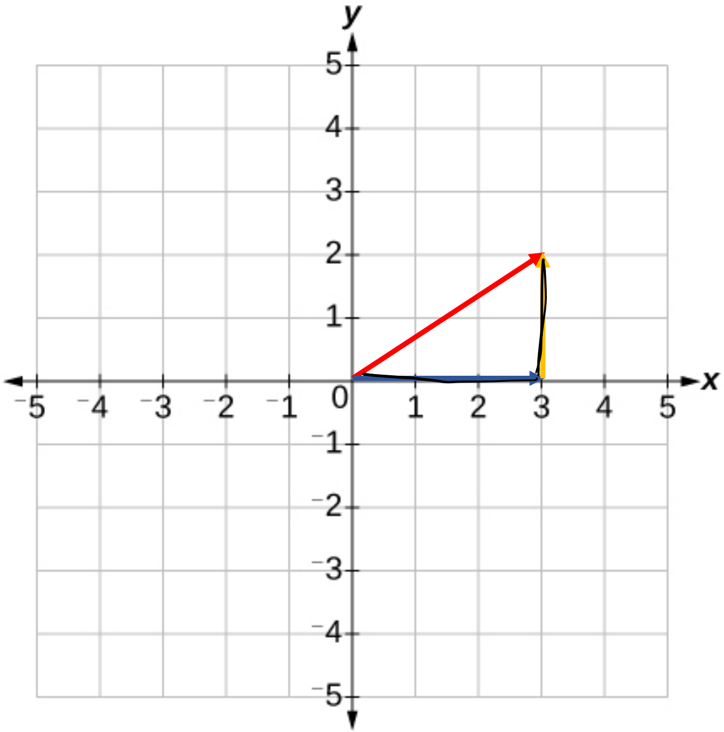
VECTOR DECOMPOSITION

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$



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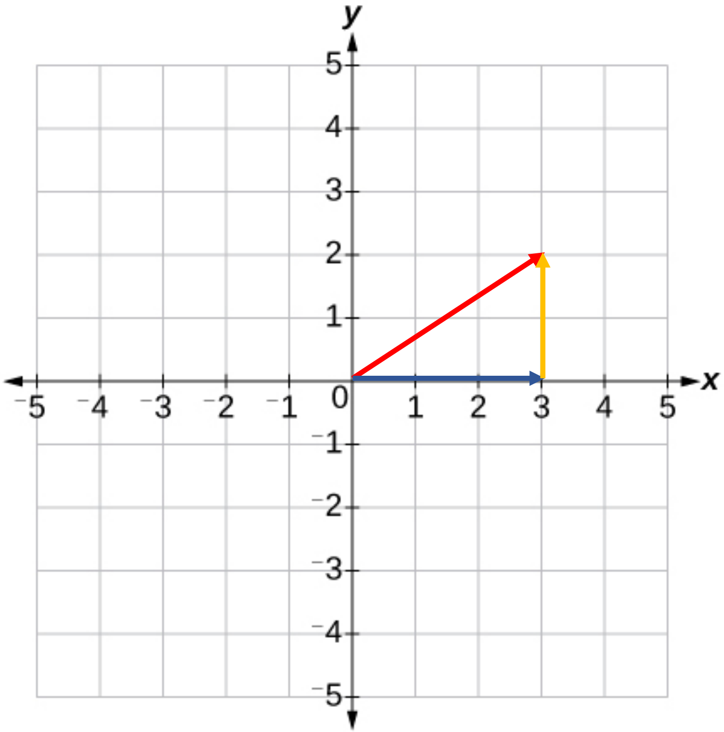
VECTOR DECOMPOSITION

$$\begin{pmatrix}3\\2\end{pmatrix} = \underbrace{\begin{pmatrix}3\\0\end{pmatrix}}_{\text{along x-axis}} + \underbrace{\begin{pmatrix}0\\2\end{pmatrix}}_{\text{along y-axis}}$$

- x-component: 3
- y-component: 2



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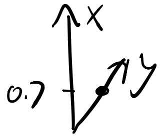
QUESTIONS

"dot product"

$$\begin{pmatrix} v_x & v_y \end{pmatrix} \cdot \begin{pmatrix} u_x \\ u_y \end{pmatrix} = u_x v_x + u_y v_y$$



$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]$$

Questions on content so far?

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COMPLEX NUMBERS

A complex number consists of both a ***real*** and ***imaginary*** component.

$$z = a\begin{pmatrix}1\\0\end{pmatrix} + \underbrace{\overset{\sqrt{-1}}{i}\; b\begin{pmatrix}0\\1\end{pmatrix}}_{\text{Imaginary Component}}$$

Complex Number

Real Component

Imaginary Unit

Imaginary Coefficient

Imaginary Component

COMPLEX NUMBER REPRESENTATION

Complex numbers can additionally be represented as ***vectors***,
in the 2D ***complex plane***!

$$z = a + i\,b$$

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COMPLEX NUMBER REPRESENTATION

Complex numbers can additionally be represented as ***vectors***, in the 2D ***complex plane***!

$$z = a + i\,b$$

$$\vec{z} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Real Component

Imag. Component

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COMPLEX NUMBER REPRESENTATION

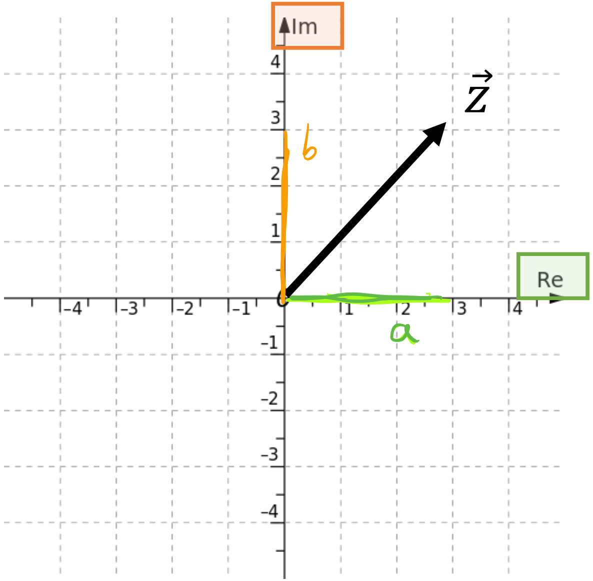
Complex numbers can additionally be represented as ***vectors***, in the 2D ***complex plane***!

$$z = a + i\,b$$

$$\vec{z} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Real Component

Imag. Component



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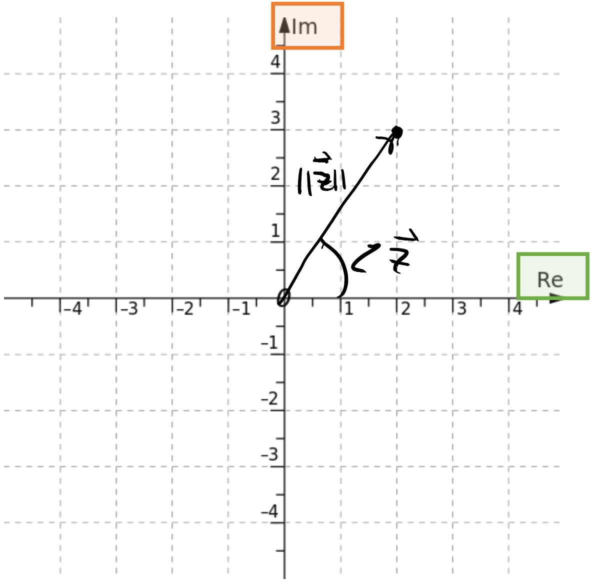
COMPLEX NUMBER REPRESENTATION

real a imag b

$$z = 2 + 3i$$

$$\vec{z} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



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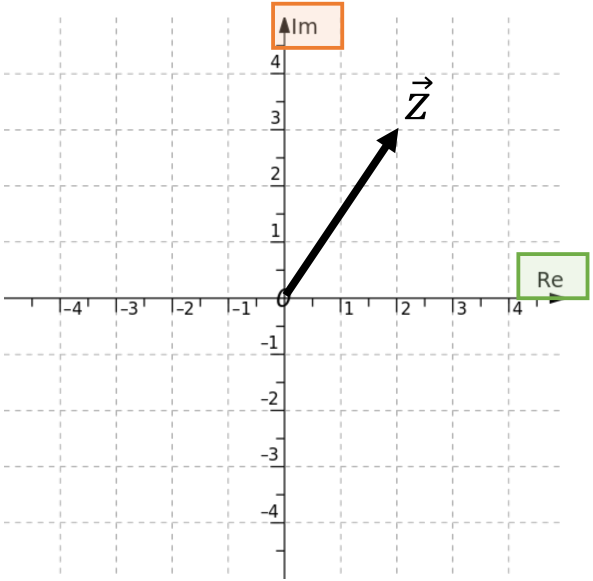
COMPLEX NUMBER REPRESENTATION

$$z = 2 + 3i$$

$$\vec{z} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



MODULUS AND CONJUGATE NOTATION

$$z = a + i\,b$$

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MODULUS AND CONJUGATE NOTATION

$$z = a + i\,b \qquad \dagger$$

c, $\|z\|$

$$\text{Modulus: } |z| = \sqrt{a^2 + b^2}$$

$$\text{Conjugate: } \bar{z} = a - ib$$

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MODULUS AND CONJUGATE

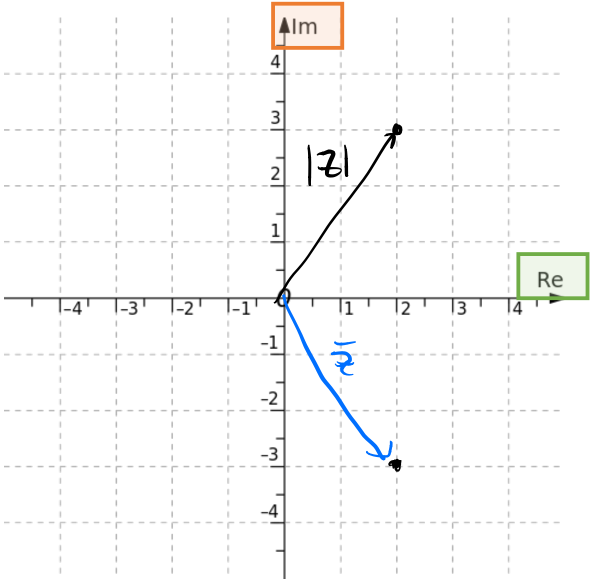
$$|a + ib| = \sqrt{a^2 + b^2}$$

$$\overline{(a + i\,b)} = (a - ib)$$

$$z = 2 + 3i$$

$$|z| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$\bar{z} = 2 - 3i$$



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COMPLEX NUMBER ADDITION

$$(a + i\,b) + (c + id) = (a + c) + i(b + d)$$

$$\left(2x + 3x^2\right) + \left(4x + x^2\right) = \begin{pmatrix} 2x \\ 3x^2 \end{pmatrix} + \begin{pmatrix} 4x \\ x^2 \end{pmatrix}$$

$$z_1 = 2 + 3i$$

$$z_2 = 3 - 2i$$

$$z_1 + z_2 = (2+3) + i(3-2)$$
$$= 5 + i$$

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COMPLEX NUMBER MULTIPLICATION

$$(a + i\,b) * (c + id) = \underbrace{(ac - bd)} + i\underbrace{(ad + bc)}$$

$$z_1 = \overset{a}{2} + \overset{b}{3}i$$

$$z_2 = \overset{c}{3} - \overset{d}{2}i$$

$i = \sqrt{-1}$ $\quad$ $i^2 = -1$

$$\underbrace{ac}_{\text{real}} + \underbrace{a\cdot id}_{\text{imag}} + \underbrace{ib\cdot c}_{\text{imag}} + \underbrace{ib\cdot id}_{-bd}$$

real

$$z_1 . z_2 = 2\cdot 3 - 3\cdot(-2) + i[2(-2) + 3\cdot 3]$$

$$= 12 + i5$$

POLAR FORM OF COMPLEX NUMBERS

Euler's formula: $$\boxed{e^{i\varphi} = \cos\varphi + i\sin\varphi}$$

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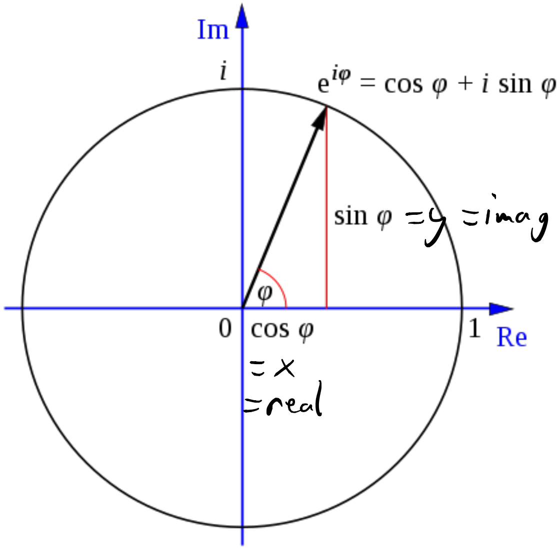
POLAR FORM OF COMPLEX NUMBERS



Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

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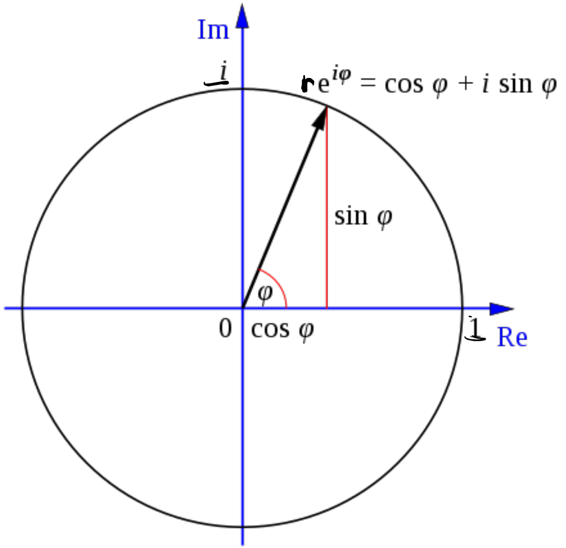
POLAR FORM OF COMPLEX NUMBERS



Source: Wikipedia (CC)

Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Polar representation of complex numbers!

angle

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

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POLAR FORM OF COMPLEX NUMBERS

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

$$z = 2 + 3i$$

$$r =$$

$$\varphi =$$

$$z\ (\text{Polar form}) =$$

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POLAR FORM OF COMPLEX NUMBERS

Steps to find the polar form of $z = x + iy$:

1. Find the vector radius $r = |z| = \sqrt{x^2 + y^2}$ $\|\vec{v}\|$
2. Find the vector angle $\varphi = \tan^{-1}\left(\frac{y}{x}\right)$
3. Combine radius and angle to get $z = re^{i\varphi}$ ← phase

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QUESTIONS

Questions on content so far?

POST-LAB CANVAS FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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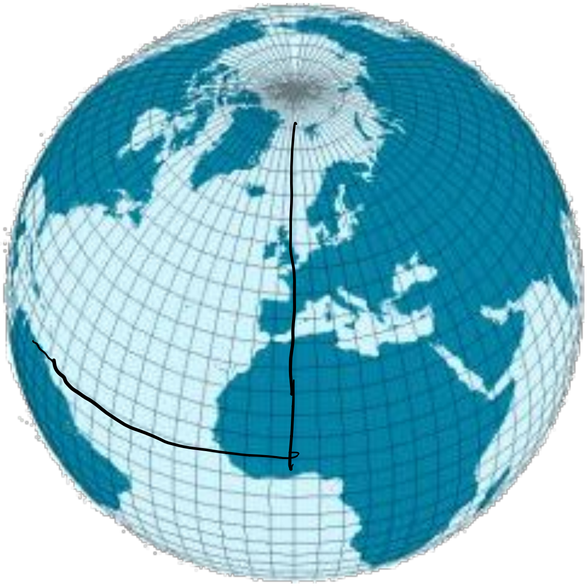
OPTIONAL CONENT

LOCATING A PLACE ON EARTH



- How would you tell me where your hometown is?

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LOCATING A PLACE ON EARTH

- How would you tell me where your hometown is?
- Latitude and longitude! Two angles

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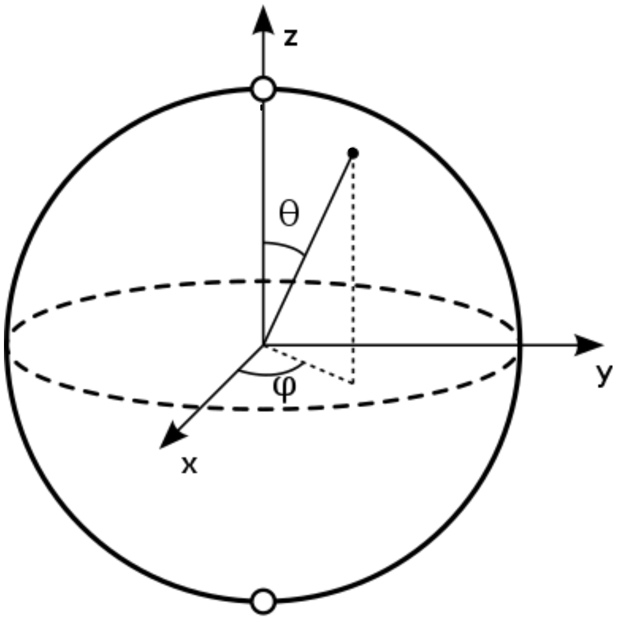
LOCATING POINTS ON A SPHERE



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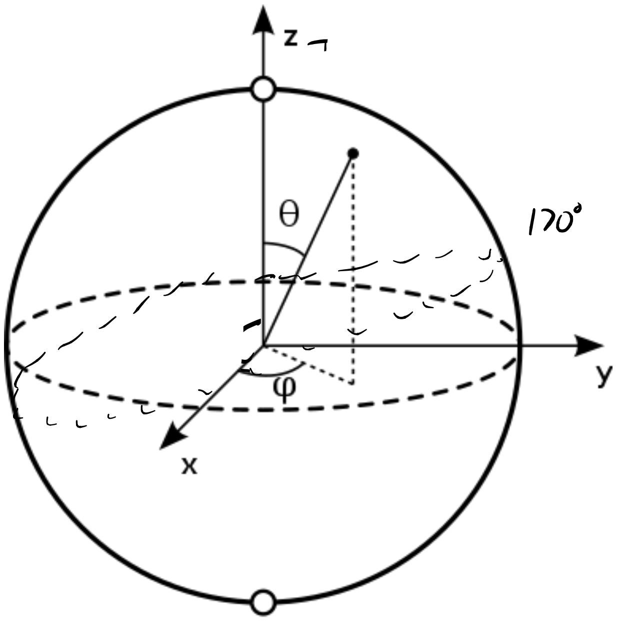
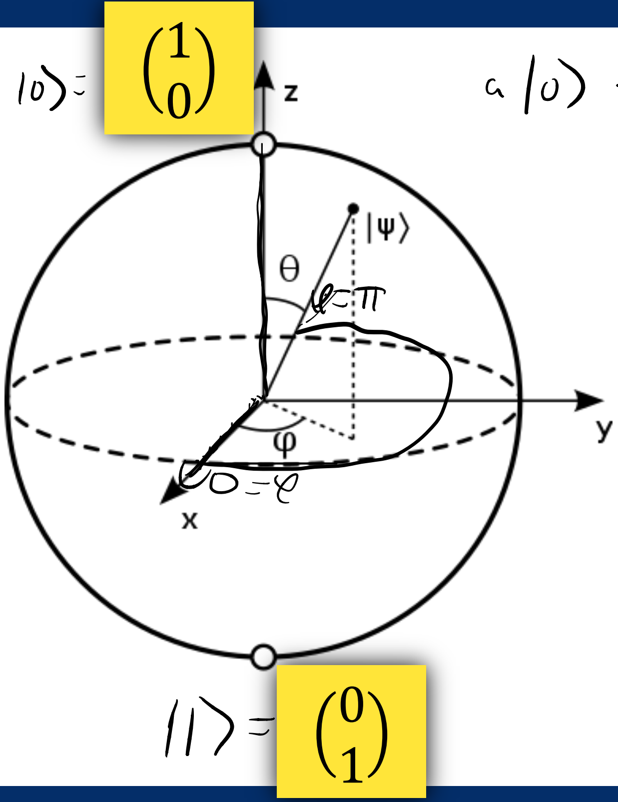
LOCATING POINTS ON A SPHERE



$\theta \in \{0, \pi\}$

φ ~~ϕ~~ $\in \{0, 2\pi\}$

THE BLOCH SPHERE



$a|0\rangle + b e^{i\phi}|1\rangle$

$H\psi = E\psi$

psi

$$\psi(\theta, \phi) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi} \sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$\psi^* \psi$

$(x - iy)(x + iy)$

$\theta \in \{0, \pi\}$

$\phi \in \{0, 2\pi\}$

MULTIPLICATION IN THE POLAR FORM

$$e^{\pi} \cdot e^{3} = e^{3+\pi}$$

$$z_1 = 1e^{i\pi/4}$$

r_1 φ_1

$$z_2 = 2e^{i\pi/4}$$

r_2 φ_2

$$r_{tot} = r_1 \cdot r_2$$

$$\varphi_{tot} = \varphi_1 + \varphi_2$$

$$z_1 . z_2 = 2e^{i\frac{\pi}{2}}$$

$$z = re^{i\varphi}$$

$$\bar{z} = re^{-i\varphi}$$

$$|z| = r$$

$$z_x = r * \cos(\varphi)$$

$$z_y = r * \sin(\varphi)$$

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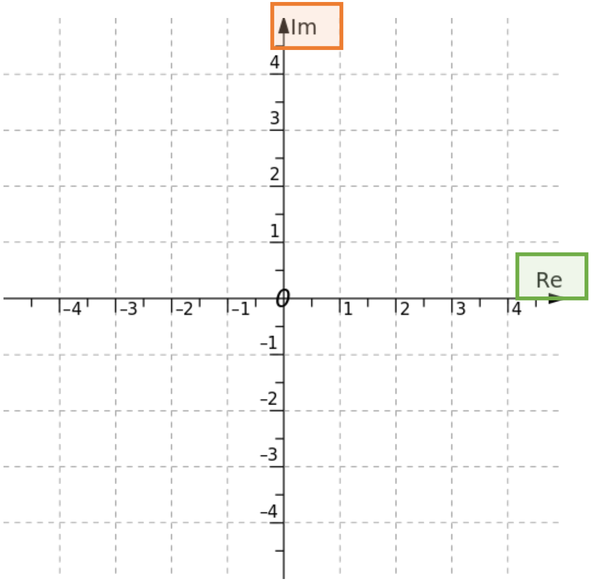
MULTIPLICATION IN THE POLAR FORM

$$z_1 = 1e^{i\pi/4}$$

$$z_2 = 2e^{i\pi/4}$$

$$z_1 . z_2 =$$

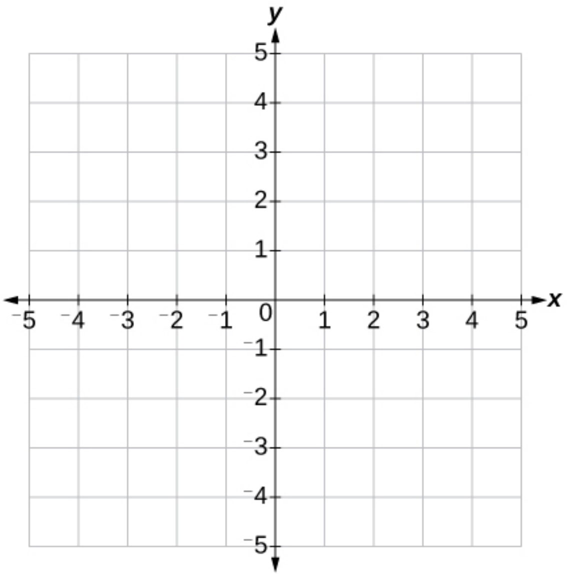


EXTRA PROBLEMS

SCALAR MULTIPLCATION

Find $2 * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $-2 * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$c * \vec{a} = \begin{pmatrix} c * w_x \\ c * w_y \end{pmatrix}$$



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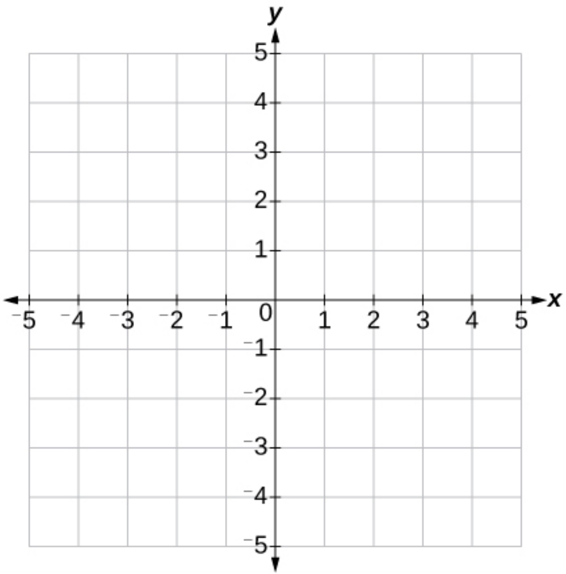
VECTOR ADDITION

Add the vectors $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$



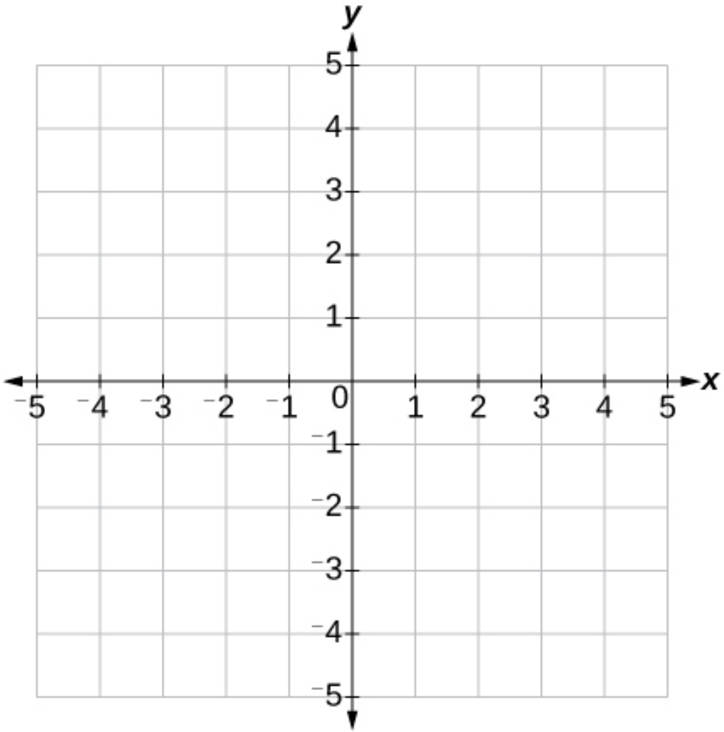
$$\vec{a} + \vec{b} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix}$$

VECTOR DECOMPOSITION

Find the x and y components of

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

- Result of adding:
- x-component:
- y-component:



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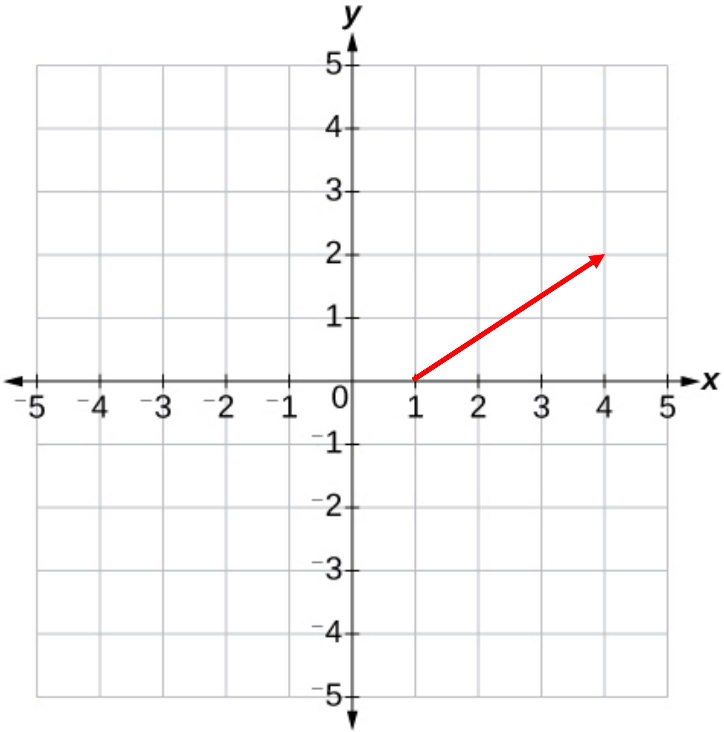
VECTOR REPRESENTATION

- What is this vector?
- What is the magnitude of this vector?
- What is the direction of this vector?

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

$$\angle \vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



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POLAR FORM OF COMPLEX NUMBERS

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$z = 3/5 + 4/5i$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

$$r =$$

$$\varphi =$$

$$z\ (\text{Polar form}) =$$

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WORKING WITH THE POLAR FORM

$$z = 1\,e^{i\pi/4}$$

$$\bar{z} =$$

$$|z| =$$

$$z(\text{Cartesian form}) =$$

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$\bar{z} = re^{-i\varphi}$$

$$|z| = r$$

$$x = r * \cos\varphi$$

$$y = r * \sin\varphi$$

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MODULUS AND CONJUGATE

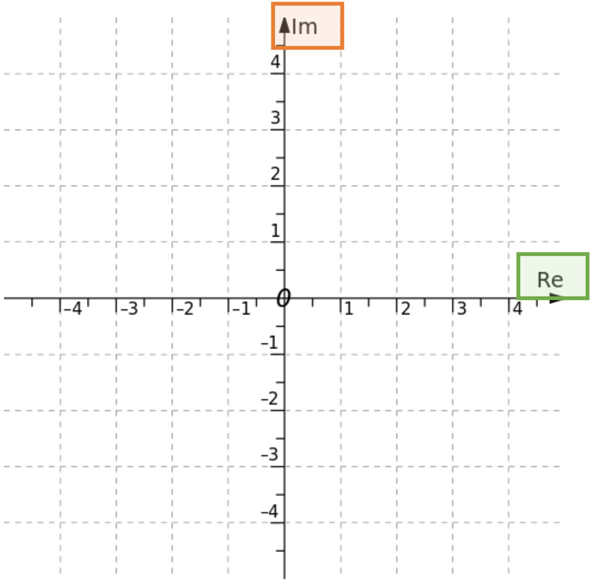
$$|a + ib| = \sqrt{a^2 + b^2}$$

$$\overline{(a + i\,b)} = (a - ib)$$

$$z = 3/5 + 4/5i$$

$$|z| =$$

$$\bar{z} =$$



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COMPLEX NUMBER ADDITION

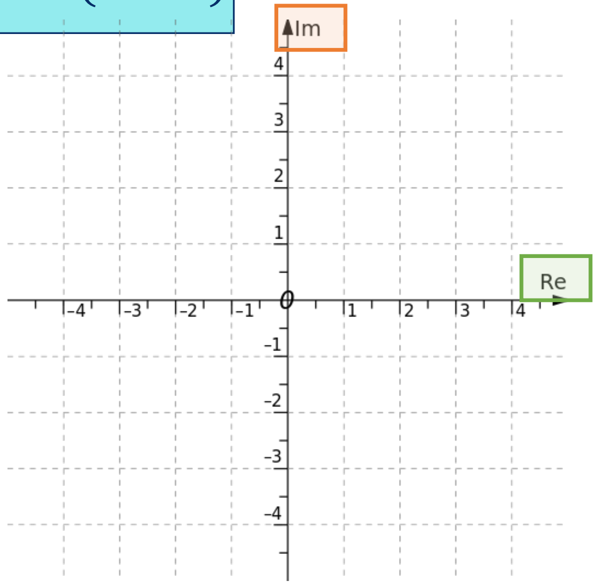
$$\boxed{(a + i\,b) + (c + id) = (a + c) + i(b + d)}$$

$$z = 2 + 3i$$

$$\bar{z} = 2 - 3i$$

$$z + \bar{z} =$$

$$z - \bar{z} =$$



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COMPLEX NUMBER MULTIPLICATION

$$(a + i\, b) * (c + id) = (ac - bd) + i(ad + bc)$$

$$z = 2 + 3i$$

$$\bar{z} = 2 - 3i$$

$$z.\bar{z} =$$

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