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PHY1002 Physics Laboratory

Lab Report

Experiment 7

Resonance Air Column

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Contents

Introduction

1.1 Physical Theory

Standing Waves

The interference of two identical sinusoidal waves moving in opposite directions produces standing waves. Standing waves are created in a vibrating string when a wave is reflected from an end of the string so that the returning wave interferes with the original wave. Standing waves also occur when a sound wave is reflected from the end of a tube. For a string with fixed ends, the standing wave is given by

$$y(x,t) = [2y_m \sin(kx)]\cos(\omega t) \tag{1.1}$$

The absolute value of the quantity $2y_m \sin(kx)$ in the brackets of (1.1) can be viewed as the amplitude of oscillation of the string element that is located at position x.

Standing waves are characterized by fixed locations of zero displacement called nodes and fixed locations of maximum displacement called antinodes. The amplitude is

zero for values of kx that give $sin(kx) = 0$. Those values are

$$kx = n\pi, for\ n = 0, 1, 2, \ldots. \tag{1.2}$$

Substituting $k = 2\pi/\lambda$ in this equation and rearranging, we get

$$x = n\lambda, for\ \ n = 0, 1, 2, \ldots. \tag{1.3}$$

as the positions of zero amplitude—the nodes—for the standing wave of equation (1.1). Note that adjacent nodes are separated by $\frac{1}{2}\lambda$, half a wavelength.

The amplitude of the standing wave has a maximum value of $2y_m$, which occurs for values of kx that give $|sin(kx)| = 1$.Those values are

$$kx = \left(n + \frac{1}{2}\right)\pi, for\ n = 0, 1, 2, \ldots. \tag{1.4}$$

Substituting $k = 2\pi/\lambda$ n this equation and rearranging, we get

$$x = \left(n + \frac{1}{2}\right)\lambda, for \ n = 0, 1, 2, \dots. \tag{1.5}$$

as the positions of maximum amplitude—the antinodes—of the standing wave of equation (1.1). Antinodes are separated by $\frac{1}{2}\lambda$ and are halfway between nodes. A standing wave on a string has nodes—points where the string does not move—and antinodes—points where the string vibrates up and down with a maximum amplitude. Analogously, a standing sound wave has displacement nodes—points where the air does not vibrate very much—and displacement antinodes—points where the amplitude of the air vibration is a maximum. Pressure nodes and antinodes also exist within the waveform. In fact, pressure nodes occur at displacement antinodes and pressure antinodes occur at displacement nodes.

Reflection of the sound wave occurs at both open and closed tube ends. If the end of the tube is closed, the air has nowhere to go, so a displacement node (a pressure antinode) must exist at a closed end. If the end of the tube is open, the pressure stays very nearly at room pressure, so a pressure node (a displacement antinode) exists at an open end of the tube.

Resonance

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a resonant frequency, and the corresponding standing wave pattern is an oscillation mode.

The resonance modes also depend on whether the ends of the tube are open or closed.

For an open tube (a tube open at both ends), resonance occurs when the wavelength of the wave (λ) satisfies the condition:

$$L = \frac{n\lambda}{2}, n = 1, 2, 3, 4 \dots. \tag{1.6}$$

where L is the tube length. These wavelengths allow a standing wave pattern such that a pressure node (displacement antinode) of the wave pattern exists naturally at each end of the tube. Another way to characterize the resonance states is to say that an integral number of half wavelengths fits between the ends of the tube. The first four resonance modes for open tubes are shown below. The first resonance mode (n = 1) is called the fundamental. Successive resonance modes are called overtones. The representation in each case shows relative displacement. A displacement node is marked **N** and a displacement antinode is marked **A**.

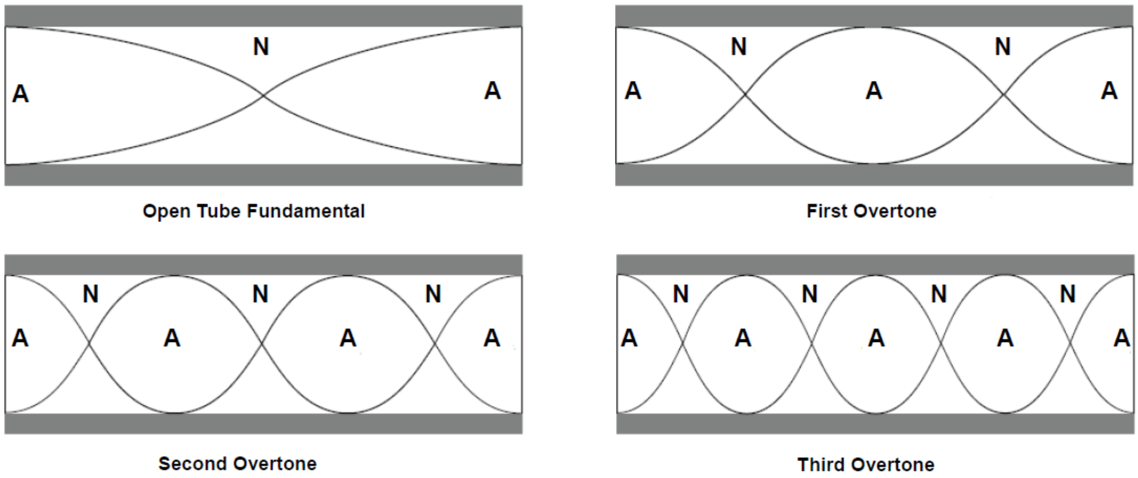

__Figure 1__ First four resonance modes for open tubes.

For a closed tube, resonance occurs when the wavelength of the wave (λ) satisfies the condition:

$$L = \frac{n\lambda}{4}, n = 1, 3, 5, 7, \dots . \quad (1.7)$$

where L is the length of the air column. These wavelengths allow a standing wave pattern such that a pressure node (displacement antinode) occurs naturally at the open end of the tube and a pressure antinode (displacement node) occurs naturally at the closed end of the tube. As for the open tube, each successive value of n describes a state in which one more half wavelength fits between the ends of the tube. The first four resonance modes for closed tubes are shown below. The first resonance mode (n = 1) is called the fundamental. Successive resonance modes are called overtones. The representation in each case shows relative displacement. A displacement node is marked **N** and a displacement antinode is marked **A**.

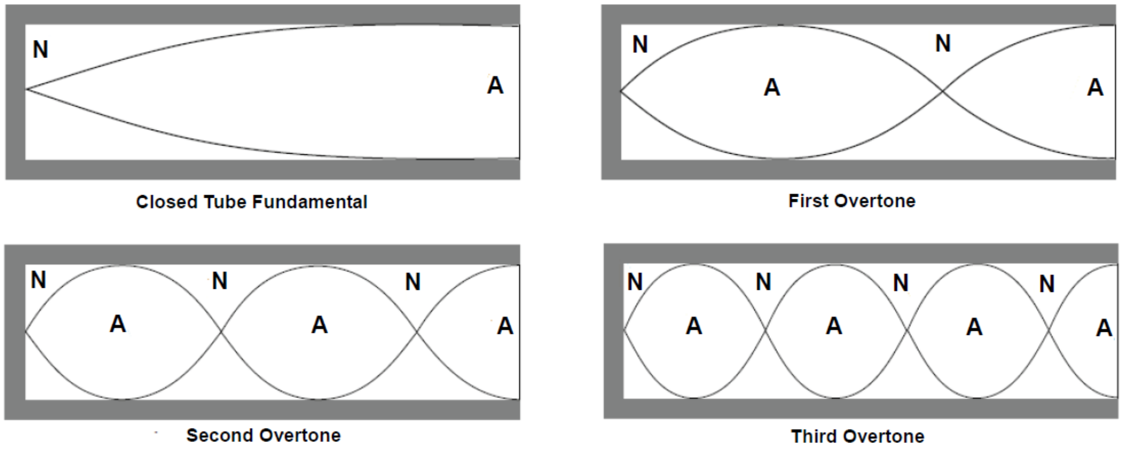

***Figure 2** First four resonance modes for closed tubes.*

1.2 Experiment Principle

In the experiment, we will explore the relationship between the wavelength, wave speed, and frequency of sound waves both in a closed tube and an open tube.

A resonating air column in a tube with one end open and the other end closed will have a node at the closed end and an anti-node at the open end. A node represents an area where the displacement of the air is a minimum (zero), and an anti-node represents an area where the displacement of the air is a maximum. If the air column is resonating in the fundamental mode (lowest possible frequency) it will have no other nodes or anti-nodes. Thus, for an air column in such tube, the length of the resonating air column, ***L***, and the wavelength, λ, are related by:

$$\lambda = 4L \tag{1.8}$$

For all types of waves, the relationship between the frequency and the velocity of the wave is:

$$\lambda f = v \tag{1.9}$$

For a resonating air column in a tube, v is the speed at which sound travels through the air, and f is the frequency of the sound.

In this experiment, the sound frequency is the frequency of the Sine Wave Generator.

Combining equations *1* and *2* yields:

$$L = \frac{v}{4}f \qquad (1.10)$$

The length of the air column is inversely proportional to the fundamental frequency. Equation (1.10) will be used to find the velocity of the air travelling in the tube and the percent difference with the actual value will be determined by:

$$\%Difference = \frac{Measured - Actual}{Actual} \times 100\% \qquad (1.11)$$

A resonating tube with both ends open will always have an anti-node at another end, and at least one node in between. The number of nodes is related the wavelength and the harmonic.

The wave at the ends of the tube behaviors differently. Due to the behavior, the effective length of the tube is slightly longer than the measured length. The following empirical formulas give an approximate description of the resonance requirements for standing waves in a tube.

For an open tube,

$$L + 0.6d = \frac{n\lambda}{2}, n = 1, 2, 3, 4 \ldots. \qquad (1.12)$$

where L is the length of the tube and d is the diameter of the tube.

For a closed tube,

$$L + 0.3d = \frac{n\lambda}{4}, n = 1, 3, 5, 7 \ldots. \qquad (1.13)$$

where L is the length of the tube and d is the diameter of the tube.

Methods

2.1 *Set up*

Two adjustable feet were put on each end of the resonance air column. Thumbscrews were applied to hold the feet in place, but not very tight. The Mini Speaker was placed at one end of the resonance air column with its housing in the end of the tube. The UI-5000 850 Universal Interface is then connected to the Mini Speaker. An acoustic sensor was then placed besides the speaker with its sensing element facing the air column. A block was put under the acoustic sensor to keep it at the similar height to the mini speaker. In the software, the amplitude was set to 3V so that the sound could be heard.

2.2 Experimental Procedure

2.2.1 Resonance in a Closed Tube

The piston was placed at the end of the air column that was opposite to the mini speaker. The position was adjusted so that the initial length of the air column is about 110 cm. Click on the “Signal Generator” in Capstone. The “Sweep Type” was set to be “Single” and “Wave Form” was “Positive Ramp”. “Duration” was set to be “50s” so that 1s represented 1Hz, “Initial Frequency” was 50Hz and “Final Frequency” was 100Hz. Click “Record” so that the mini speaker started working at the same time when the program started recording data. The frequency when resonance occurred was determined on the data figure. The same procedure was repeated seven times with the air column decreasing 10cm every time. When the length of the air column became shorter, the frequency when resonance occurred would increase and exceeded 100Hz. Therefore, the initial frequency and final frequency should be adjusted appropriately in order to capture the resonance frequency. The graph of Air Column Length versus Inverse Frequency (L vs.1/f) was plotted. both the slope and the y-intercept of the best-fit line was determined through this data. After that, according to equation (1.10), the speed of sound was calculated. This value then was compared to the actual speed of sound (340m/s) using equation (1.11).

***Further inspections**

After finishing the above procedure, the frequency was set to be 230 Hz and kept unchanged. The piston was placed at the end of the tube so that the air column was 110 cm again. Click on the mini speaker and started recording. Move the piston to shorten the length of the air tube until the first resonance occurred. The length of the air in the tube was recorded was recorded to represented the position of the first node. The same procedure was repeated again to record the position of the second node. Two times of the distance between the two nodes was the wavelength of the wave. According to equation (1.9), the speed of the sound was calculated and compared to the previous results.

2.2.2 Resonance in an open Tube

The piston was removed out of the tube with other equipment remaining unchanged. The frequency was increased slowly. When the frequency was increasing, the frequencies when the fundamental, second, and third harmonic occurred were recorded. With the frequency of the fundamental harmonic, the velocity obtained in the previous experiment, the effective length of the tube was calculated by using equation (1.6).

Next, set the frequency to be the fundamental frequency. The piston was placed at the end of the tube so that the tube was closed. The frequency was then slowly decreasing until the fundamental resonance of the closed tube occurred. Record the frequency and the ratio of the open-tube frequency and closed-tube frequency was calculated.

Raw Data

3.1 Raw Data Figures

3.1.1 Resonance in a Closed Tube

The graphs of Sound Intensity (mV) vs. Time (t) for each length of air column are shown in the following figures, with 1 second standing for 1 Hz. The time (frequency) when resonance occurred is highlighted in each figure. Besides, the frequency is determined to the nearest 1Hz.

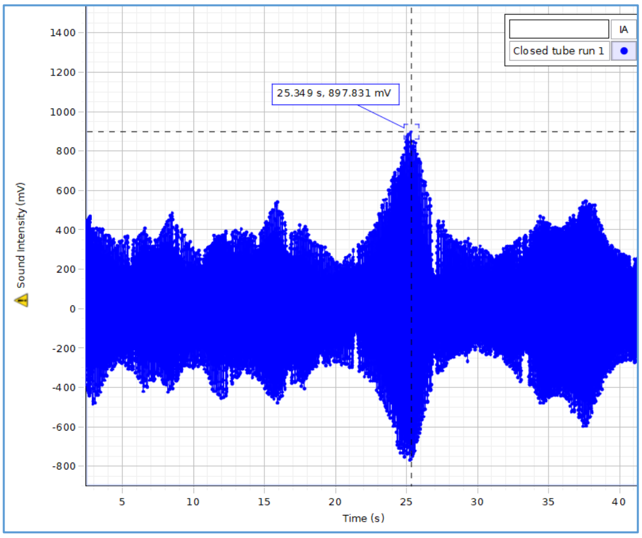

Figure 3 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 110cm, the initial frequency is 50Hz and the final frequency is 100Hz.

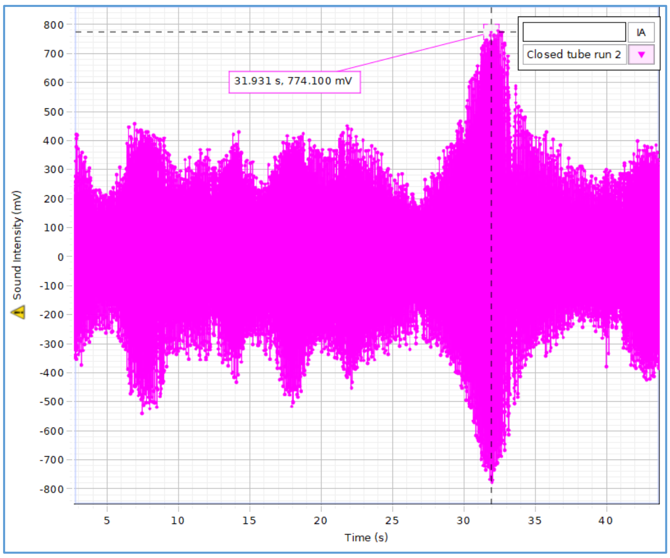

Figure 4 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 100cm, the initial frequency is 50Hz and the final frequency is 100Hz.

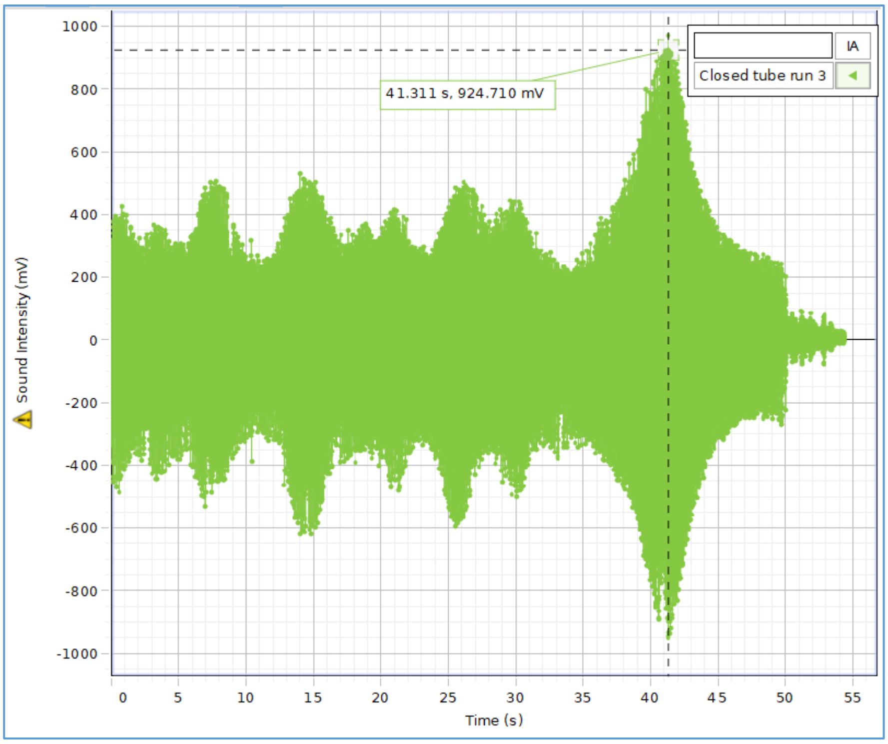

Figure 5 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 90cm, the initial frequency is 50Hz and the final frequency is 100Hz.

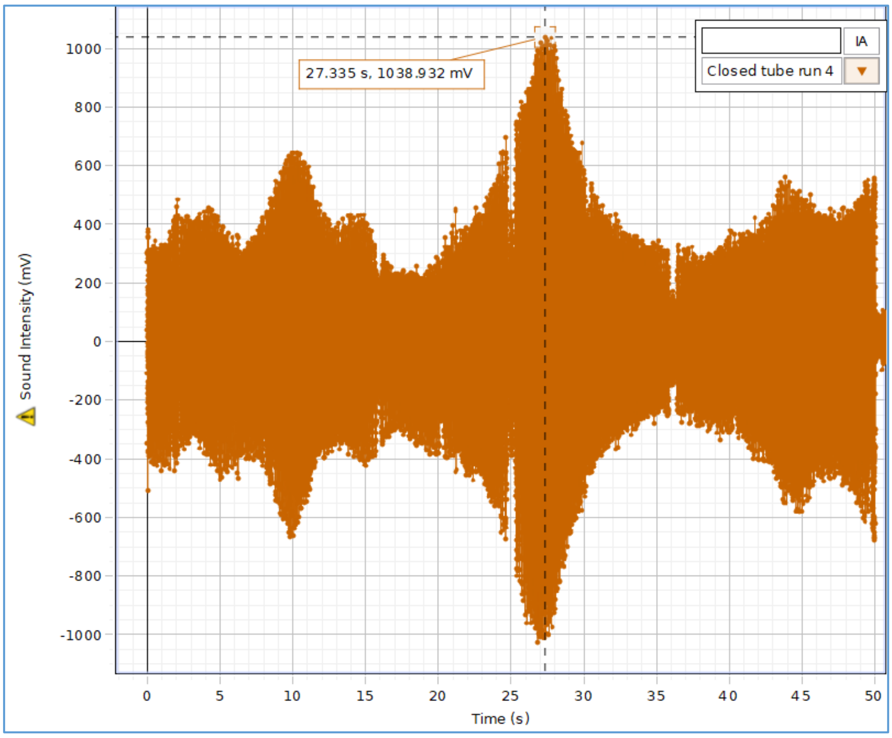

Figure 6 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 80cm, the initial frequency is 75Hz and the final frequency is 125Hz.

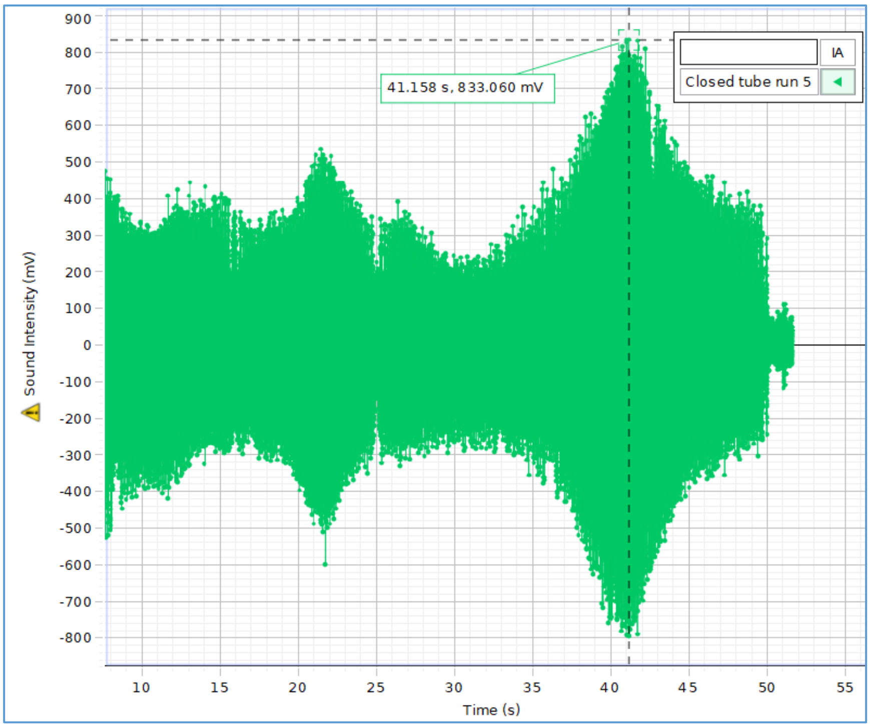

Figure 7 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 70cm, the initial frequency is 75Hz and the final frequency is 125Hz.

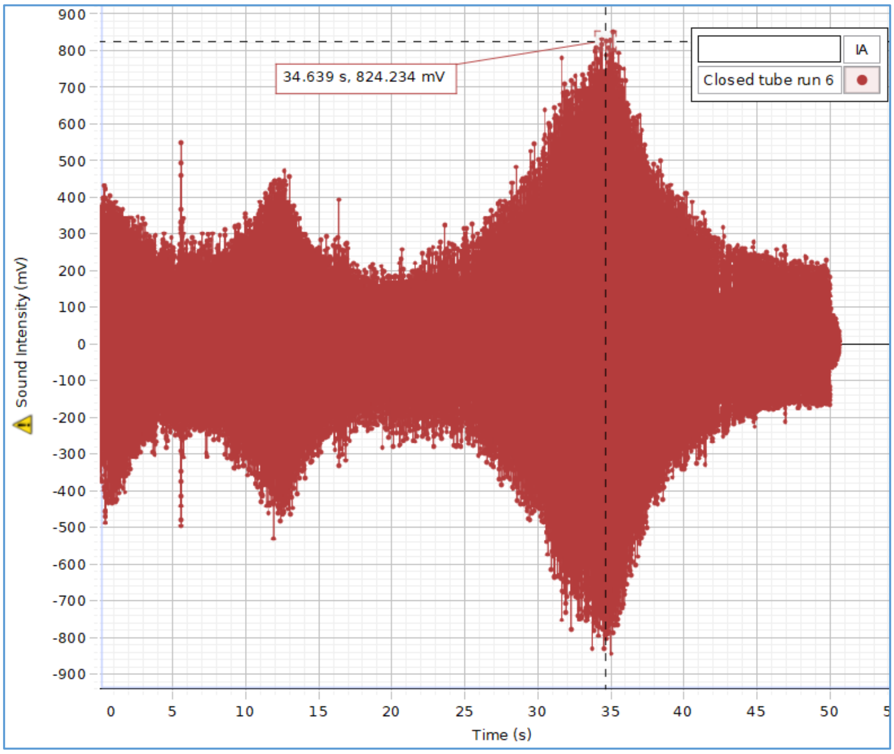

Figure 8 *Sound Intensity (mV) vs. Time (t).*

The length of the tube is 60cm, the initial frequency is 100Hz and the final frequency is 150Hz.

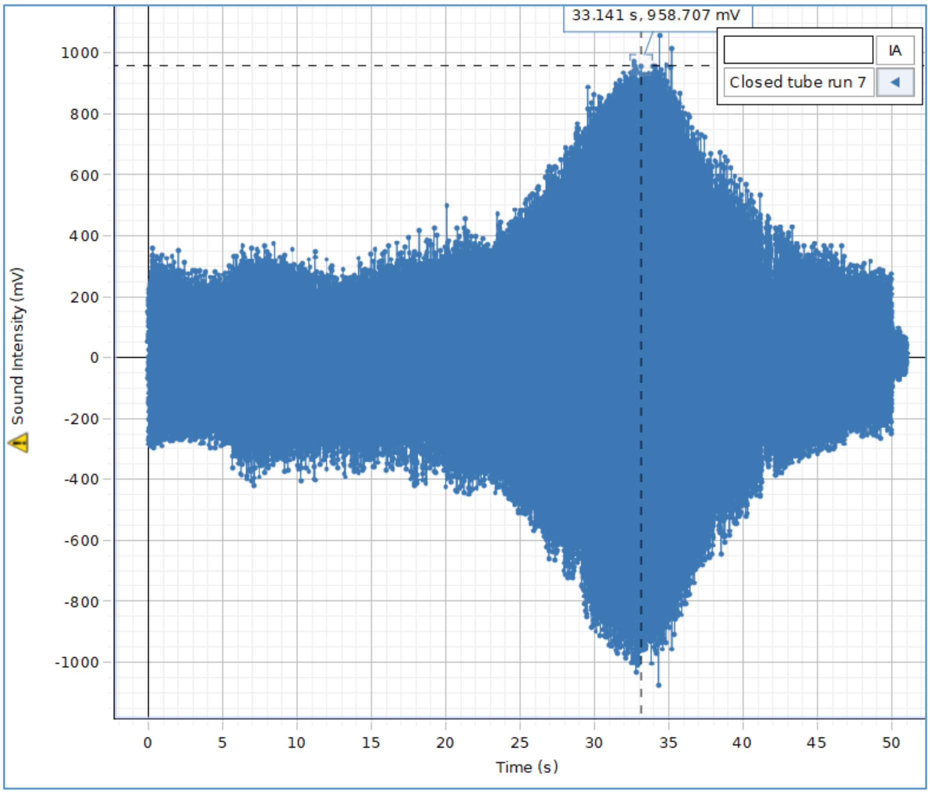

__Figure 9__ Sound Intensity (mV) vs. Time (t).

The length of the tube is 50cm, the initial frequency is 125Hz and the final frequency is 175Hz.

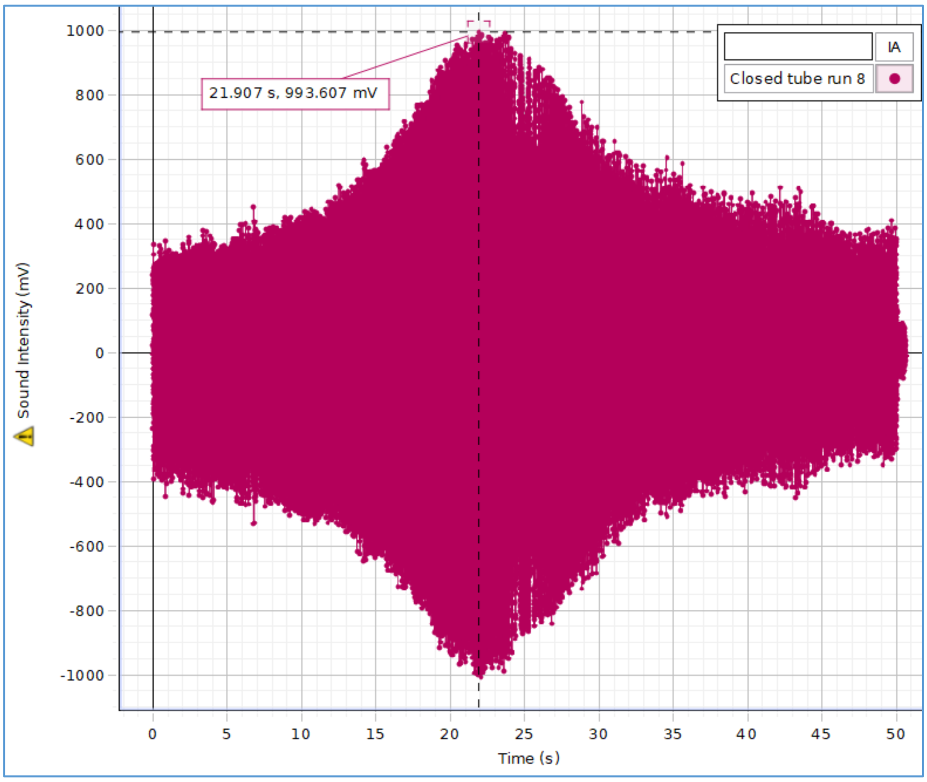

__Figure 10__ Sound Intensity (mV) vs. Time (t).

The length of the tube is 40cm, the initial frequency is 175Hz and the final frequency is 225Hz.

3.1.2 Resonance in an Open Tube

The graphs of Sound Intensity (mV) vs. Time (t) for each length of air column are shown below, with 1 second standing for 1 Hz. The time (frequency) when resonance occurred is highlighted in each figure. Besides, the frequency is determined to the nearest 1Hz.

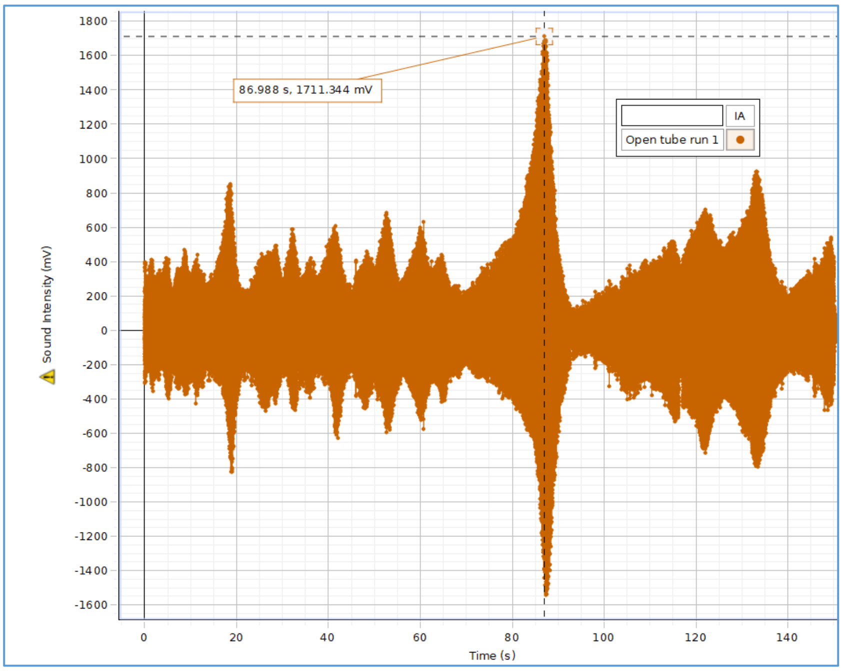

Figure 11 *Sound Intensity (mV) vs. Time (t).*

Fundamental resonance. The initial frequency is 50Hz and the final frequency is 200Hz.

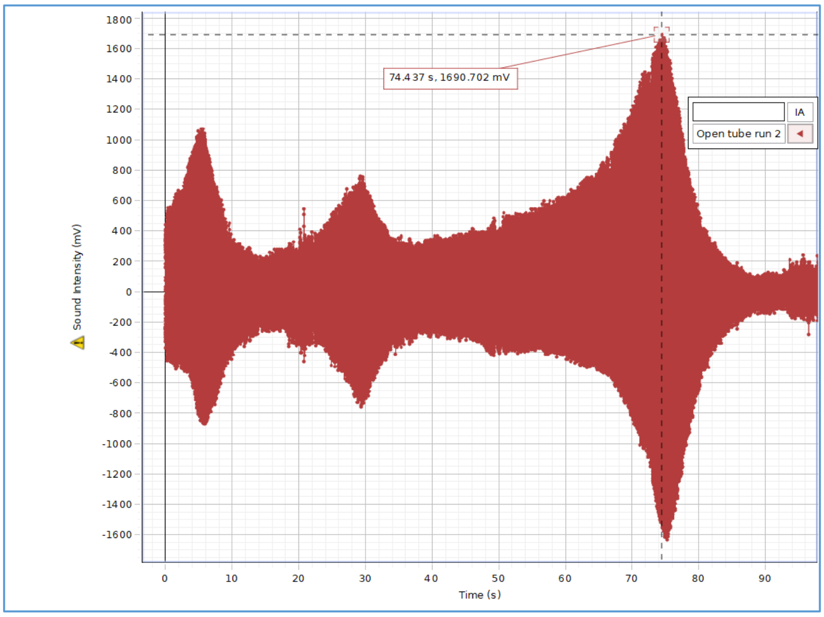

Figure 12 *Sound Intensity (mV) vs. Time (t).*

Second resonance. The initial frequency is 200Hz and the final frequency is 300Hz.

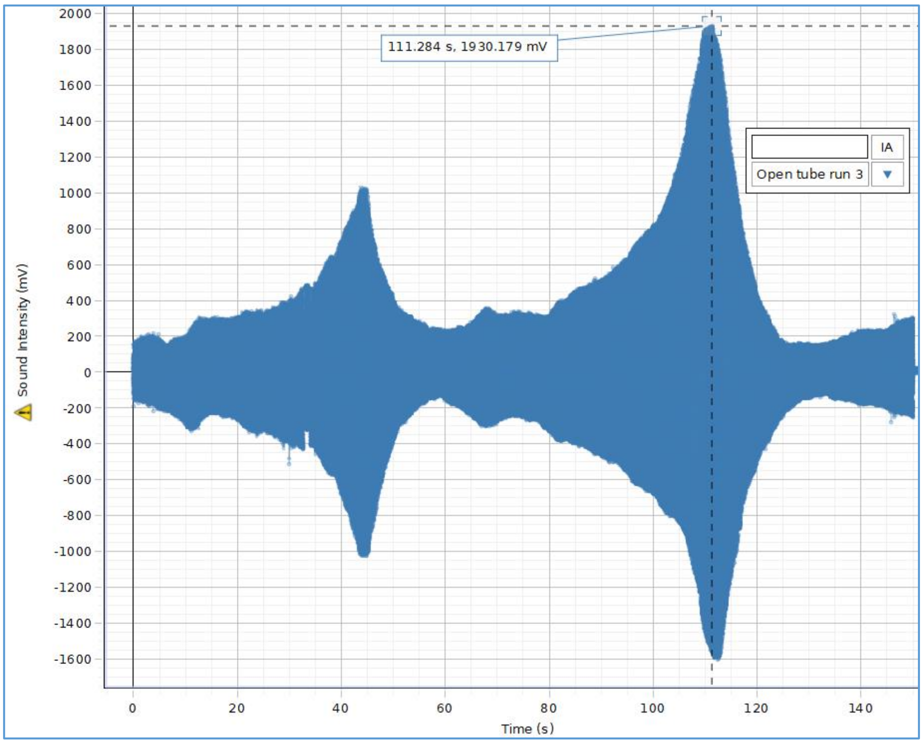

Figure 13 *Sound Intensity (mV) vs. Time (t).*

Third resonance. The initial frequency is 300Hz and the final frequency is 400Hz.

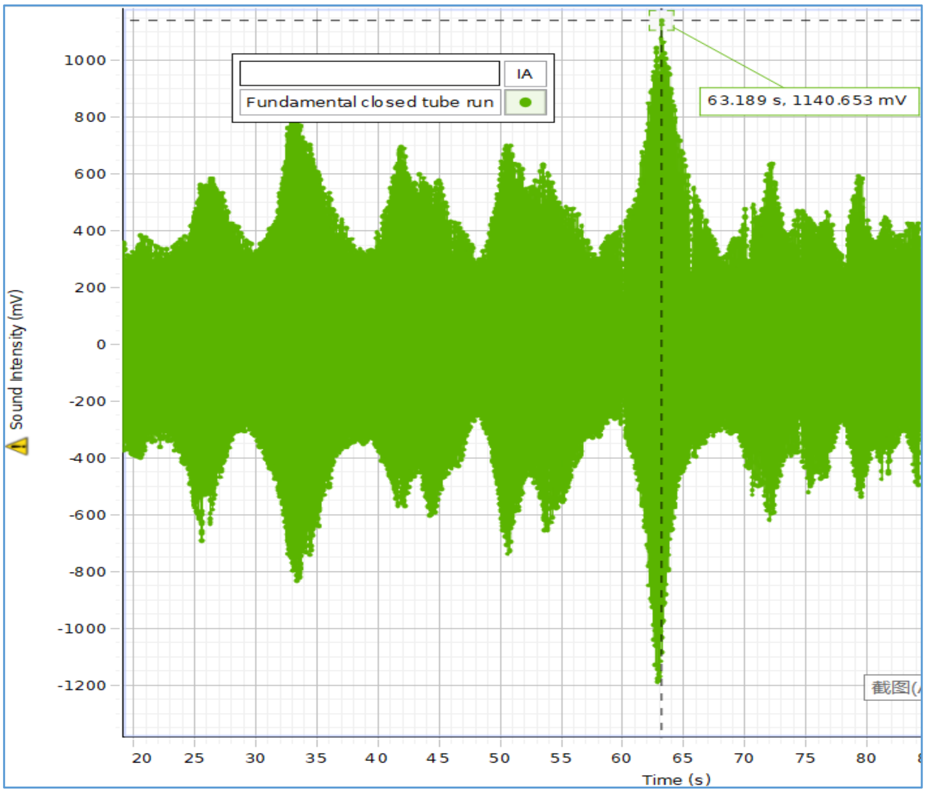

Figure 14 *Sound Intensity (mV) vs. Time (t).*

Fundamental resonance in a closed tube. The initial frequency is 137Hz and the final frequency is 50Hz.

3.2 Raw Data Tables

To calculate the velocity of sound and do further analysis, all the magnitudes above are required, including frequencies at different resonance, length of the air column at each frequency. This section contains four tables, two for the resonance in the closed tube and the other two for the resonance in the open tube. All these tables contain all these required magnitudes.

In this experiment, the length of the air column was measured by a common ruler, so the estimated error is ± 0.05cm. As for the frequency, the estimated error is $5 \times 10^{-4} Hz$, which is half of the resolution.

3.2.1 Resonance in a Closed Tube

***Table 1** Resonance frequencies at different length of tube.*

Number of trials	Length of Tube (m)	Resonance Frequency (Hz)
1	$1.1 \pm 5 \times 10^{-4}$	$75.349 \pm 5 \times 10^{-4}$
2	$1.0 \pm 5 \times 10^{-4}$	$81.931 \pm 5 \times 10^{-4}$
3	$0.9 \pm 5 \times 10^{-4}$	$91.311 \pm 5 \times 10^{-4}$
4	$0.8 \pm 5 \times 10^{-4}$	$102.335 \pm 5 \times 10^{-4}$
5	$0.7 \pm 5 \times 10^{-4}$	$116.158 \pm 5 \times 10^{-4}$
6	$0.6 \pm 5 \times 10^{-4}$	$134.639 \pm 5 \times 10^{-4}$
7	$0.5 \pm 5 \times 10^{-4}$	$158.141 \pm 5 \times 10^{-4}$
8	$0.4 \pm 5 \times 10^{-4}$	$196.907 \pm 5 \times 10^{-4}$

The data of further investigation is shown below.

***Table 2** Position of two successive nodes.*

	Length of Tube (m)

The First Node	$0.903 \pm 5 \times 10^{-4}$
The Second Node	$0.140 \pm 5 \times 10^{-4}$

3.2.2 Resonance in an Open Tube

Table 3 *Frequencies of different harmonic orders.*

Harmonic Order	Resonance of Frequency (Hz)
1^{st}	$136.988 \pm 5 \times 10^{-4}$
2^{nd}	$274.437 \pm 5 \times 10^{-4}$
3^{rd}	$411.284 \pm 5 \times 10^{-4}$

Table 4 *Fundamental resonance frequencies.*

	Resonance Frequency (Hz)
Open Tube	$136.988 \pm 5 \times 10^{-4}$
Closed Tube	$73.811 \pm 5 \times 10^{-4}$

Besides, the measured diameter of the tube is shown below.

Table 5 *Diameter of tube.*

Diameter of Tube (m)	$0.0366 \pm 5 \times 10^{-4}$

Data and Error Analysis

4.1 Error Analysis

We can use the following equation to calculate the propagated error if all variables are independent.

$$\delta k = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial f}{\partial p_i}\right)^2 (\delta p_i)^2} \tag{4.1}$$

4.1.1 Error Analysis of Resonance in a Closed Tube

First a length of tube vs. the inverse of frequency (L vs. $\frac{1}{f}$) is needed. The error of the inverse of frequency is determined by:

$$\delta\left(\frac{1}{f}\right) = \frac{\delta f}{f^2} \tag{4.2}$$

Using this function, we modify Table 1 to the table below.

Table 6 *Resonance frequencies and their inverse at different length of tube.*

Number of trials	Length of Tube (m)	Resonance Frequency (Hz)	Inverse of Frequency (Hz^{-1})
1	$1.1 \pm 5 \times 10^{-4}$	$75.349 \pm 5 \times 10^{-4}$	$0.013272 \pm 8.9 \times 10^{-8}$
2	$1.0 \pm 5 \times 10^{-4}$	$81.931 \pm 5 \times 10^{-4}$	$0.012205 \pm 7.5 \times 10^{-8}$
3	$0.9 \pm 5 \times 10^{-4}$	$91.311 \pm 5 \times 10^{-4}$	$0.010952 \pm 6.0 \times 10^{-8}$
4	$0.8 \pm 5 \times 10^{-4}$	$102.335 \pm 5 \times 10^{-4}$	$0.009772 \pm 4.8 \times 10^{-8}$
5	$0.7 \pm 5 \times 10^{-4}$	$116.158 \pm 5 \times 10^{-4}$	$0.008609 \pm 3.8 \times 10^{-8}$
6	$0.6 \pm 5 \times 10^{-4}$	$134.639 \pm 5 \times 10^{-4}$	$0.007427 \pm 2.8 \times 10^{-8}$
7	$0.5 \pm 5 \times 10^{-4}$	$158.141 \pm 5 \times 10^{-4}$	$0.006323 \pm 2.0 \times 10^{-8}$
8	$0.4 \pm 5 \times 10^{-4}$	$196.907 \pm 5 \times 10^{-4}$	$0.005079 \pm 1.3 \times 10^{-8}$

The L vs. $\frac{1}{f}$ figure can be plotted as follows using data in Table 5. A linear fitting was applied to fit the data. The fitting curve is also shown in the figure.

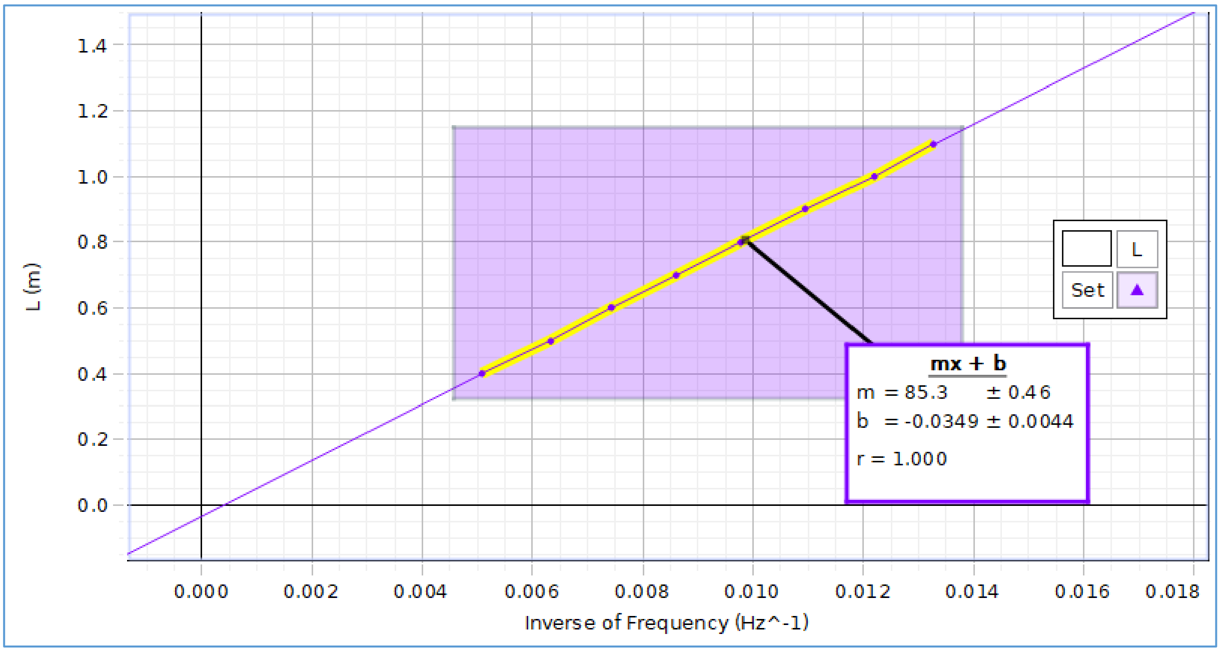

Figure 15 L (m) vs. $\frac{1}{f}$ (Hz^{-1}).

According to equation (1.10), the slope of the fitting curve is $\frac{1}{4}v$, where v is the speed of sound. Therefore, we can calculate the speed of sound as four times the slope of the curve. In the figure, the slope is 85.3 ± 0.46. Therefore, the velocity is given by:

$$v = (341.20 \pm 1.9)\ m/s \tag{4.3}$$

***Further Investigation**

The distance between the two resonance positions is half of the wave length. Therefore, the wavelength can be calculated by:

$$\lambda = 2(x_1 - x_2) \tag{4.4}$$

where λ is the wavelength, x_1 and x_2 are the length of air column of the first and second resonance position, respectively.

The error can be determined by:

$$\delta\lambda = 2\sqrt{(\delta x_1)^2 + (\delta x_2)^2} \tag{4.5}$$

The calculated result is given as follow.

$$\lambda = (1.526 \pm 0.0015)m \tag{4.6}$$

The speed of sound can then be determined by equation (1.9), with the frequency being 230Hz.

The error is

$$\delta v = \sqrt{f^2 \delta\lambda^2 + \lambda^2 \delta f^2} \tag{4.7}$$

Therefore, the speed of sound is:

$$v = (350.98 \pm 0.35)\ m/s \tag{4.8}$$

Equation (1.11) is used to calculated the percent difference.

Table 7 *Table of Speed of Sound.*

	Actual speed	Experimental result	Further Investigation
Speed of Sound (m/s)	340	341.20 ± 1.9	350.98 ± 0.35
Percent Difference	0	0.353%	3.23%

4.1.2 Error Analysis of Resonance in an Open Tube

With the frequency of each resonance and the velocity of sound obtained in equation (4.3), the wave length of each resonance can be determined by:

$$\lambda = \frac{v}{f} \tag{4.9}$$

The estimated error is

$$\delta\lambda = \sqrt{\left(\frac{1}{f}\right)^2 \delta v^2 + \left(\frac{v}{f^2}\right)^2 \delta f^2} \tag{4.10}$$

According to equation (1.6), the effective length of thee tube is given by:

$$L = \frac{n\lambda}{2}, n = 1, 2, 3 \tag{4.11}$$

The estimated error is

$$\delta L = \frac{n}{2}\delta\lambda^2, n = 1, 2, 3 \tag{4.12}$$

With equation (4.9), (4.10), (4.12), and the speed of sound, we can calculate the wave length of each resonance and effective length of the tube with errors, as shown below. In this table, the speed of sound is the value in equation (4.3).

Table 8 Wave length and effective length of tube.

Harmonic Order	Resonance of Frequency (Hz)	Wave Length (m)	Effective Length of Tube (m)
1st	$136.988 \pm 5 \times 10^{-4}$	2.491 ± 0.014	1.246 ± 0.007
2nd	$274.437 \pm 5 \times 10^{-4}$	1.243 ± 0.007	1.243 ± 0.007
3rd	$411.284 \pm 5 \times 10^{-4}$	0.8296 ± 0.005	1.244 ± 0.008

For the fundamental resonance frequency in open and closed tube, the ratio is given below.

$$ratio = \frac{f_{Open-tube}}{f_{Closed-tube}} \tag{4.13}$$

The estimated error is

$$\delta ratio = \sqrt{\left(\frac{1}{f_{Closed-tube}}\right)^2 \delta f_{Open-tube}{}^2 + \left(\frac{f_{Open-tube}}{f_{Closed-tube}{}^2}\right)^2 \delta f_{Closed-tube}{}^2} \tag{4.14}$$

Using the data in Table 4, the ratio is given by:

$$ratio = 1.86 \pm 2 \times 10^{-5} \tag{4.15}$$

Table 9 Fundamental resonance frequency.

Closed Tube	Open Tube	ratio
$136.988 \pm 5 \times 10^{-4}$	$73.811 \pm 5 \times 10^{-4}$	$1.86 \pm 2 \times 10^{-5}$

4.2 Data Analysis

4.2.1 Analysis of Resonance in a closed tube

From equation (4.3) or Table 6, the measured speed of sound is $(341.20 \pm 1.9)\ m/s$, the difference between the measured error is within 1%. As for the speed measured in **Further Investigation** part, the value $(350.98 \pm 0.35)\ m/s$, with an error around 3%. Both the errors are within the acceptable range. Some factors may contribute to the errors.

Human error had significant effects in this experiment. Human error mainly effected two measurements: (1). Measuring the length of the length of the air column. In this experiment, a common ruler was applied to measure the length of the air column. However, during the experiment, it was discovered that it was difficult for the ruler to be parallel with the tube. The two adjustable feet separated the tube and the ruler, making it hard for the students to measure an accurate length. In the **further Investigation** part, it was difficult to determine the exact point of resonance node because the sound intensity remained almost the same within a certain interval around the resonance node. Therefore, the measured length of air column may have a relatively large error. (2) Finding the resonance frequencies. Even though records of sound intensity can be obtained in the software, the point of time when the resonance happened was decided by people. If the record, like what is shown in Figure 16, does not have an extreme peak, there may be a relatively large error when choosing the resonance point.

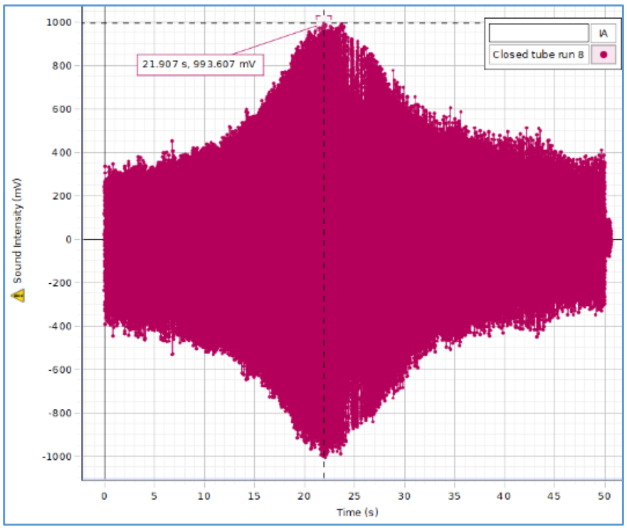

Figure 16 *Sound Intensity (mV) vs. Time (t)*

An example of recorded figure which is difficult to determine the resonance frequency.

Environmental noise had a strong effect on the recorded intensity figure. During the experiment, it was observed that the sound intensity increased suddenly when some environmental noise occurred. Such sudden increase is similar to the sudden increase when resonance occurs. This would make it hard for people to distinguish the time when resonance happened. In addition, environmental noise is the main outer disturbance for the problem mentioned before.

(3). The environmental temperature and pressure will affect the speed of sound. The speed of sound is 340m/s under the condition that the temperature is 15℃ and the pressure is one atmosphere. However, the temperature when the experiment was carried out was above $20℃$. This contributes a slightly higher speed of sound.

As a conclusion, the measured and actual speed of sound in the air are equal with an acceptable range of error. It can be verified that the speed of sound in the air equals the product of its wavelength and frequency in a closed tube.

4.2.2 Analysis of Resonance in an open tube

From Table 8, it is observed that the effective lengths of the tube calculated from the first three resonant frequencies are equal within the range of uncertainty. The results have a high precision. In the experiment, the actual length of the tube is 1.2 m. However, the effective length of the tube calculated in this experiment is slightly larger than this value. This is due to the end effect of the wave. According to equation (1.12) and the value in Table 5, the effective length of the tube is approximately $1.222 \pm 3 \times 10^{-4}$ m. The difference of the measured value between the calculated value is within 2%. Considering the possible error mentioned before, the measured value is acceptable.

From Table 9, the ratio of fundamental frequency in a closed tube to that in an open tube is 1.86. Theoretically, this ratio can be calculated from Equation (1.6), (1.7), and (1.9) with n = 1 in both Equation (1.6) and (1.7). The theoretical value should be 2. However, the experimental value is slightly smaller than the calculated. This could be explained by the following reason. An unavoidable decrease of the length of the tube exists when the open tube was changed into the closed tube. During the experiment, it was observed that the air still behaved like a resonance in an open tube and had a fundamental frequency around 137Hz, when the piston was just put into the tube. This was because the piston did not match with the tube exactly. There was a small gap exiting between them. Consequently, the piston should be put a little bit deeper inside the tube (about 5cm) to make the tube a closed tube, which will cause a decrease in the length of the air column. According to equation (1.7), when n equals to 1

$$L = \frac{\lambda}{4} \tag{4.16}$$

A decreased air column makes the fundamental wavelength of the closed tube decrease. According to equation (1.9),

$$v = \lambda f \tag{1.9}$$

A decreased wavelength will make the fundamental frequency of the closed tube increase. Therefore, the measured fundamental frequency in a closed tube is slightly higher than the actual value. This will further make the measured ratio a little bit less than 2. Therefore, the measured ratio is acceptable considering the possible error.

As a conclusion, it can be verified that the speed of sound in the air equals the product of its wavelength and frequency in an open tube by analyzing the data in Table 8. The ratio of fundamental frequency in an open tube to that in a closed tube with the same length is 2.

Conclusion

5.1 Summary

This experiment verified the relationship between the wavelength, wave speed, and frequency of sound waves both in a closed tube and in an open tube. For the closed tube, the relationship between the wavelength, wave speed and frequency was verified by curve fitting for fundamental resonance. For the open tube, the relationship between the wavelength, wave speed and frequency was verified by calculating the effective length of the tube. In this experiment, The ratio of fundamental frequency in an open tube to that in a closed tube with the same length is also get verified.

5.2 Answers to the Questions

5.2.1 Questions of Resonance in a Closed Tube

1. *On your graph of L vs. 1/f, why isn't the y-intercept zero?*

Answer: Because of end effect, the effective length of the tube is slightly larger than the actual length, resulting a nonzero y-intercept. Considering the end effective in the closed tube, according to equation (1.13), the relationship between the length and the inverse of frequency (equation (1.10)) can be modified as:

$$L = \frac{v}{4} f - 0.3d \tag{5.1}$$

which indicates a negative y-intercept.

***2.** Is the intercept negative?*

Answer: According to equation (5.1), the intercept is negative. This is the exact situation in Figure15. From this figure, we can see that the intercept has negative value of 0.0349 ± 0.0044.

***3.** Measure the diameter of the resonance air column and use this equation to calculate the end effect. How does this value for the extra end-effect length compare with the y-intercept of your graph?*

Answer: The measured diameter of the resonance air column is $0.0366 \pm 5 \times 10^{-4} m$. The calculated end effect is $0.0110 \pm 2 \times 10^{-4} m.$ This value is lower than the y-intercept value.

However, as mentioned before (section 4.2.1), some possible error exists during the experiment, which may contribute to some discrepancy.

***Further Investigation**

***1.** The distance between the two resonance positions (the distance between adjacent nodes) is 1/2 λ. Why?*

Answer: According to Figure 2, and the discussion in the **theory** section, it can be observed that the distance between adjacent nodes is equal to half of the wavelength, which is *1/2 λ*.

***2.** Calculate the wavelength from the distance between the nodes. From this wavelength and the frequency of the Signal Generator, calculate the speed of sound. How does it compare with your earlier value?*

Answer: The related values have already been calculated before. For reference, check equation (4.6), (4.8) and Table 7. The speed of sound is $(350.98 \pm 0.35)\ m/s$, which is larger than my earlier value. Since the measurement of the wavelength is only done once, there may be a relatively large random error. Considering the possible reasons mentioned in section 4.2.1, the value is acceptable within the error range.

***3.** Draw a companion sketch of the waveform diagram on the first page of this experiment, showing two nodes and the same frequency. Remember that there must be a node at the closed end and an anti-node at the open end. Hint: the tubes in the two drawings should not be the same length, but the wavelengths are the same.*

Answer: The figure is shown below.

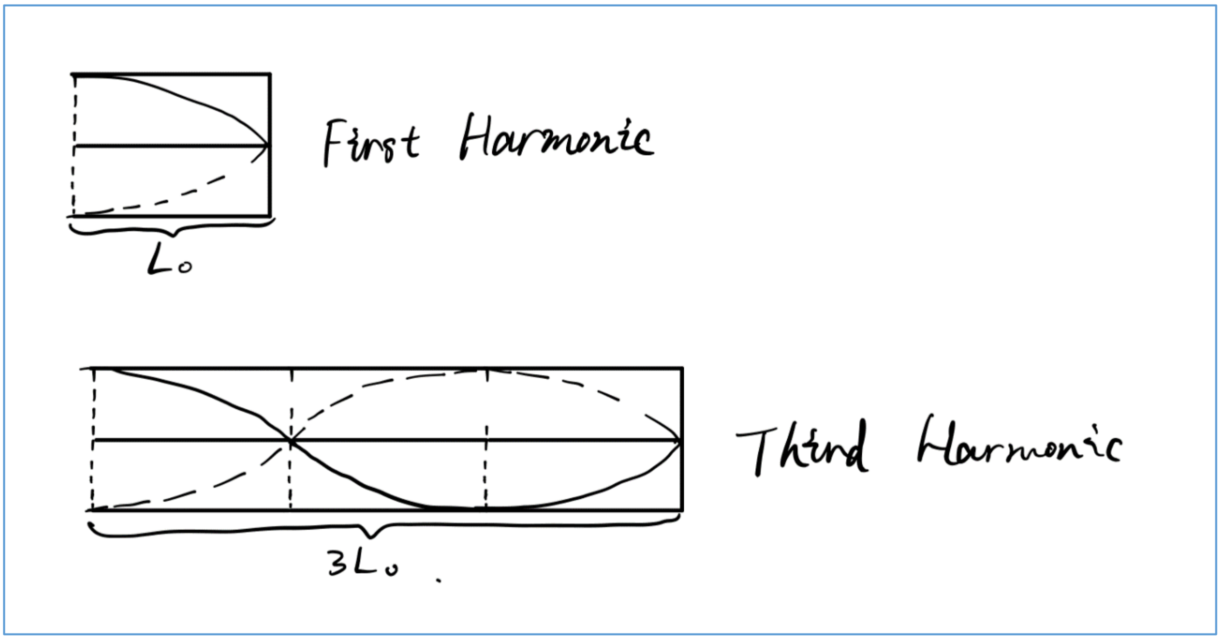

Figure 17 First and Third Harmonic.

5.2.2 Questions of Resonance in an Open Tube

***1.** Why is the frequency of the fundamental higher for the open tube than it was for the closed tube?*

Answer: According to equation (1.6) and equation (1.7), the fundamental harmonic will occur with a smaller wavelength in an open tube with the same length of tubes, the fundamental frequency in the open tube will be higher according to Equation (1.9).

***2.** How does the actual tube length compare to the effective length? (Hint: The effect is about twice as big compared to the previous tube because there are two open ends.)*

Answer: According to Table 8, the average effective length is 1.244 ± 0.008m. The actual tube length is 1.2m, which is smaller than the effective length.

***3.** When the frequency was returned to the fundamental, and the end of the tube was closed, was it still in resonance?*

Answer: It was not in resonance because the fundamental frequencies for the open tube and the closed tube with the same length are different.

***4.** What should the ratio of the open-tube frequency to the closed-tube frequency be? Why?*

Answer: The ratio should be 2. Here is the detailed calculation. With n equaling to 1,

For an open tube:

$$L = \frac{\lambda_{Open}}{2} \tag{5.2}$$

For the closed tube:

$$L = \frac{\lambda_{Closed}}{4} \tag{5.3}$$

Because the length is the same, we have

$$\frac{\lambda_{Open}}{\lambda_{Closed}} = \frac{1}{2} \tag{5.4}$$

According to equation (1.9),

$$\frac{f_{Open}}{f_{Closed}} = \frac{\lambda_{Closed}}{\lambda_{Open}} = 2 \tag{5.5}$$

***5.** Why does a tube open at both ends play all the harmonics, but a tube with one end closed only plays the odd harmonics (1, 3, 5, etc.). What is the relationship between the tube length and the wavelength for the third harmonic of a closed tube?*

Answer: There should be a node at the closed end and an antinode at the open end for the tube with one end closed, resulting that the length of the tube should satisfy

$$L = \frac{n}{2}\lambda + \frac{1}{4}\lambda, n = 0,1,2, \dots \tag{5.6}$$

which is equivalent to equation (1.7). Therefore, a tube with one end closed only harmonic when the length is odd times the wave length. When *n* equals to 3, the relationship is

$$L = \frac{3}{4}\lambda \tag{5.7}$$

***6.** Draw a companion sketch of the waveform diagram in the first page of Experiment 1 (closed tube) showing the third harmonic in a tube of the same length. Remember that you must still have a node at the closed end and an anti-node at the open end. Why is this set of pictures different from what you drew in Experiment 1? In each case, what is forced to stay constant, and what is allowed to change?*

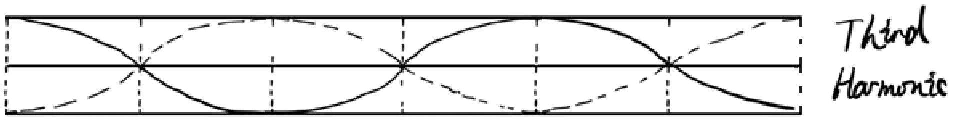

***Figure 18** Third Harmonic.*

In experiment 1, the wavelength is forced to stay constant, and the length of the tube is allowed change. In this part of experiment, the length of the tube is forced to stay constant, and the wavelength is allowed to change. Therefore, the two sets of pictures are different.

5.3 Improvements to the experiment

Most of the error in this experiment comes from human error. Therefore, to improve the accuracy of the experiment, we should use equipment with higher resolution.