INTRO TO QUANTUM COMPUTING

Week 4 Lab

MATRICES AND LINEAR ALGEBRA

Sarah Muschinske

11-10-2020

PROGRAM FOR TODAY

- Announcement
- Attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz passcode: 5517

ANNOUNCEMENT

Student Assistant Virtual Office Hours

- **Every Friday**, from **8am-8pm EST (UTC-5)**
- Student Assistants are available to review lab and lecture materials, walk through homework problems, or answer any other content-related questions you might have at the end of each week
- You can find a link to office hours under the "Course Materials" module on Canvas.

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CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number : **3**
  - Passcode: **5517**
- The magnitude of the vector (1 0) is:
  - 1
  - 0
  - ½
  - -1

$||\vec{x}||$

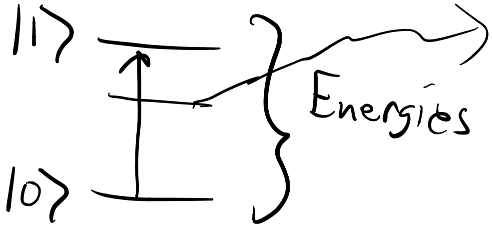
- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

QUESTIONS FROM LAST WEEK



Questions about content from last week?

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LEARNING OBJECTIVES FOR LAB 4

- Learning how to compare vectors
  - Normalization
  - Unit vectors
- Getting comfortable with vector inner products
- Relating matrices and vectors to quantum computing
  - Matrix notation
  - Multiplying matrices with vectors
  - Multiplying matrices*
  - Inverting matrices*

*Optional content

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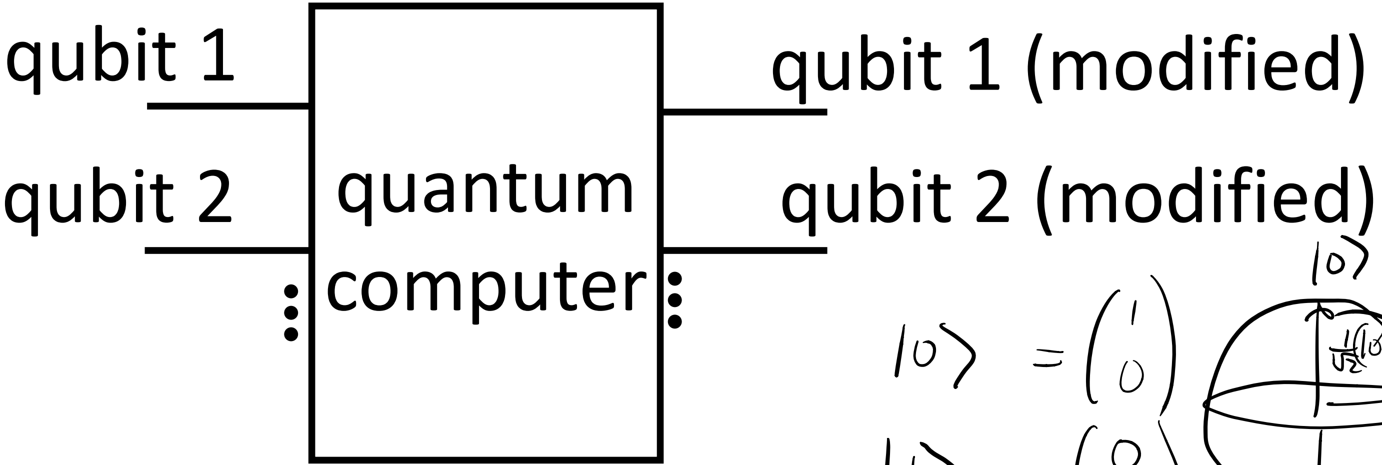
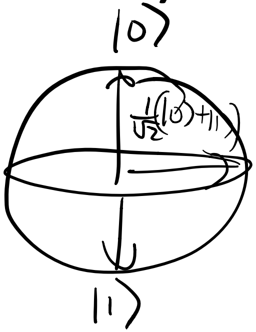
SCHEMATIC OF A QUANTUM COMPUTER



$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



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REPRESENTING QUBITS

classical bit

either 0 or 1

qubit

Superposition of 0 and 1

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VECTOR NOTATION

scalar

$$v = 2$$

vector

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Big] 2$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \Big] 4$$

4, 2 particles / qubits

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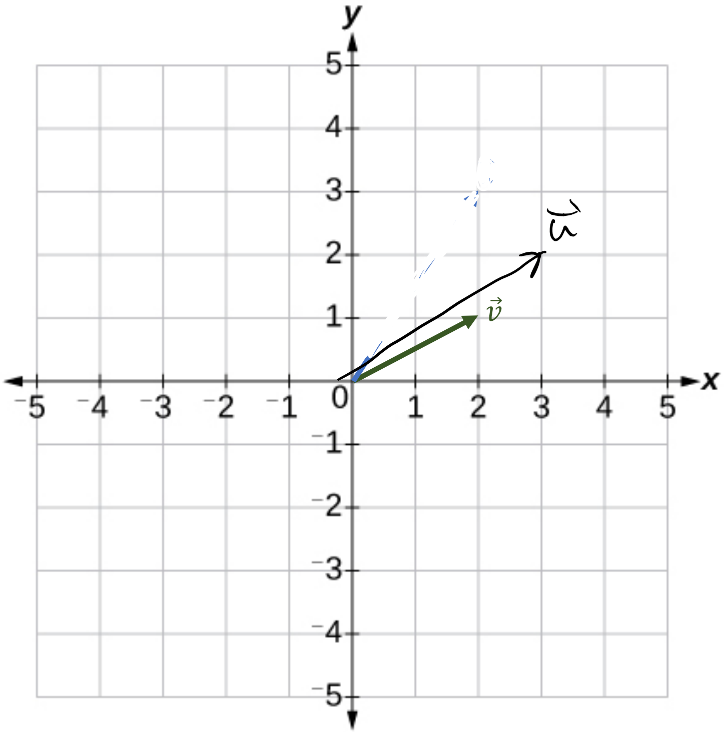
COMPARING VECTORS

Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$||\vec{u}|| = \sqrt{u_x^2 + u_y^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$

$||\vec{v}|| = \sqrt{2^2 + 1^2} = \sqrt{5}$



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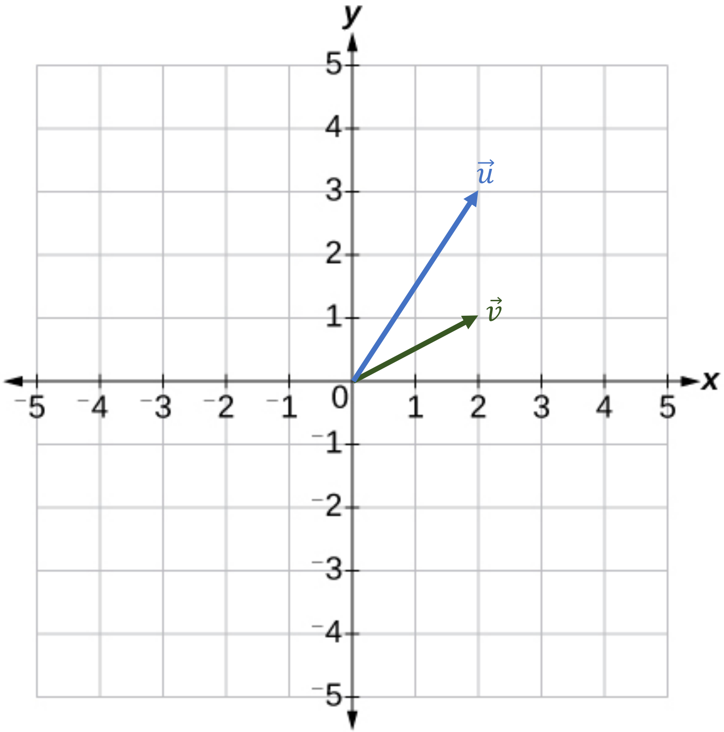
COMPARING VECTORS

Problem: Compare the vectors $\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Idea 1: We can find the lengths of the two vectors

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$



COMPARING VECTORS

Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

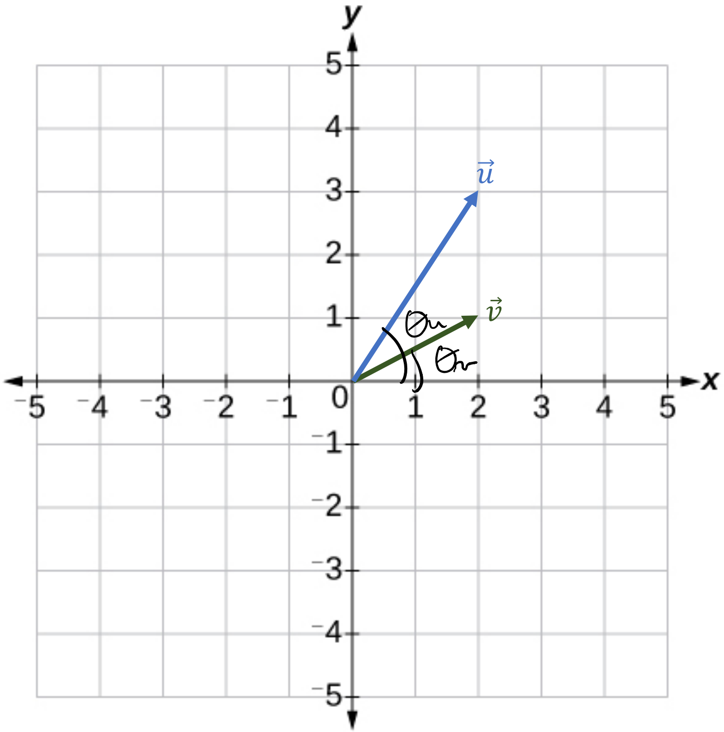
Idea 2: We can now compare the directions of the two vectors

- Divide out the lengths of both vectors to form two new **unit vectors**, $\hat{u}$ and $\hat{v}$:

$$\hat{u} = \frac{\vec{u}}{||\vec{u}||}, \quad \hat{v} = \frac{\vec{v}}{||\vec{v}||}$$

$\sqrt{13}$ $\sqrt{5}$

$$\hat{u} = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{13}} \\ \frac{3}{\sqrt{13}} \end{pmatrix} \quad \hat{v} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$



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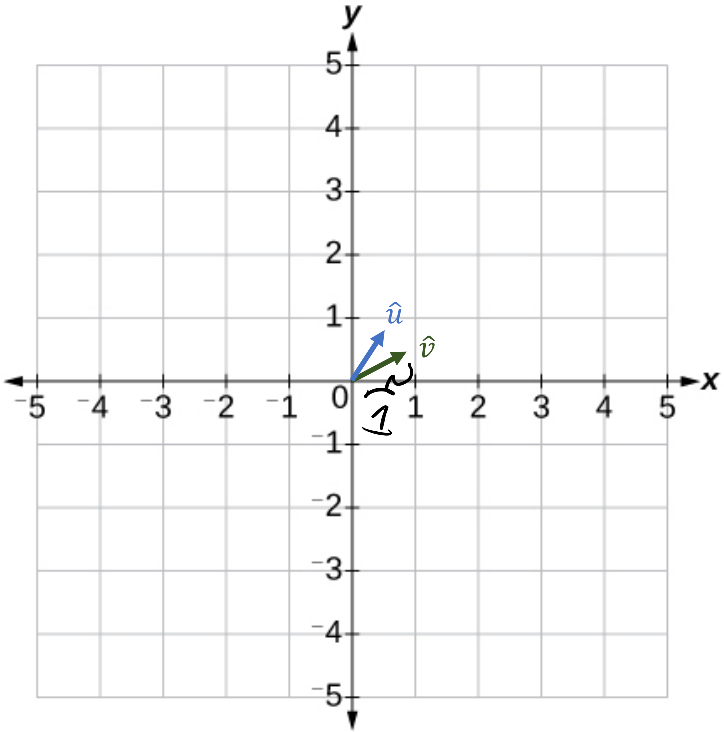
Problem: Compare the vectors

$$\vec{u} = \begin{pmatrix} \cancel{3}^{\,2} \\ \cancel{2} \end{pmatrix} \text{ and } \vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Idea 2: We can now compare the directions of the two vectors

- Divide out the lengths of both vectors to form two new **unit vectors**, $\hat{u}$ and $\hat{v}$:

$$\hat{u} = \frac{\vec{u}}{||\vec{u}||}, \quad \hat{v} = \frac{\vec{v}}{||\vec{v}||}$$



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COMPARING VECTORS

Problem: Compare the vectors

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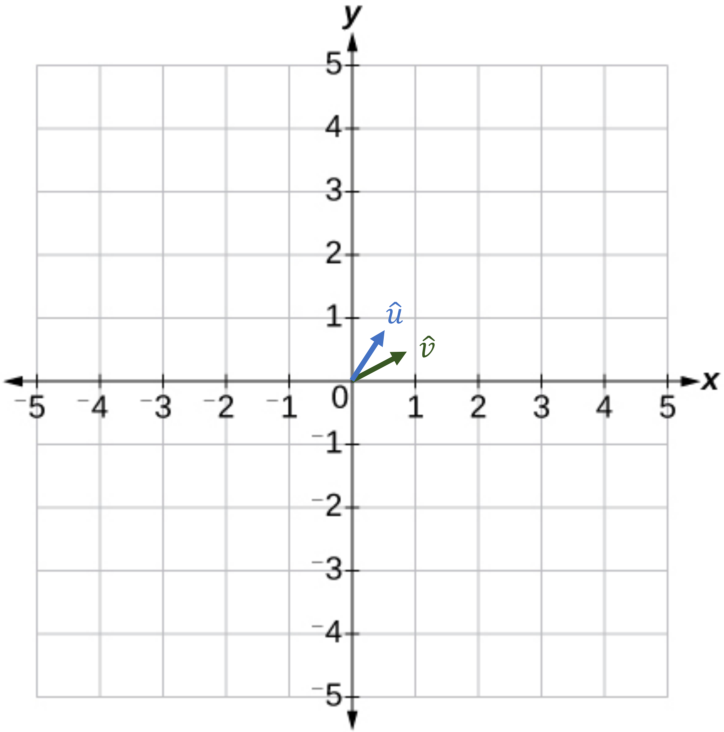
- Divide out the lengths of both vectors to form two new **unit vectors**, $\hat{u}$ and $\hat{v}$:

$$\hat{u} = \frac{\vec{u}}{||\vec{u}||}, \hat{v} = \frac{\vec{v}}{||\vec{v}||}$$

Normalization



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COMPARING VECTORS

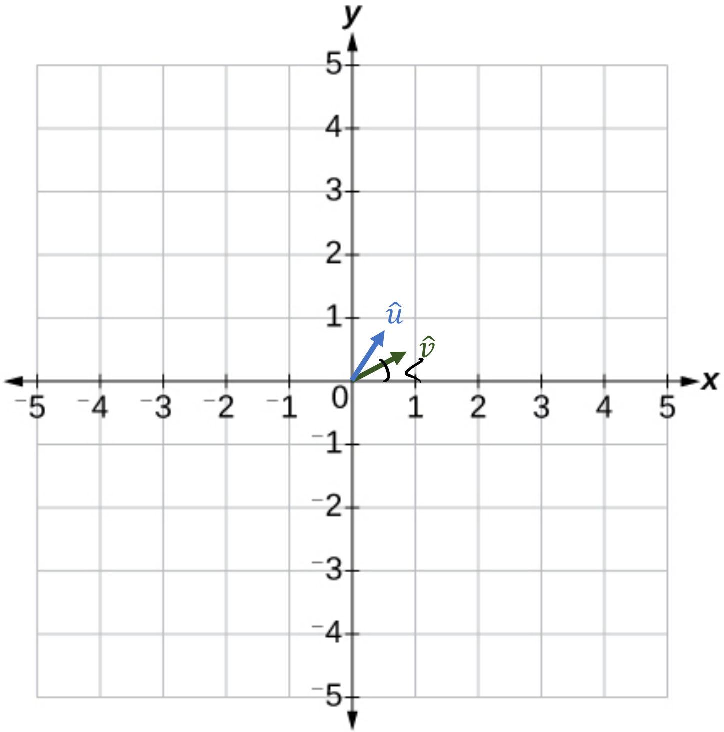
Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

$\hat{u} = \frac{\vec{u}}{||\vec{u}||}$, $\hat{v} = \frac{\vec{v}}{||\vec{v}||}$

$$\angle \vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\sqrt{u_x^2 + u_y^2} = ||\vec{u}|| \qquad \left|\left|\frac{u}{\underset{\underset{\text{scalar}}{\mathbb{R}}}{||\vec{u}||}}\right|\right| = \frac{||\vec{u}||}{||\vec{u}||} = 1$$



$P_{rob} = 1$

$\sum_i \left(P_1 \cdot \text{state}1 + P_2 \cdot \text{state}2 + \ldots \right) = 1$

Questions about content so far?

INNER PRODUCT OF TWO VECTORS

$\vec{v}^* = Re\{v\} - i Im(\vec{v})$ $\vec{v} = Re\{\vec{v}\} + i Im(\vec{v})$ $\begin{pmatrix} Re \\ Im \end{pmatrix}$

conjugate transpose

$$\langle \vec{v}, \vec{w} \rangle = \vec{v}^\dagger \vec{w} = v_1^* w_1 + \cdots + v_n^* w_n = \sum_{i=1}^{n} v_i^* w_i$$

$\vec{v} \cdot \vec{w}$

where
$\vec{v}, \vec{w} \in \mathbb{C}^n$

$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$ $v^T = (v_x \ \ v_y)$

$(n \times m)(m \times p)$

same

$\begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}^T = \begin{pmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{pmatrix}$

on diagonals stay

off-diagonals swap

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INNER PRODUCT OF TWO VECTORS

Find the inner product of

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

u_1, u_2

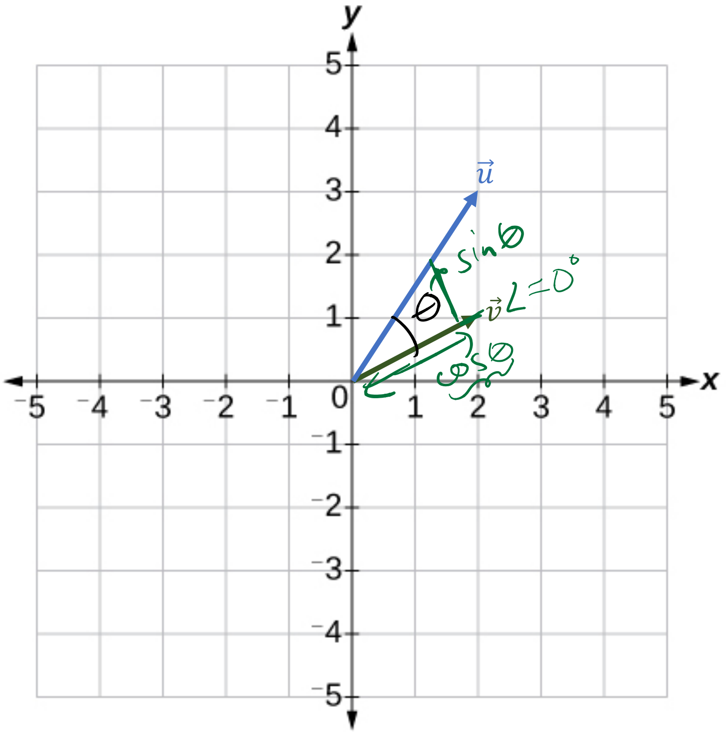
$$\langle \vec{u}, \vec{v} \rangle = u_1^* v_1 + u_2^* v_2$$

$$\langle \vec{u}, \vec{v} \rangle = 3 \cdot 2 + 2 \cdot 1 = 8$$

$$\langle \vec{u}, \vec{v} \rangle = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\sqrt{5} \quad \sqrt{13}$$

$$= \sqrt{65} \cos\theta$$



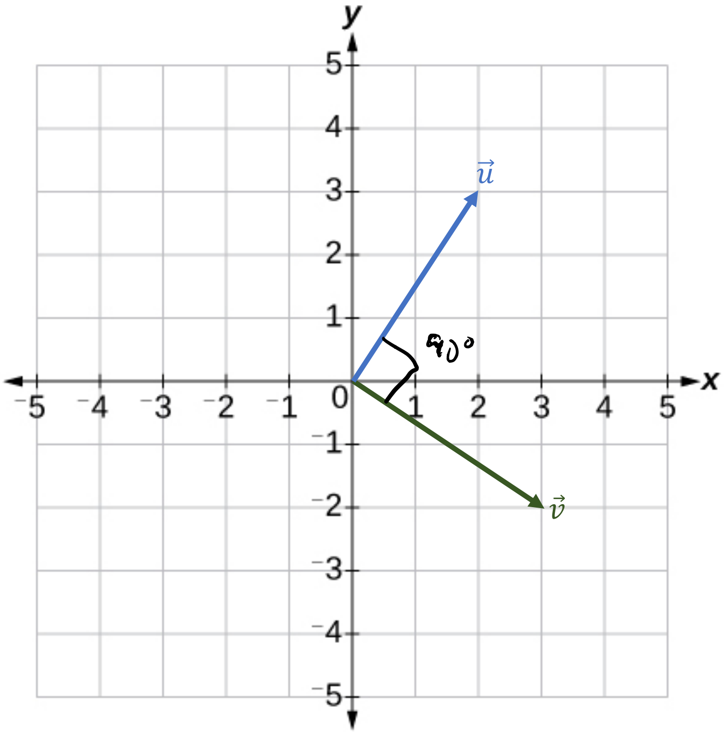
ORTHOGONALITY OF VECTORS

Find the inner product of

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$$\langle \vec{u}, \vec{v} \rangle = u_1^* v_1 + u_2^* v_2$$

$$= -6 + 6 = 0$$



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QUESTIONS

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ i \end{pmatrix}$$

Real vector　　imag vector

Basis choice

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$(10+3i)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + (5-6i)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Questions about content so far?

$$\Rightarrow \begin{pmatrix} 10+3i \\ 5-6i \end{pmatrix}$$

$|1\rangle e^{i\phi}$ phase

$$\begin{bmatrix} \hat{x}+\hat{y}+\hat{z} \\ \hat{x}-\hat{y}+\hat{z} \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} \begin{pmatrix} 10 \\ 3 \end{pmatrix} \\ \begin{pmatrix} 5 \\ -6 \end{pmatrix} \end{pmatrix}$$

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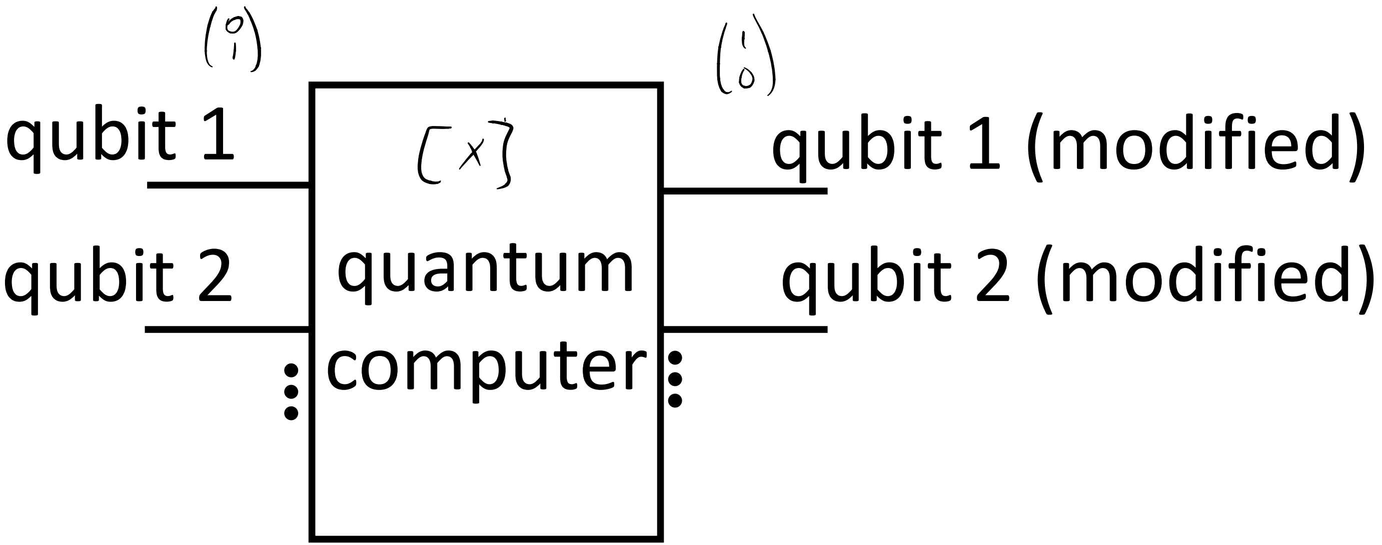
SCHEMATIC OF A QUANTUM COMPUTER



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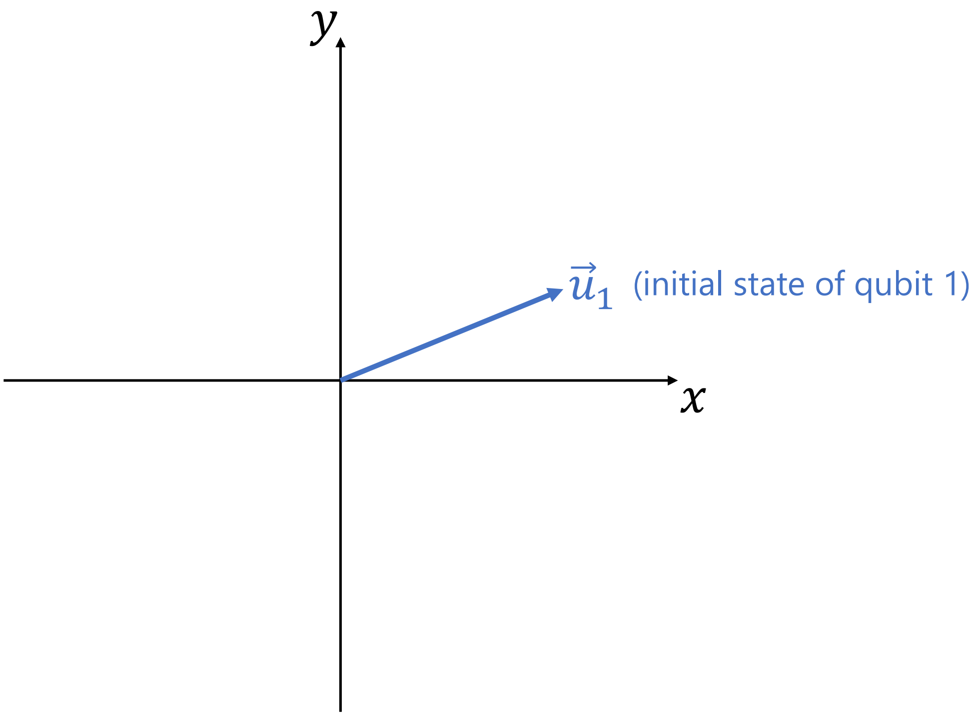
TRANSFORMING VECTORS



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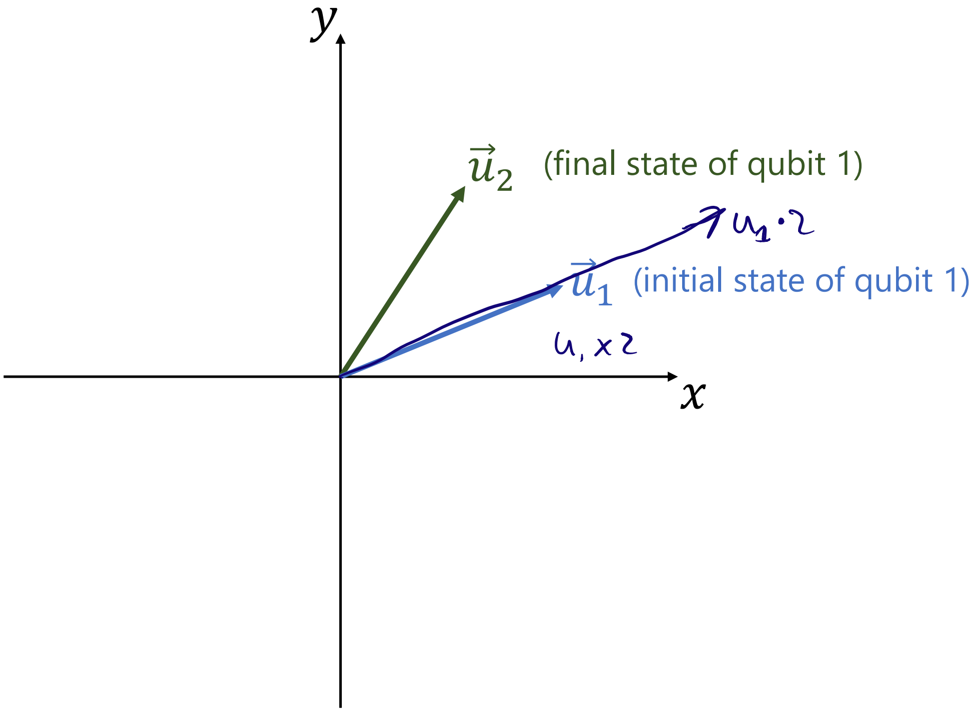
TRANSFORMING VECTORS



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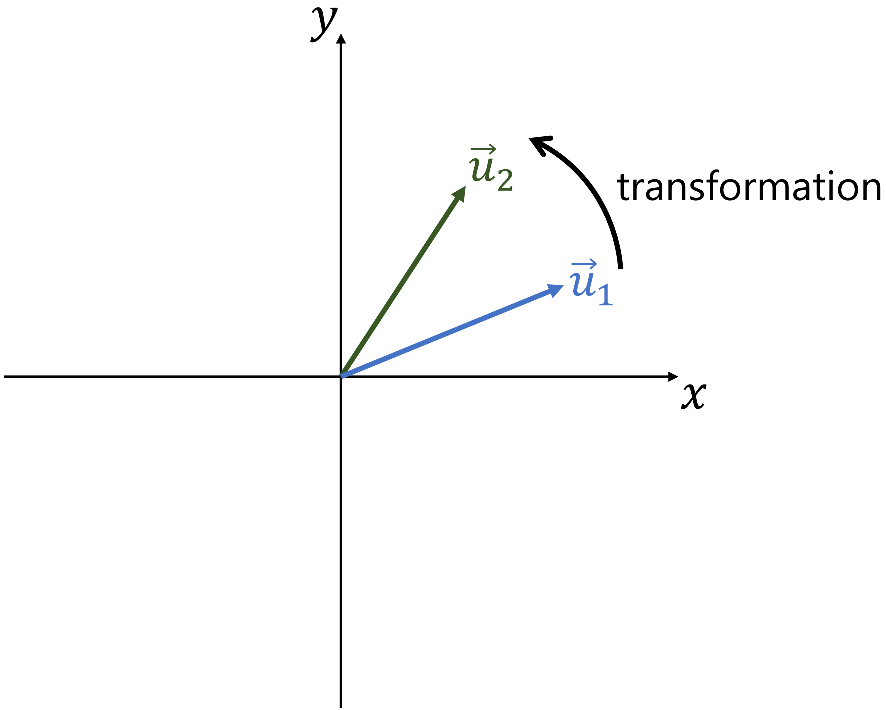
TRANSFORMING VECTORS



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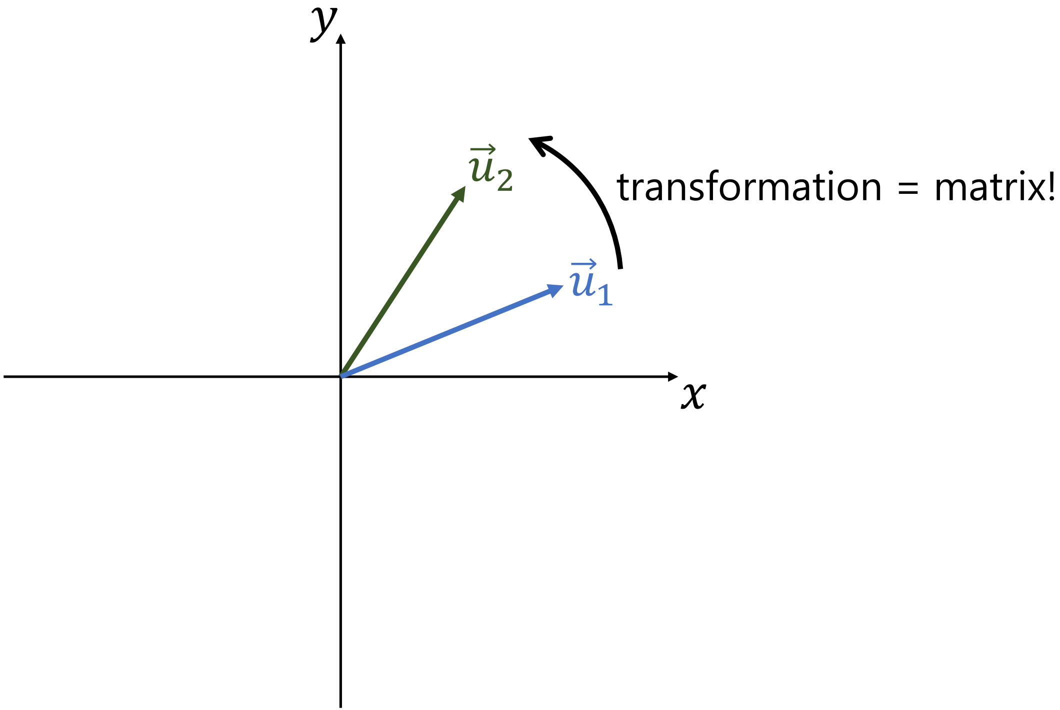
TRANSFORMING VECTORS



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MATRIX NOTATION

Vector

$$\vec{v} = \overbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}^{1} \left.\vphantom{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}\right\} 2$$

Matrix

$$\boldsymbol{S} = \overbrace{\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}}^{2} \left.\vphantom{\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}}\right\} 2$$

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USING MATRICES TO TRANSFORM VECTORS

$$A\vec{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} a_{11} * x_1 + a_{12} * x_2 + \cdots + a_{1m} * x_m \\ a_{21} * x_1 + a_{22} * x_2 + \cdots + a_{2m} * x_m \\ \vdots \\ a_{n1} * x_1 + a_{n2} * x_2 + \cdots + a_{nm} * x_m \end{pmatrix}$$

Note: The vector height must match the matrix width.

$$(n \times m) \times (m \times 1)$$

$$\downarrow$$

$$(n \times 1)$$

$(m \times m) \times (m \times 1)$

square matrix

$(2 \times 2) \times (2 \times 1)$

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USING MATRICES TO TRANSFORM VECTORS

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} a_{11} * x_1 + a_{12} * x_2 + \cdots + a_{1m} * x_m \\ a_{21} * x_1 + a_{22} * x_2 + \cdots + a_{2m} * x_m \\ \vdots \\ a_{n1} * x_1 + a_{n2} * x_2 + \cdots + a_{nm} * x_m \end{pmatrix}$$

Note: The vector height must match the matrix width.

$$(n \times m) \times (m \times 1)$$

$$\downarrow$$

$$(n \times 1)$$

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

matrix

transformation input output

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APPLYING MATRICES TO VECTORS

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = ? \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

X gate!

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

APPLYING MATRICES TO VECTORS

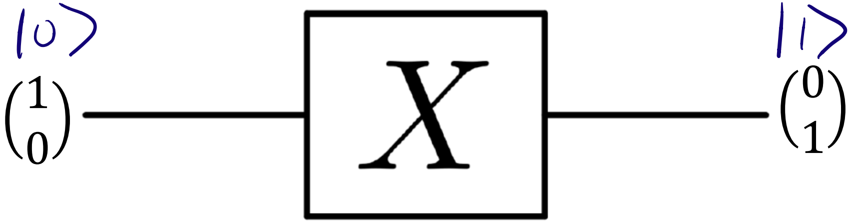
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

APPLYING MATRICES TO VECTORS

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

input qubit   output qubit



APPLYING MATRICES TO VECTORS

Multiplying constant with matrix

$$c * \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} c * a_{11} & c * a_{12} \\ c * a_{21} & c * a_{22} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =? \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

normalization

$e^{i\pi} = -1$

Hadamard gate

equal superposition

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

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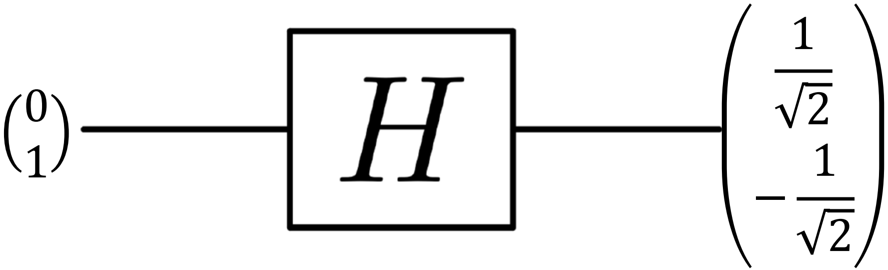
APPLYING MATRICES TO VECTORS

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$



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IMPORTANT TAKEAWAYS

- Qubit → vector
- Quantum gate → matrix
- Operating a quantum gate on a qubit → multiplying matrix with vector

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QUESTIONS

normalized

$$\left\| \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\| = \sqrt{1^2+1^2+1^2+1^2} = \sqrt{4} \qquad \frac{1}{\sqrt{4}}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Questions about content so far?

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POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
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OPTIONAL CONTENT

MATRIX MULTIPLICATION

- Qubit → vector
- Quantum gate → matrix
- Operating a quantum gate on a qubit → multiplying matrix with vector
- Operating multiple gates on a qubit → matrix multiplication and multiplying matrix with vector

MATRIX MULTIPLICATION

$$AB = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{m} a_{1i} * b_{i1} & \sum_{i=1}^{m} a_{1i} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{1i} * b_{ik} \\ \sum_{i=1}^{m} a_{2i} * b_{i1} & \sum_{i=1}^{m} a_{2i} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{2i} * b_{ik} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{m} a_{ni} * b_{i1} & \sum_{i=1}^{m} a_{ni} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{ni} * b_{ik} \end{pmatrix}$$

Remember to always check your shapes! : $\underbrace{(n \times m) \times (m \times k)} \longrightarrow (n \times k)$

Note: The first matrix width must match the second matrix height!

$(2 \times 2) \times (2 \times 2)$

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MATRIX MULTIPLICATION

$$\boldsymbol{AB} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

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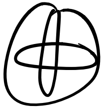
MATRIX MULTIPLICATION

$$\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{X}\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{H} \stackrel{?}{=} \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}-1\\ 1\end{pmatrix}\frac{1}{\sqrt{2}}$$

$e^{i\pi}$



$$\boldsymbol{AB} = \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{pmatrix} = \begin{pmatrix}a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22}\\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22}\end{pmatrix}$$

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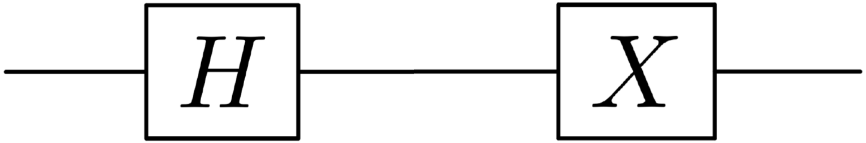
MATRIX MULTIPLICATION

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\boldsymbol{AB} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

MATRIX MULTIPLICATION



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

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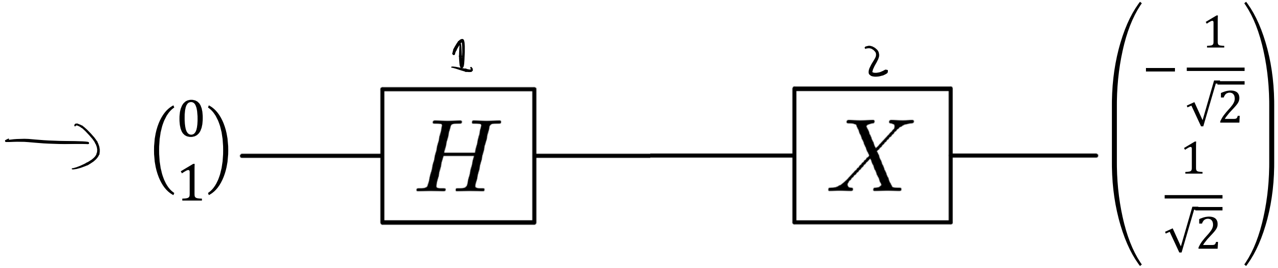
MATRIX MULTIPLICATION



$$\longrightarrow \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{X} \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\text{state}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

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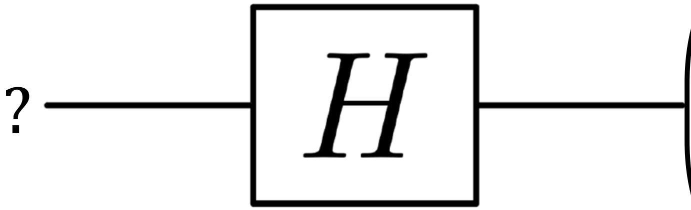
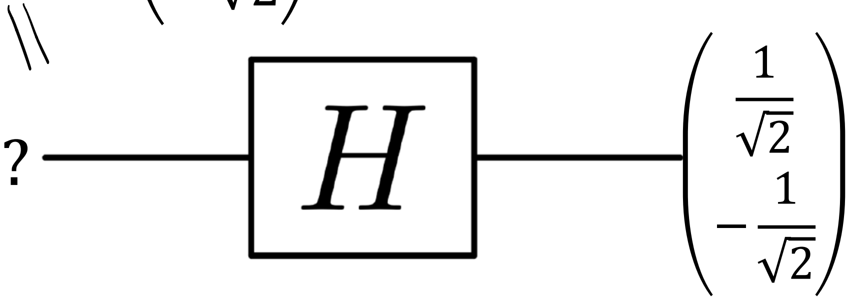


$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

final state

MATRIX INVERSION

$$\underset{\boldsymbol{H}}{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}\begin{pmatrix}?\\ ?\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$



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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

$$\vec{?} = \vec{y}/\boldsymbol{H}$$

MATRIX INVERSION

$$H^{-1}\boldsymbol{H}\vec{?} = H^{-1}\vec{y}$$

$$\Downarrow$$

inverse

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

$$H^{-1}H = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

diagonal 1's

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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \dfrac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

MATRIX INVERSION

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

scalars

$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$; $\boldsymbol{H}^{-1} = ?$ $\frac{1}{-\frac{1}{2}-\frac{1}{2}}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

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MATRIX INVERSION

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad ; \boldsymbol{H}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

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MATRIX INVERSION

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

$$\begin{pmatrix} ? \\ ? \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\cdot 1 - \frac{1}{\sqrt{2}}\cdot 1 \\ +\frac{1}{\sqrt{2}}\cdot 1 - \left(-\frac{1}{\sqrt{2}}\right)\cdot 1 \end{pmatrix}\cdot\frac{1}{\sqrt{2}}$$

$$= \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}\frac{1}{\sqrt{2}} = \begin{pmatrix} 0 \\ \frac{2}{2} \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

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MATRIX INVERSION

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

$$\begin{pmatrix} ? \\ ? \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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