Image processing in physics

Resolution and Noise

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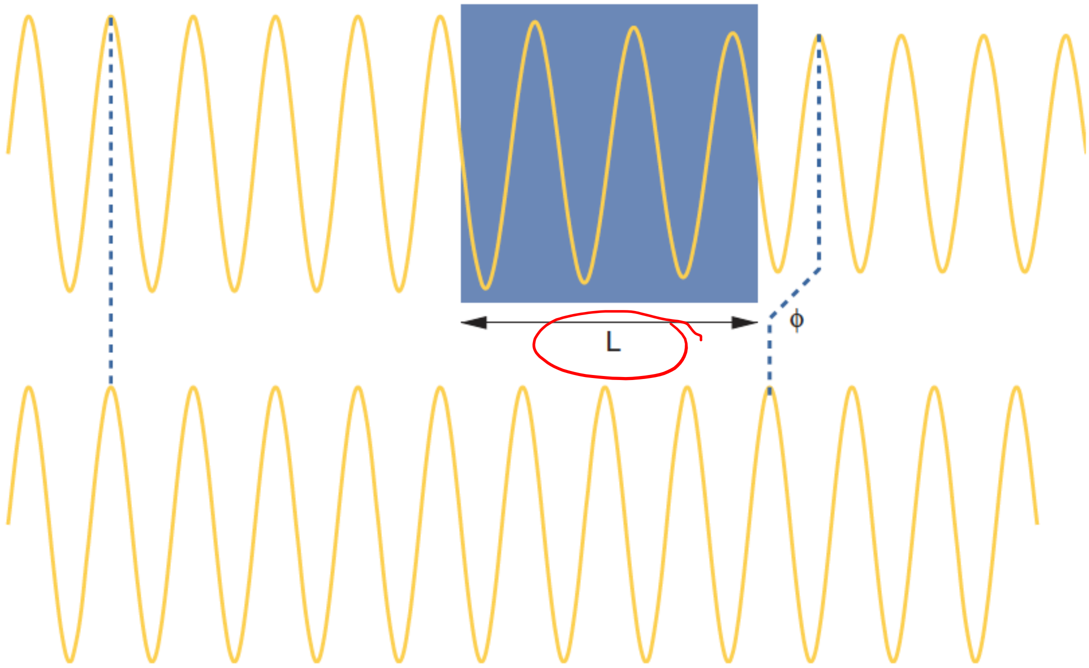
Organizational: evaluation next lecture

Review of Last Lecture

Phase Retrieval

What is Phase Contrast?

Phase Contrast



Willmott, Fig. 7.11

- phase velocity is changing inside the object

$$v_{ph} = \frac{c}{n} > c_{vacuum}$$

→ phase shift $\phi = \frac{2\pi\delta}{\lambda} \cdot L$

↑ object thickness

What are the Advantages/Problems of Phase Contrast Imaging?

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Main advantage: high soft-tissue contrast;

Main problem: phase not directly measuralbe

Which methods exist to measure the phase shift?

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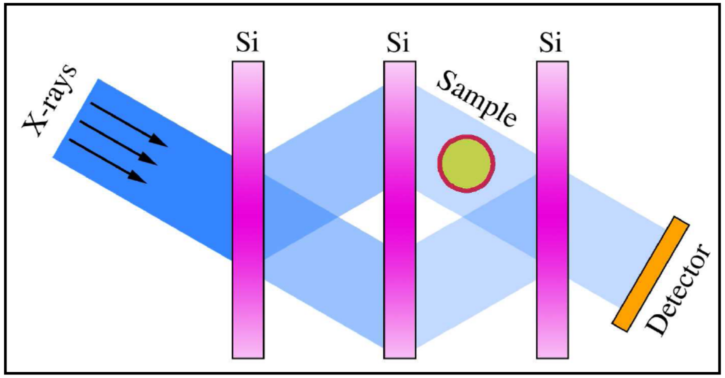
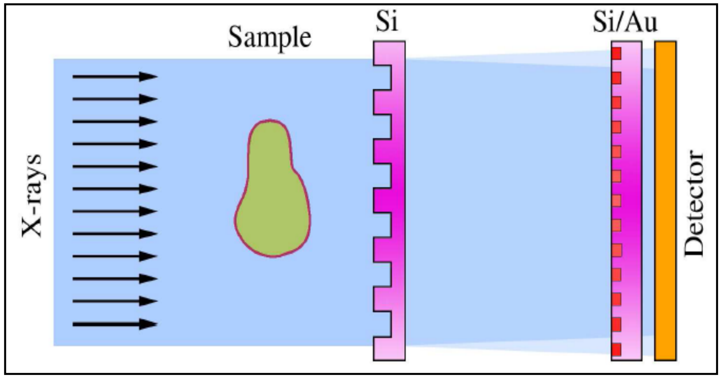
Φ Cristal Interferometer



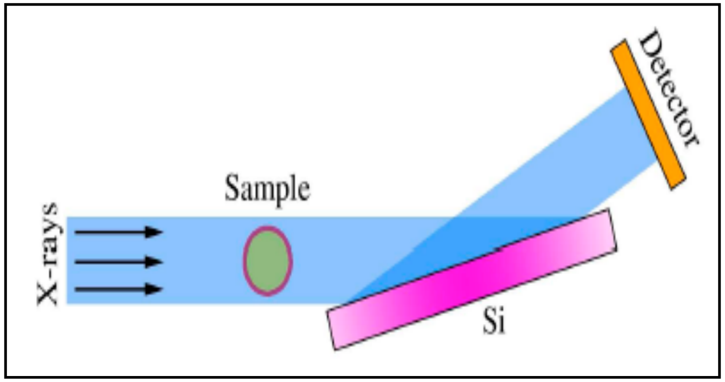
Bonse & Hart 1965

$d\Phi/dx$ Grating Interferometer



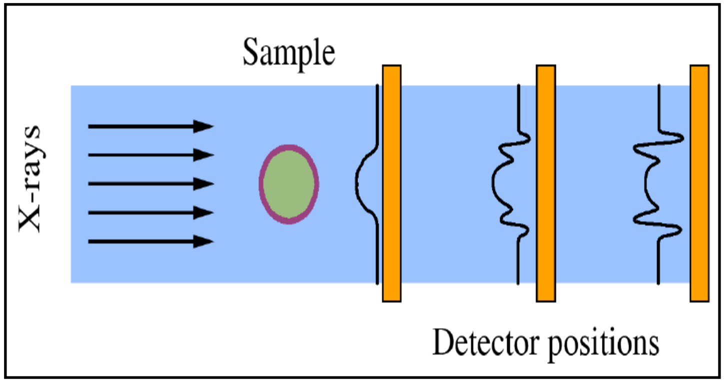
Momose 2003 & David 2002

$d\Phi/dx$ Cristal Analyzor



Förster 1980 & Davis 1995

$\Delta\Phi$ Propagation-based Method



Snigirev 1995, Cloetens & Wilkens 1996

What is CDI and what are its advantages/problems?

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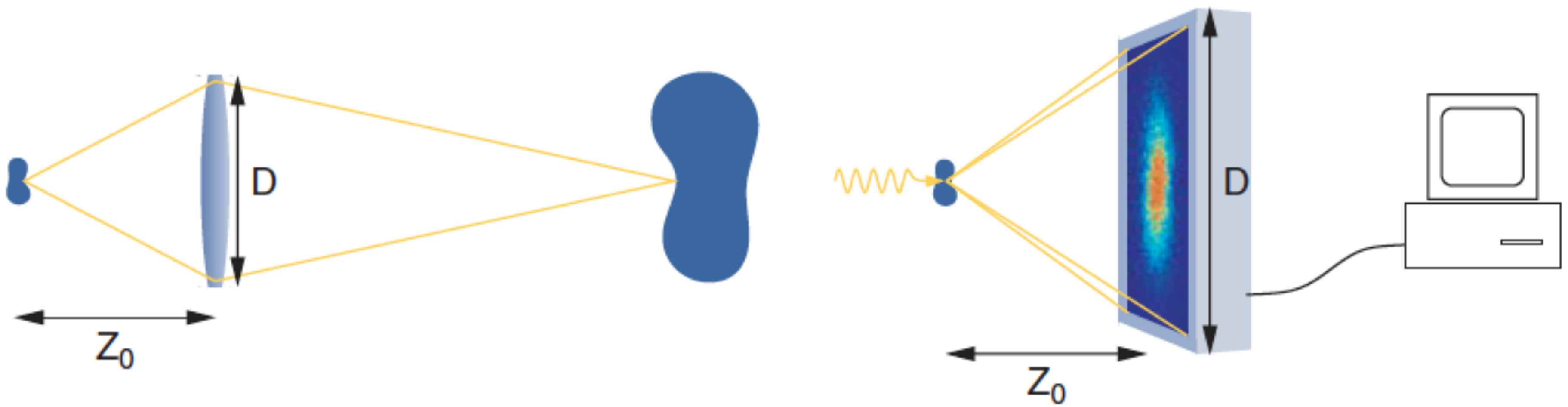


CDI: Coherent Diffractive Imaging is a lensless imaging method working in the far-field regime.

Main advantages: high soft-tissue contrast & atomic resolution

Main problem: phase retrieval required (not trivial) – either with constraints like isolated objects or with ptychography

Overview

- Resolution in imaging
- Linear imaging systems and transfer functions
- Noise and denoising
- Correlation theorem
- Wiener filter

- Chapter 5 in Gonzalez, “Digital Image Processing” covers the noise, correlation, filtering stuff but not the optics parts

Definition of resolution

- no unified definition for resolution
- criteria depend on context e.g.
  - detector characterization: FWHM of PSF, 10% MTF
  - microscopy: numerical aperture

We will use the general definition:

smallest detail that can be distinguished

- need understanding of “detail”
- need understanding of “distinguished”

Definition of resolution

1280 x 1280

640 x 640

- resolution is not simply given by pixel size (i.e. sampling rate)
- light quality, optics quality, detector quality, algorithm quality, noise, ...

Linear translation-invariant (LTI) systems

- Model the system as operator

input f operator S output g

$$S(f) = g$$

- Linearity

$$S(af_1 + bf_2) = ag_1 + bg_2$$

- Translation-invariance

$$S(f(x-x_0)) = g(x-x_0)$$

Impulse response function

- Impulse response

$$f(x) = \delta(x) \begin{cases} \infty & x=0 \\ 0, & \text{else} \end{cases}$$

$$S(f(x)) = g$$

$$S(\delta(x-x_0)) = \underbrace{h(x-x_0)}_{\text{impulse function}}$$

- Impulse response in LTI systems

$$\underset{\text{output}}{g(x)} = S(f(x)) = \underset{\text{input}}{f(x)} \underset{\text{convolution}}{\otimes} \underset{\text{impulse function}}{h(x)}$$

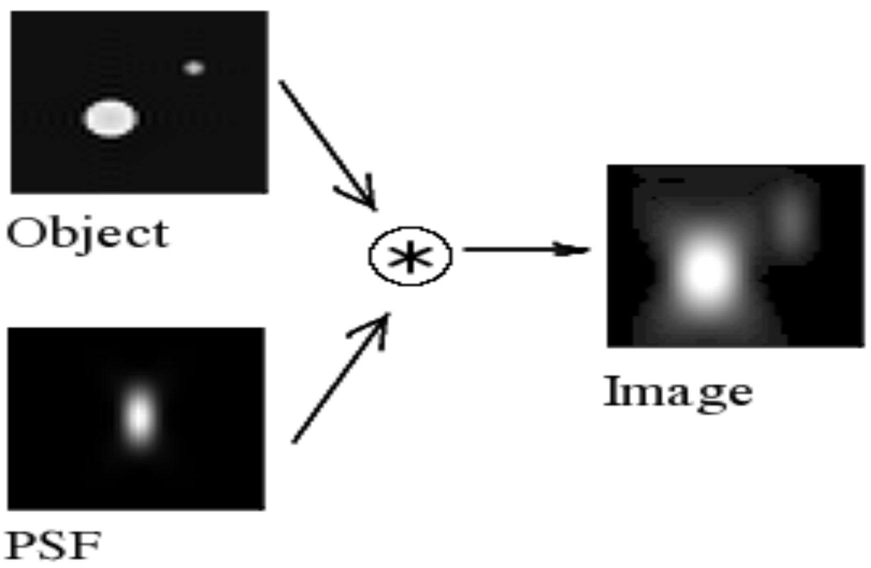
- LTI systems can be characterized by h(x)

fully

Point Spread Function (PSF)

- PSF is the impulse response of an imaging system (response to a point source)
- PSF describes the blurring (spreading) of an optical system

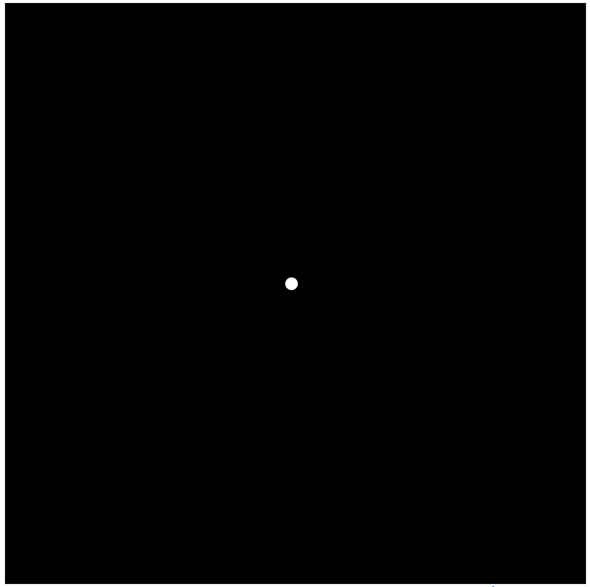
image = object $\otimes$ PSF

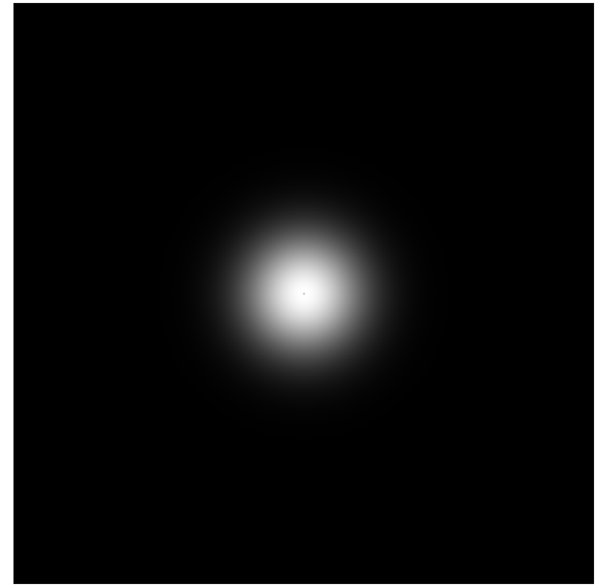


linear image = $\text{object}_1 \otimes \text{PSF} + \text{object}_2 \otimes \text{PSF} + \ldots$

Point Spread Function (PSF)

point source

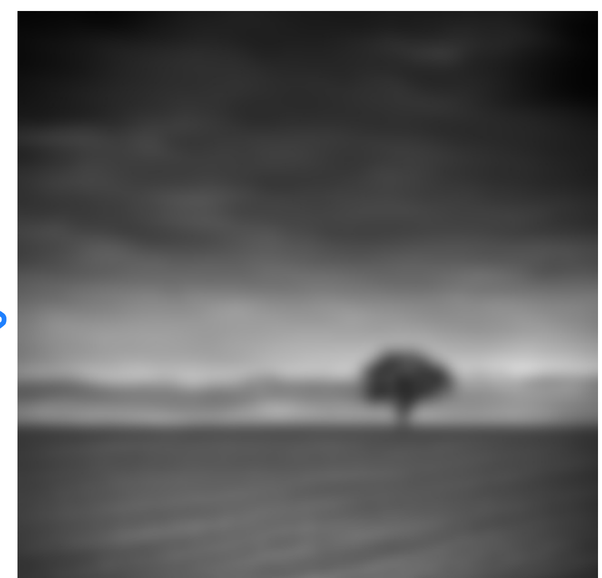




PSF

optical system

object



$f(x) \otimes PSF$

PSF examples

- Isolated stars are essentially PSFs
- Each star gives a PSF
- PSF can depend on position in the field (lens aberrations etc)

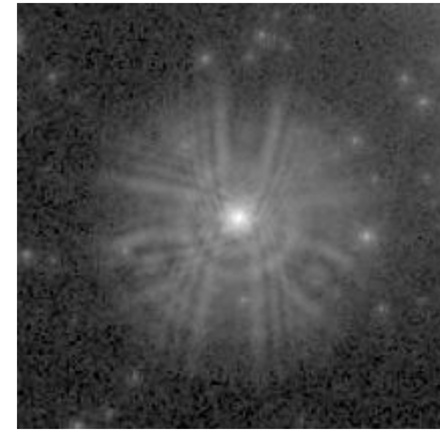
source: www.apod.nasa.gov

Hubble Space Telescope PSF

- Primary mirror of Hubble mirror had too much spherical aberration => PSF much broader than expected
- Extra optic (with inverse aberration) added on a later shuttle mission

first mirror

second mirror



PSF

see lecture „adaptive optics"

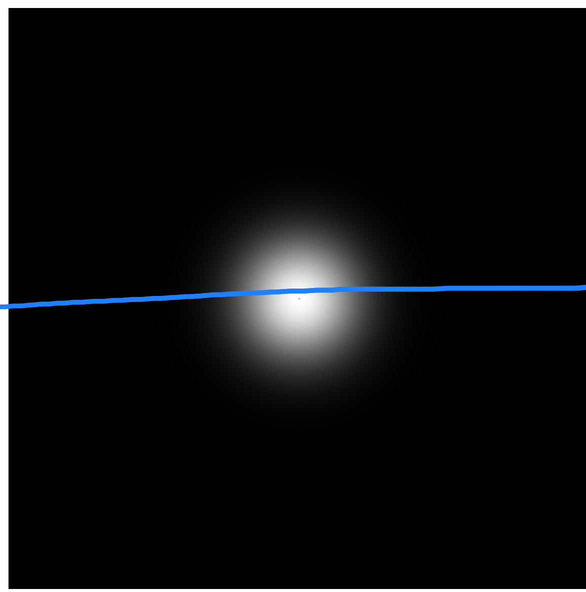
Before correction

After correction

source: www.wikipedia.org

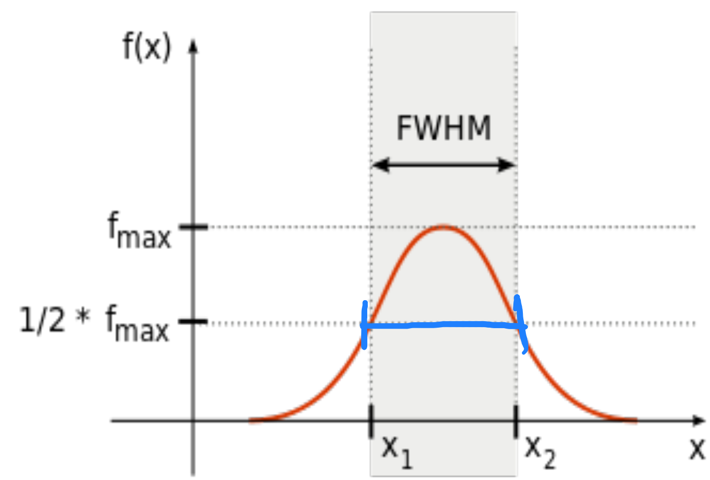
PSF and resolution

- FWHM (Full Width at Half Maximum) of the PSF is a measure of resolution (used in optics/astronomy)
- Often estimated by fitting a (2D) Gaussian to the PSF



PSF

Profile



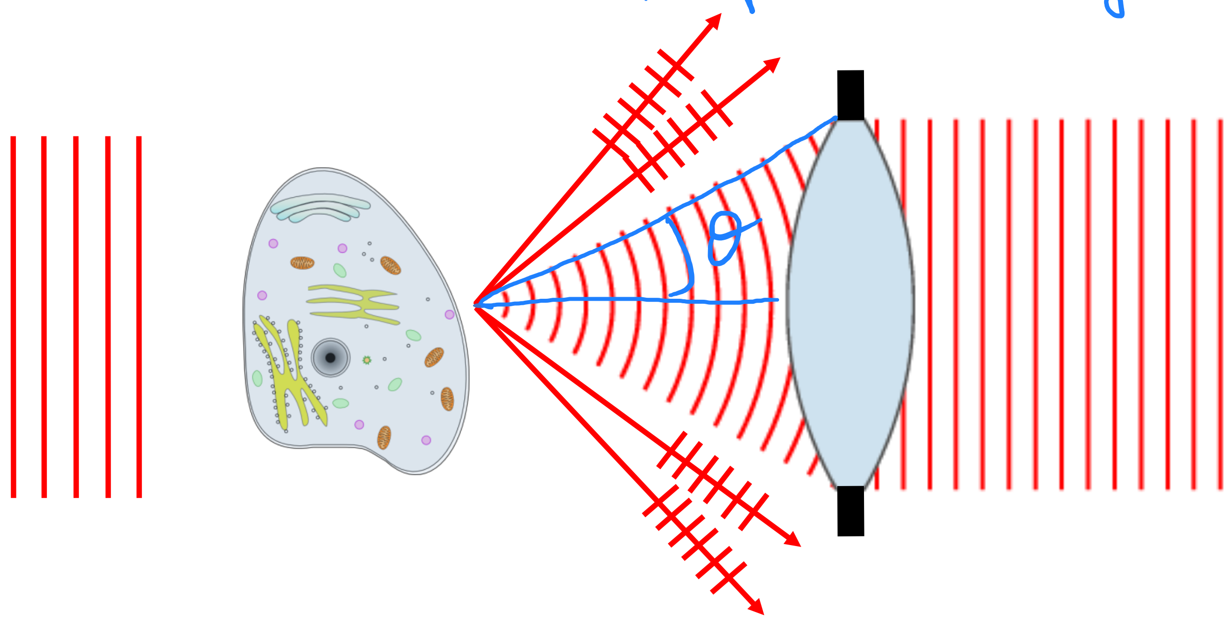
PSF

resolution of imaging system

Numerical aperture (NA)

- Commonly used in microscopy
- NA gives the range of angles which the system can accept or emit light

$$NA = \underbrace{n}_{\text{refractive index}} \cdot \sin \underbrace{\theta}_{\text{acceptence angle}}$$



Optical Transfer Function (OTF)

- OTF describes how system affects an oscillating signal with well-defined frequency
- OTF is Fourier transform of PSF

$$OTF = \mathcal{F}\{PSF\}$$

- Amplitude of OTF is known as modulation transfer function (MTF)

$$MTF = |OTF|$$

- Phase of OTF is known as phase transfer function (PTF)

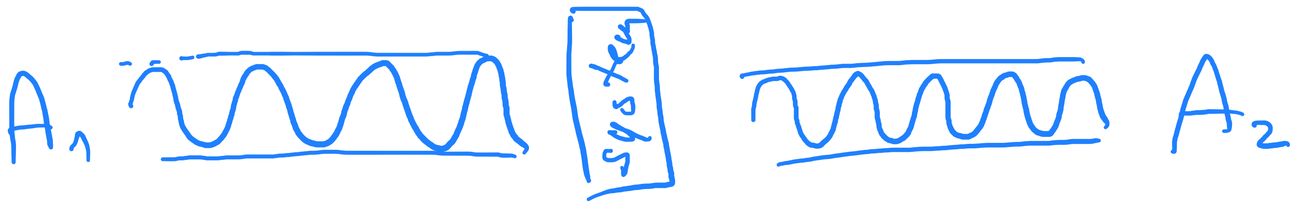
$$PTF = \text{angle}(OTF)$$

- OTF can therefore also be written as

$$OTF = MTF \cdot e^{-i(PTF)}$$

Modulation transfer function (MTF)

- MTF describes how an oscillating signal changes in amplitude due to system
- Resolution sometimes defined as spatial frequency at 10% of MTF



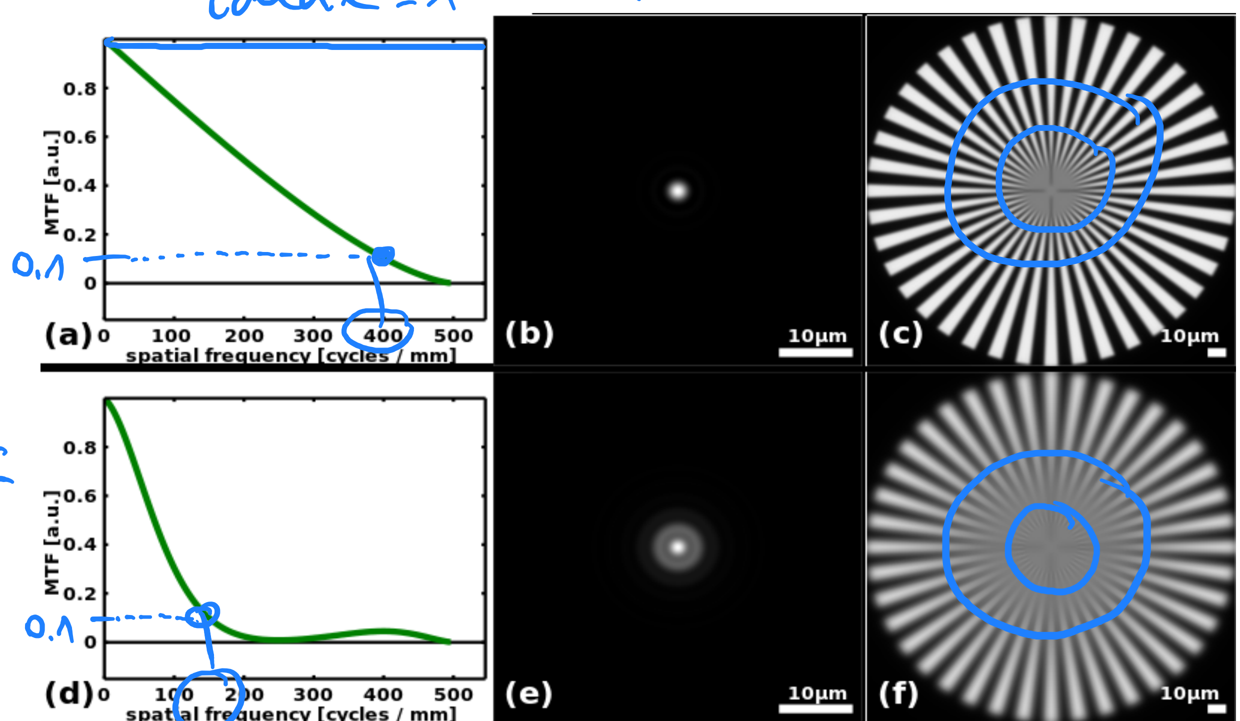
same frequency, but reduced amplitude

$$MTF = \frac{A_2}{A_1}$$

ideal = 1

PSF

10% MTF



Well-focused

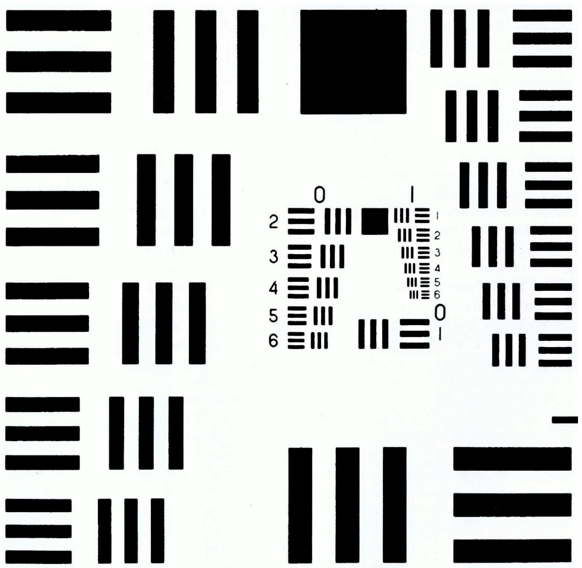
Out of focus

Point source

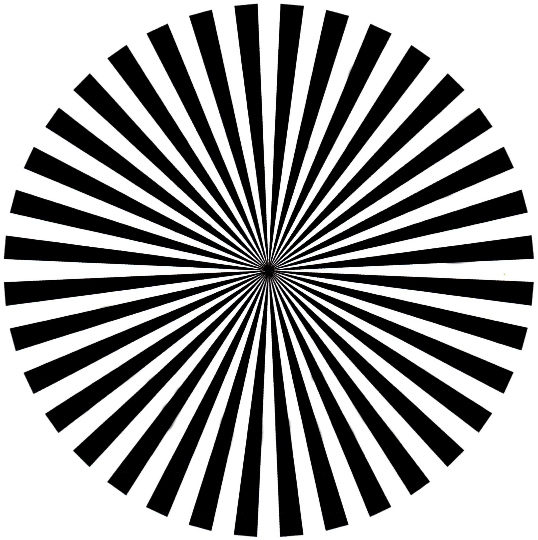
Siemens star

source: www.wikipedia.org

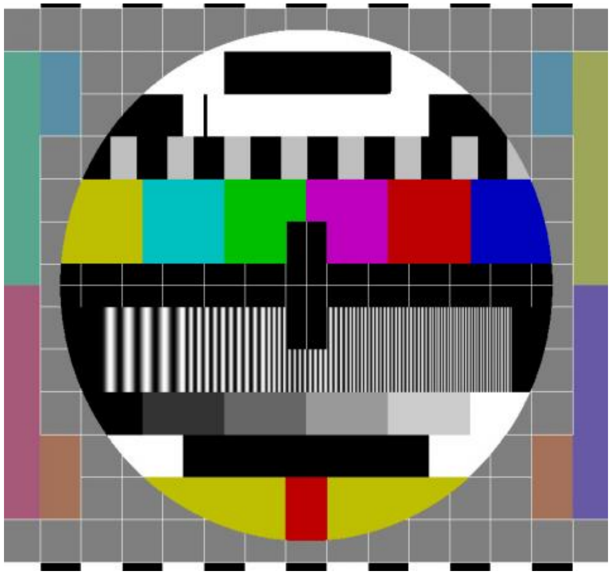
Measurement of MTF



USAF 1951 chart



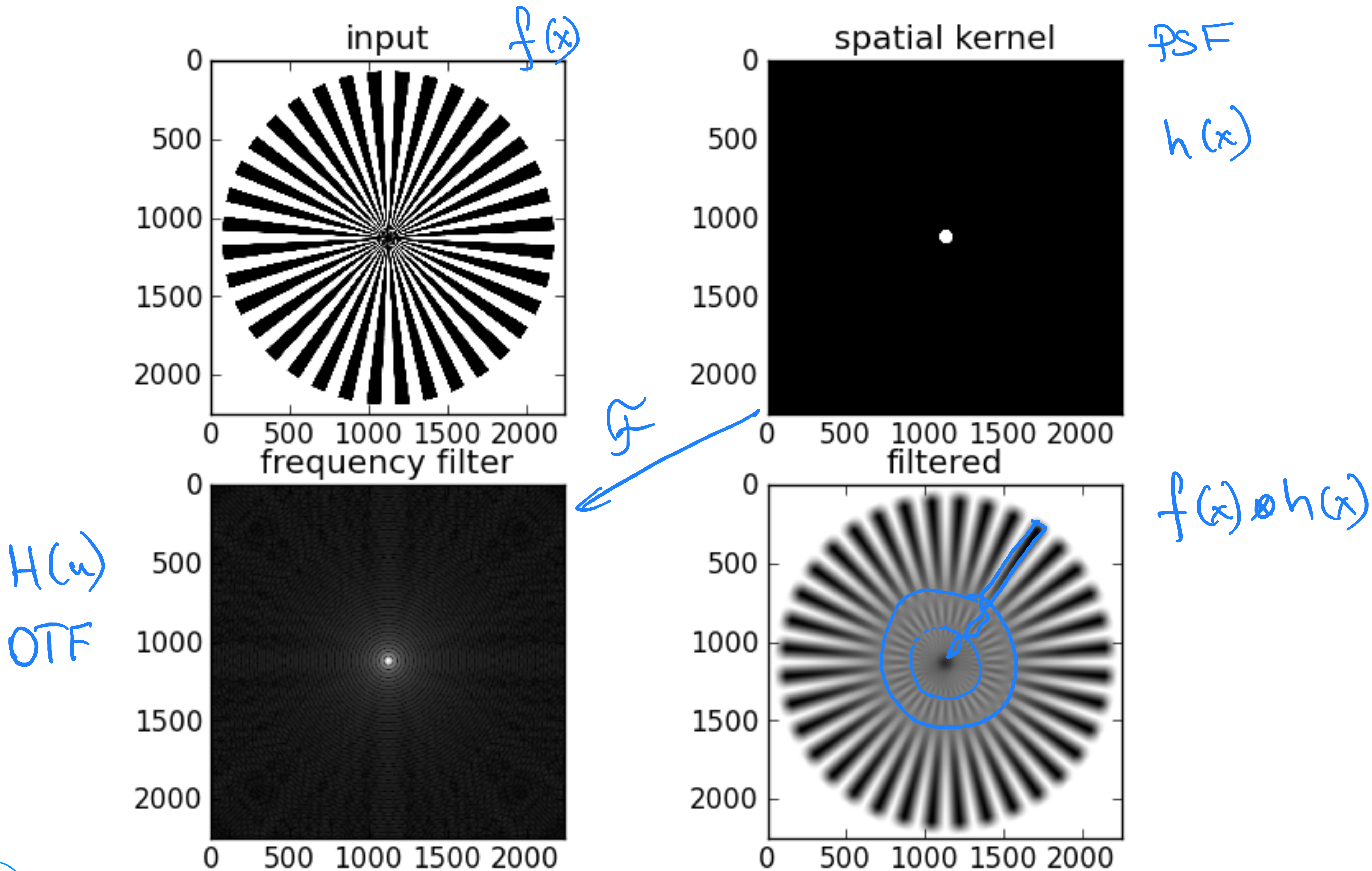
Siemens star



Old TV

Phase transfer function (PTF)

- Describes how an oscillating signal changes in phase due to system



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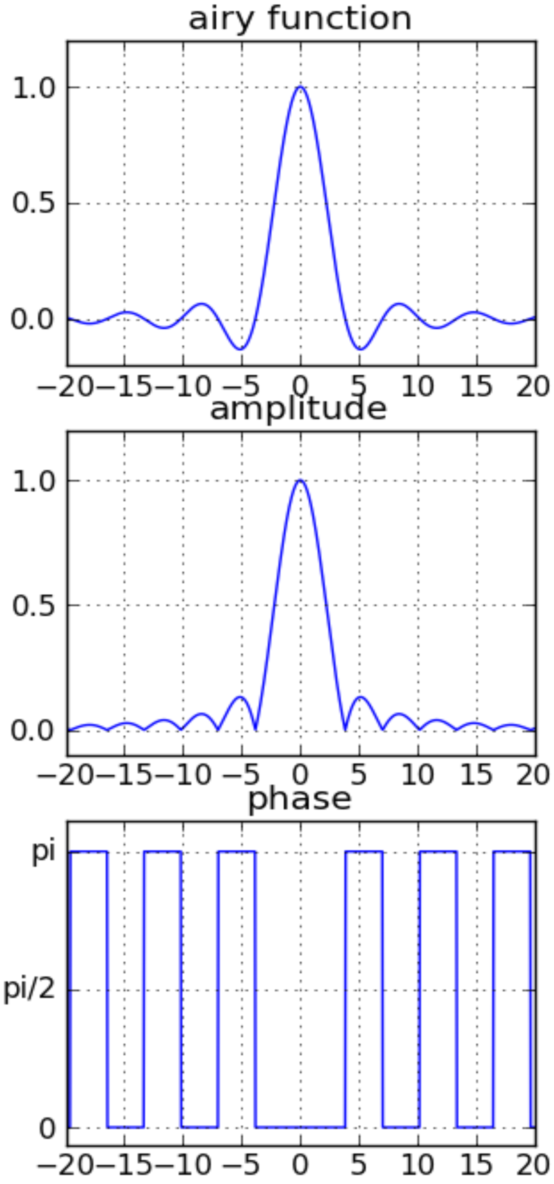
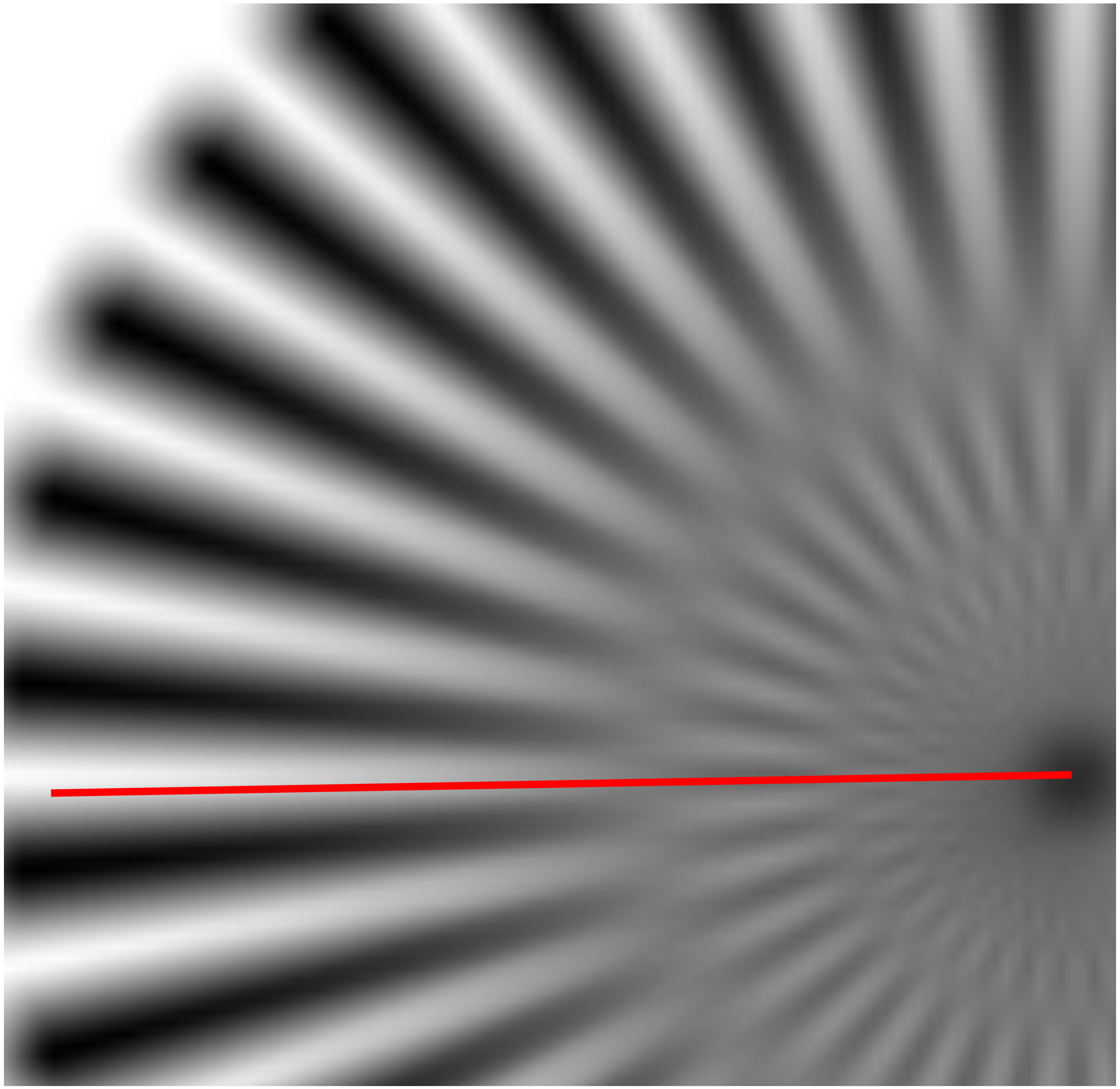




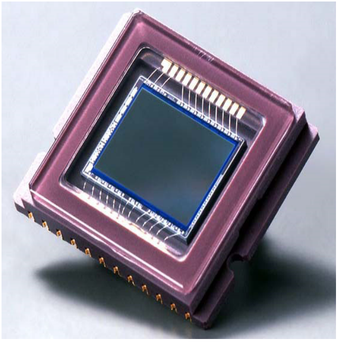
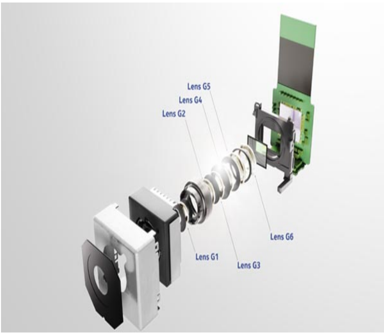
Imaging as a linear filter

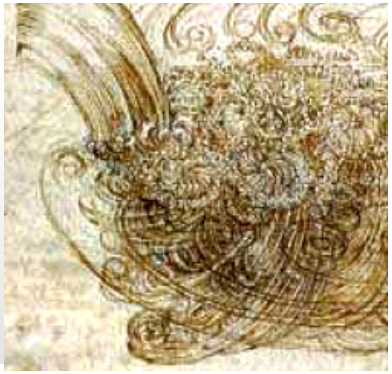
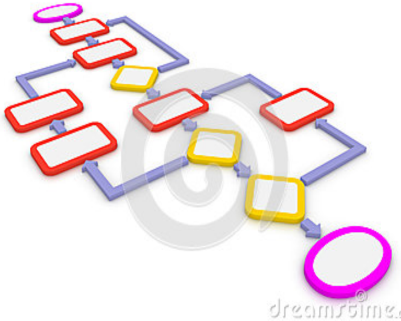
- MTF of system is the product of the MTFs of the individual subsystems

output (u) = input (u) · MTF (optics) · MTF (detector) ·

MTF (atmosphere) · MTF (algorithm) · ...

imaging → linear filtering in frequency domain

Definition of noise

The general definition of noise is:

any unwanted signal (image)

Here, we will use the following definition of noise:

random, uncorrelated signal (image)

- need understanding of statistics and random processes
- need understanding of correlation

Random variables (RV)

X is a RV

- Probability Density Function (PDF)

$$P(a<x<b) = \int_a^b p(x)\,dx$$

- Expectation Value

$$E[x] = \int_{-\infty}^{\infty} x\,p(x)\,dx = \mu$$

- Variance

$$Var[x] = E[(x-E[x])^2]$$

- Skewness (symmetry), kurtosis (peakedness),= higher central moments

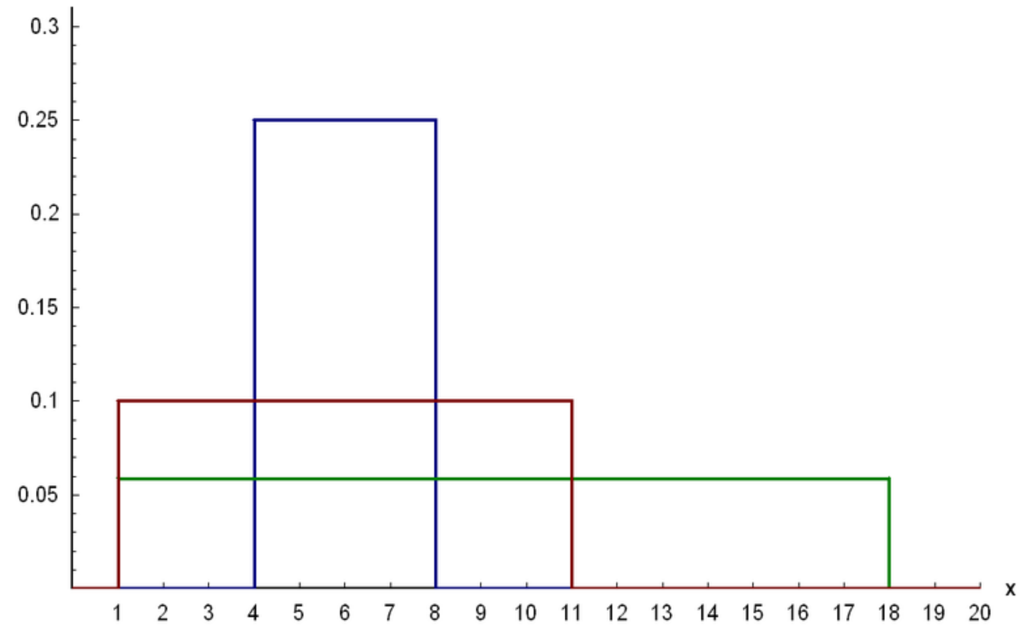
$$S[x] = E[(x-E[x])^3]$$

$$K[x] = E[(x-E[x])^4]$$

Uniform distribution

- Probability density function

$$P(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0, & \text{else} \end{cases}$$



- Expectation value

$$E[x] = \frac{1}{2}(a+b)$$

- Variance

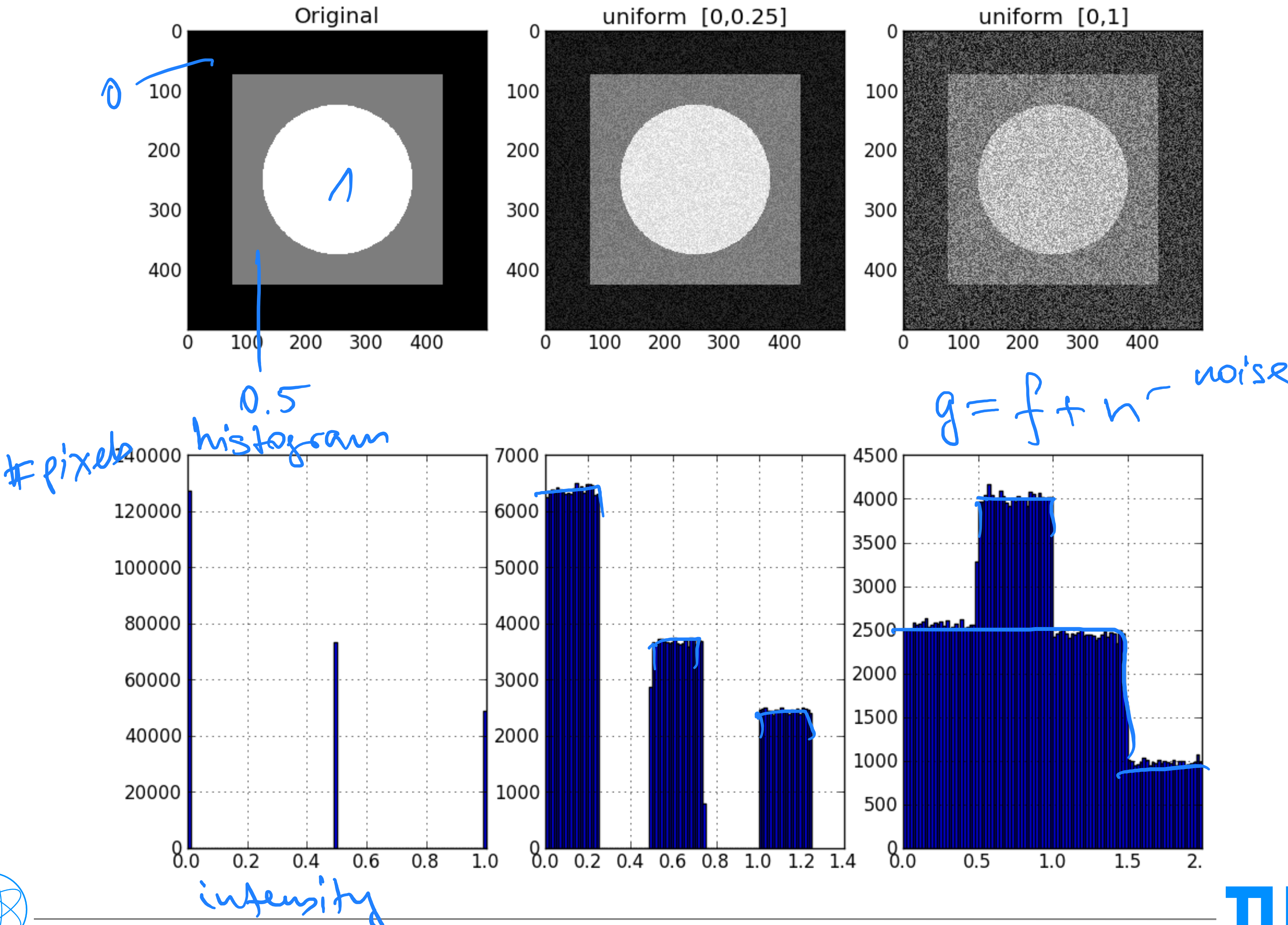
$$Var[x] = \frac{(b-a)^2}{12}$$

- Occurrence

„ideal"

no real applications in physics

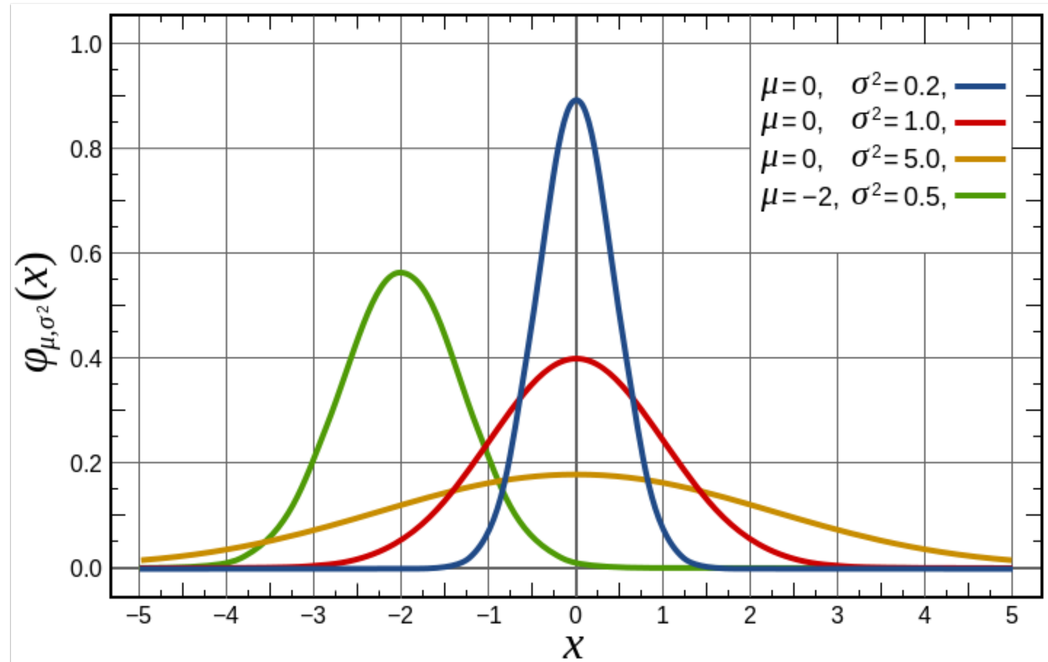
Uniform distribution



Gaussian distribution

- Probability density function

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



- Expectation value

$$E[x] = \mu$$

- Variance

$$Var[x] = \sigma^2$$

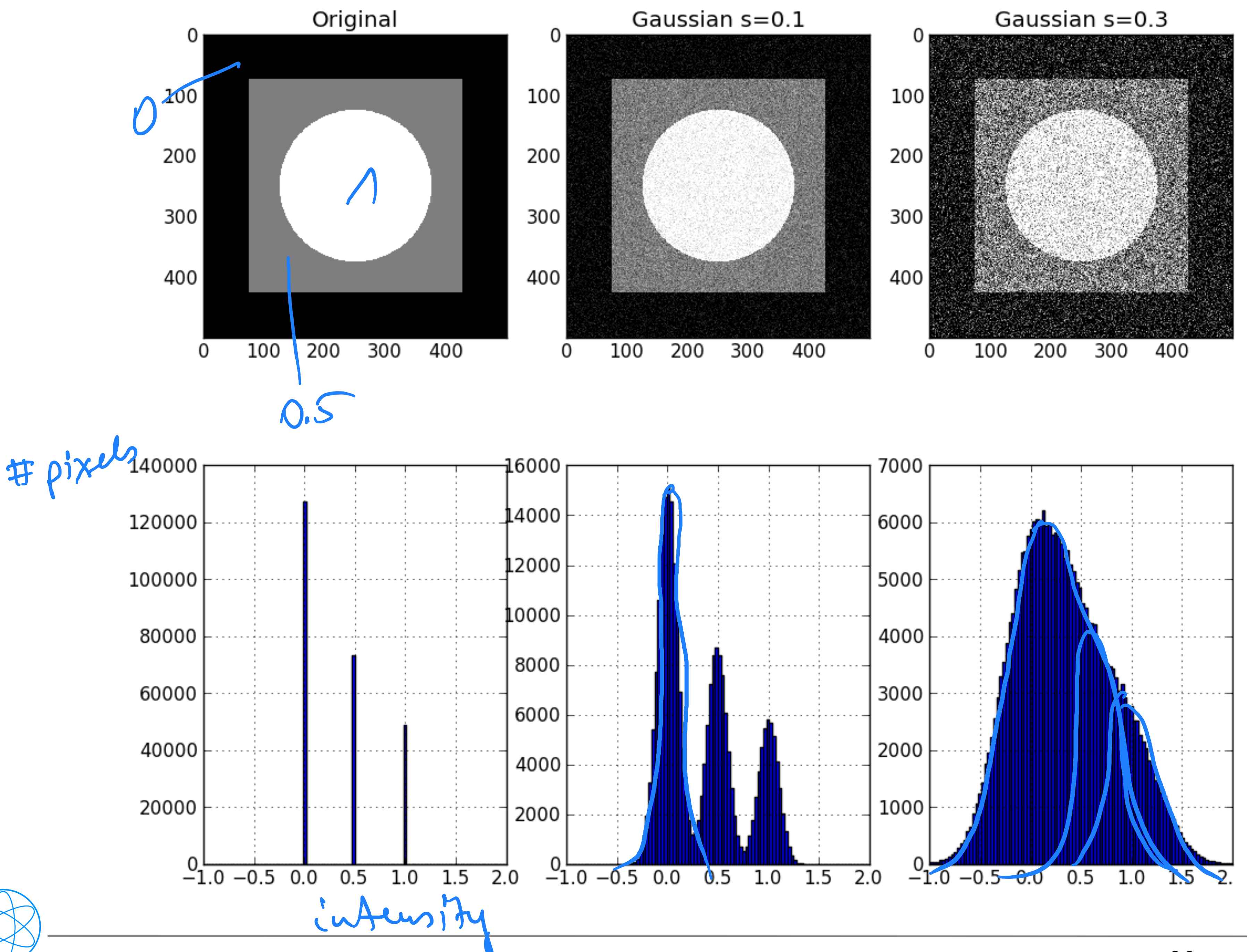
- Occurrence

— normal distribution

e.g. read-out noise of detectors

Central Limit Theorem: sum of independent and identically distributed variables converge towards the normal distribution

Gaussian distribution



Poisson distribution

- Probability mass function

$$P(x) = \frac{\lambda^{x} \cdot e^{-x}}{\lambda !}$$

- Expectation value

$$E[x] = \lambda$$

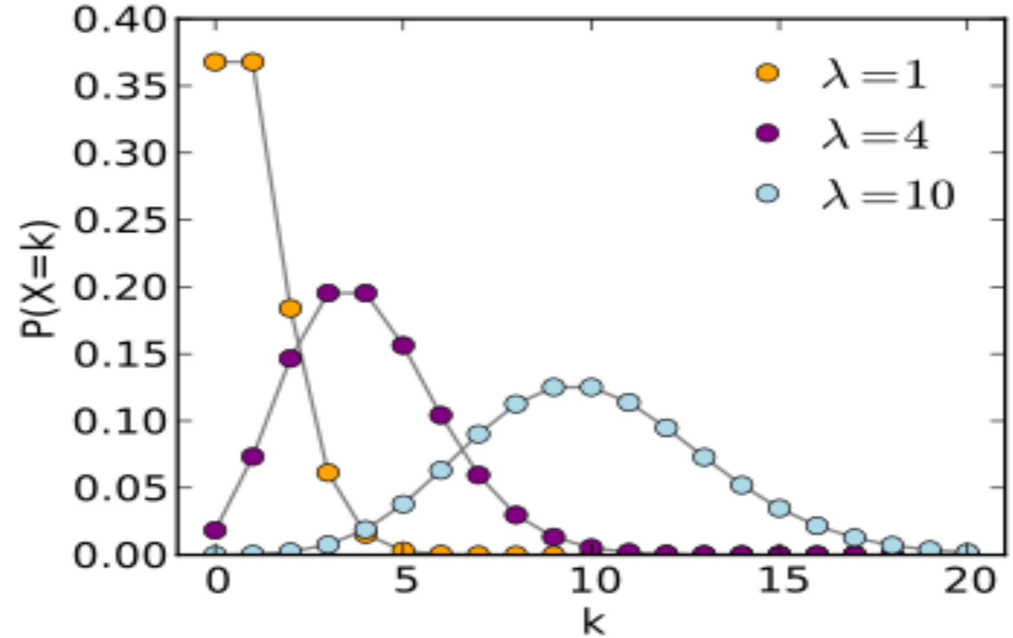
- Variance

$$Var[x] = \lambda$$

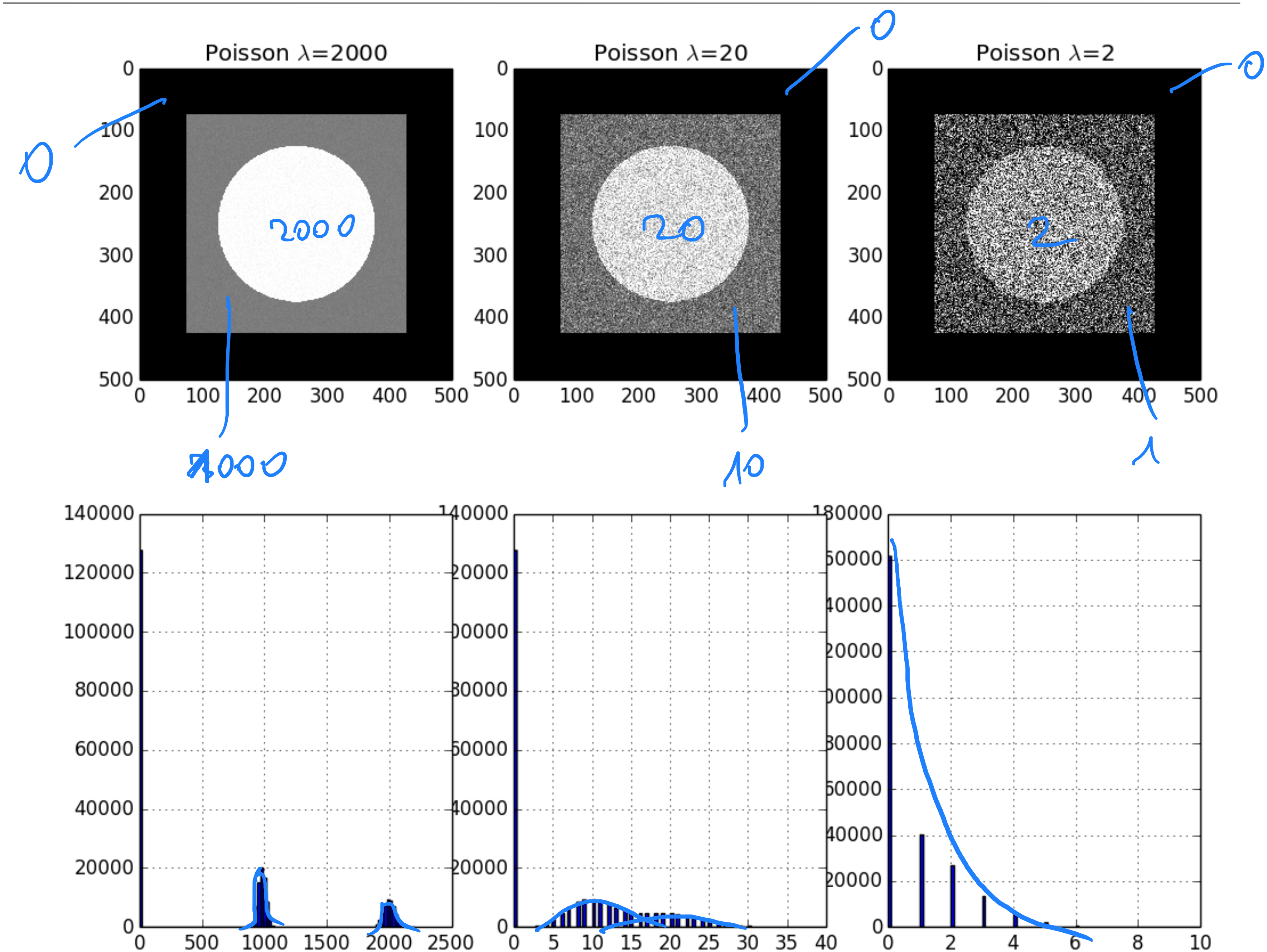
- Occurrence

photon counting

(e.g. phone calls, emails counting)

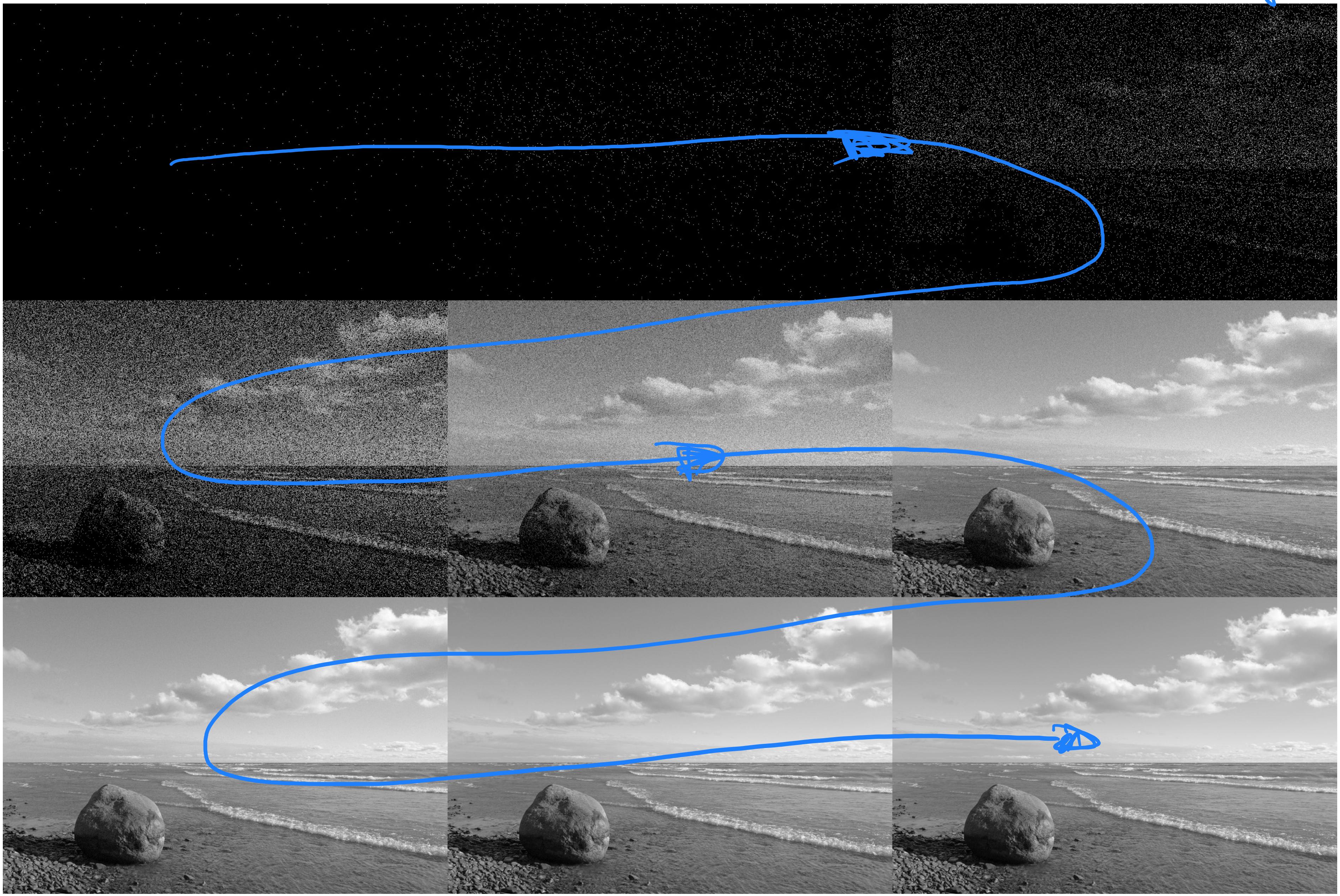


Poisson distribution



Poisson distribution

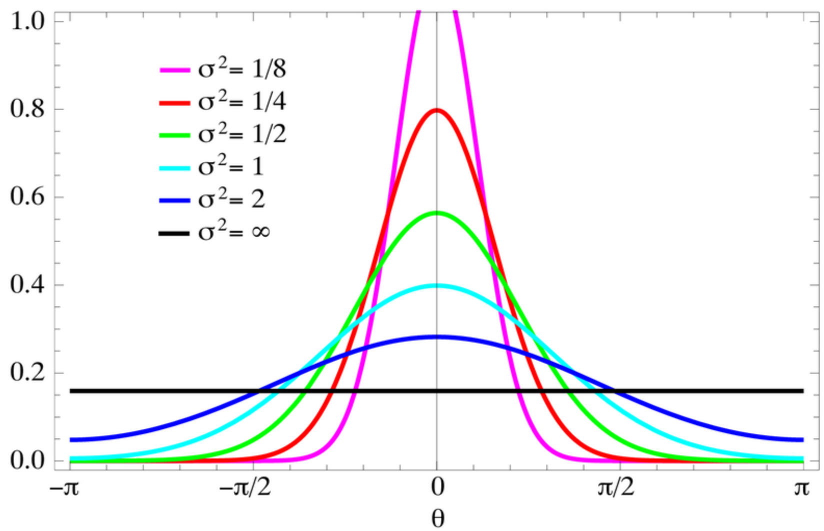
increasing λ



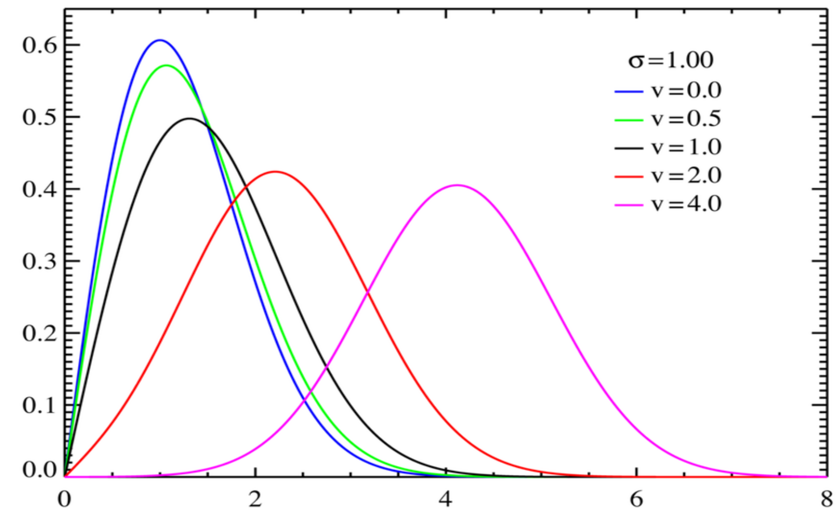
Source: wikipedia.org

Many other distributions!

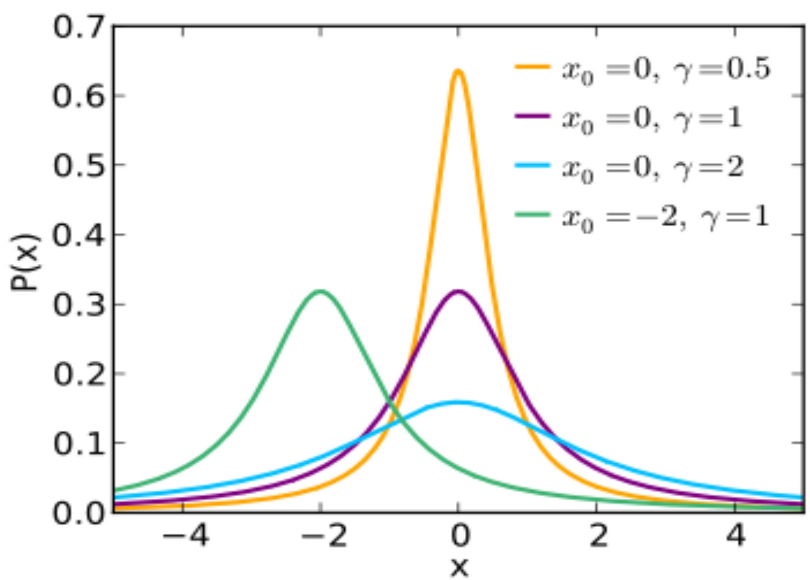
Wrapped normal distribution



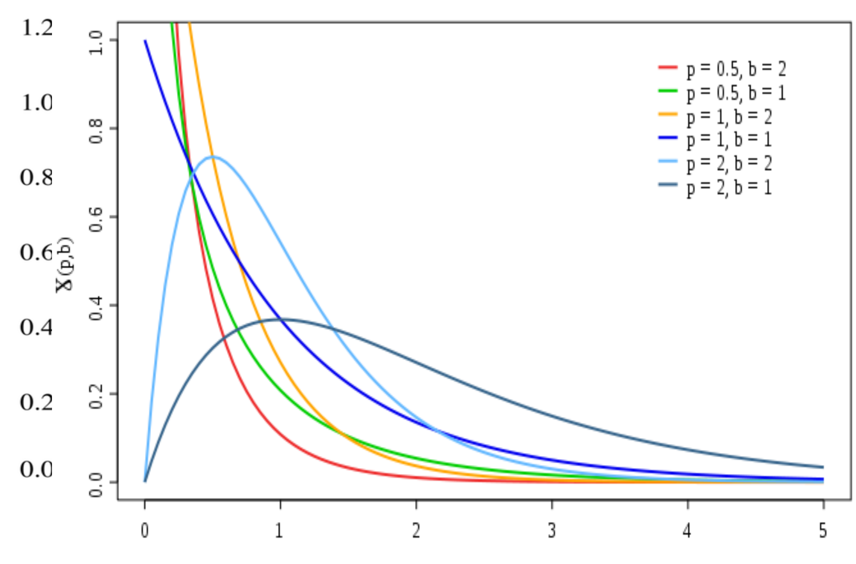
Rice distribution



Lorentz distribution



Gamma distribution

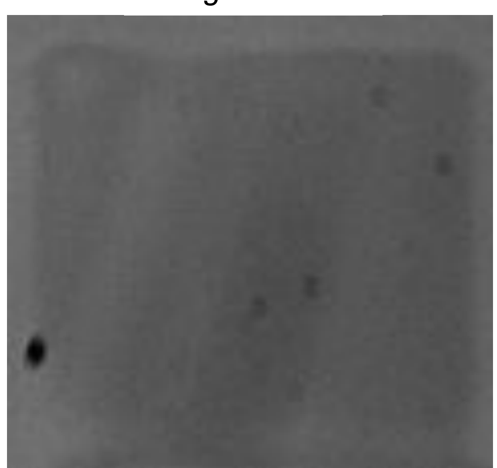


Detector noise (for CCDs)

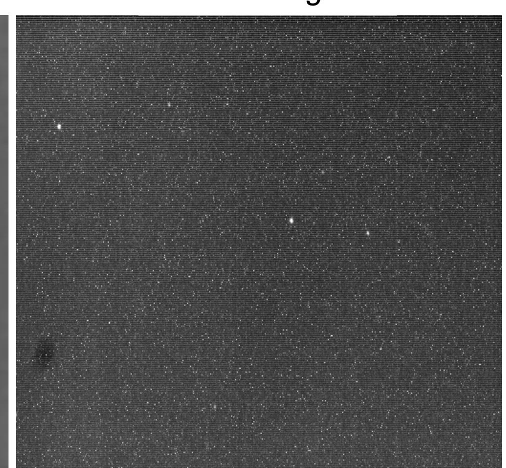
- CCD = Charge Coupled Device
- Various sources including:
  - shot noise (photon statistics, Poisson)
  - dark current (thermal electronic fluctuations in semiconductor, Poisson)
  - readout noise (fluctuations during amplification and digitization, Gaussian)
- dark frame measures detector noise, hot pixels, dead pixels
- bright frame measures gain differences and imperfections (dust, etc)

dark frame

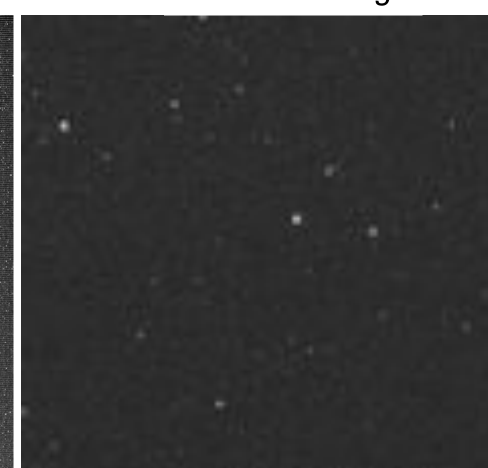
bright frame



raw image

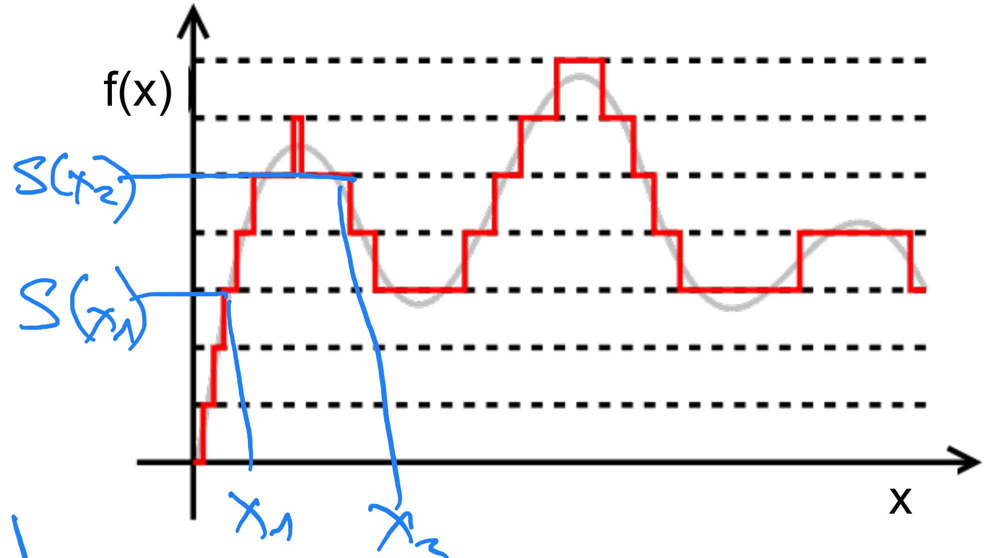


calibrated image



source: H. Raab, Johannes-Kepler-Observatory, Linz

Signal and contrast



Signal

$S(x)$

Contrast

$c = |S(x_2) - S(x_1)|$

Visibility (Michelson contrast)

$$V = \frac{|S(x_2) - S(x_1)|}{S(x_2) + S(x_1)}$$

SNR and CNR

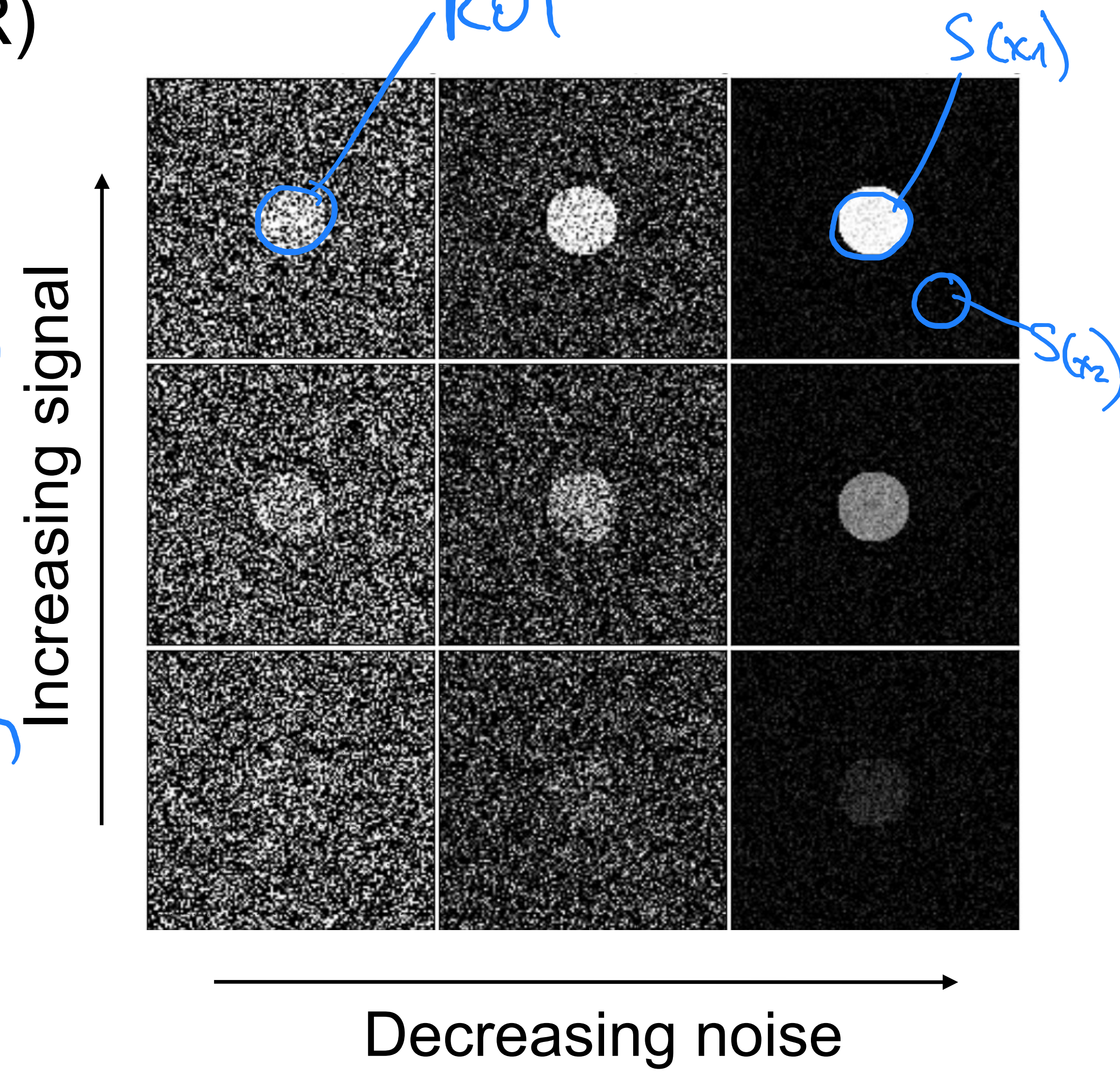
ROI – region of interest

Signal-to-noise ratio (SNR)

$$SNR = \frac{\text{signal}}{\text{noise}} = \frac{s(x)}{\sigma(x)}$$

Contrast-to-noise (CNR)

$$CNR = \frac{|S(x_2) - S(x_1)|}{\sqrt{\sigma(x_2)^2 + \sigma(x_1)^2}}$$



Noise power spectrum (NPS)

- Power spectrum of pure noise image

$$n(x,y) \Longleftrightarrow N(u,v)$$

$$NPS = E\left[\,|N(u,v)|^2\,\right]$$

- Connection to auto-correlation

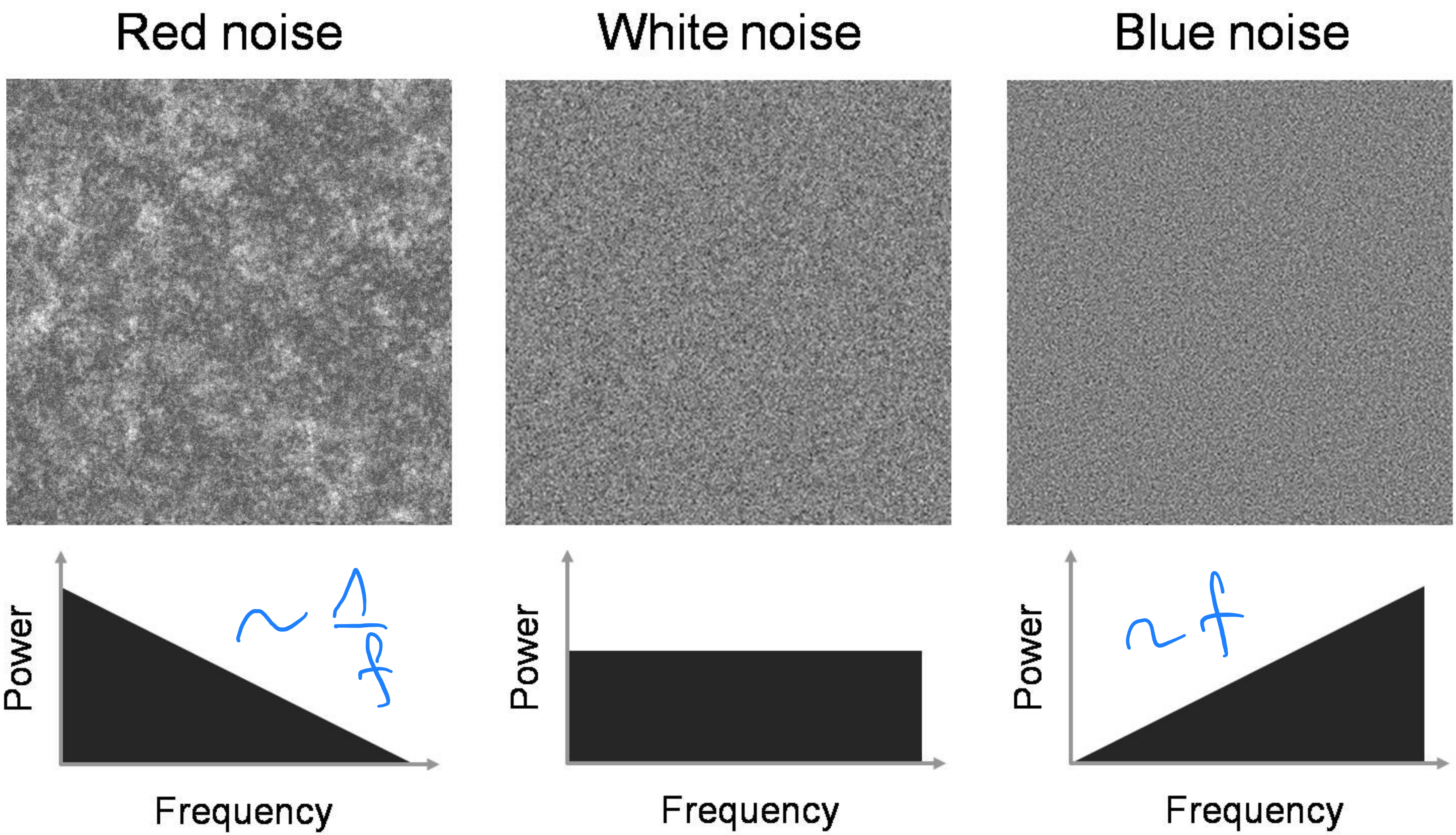
$$n(x,y) * n(x,y) = \mathcal{F}^{-1}\left\{N(u,v) \cdot N^*(u,v)\right\}$$

$$= \mathcal{F}^{-1}\left\{|N(u,v)|^2\right\}$$

- Weiner-Khinchin theorem

auto-correlation of noise $\Longrightarrow$ noise spectral density

Noise power spectrum (NPS)

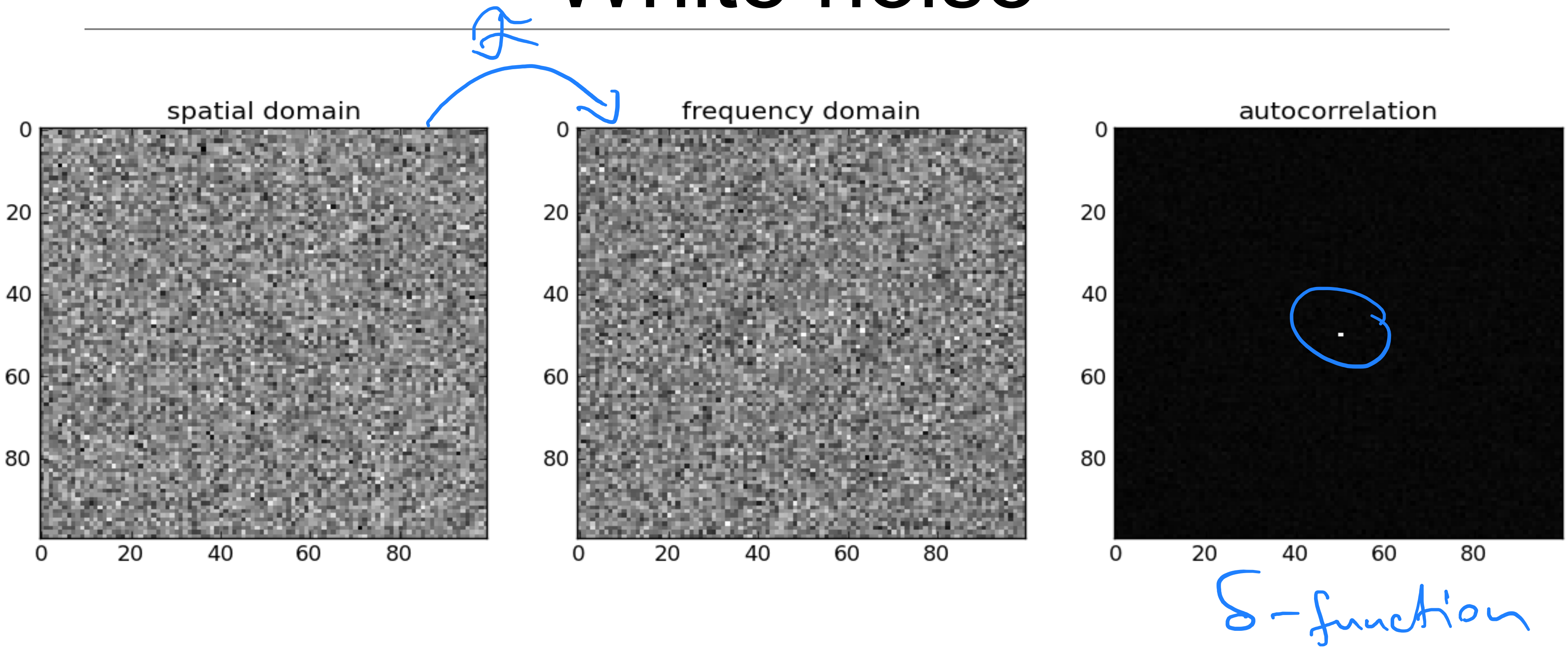


low-frequency noise

high-frequency noise

sourcre: http://scien.stanford.edu/pages/labsite/2008/psych221/projects/08/AdamWang/project_report.htm

White noise

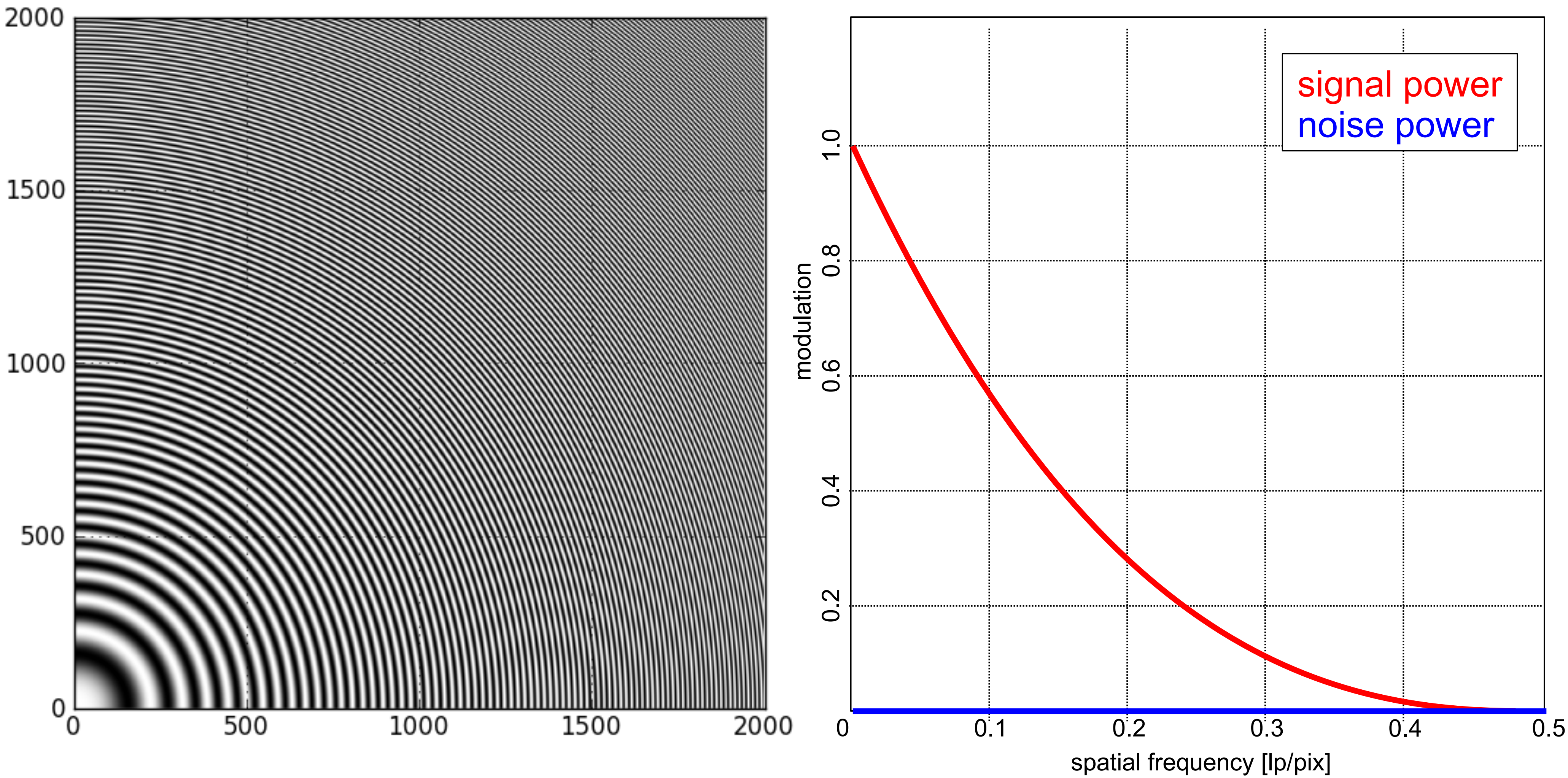


- White noise in spatial domain means white noise in frequency domain
- White noise is perfectly uncorrelated

(Auto-correlation of white noise is the delta function)

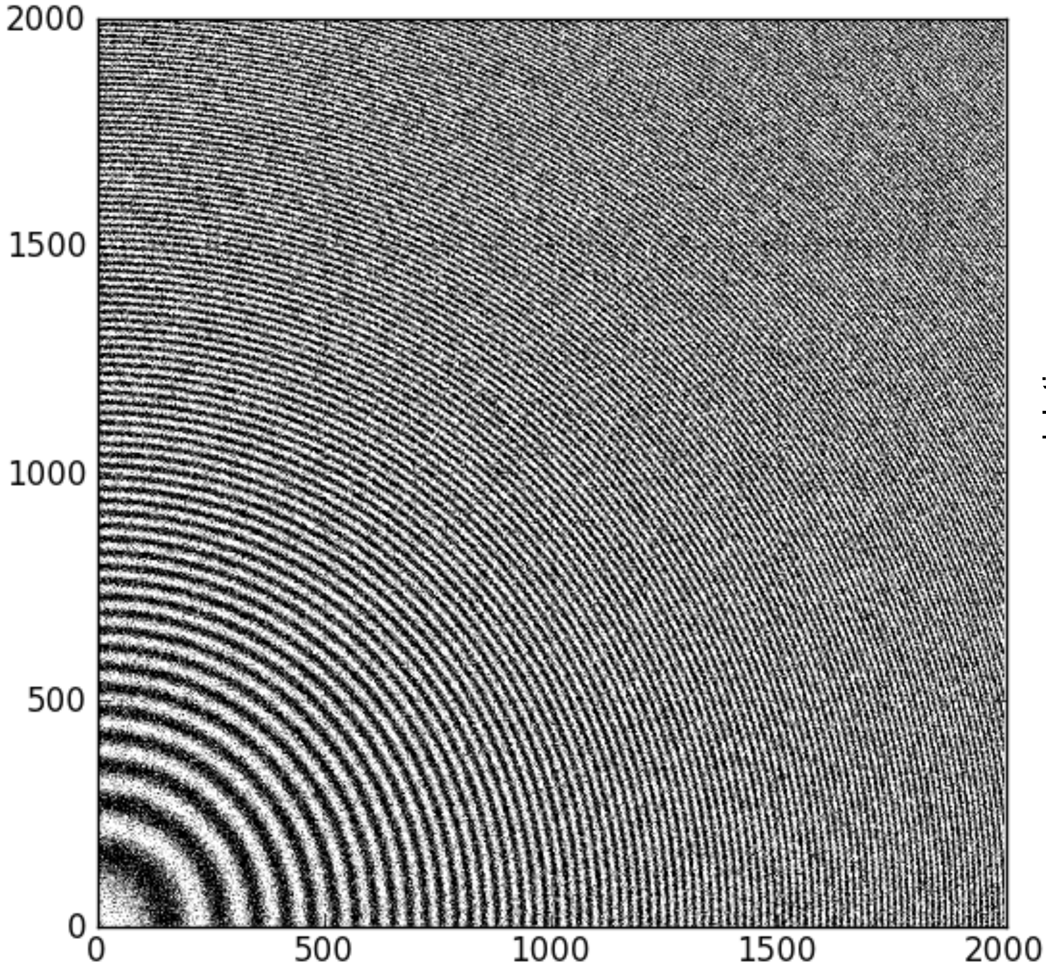
- All other types of noise are correlated to some degree

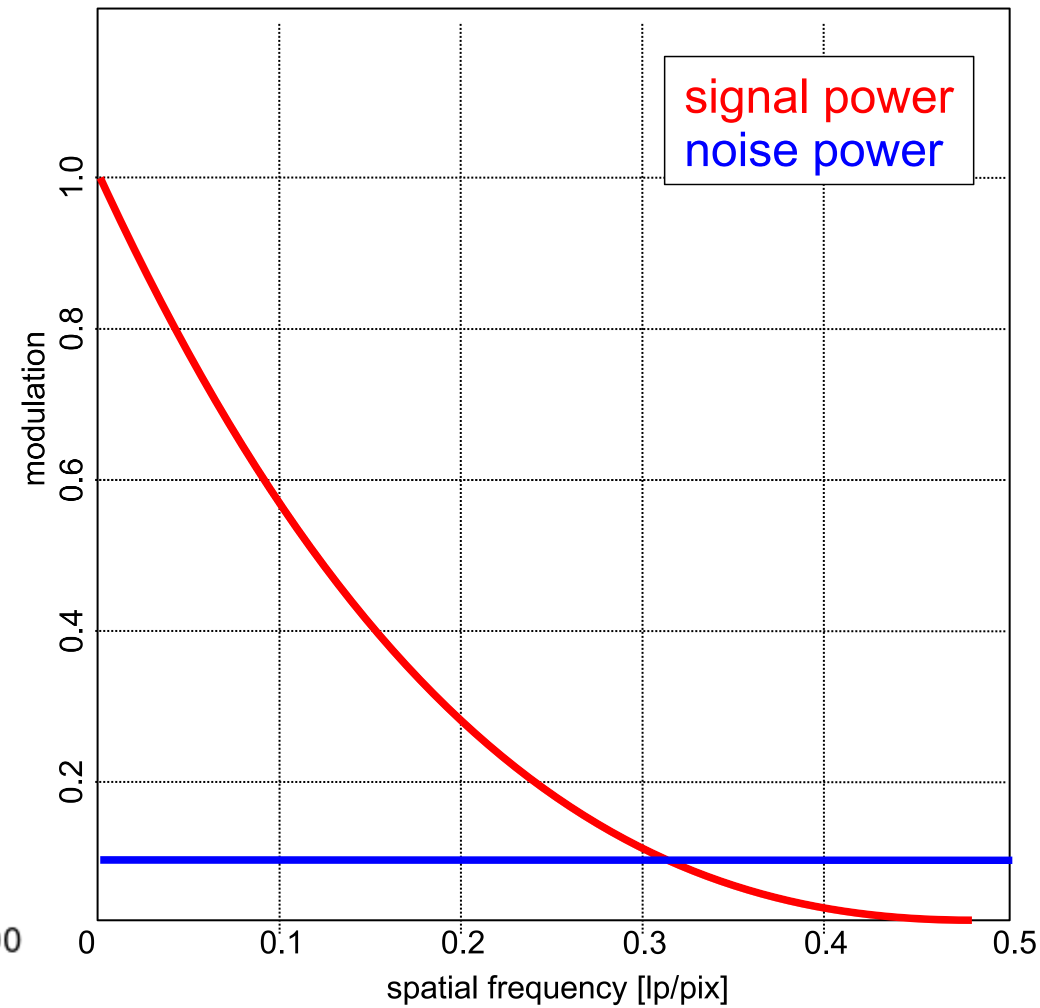
Signal power vs. noise power



Adding white noise to a Fresnel test pattern

Signal power vs. noise power





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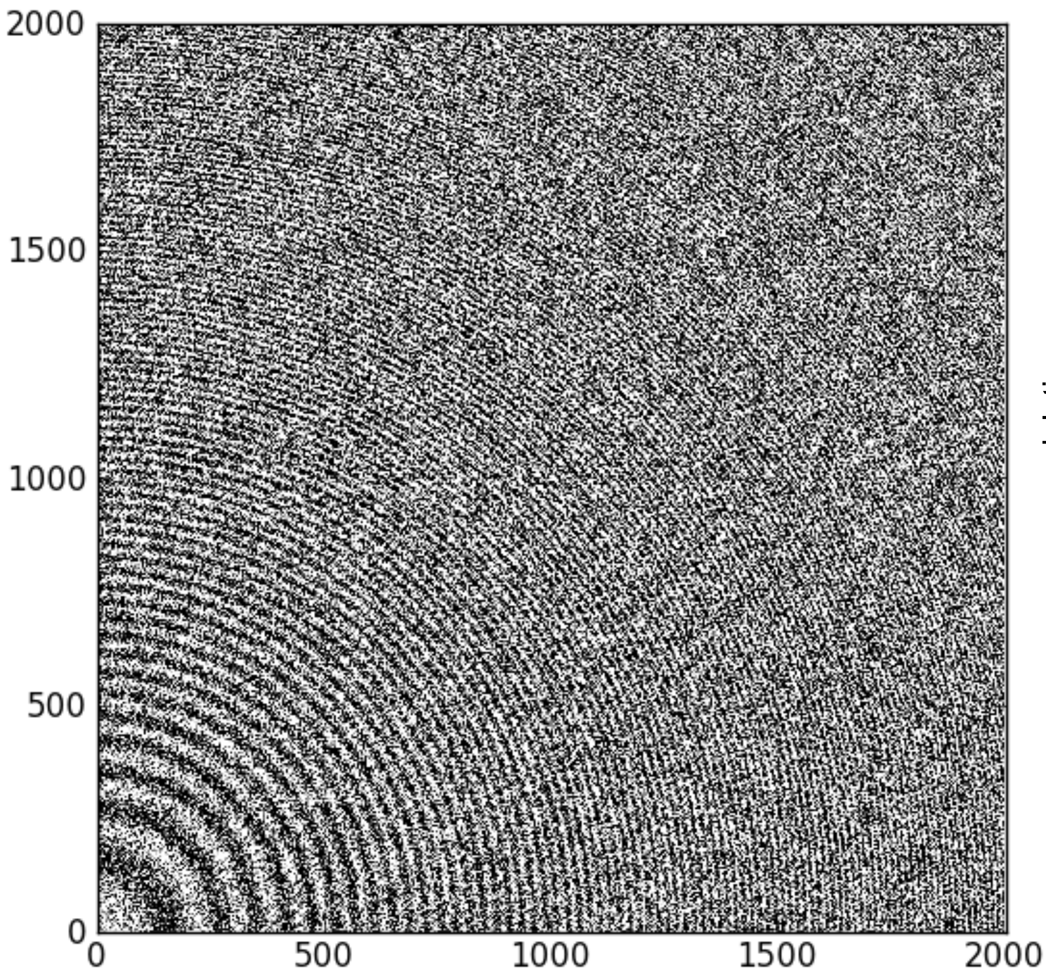
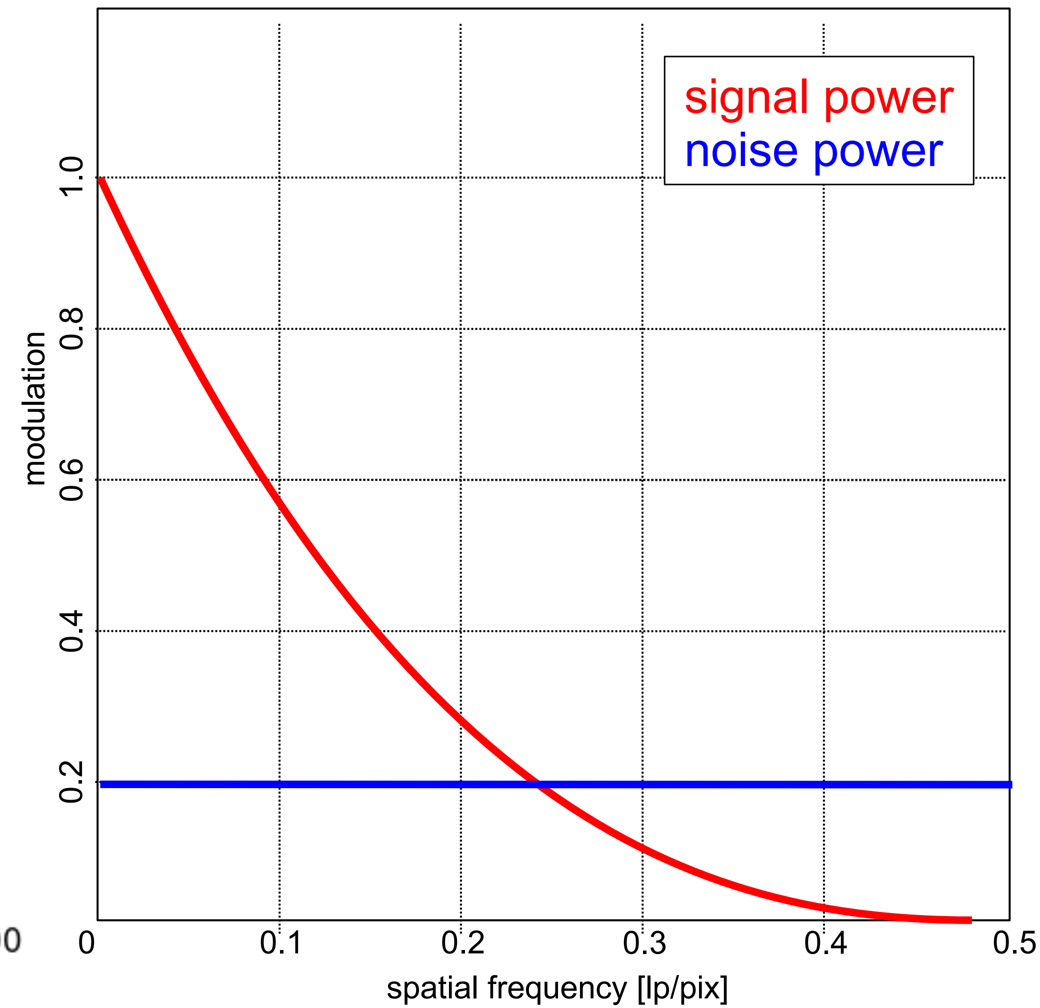
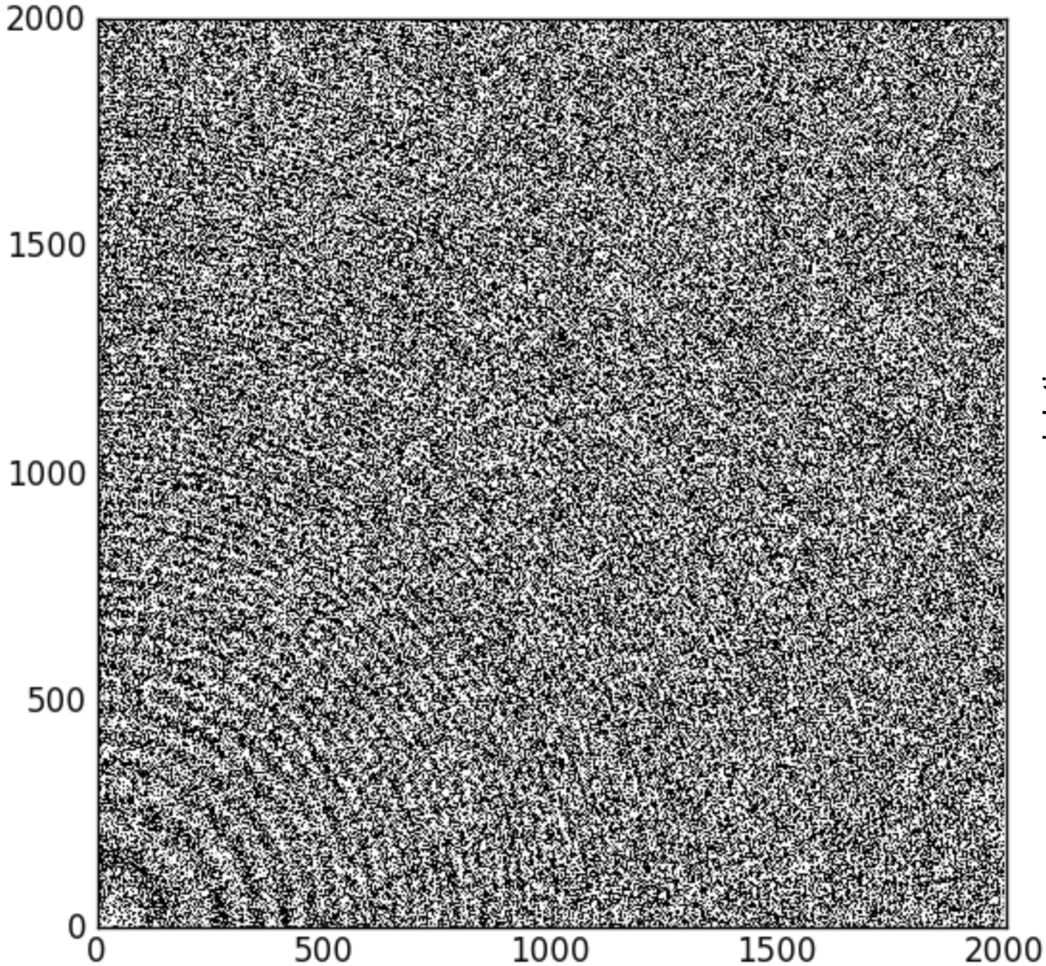
Signal power vs. noise power

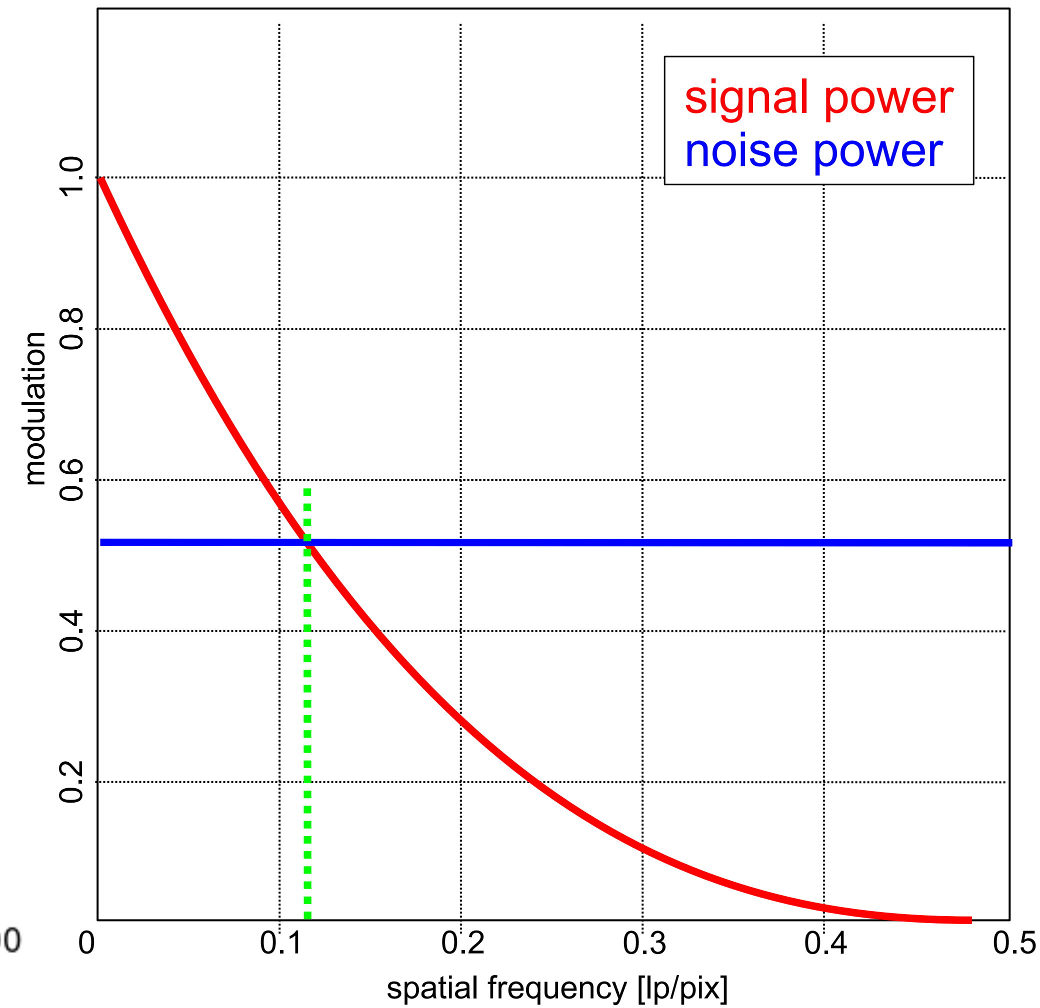




Adding white noise to a Fresnel test pattern

Signal power vs. noise power





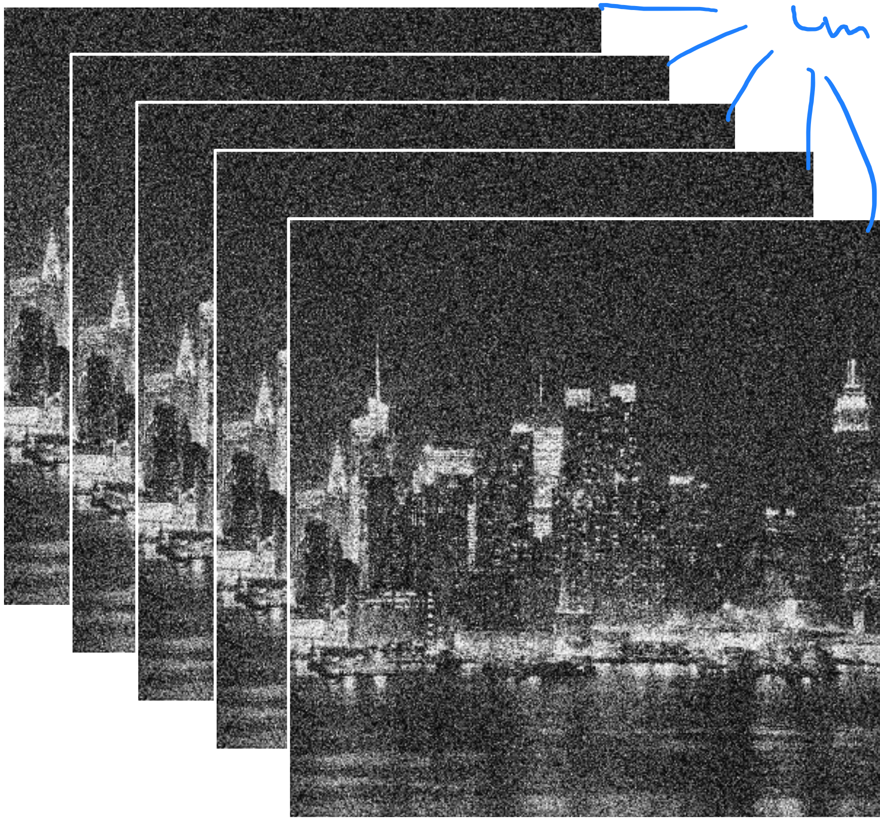
Adding white noise to a Fresnel test pattern

- Noise power exceeds signal power for high frequencies
- Small scale image details are lost in noise first

Noise reduction by averaging

- Average multiple images

$i_m(x,y)$



$$i = \frac{1}{M} \sum_{m=1}^{M} i_m(x,y)$$

- Requirement: additive noise, zero mean

$$\overline{i(x,y)} = \overline{g(x,y) + n(x,y)} = \overline{g(x,y)} + \underbrace{\overline{n(x,y)}}_{=0}$$

Denoising by linear filtering

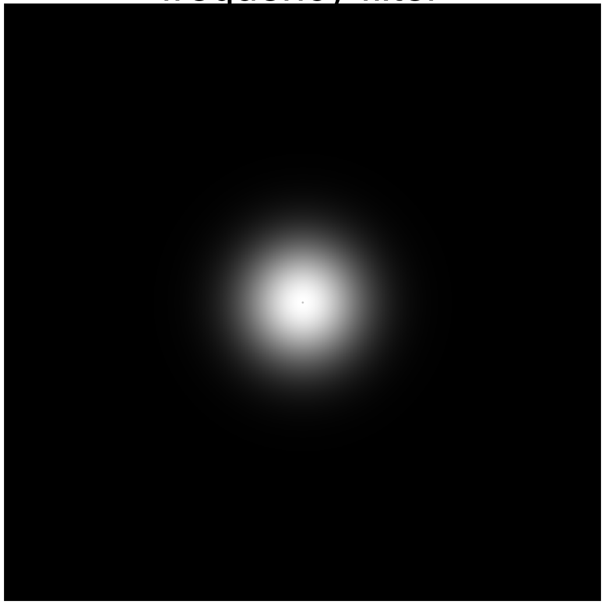
- Use spatial convolution or frequency filtering to reduce noise
- Noise reduction possible, but at cost of sharpness (blurred)
- Trade-off between noise reduction and resolution
- Need better method

original

Gaussian convolution kernel

0	1	0
1	4	1
0	1	0

frequency filter



Resulting image

Median filtering

- Good for getting rid of 'outliers'

- Less sensitive to outliers in pixel ensemble, better edge preservation

Salt and pepper noise

Gauss sigma=1 pixel

Median 1 pixel

PSF deconvolution

Image formation:

$$g = f \otimes \underbrace{h}_{\text{PSF}}$$

In Fourier domain:

$$G = F \cdot H$$

Estimate the object

$$F = \frac{G}{H} \qquad \hat{f} = \mathcal{F}^{-1}\left\{\frac{G}{H}\right\}$$

estimated function

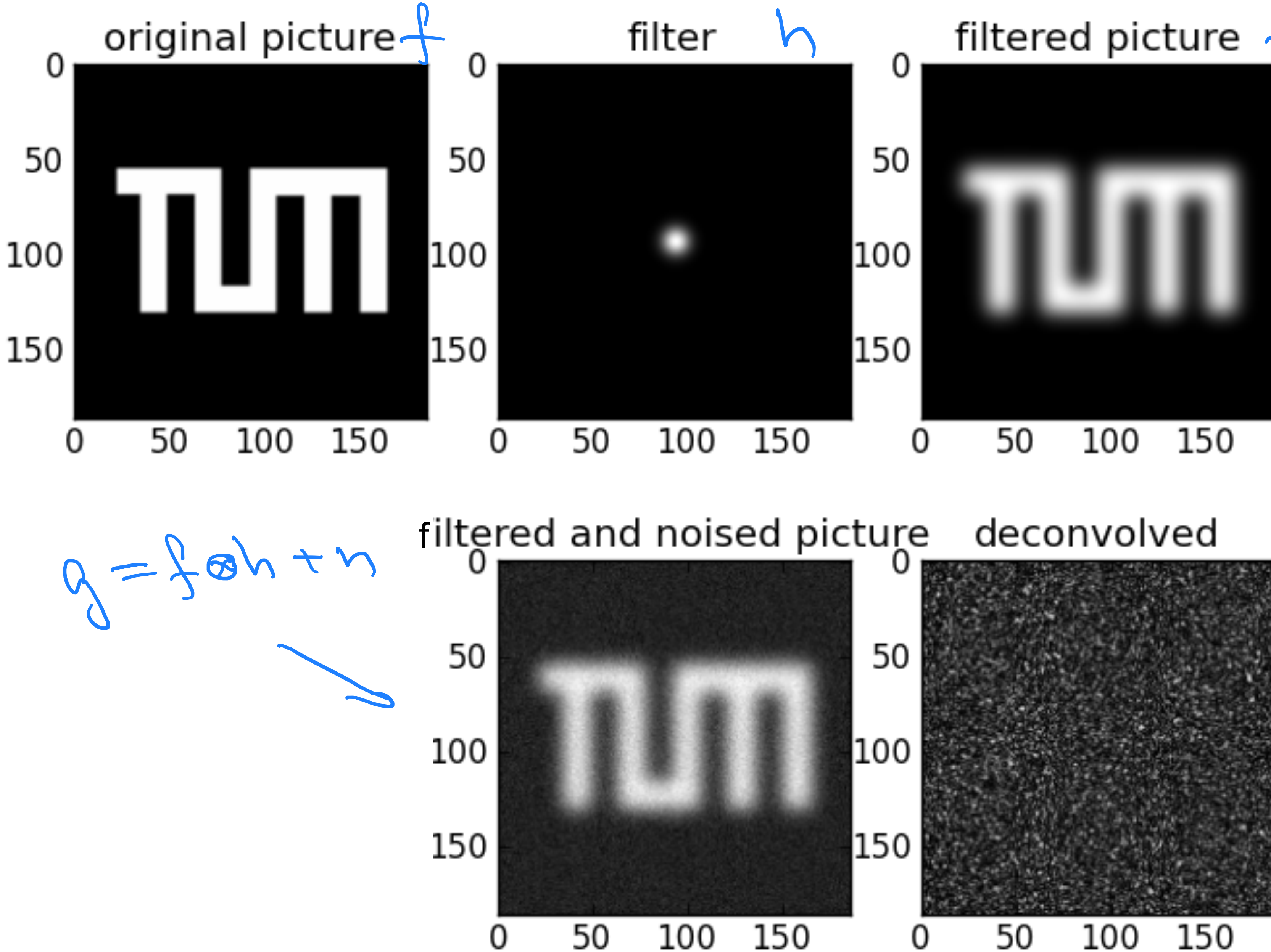
This is known as 'naive' deconvolution. In reality, we also have noise:

$$g = f \otimes h + n \quad \text{noise}$$

$$F = \frac{G}{H} + \frac{N}{H}$$

The noise term dominates for high frequencies

PSF deconvolution example



f h $f \otimes h$

$g = f \otimes h + n$

$f = \mathcal{F}^{-1}\left\{\frac{G}{H}\right\}$

„naive" deconvolution fails because of noise!

PSF deconvolution

Problems of “naive” deconvolution

- numerically unstable (division by zero)
- artifacts (ringing, ripples, ghosts)
- no accounting for SNR (increase of noise at high frequencies)

=> We need more sophisticated methods

standard methods used:

Wiener deconvolution (Least squares optimization, with known MTF and NPS)

Richardson-Lucy deconvolution (iterative, known PSF)

Blind deconvolution (unknown PSF)

Wiener deconvolution

$SPS = |G|^2$

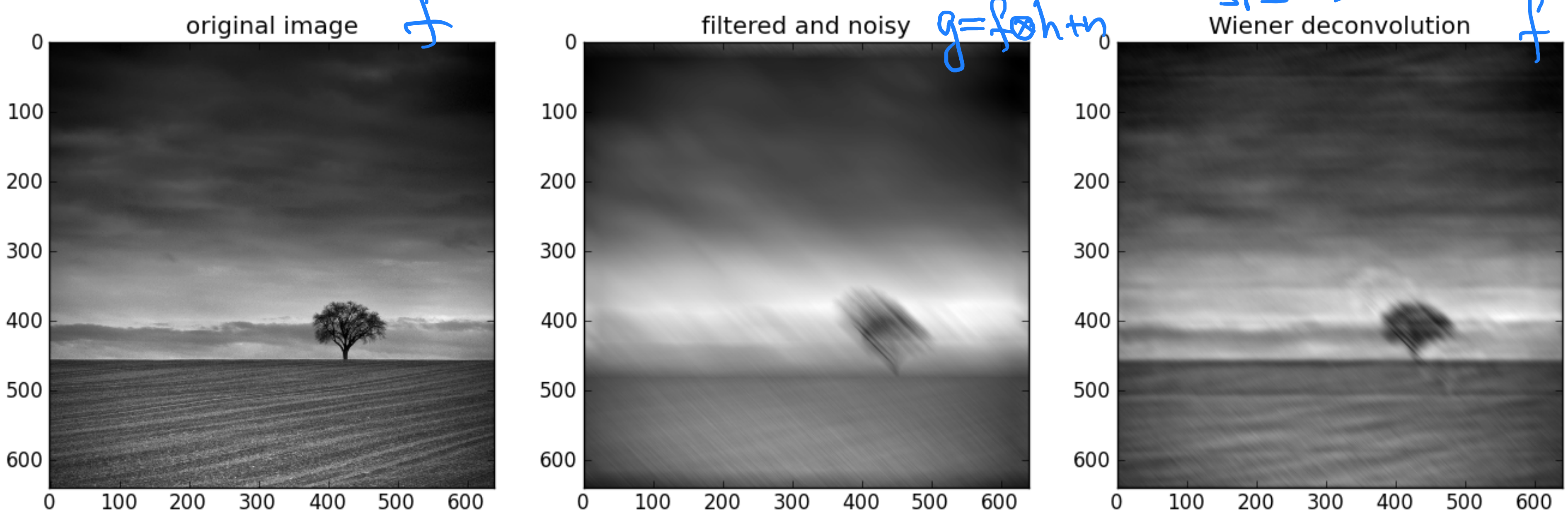
- Assume we know PSF and the signal power spectrum(SPS), guess NPS
- Optimal filter that minimizes least squares
- Physically we minimize impact of noise for frequencies with poor SNR

$\hat{F} = G \cdot W$

$G = \mathcal{F}\{g\}$

$\hat{f} = \mathcal{F}^{-1}\{\hat{F}\}$

$W = \frac{1}{H}\left\{\frac{|H|^2}{|H|^2 + \frac{NPS}{SPS}}\right\}$



Summary

Resolution:

- Imaging systems can be modeled by the convolution with a PSF
- OTF, PTF, MTF are the frequency representations of the PSF
- PTF and MTF describe how phase and amplitude of the signal are transmitted by the system
- Detector resolution is (usually) defined as 10% MTF or FWHM PSF

Noise:

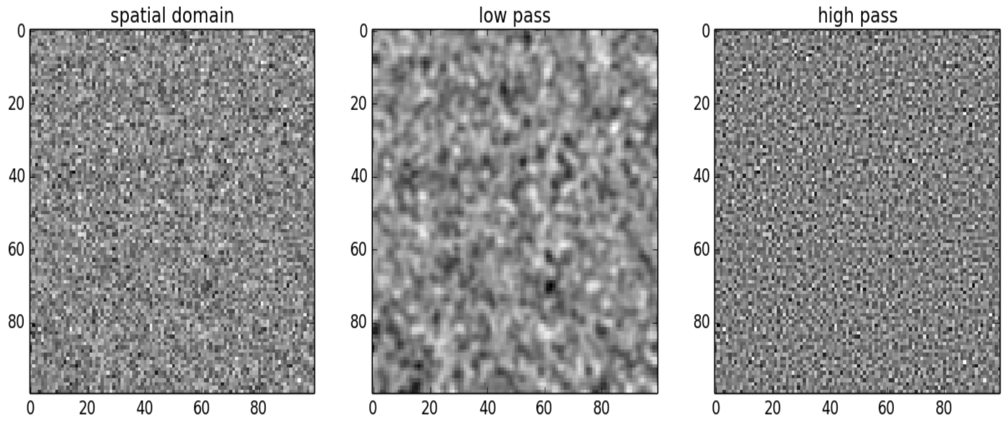
- Noise is an uncorrelated signal, characterized by its probability density function, mean & variance (uniform, Gaussian, Poission most commonly used)
- Noise can be characterized by its noise power spectrum, which is related to the auto-correlation of the noise

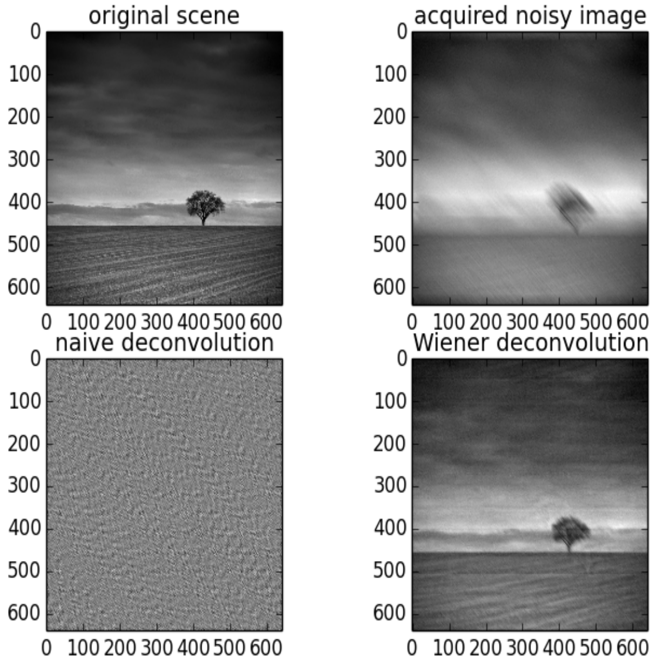
Deconvolution:

- “Naive” deconvolution fails when noise is present
- Wiener filter optimal least squares filter for deconvolution

Exercises

- Two exercises are on Moodle
- One on properties of noise & correlation, one on Wiener deconvolution





Next lecture (14th June) by Julia:

Tomography