

Design and Analysis of Algorithms I

Data Structures

Heaps: Some Implementation Details

Heap: Supported Operations

- A container for objects that have keys
- Employer records, network edges, events, etc.

```
Insert: add a new object to a heap.

Running time: O(log(n))

Equally well,
EXTRACT MAX
```

Extract-Min: remove an object in heap with a minimum key value. [ties broken arbitrarily]

Running time : O(log n) [n = # of objects in heap]

```
Also: HEAPIFY (n batched Inserts), DELETE(O(log(n)) time)
```

The Heap Property

alternatively

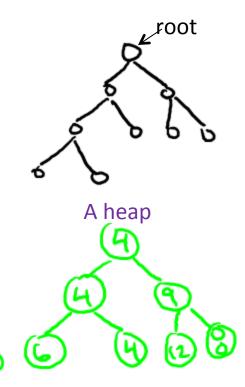
Conceptually: think of a heap as a tree. -rooted, binary, as complete as possible

Heap Property: at every node x,

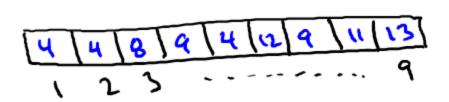
Key[x] <= all keys of x's children</pre>

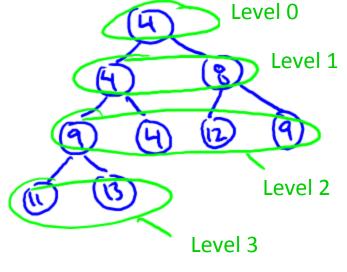
<u>Consequence</u>: object at root must

have minimum key value



Array Implementation





and children of i are 2i, 2i+1

Insert and Bubble-Up

Implementation of Insert (given key k)

Step 1: stick k at end of last level.

Step 2: Bubble-Up k until heap property

is restored (i.e., key of k's parent

 $ls \le k$

~ log2n levels (n = # of items in heap)

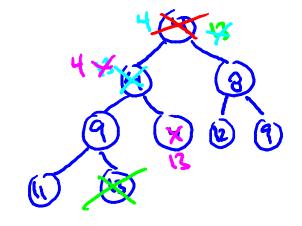
Check : 1.) bubbling up process must stop, with
heap property restored
2.) runtime = O(log(n))

Extract-Min and Bubble-Down

Implementation of Extract-Min

- 1. Delete root
- Move last leaf to be new root.
- 3. Iteratively Bubble-Down until heap property has been restored

[always swap with smaller child!]



- Check: 1.) only Bubble-Down once per level, halt with a heap
 - 2.) run time = O(log(n))