

Universität Stuttgart

# B-Splines for Sparse Grids: Algorithms and Application to Higher-Dimensional Optimization

Vom Stuttgarter Zentrum für Simulationswissenschaften der  
Universität Stuttgart zur Erlangung der Würde eines Doktors der  
Naturwissenschaften (Dr. rer. nat.) genehmigte Abhandlung

Vorgelegt von

**Emmett Brown**  
aus Hill Valley, California

Hauptberichter: Prof. Dr. Albert Einstein

Mitberichter: Prof. Dr. Blaise Pascal  
Prof. Dr. Marie Curie  
Prof. Dr. Charles Darwin

Tag der mündlichen Prüfung: 21. Oktober 2015

Institute for Advanced Time Travel

2015

Draft v206 (Apr 02, 9:51am)

Commit 23febca\* (Apr 02, 9:51am)

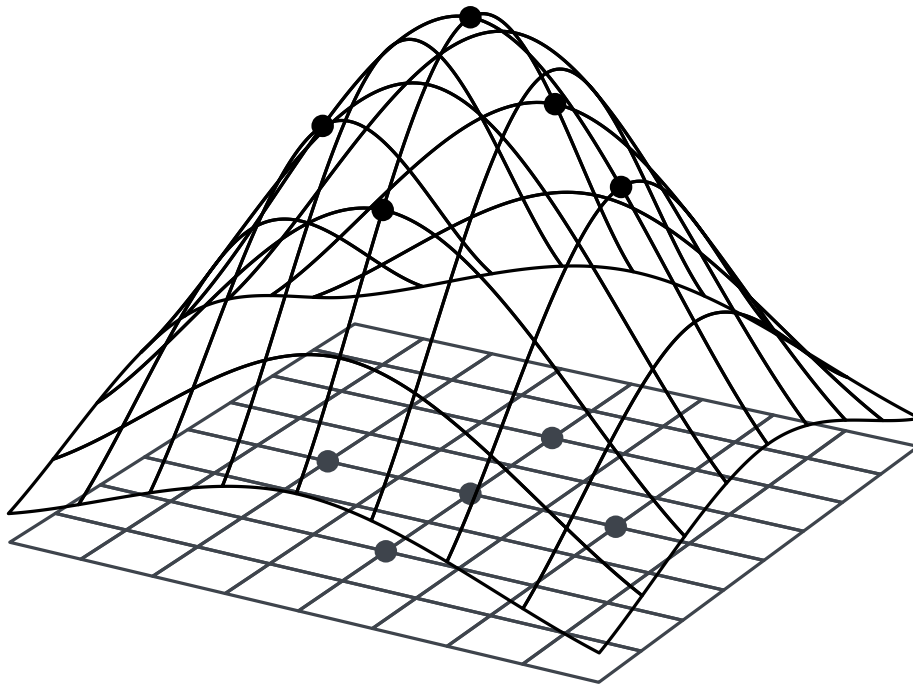
1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

*Emmett Brown*

# INVENTING TIME TRAVEL

Theory and Applications of the  
Flux Capacitor



University of Stuttgart  
Germany

SimTech

IPVS



Draft v206 (Apr 02, 9:51am)

Commit 23febca\* (Apr 02, 9:51am)

“ *It seems that it is not enough to have a good idea or insight. One needs, like Schoenberg, the appreciation and courage to develop the idea systematically, make its objects mathematically presentable by giving them names, and give them much exposure in many papers.*

— Carl de Boor [Boo16]

**COVER FIGURE** A regular sparse grid in two dimensions (*dots*) as a subset of the full grid (*mesh, bottom*) with a bicubic B-spline (*mesh, top*).

Compiled as version v206 on April 2, 2018 at 9:51am.

Committed as 23febca\* on April 2, 2018 at 9:51am.



Copyright © 2015 Emmett Brown. This work is licensed under the *Creative Commons Attribution-ShareAlike 4.0 International License*.

Although this thesis was written with uttermost care, it cannot be ruled out that it contains errors. Please send any corrections and mistakes to [thesis@example.com](mailto:thesis@example.com).

Draft v206 (Apr 02, 9:51am)

Commit 23febca\* (Apr 02, 9:51am)

# Contents

		1
		2
		3
		4
		5
		6
		7
		8
		9
Lists of Figures, Tables, Algorithms, and Theorems	7	10
Abstract/Kurzzusammenfassung	9	11
Preface	11	12
		13
1 Introduction	13	14
1.1 Bla . . . . .	13	15
		16
2 The Flux Capacitor	19	17
		18
3 Conclusion	23	19
		20
A Proofs	25	21
		22
B Bibliography	27	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35
		36
		37

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

# Lists of Figures, Tables, Algorithms, and Theorems

## List of Figures

- 1.1 This is a test caption. . . . . 15
- 1.2 This is a test caption. This is a test caption. This is a test caption. This is a  
test caption. This is a test caption. This is a test caption. This is a test caption.  
This is a test caption. This is a test caption. . . . . 15

## List of Tables

- 1.1 This is a test table. . . . . 14

## List of Algorithms

- 1.1 Approximative Auswertung von Linearkombinationen auf dünnen Gittern,  
Zeilen 5, 6, 9, 10 nicht für stückweise lineare Basisfunktionen, *input*: Gitter  
 $X = \{\mathbf{x}_i\}_i$ , Koeffizienten  $\boldsymbol{\alpha} = (\alpha_i)_i$ , Auswertungspunkt  $\mathbf{x} \in [0, 1]^d$ , aktuelle  
Dimension  $t \in \{1, \dots, d\}$  (anfangs 1), Level und Index  $(\ell, j)$  des aktuellen  
Punkts (für randlose Gitter anfangs  $(\mathbf{e}, \mathbf{e})$ ) und aktuelles Produkt  $b$  von 1D-  
Auswertungen (anfangs 1), *output*:  $a \approx \tilde{f}(\mathbf{x}) = \sum_{k=1}^N \alpha_k \varphi_k(\mathbf{x})$  (für stück-  
weise lineare Funktionen sogar  $a = \tilde{f}(\mathbf{x})$ ) . . . . . 16

## List of Theorems

- 1.1 Theorem (TODO Theorem) . . . . . 14
- 1.2 Lemma (TODO Lemma) . . . . . 16

1.3	Definition (TODO Definition) . . . . .	16
-----	--	----



# Abstract/Kurzzusammenfassung

## Abstract

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like „Huardest gefburn“? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

## Kurzzusammenfassung

**TODO: write** Dies hier ist ein Blindtext zum Testen von Textausgaben. Wer diesen Text liest, ist selbst schuld.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . Der Text gibt lediglich den Grauwert der Schrift an  $E = mc^2$ . Ist das wirklich so? Ist es gleichgültig, ob ich schreibe: „Dies ist ein Blindtext“ oder „Huardest gefburn“? Kjift – mitnichten! Ein Blindtext bietet mir wichtige Informationen.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . An ihm messe ich die Lesbarkeit einer Schrift, ihre Anmutung, wie harmonisch die Figuren zueinander stehen und prüfe, wie breit oder schmal sie läuft.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . Ein Blindtext sollte möglichst viele verschiedene Buchstaben enthalten und in der Originalsprache gesetzt sein.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ . Er muss keinen Sinn ergeben, sollte aber lesbar sein.  $d\Omega = \sin\vartheta d\vartheta d\varphi$ . Fremdsprachige Texte wie „Lorem ipsum“ dienen nicht dem eigentlichen Zweck, da sie eine falsche Anmutung vermitteln.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

# Preface

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

Stuttgart, October 21, 2015

Emmett Brown

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

# 1 Introduction

“ Ah, Jesus Christ! Jesus Christ, Doc, you  
disintegrated Einstein!

— Marty McFly

TODO: write

Citations: [Boo72]

Hello World! Hello World!

Now I'm citing all references for demonstration purposes. TODO: don't cite everything

Here are some umlauts: äöüß

I'm testing the glossary: non-uniform rational B-splines (NURBS) are very cool.



## 1.1 Bla

This is TODO: write defined TODO: write as  $a := 2b$ . This is the function  $f$  (which is defined as  $y =: f(x)$ ).

Header 1	Header 2	Header 3	Header 4
bla	bla	bla	bla
bla	bla	bla	bla
bla	bla	bla	bla

TABLE 1.1 This is a test table.

(1.1)
$$X \times Y$$

(1.2)
$$A \cdot \boldsymbol{x} = \boldsymbol{b}$$

(1.3)
$$\min_{\boldsymbol{x} \in [0,1]} \int_{\Omega} f(\boldsymbol{x}, \boldsymbol{y}) d\boldsymbol{y}$$

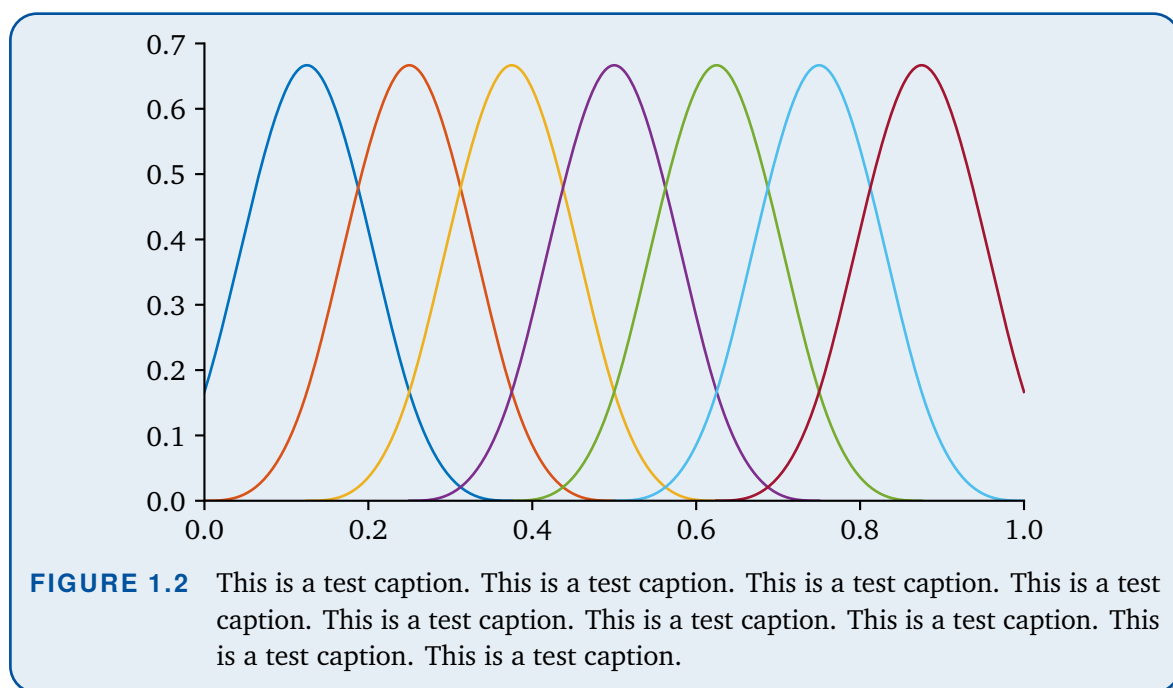
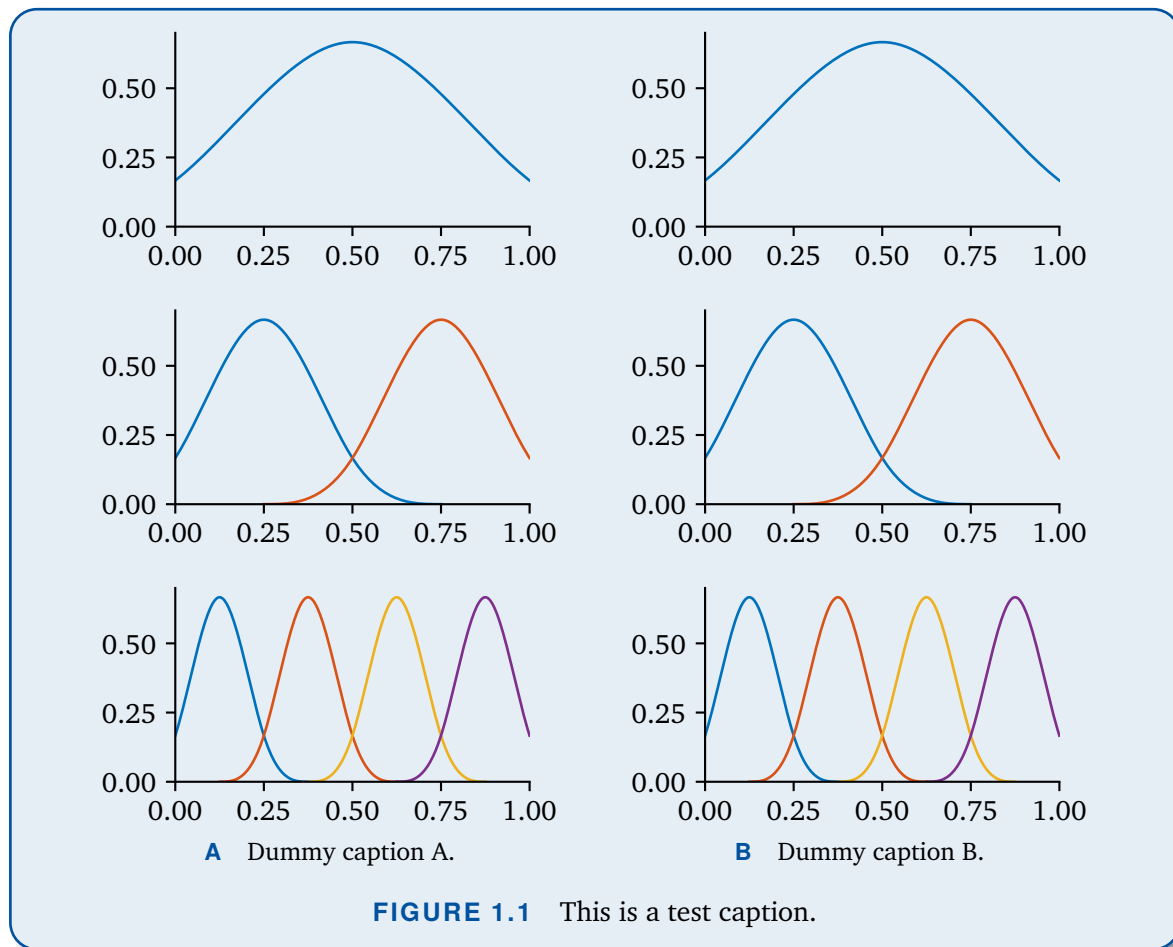
(1.4)
$$4(a+b)f(x)g(x)h(x)p(x)(c+d)fghf'g'h'$$

(1.5)
$$f(x)\cos(x)g(x)$$

(1.6)
$$\mathrm{f}(x)\cos(x)g(x)$$

Tab. 1.1  
Fig. 1.1  
Fig. 1.1a  
Fig. 1.1b  
Fig. 1.2  
Alg. 1.1

**THEOREM 1.1** (TODO Theorem)  
*TODO: write* Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^nb}$ .



```

1 function  $a = \text{GetAffectedBasisFunctions}(X, \alpha, x, t, \ell, j, b)$ 
2 if  $x_{\ell,j} \notin X$  then return 0  $\rightsquigarrow$  nichts tun, falls Gitterpunkt nicht vorhanden
3 if  $t = d$  then
4    $a \leftarrow \alpha_{\ell,j} \cdot (b \cdot \varphi_{\ell_d,j_d}(x_d))$   $\rightsquigarrow$  letzte Dimension: Summanden zu Ergebnis addieren
5   if  $x_{\ell,j}^{(\text{rn}(d))} \in X$  then  $a \leftarrow a + \alpha_{\ell,j}^{(\text{rn}(d))} \cdot (b \cdot \varphi_{\ell_d,j_d}^{(\text{rn}(d))}(x_d))$ 
6   if  $x_{\ell,j}^{(\text{ln}(d))} \in X$  then  $a \leftarrow a + \alpha_{\ell,j}^{(\text{ln}(d))} \cdot (b \cdot \varphi_{\ell_d,j_d}^{(\text{ln}(d))}(x_d))$ 
7 else
8    $a \leftarrow \text{GABF}(X, \alpha, x, t+1, \ell, j, b \cdot \varphi_{\ell_t,j_t}(x_t))$   $\rightsquigarrow$  nächste Dimension
9   if  $x_{\ell,j}^{(\text{rn}(t))} \in X$  then  $a \leftarrow a + \text{GABF}(X, \alpha, x, t+1, \ell, j^{(\text{rn}(t))}, b \cdot \varphi_{\ell_t,j_t}^{(\text{rn}(t))}(x_t))$ 
10  if  $x_{\ell,j}^{(\text{ln}(t))} \in X$  then  $a \leftarrow a + \text{GABF}(X, \alpha, x, t+1, \ell, j^{(\text{ln}(t))}, b \cdot \varphi_{\ell_t,j_t}^{(\text{ln}(t))}(x_t))$ 
11  if  $x_t > j_t h_{\ell_t}$  then  $a \leftarrow a + \text{GABF}(X, \alpha, x, t, \ell^{(\text{rc}(t))}, j^{(\text{rc}(t))}, b)$   $\rightsquigarrow$ 
    nächster Level
12  else  $a \leftarrow a + \text{GABF}(X, \alpha, x, t, \ell^{(\text{lc}(t))}, j^{(\text{lc}(t))}, b)$ 
13  return  $a$ 

```

**ALGORITHM 1.1** Approximative Auswertung von Linearkombinationen auf dünnen Gittern, Zeilen 5, 6, 9, 10 nicht für stückweise lineare Basisfunktionen,  
input: Gitter  $X = \{x_i\}_i$ , Koeffizienten  $\alpha = (\alpha_i)_i$ , Auswertungspunkt  $x \in [0,1]^d$ , aktuelle Dimension  $t \in \{1, \dots, d\}$  (anfangs 1), Level und Index  $(\ell, j)$  des aktuellen Punkts (für randlose Gitter anfangs  $(e, e)$ ) und aktuelles Produkt  $b$  von 1D-Auswertungen (anfangs 1),  
output:  $a \approx \tilde{f}(x) = \sum_{k=1}^N \alpha_k \varphi_k(x)$  (für stückweise lineare Funktionen sogar  $a = \tilde{f}(x)$ )

**LEMMA 1.2** (TODO Lemma)

TODO

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a \sqrt[n]{b} = \sqrt[n]{a^n b}$ .

**DEFINITION 1.3** (TODO Definition)

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text,



you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{i=n} x_i = \frac{x_1 + x_2 + \dots + x_n}{n}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\int_{-\infty}^\infty e^{-ax^2} dx \int_{-\infty}^\infty e^{-ay^2} dy} = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the

alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\sum_{k=0}^{\infty} a_0 q^k = \lim_{n \rightarrow \infty} \sum_{k=0}^n a_0 q^k = \lim_{n \rightarrow \infty} a_0 \frac{1 - q^{n+1}}{1 - q} = \frac{a_0}{1 - q}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

# 2 The Flux Capacitor

“ If my calculations are correct, when this baby hits 88 miles per hour...you’re gonna see some serious shit.

— Emmett Brown

TODO: write

TODO: write Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

TODO: write Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the

alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$a\sqrt[n]{b} = \sqrt[n]{a^n b}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{i=n} x_i = \frac{x_1 + x_2 + \dots + x_n}{n}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need

for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\int_{-\infty}^\infty e^{-ax^2} dx \int_{-\infty}^\infty e^{-ay^2} dy} = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

# 3 Conclusion

**TODO: write** Hello, here is some text without a meaning. This text should show what a printed text will look like at this place.  $\sin^2(\alpha) + \cos^2(\beta) = 1$ . If you read this text, you will get no information  $E = mc^2$ . Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look.  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ . This text should contain all letters of the alphabet and it should be written in of the original language.  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ . There is no need for special contents, but the length of words should match the language.  $a\sqrt[n]{b} = \sqrt[n]{a^n b}$ .

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37



# A Proofs

This is an appendix chapter.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

# Bibliography

- [Boo16] **Boor**, C. de: *A comment on Ewal Quak's "About B-splines"*. Journal of Numerical Analysis and Approximation Theory 45.1 (2016), pp. 84–86.
- [Boo72] **Boor**, C. de: *On calculating with b-splines*. Journal of Approximation Theory 6.1 (1972), pp. 50–62. ISSN: 0021-9045. 10.1016/0021-9045(72)90080-9.
- [Bun04] **Bungartz**, H.-J.; **Griebel**, M.: *Sparse grids*. Acta Numerica 13 (2004), pp. 147–269. ISSN: 1474-0508. 10.1017/S0962492904000182.

All URLs have last been checked on October 21, 2015.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37