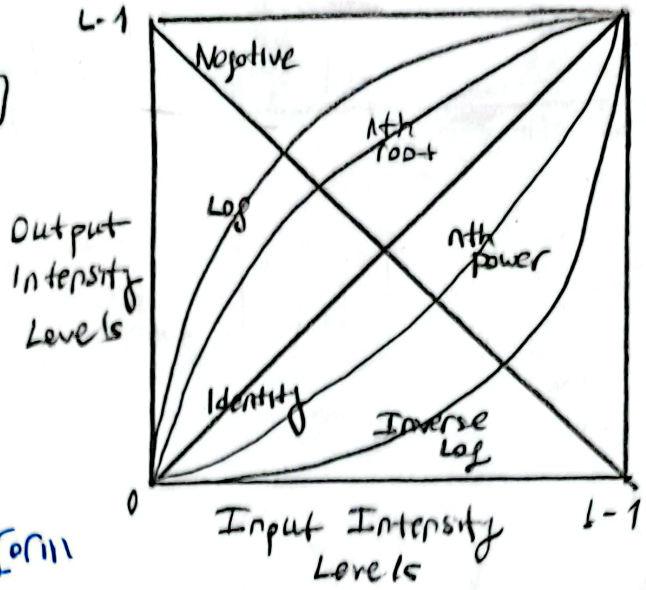
Image Negatives

Intensity levels $[0, L-1]$

$$\boxed{s = L-1-r}$$



Log Transformations

General form

$$\boxed{s = c \cdot \log(1+r)}$$

Power-Low (Gamma) Transform

$$\boxed{s = c \cdot r^{\gamma}}$$

$$s = c(r+\varepsilon)^{\gamma}$$

$\gamma < 1$ ise küçük detaylar görülmeye başlar

Beyaz seviyesi artar

$\gamma > 1$ ters durum

Beyaz seviyesi azalır

Alçak Geçiren Filtre

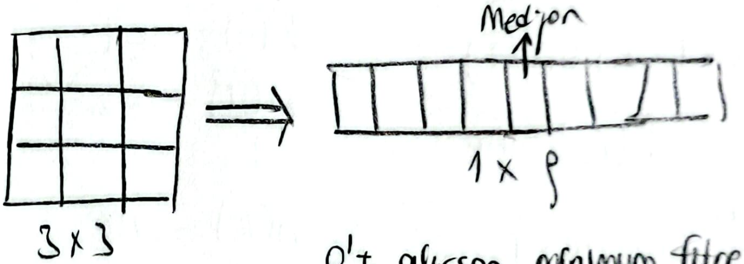
Yumuşatma

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filtre

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} \underbrace{w(s,t)}_{\text{Filtre}} f(x+s, y+t)$$

→ Filtre Uygulama



0'ı alırsan minimum filtre

8'i alırsa maximum filtre

Salt-Pepper Noise için Median Filter

Gauss Noise için Mean Filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

ile Normalize   ile normalize

Sharpening Filter

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y) \Rightarrow \begin{bmatrix} & 1 & \\ & -2 & \\ & 1 & \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y) \Rightarrow \begin{bmatrix} & & \\ 1 & -2 & 1 \\ & & \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Daha fazla keskinleştirme

Türevlerde toplam 0 yapılır

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Unsharp Masking

$$\underbrace{f_s(x,y)}_{\text{maske}} = \underbrace{f(x,y)}_{\text{original görüntü}} - \underbrace{\bar{f}(x,y)}_{\text{gumusfalmış görüntü}}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{9} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \frac{1}{9}$$

$$\underbrace{g(x,y)}_{\text{keskinleşmiş görüntü}} = \underbrace{f(x,y)}_{\text{original görüntü}} + \underbrace{f_s(x,y)}_{\text{maske}}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \frac{1}{9} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \cdot \frac{1}{9}$$

Highboost Filtering

$$f_{HB} = A \cdot f(x,y) - \bar{f}(x,y) \Rightarrow \begin{bmatrix} -1 & -1 & -1 \\ -1 & A & -1 \\ -1 & -1 & -1 \end{bmatrix} \Rightarrow g(x,y)$$

$A \geq 1$

Histogram Processing

gray level $\Rightarrow [0, L-1]$

Histogram $h(r_k) = n_k$

$r_k = k^{th}$ gray level

$n_k =$ number of pixel in the image having gray level r_k

$h(0) = 27$

$h(1) = 42$

$\vdots$

$h(k) = n_k$

$\vdots$

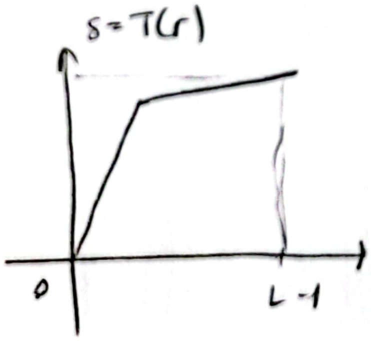
$h(255) = 8$

$$\sum_{k=0}^{L-1} h(k) = M \times N \Rightarrow \text{PDF} \left(\text{Prob. Density Func.} \right)$$

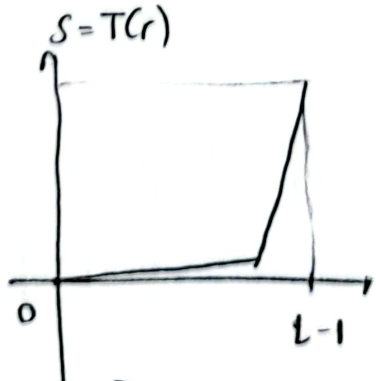
$p(k) = n_k / n \Rightarrow$ Normalize Histogram

$n = M \times N$

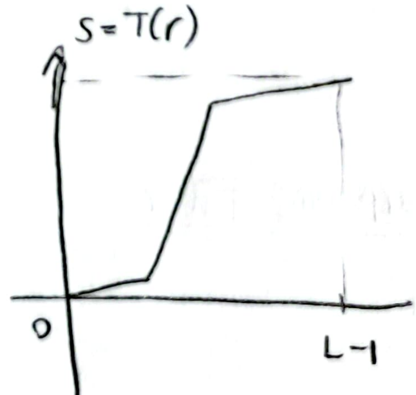
$$\sum_{k=0}^{L-1} p(k) = 1$$



Karanlık

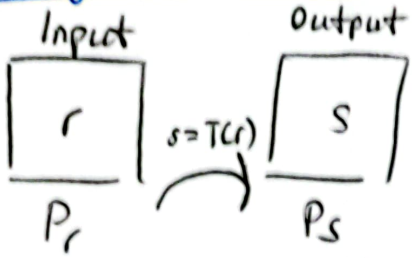


Parlak



Low Contrast

Histogram Equalization



$0 \le r \le L-1$

$0 \le P_r \le 1$

$s = T(r) = \int_0^r P_r(w)\,dw$

Histogram uniform yapılmaya çalışılıyor

Örn:

Input image

0	3	3	1
1	3	1	3
0	3	3	3
1	1	1	2

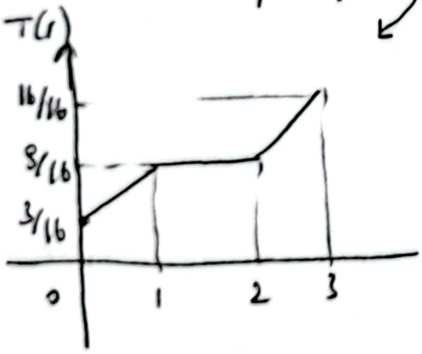
$T(r)\cdot(L-1)$

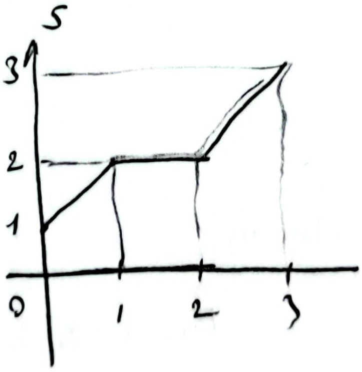
r	$h_r(r)$	$P_r(r)$	$T(r) = p(r)$	s
0	3	3/16	3/16	9/16 → 1
1	6	6/16	9/16	27/16 → 2
2	0	0/16	9/16	27/16 → 2
3	7	7/16	16/16	48/16 → 3
		+ = 1		

=

Output Image

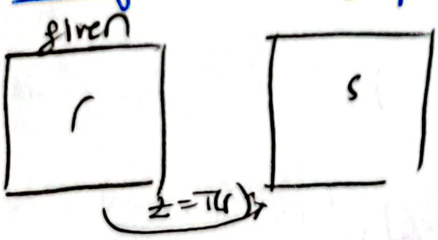
1	3	3	2
2	3	2	3
1	3	3	3
2	2	2	1

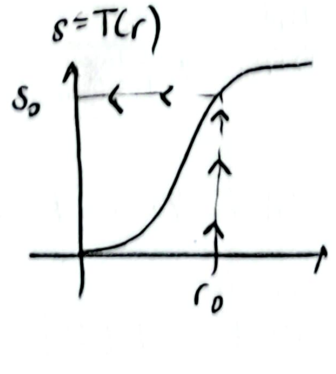


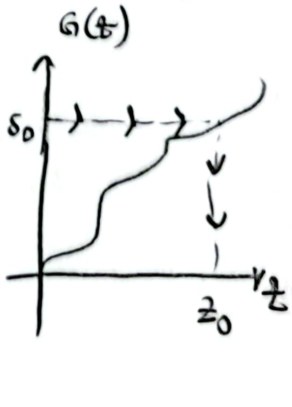


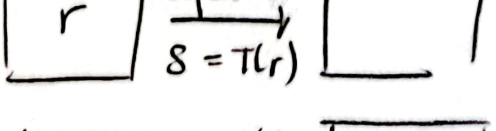
Uniform Histogram
Her zaman iyi değil
Bazen iyi sonuç
vermez.

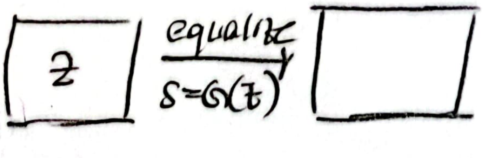
Histogram Matching











Örn:

Input Image

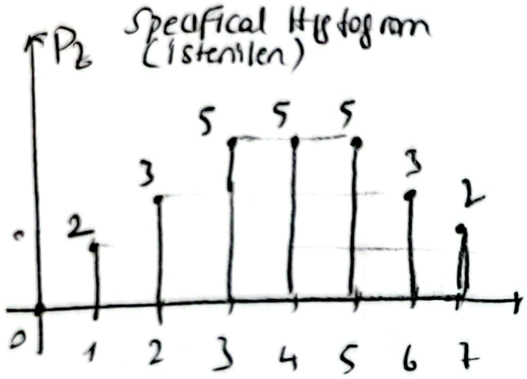
0	1	1	0	0
1	1	0	0	1
2	2	5	6	7
5	5	7	6	6
7	6	7	7	7

5 x 5

r	$h_r(r)$	$s=T(r)$	$s=G(z)$	$z=G^{-1}(T_r)$
0	5	5/25	0/25	2
1	5	10/25	2/25	3
2	2	12/25	5/25	3
3	0	12/25	10/25	3
4	0	12/25	15/25	3
5	3	15/25	20/25	4
6	4	18/25	23/25	5
7	6	25/25	25/25	7

T

= 1



Output Image

2	3	3	2	2
3	3	2	2	3
3	3	4	5	7
4	4	7	5	5
7	5	7	7	7

Filtering in Frequency Domain

Complex Numbers

$$c = a + jb$$
$$= re^{j\theta}$$

$$r = \sqrt{a^2+b^2} \quad \theta = \arctan\left(\frac{-b}{a}\right)$$

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t\neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$\delta[x] = \begin{cases} 1, & x=0 \\ 0, & x\neq 0 \end{cases}$$

$$\text{DTFT} \rightarrow x[n] \longrightarrow X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$$

Yeni

$$\text{DTFT} \rightarrow f[x] = \frac{1}{M}\sum_{u=0}^{M-1} F[u]\, e^{j\frac{2\pi}{M}ux}, \quad x=0, \ldots M-1$$

$$\text{DFT} \rightarrow X[k] \rightarrow \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}, \quad k=0 \ldots N-1$$

YENİ

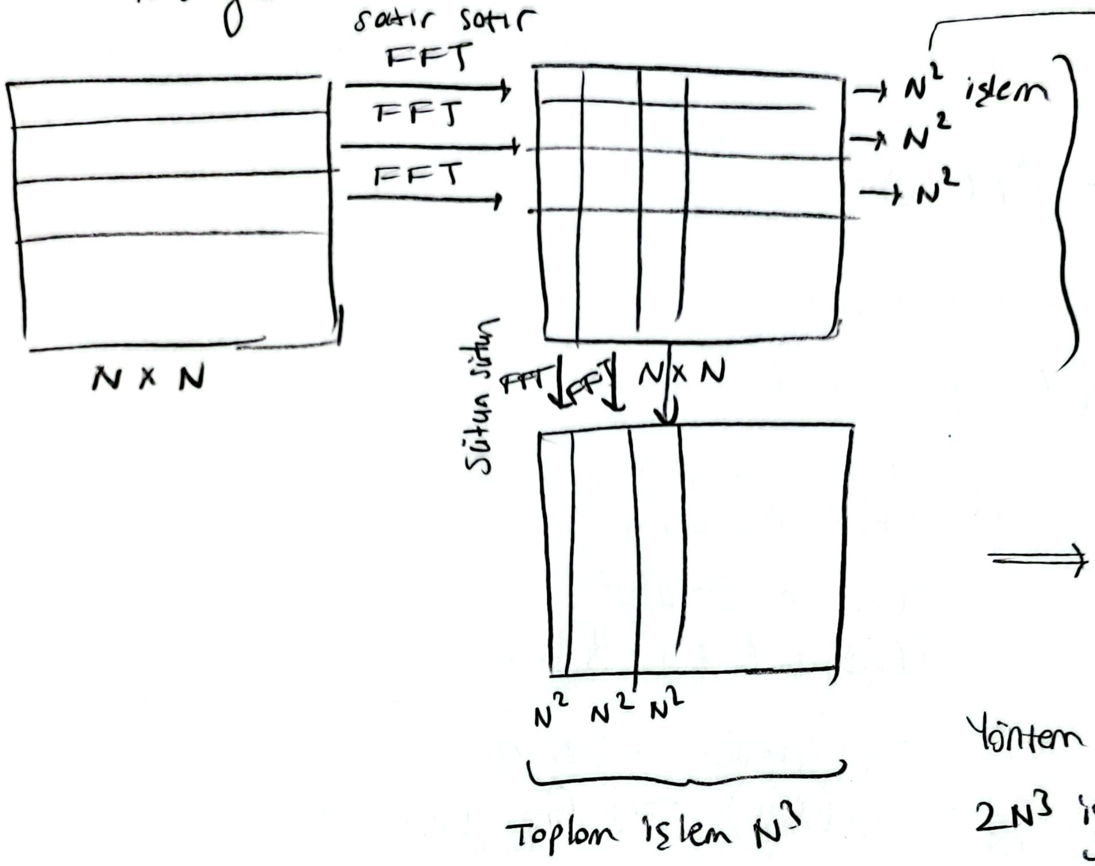
$$\text{DFT} \rightarrow F[u] = \sum_{x=0}^{M-1} f[x]\, e^{-j\frac{2\pi}{M}ux}, \quad x=0, \ldots M-1$$

DFT(2D)

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$$

$u = 0,1,2 \ldots M-1$

$v = 0,1,2 \ldots N-1$



N^3 toplam işlem

yöntem sayesinde yerine $N \log_2 N$ işlem

$\Longrightarrow$ $2N^3$ toplam 2D FFT işlemi

Yöntem sayesinde $2N^3$ işlem yerine $2N^2 \log_2 N$ işlem

FFTSHIFT

$f(x,y) \rightarrow F(u,v)$

$f(x,y)\, e^{2\pi j(u_0 x + v_0 y)} \overset{F}{\longleftrightarrow} F(u-u_0, v-v_0) \Rightarrow$ FFT kuralı

$u_0 = \frac{N}{2}$, $v_0 = \frac{M}{2}$ $\Rightarrow f(x,y)\, e^{2\pi j\left(\frac{N}{N2}x + \frac{M}{2M}y\right)}$

$= f(x,y)\, e^{j\frac{2\pi}{2}x}\, e^{j\frac{2\pi}{2}y}$

$(-1)^x (-1)^y = (-1)^{x+y}$

FFTShift $\underbrace{f(x,y)(-1)^{x+y}}_{\text{DC Merkeze alınmış olur}} \overset{F}{\longleftrightarrow} F\left(u-\frac{N}{2}, v-\frac{M}{2}\right)$

$*$ $x[t]$ reel ise $F(u,v) = F^*(-u,-v)$

$g(x,y) = f(x,y) * h(x,y)$

($h(x,y)$ simetrik ise
konvolusyon = korelasyon)

Keskinleştirme

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$G(u,v) = F(u,v) - 16\pi^4 u^2 v^2 F(u,v)$$

$$= F(u,v) \underbrace{\left(1 - 16\pi^4 u^2 v^2\right)}_{H(u,v)}$$

$H(u,v) \rightarrow$ $F(u,v)$ merkezde ise Bunun da taşınması lazım

$$H(u,v) = 1 - u^2 v^2$$

$$H(u,v) = 1 - \left(u - \tfrac{M}{2}\right)\left(v - \tfrac{N}{2}\right)^2 \Longleftarrow$$ Merkeze Taşınmış Hali

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y) \Longrightarrow G_{mask}(u,v) = F(u,v) - F(u,v)H_{LP}(u,v)$$

$$= F(u,v)\left[1 - H_{LP}(u,v)\right]$$

$$= F(u,v) \cdot H_{HP}(u,v)$$

$$g(x,y) = f(x,y) + k \cdot g_{mask}(x,y)$$

$\Downarrow$

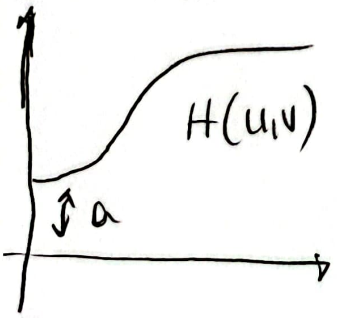
$$G(u,v) = F(u,v) + k \cdot F(u,v) H_{HP}(u,v)$$

$$= F(u,v) \underbrace{\left[1 + k \cdot H_{HP}(u,v)\right]}_{H(u,v)}$$

Homomorphic Filtering

x,y noktasındaki ışık $\rightarrow$ $f(x,y) = i(x,y)\, r(x,y)$

illumination reflection



$$\ln(f(x,y)) = \ln(i(x,y)) + \ln(r(x,y))$$

$\mathcal{F} \downarrow$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

$$Z(u,v).H(u,v) = F_i(u,v)H(u,v) + F_r(u,v)H(u,v)$$

$$S(u,v) = S_i(u,v) + S_r(u,v)$$

$\mathcal{F}^{-1} \downarrow$

$$s(x,y) = s_i(x,y) + s_r(x,y)$$

$$e^{s(x,y)} = e^{s_i(x,y)} + e^{s_r(x,y)}$$

$$g(x,y) = e^{s(x,y)}$$

$$f(x,y) \rightarrow \boxed{\ln} \rightarrow \boxed{FT} \rightarrow \boxed{H} \rightarrow \boxed{IFFT} \rightarrow \boxed{e^{(\,)}} \rightarrow g(x,y)$$

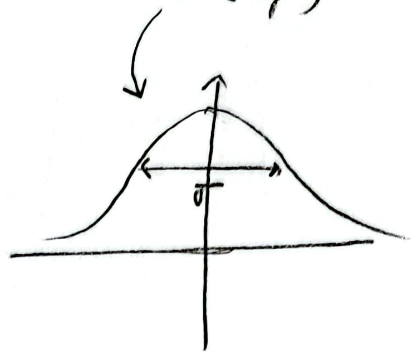
Match Filter

Filtrelene görselin konjuge simetrik olması lazım. $f(x,y)$ filtreler ise $f(-x,-y)$'de filtrelenmeli

Canny Edge Detection

$f(x,y) \longrightarrow G_\sigma(x,y)$ $\qquad g(x,y) = f(x,y) * G_\sigma(x,y)$

1. Smooth Image



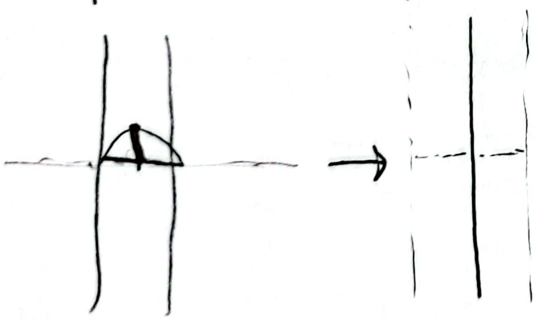
2. Compute gradient of $g(x,y)$

$g_x(x,y) \quad g_y(x,y)$ $\qquad M(x,y) = \sqrt{g_x^2(x,y) + g_y^2(x,y)}$

$$\theta = \tan^{-1}\left(\frac{g_y(x,y)}{g_x(x,y)}\right)$$

3. $M_t(x,y) = \begin{cases} M(x,y) & \text{if } M(x,y) > T \\ 0 & \text{o.w.} \end{cases}$

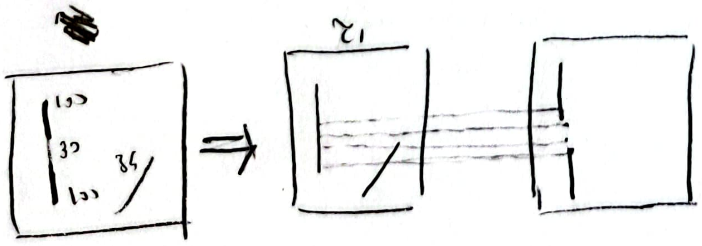
4. Supress non-maxima pixels



$\rightarrow M_s(x,y)$

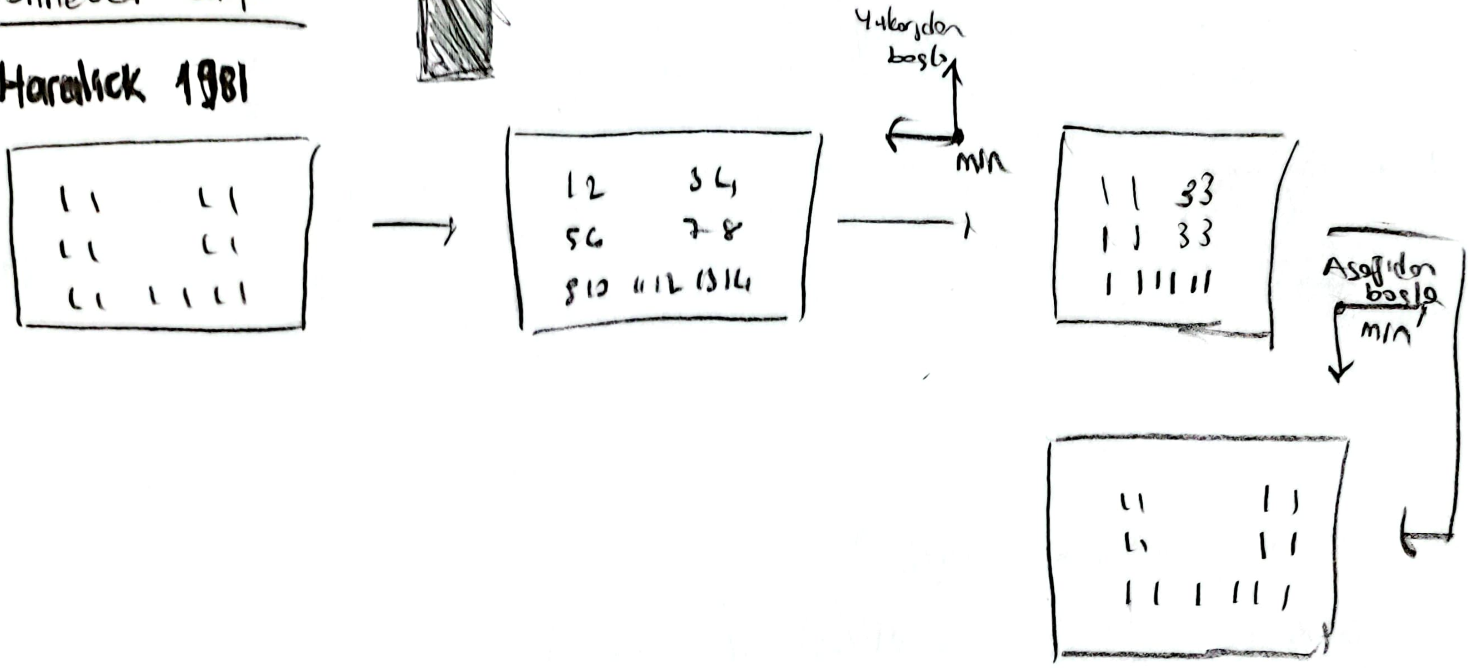
5. Threshold M_s by two thresholds τ_1, τ_2

$\tau_1 < \tau_2$



Conneted Component

Haralick 1981



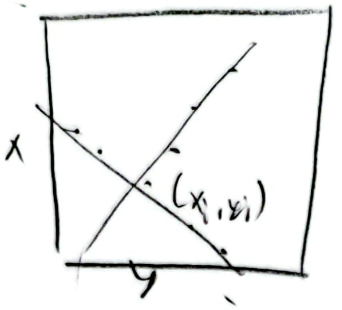
Recursive Component Labeling Algorithm

while end of image not reached
do { continue to scan image until an unlabeled "1"
pixel if found or end of image reached;
if (not end of image) { assign new label to pixel;
recursively assign same label to "1"
neighbours }

Hough Transform

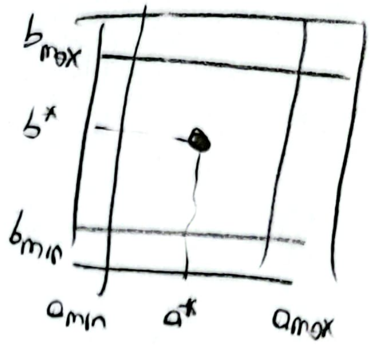
8. Hafta

Input Image : Binary Image

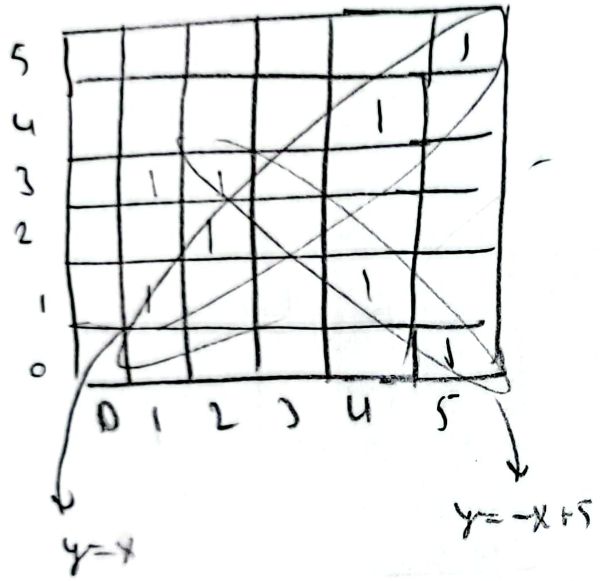


$y = a_1 x + b_1$

$y = a_2 x + b_2$



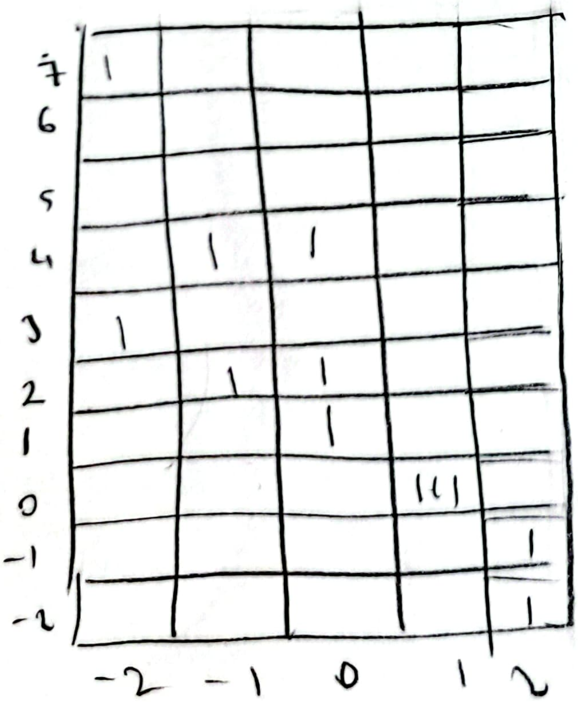
$b^* = -a^* x_i + y_i$



$(x_1, y_1) = (1,1)$

$b = -ax + y = -a + 1$

a \ b	
-2	3
-1	2
0	1
1	0
2	-1



$(x_i, y_i) = (2,2)$

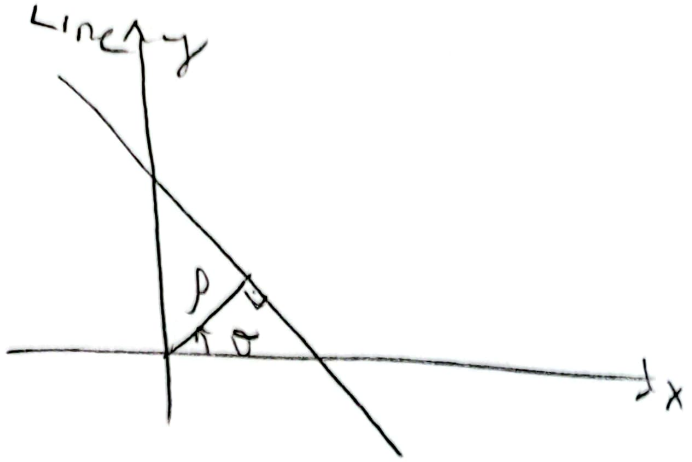
$b = -2a + 2$

a \ b	
-2	6
-1	4
0	2
1	0
2	-2

$(x_i, y_i) = (4,4)$

$b = -4a + 4$

a \ b	
-2	12
-1	8
0	4
1	0
2	-4

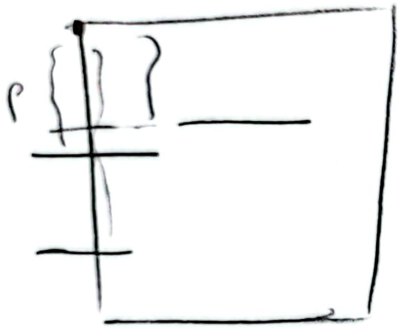


$$x\cos\theta + y\sin\theta = \rho$$

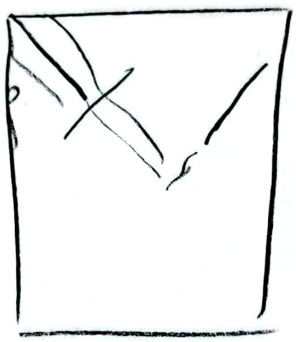
θ: 0, 180 arası sınırlı

ρ: $\sqrt{x^2+y^2}$ sınırlı

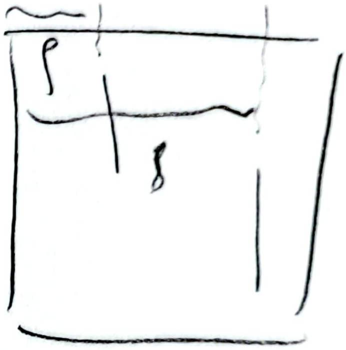
$\theta = 0$



$\theta = 45$



$\theta = 90$



daire için

$\hookrightarrow (x-x_0)^2 + (y-y_0)^2 = r^2$