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4.2 Directed Graphs

- introduction
- *digraph API*
- digraph search
- topological sort
- strong components

Digraph API

`public class Digraph`		
	`Digraph(int V)`	*create an empty digraph with V vertices*
	`Digraph(In in)`	*create a digraph from input stream*
`void`	`addEdge(int v, int w)`	*add a directed edge v→w*
`Iterable<Integer>`	`adj(int v)`	*vertices pointing from v*
`int`	`V()`	*number of vertices*
`int`	`E()`	*number of edges*
`Digraph`	`reverse()`	*reverse of this digraph*
`String`	`toString()`	*string representation*

```
In in = new In(args[0]);
Digraph G = new Digraph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(v))
      StdOut.println(v + "->" + w);
```

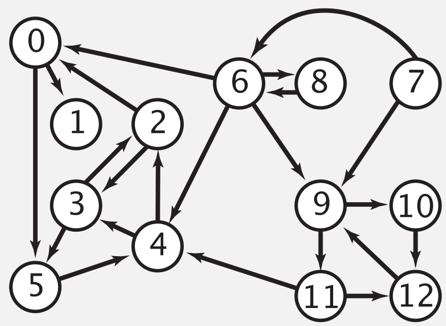
read digraph from input stream

print out each edge (once)

Digraph API

tinyDG.txt

```
V → 13
    22 ← E
     4  2
     2  3
     3  2
     6  0
     0  1
     2  0
    11 12
    12  9
     9 10
     9 11
     7  9
    10 12
    11  4
     4  3
     3  5
     6  8
     8  6
     ⋮
```




```
% java Digraph tinyDG.txt
0->5
0->1
2->0
2->3
3->5
3->2
4->3
4->2
5->4
⋮
11->4
11->12
12-9
```

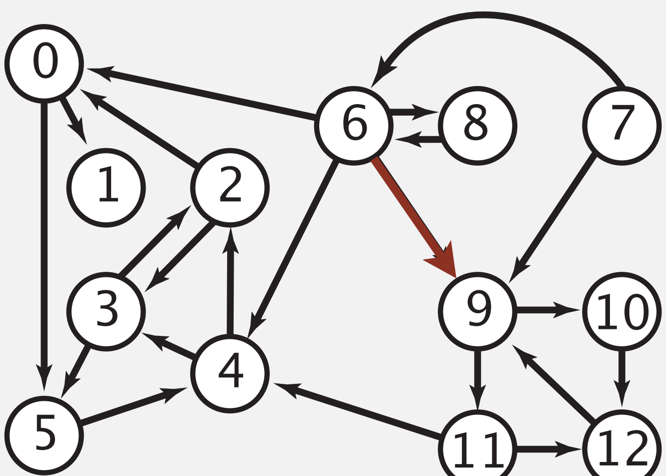
```
In in = new In(args[0]);
Digraph G = new Digraph(in);

for (int v = 0; v < G.V(); v++)
   for (int w : G.adj(v))
      StdOut.println(v + "->" + w);
```

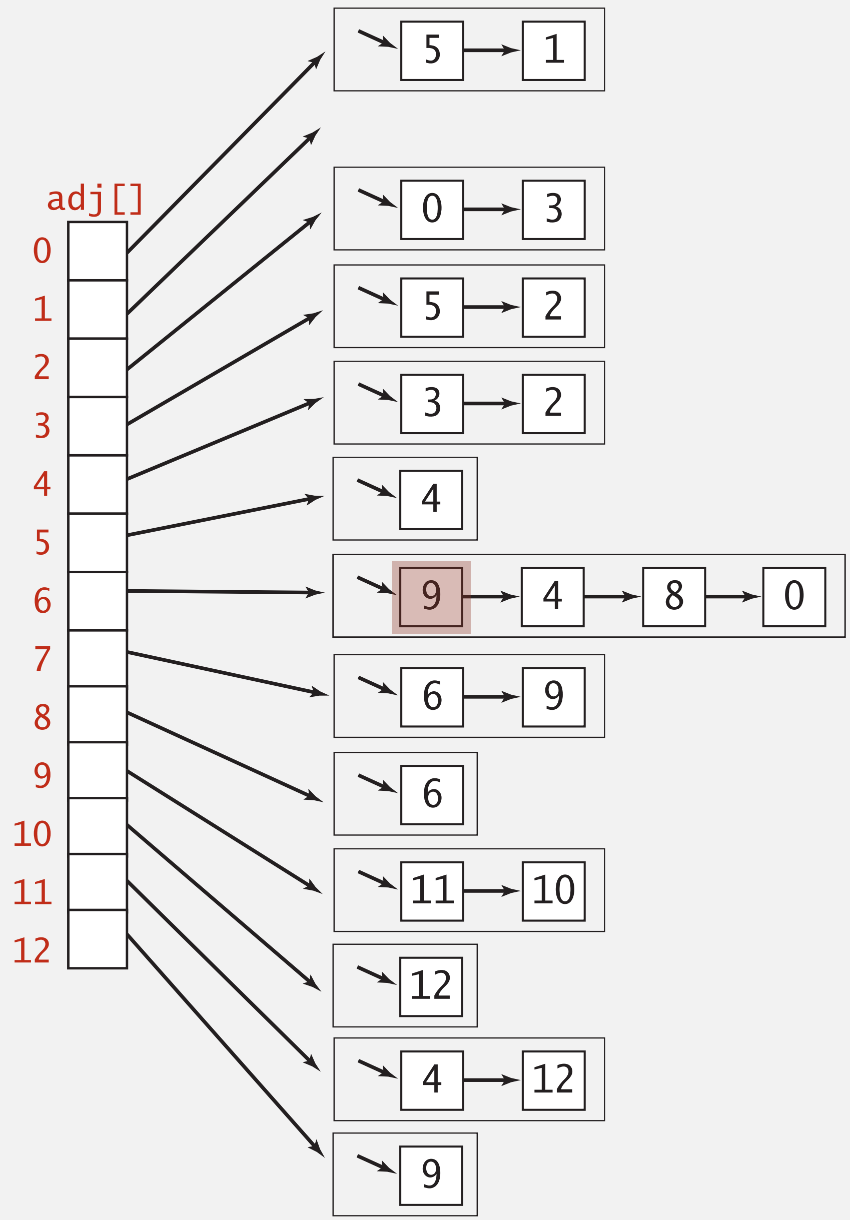
read digraph from input stream

print out each edge (once)

Adjacency-lists digraph representation

Maintain vertex-indexed array of lists.





Adjacency-lists graph representation (review): Java implementation

```
public class Graph
{
   private final int V;
   private final Bag<Integer>[] adj;

   public Graph(int V)
   {
      this.V = V;
      adj = (Bag<Integer>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);
      adj[w].add(v);
   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists

create empty graph with V vertices

add edge v–w

iterator for vertices adjacent to v

Adjacency-lists digraph representation: Java implementation

```
public class Digraph
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      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Integer>();
   }

   public void addEdge(int v, int w)
   {
      adj[v].add(w);

   }

   public Iterable<Integer> adj(int v)
   {  return adj[v];  }
}
```

adjacency lists

create empty digraph with V vertices

add edge v→w

iterator for vertices pointing from v

Digraph representations

In practice. Use adjacency-lists representation.

- Algorithms based on iterating over vertices pointing from v.
- Real-world digraphs tend to be sparse.

huge number of vertices,
small average vertex degree

representation	space	insert edge from v to w	edge from v to w?	iterate over vertices pointing from v?
list of edges	E	1	E	E
adjacency matrix	V^2	1 †	1	V
adjacency lists	E + V	1	outdegree(v)	outdegree(v)

† disallows parallel edges

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4.2 Directed Graphs

- introduction
- **digraph API**
- digraph search
- topological sort
- strong components

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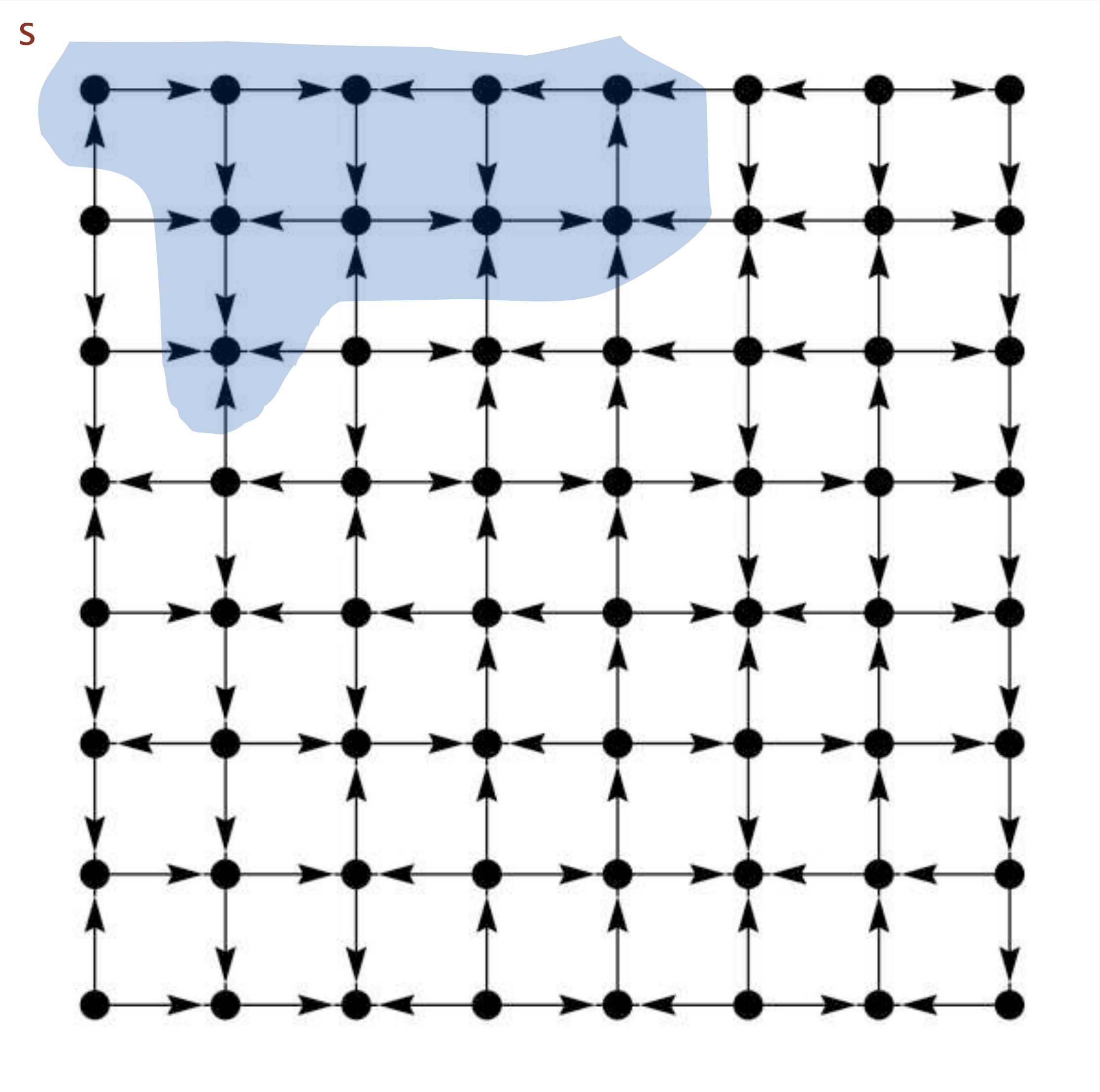
Reachability

Problem. Find all vertices reachable from s along a directed path.



Depth-first search in digraphs

Same method as for undirected graphs.

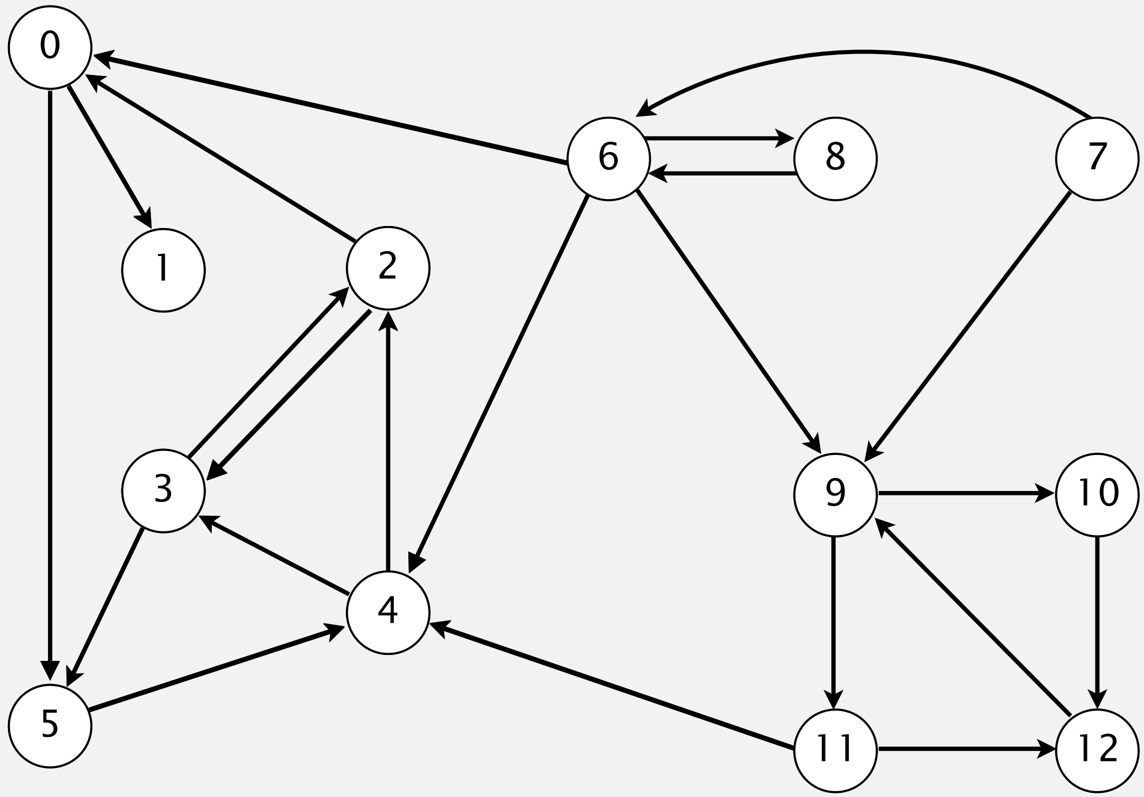
- Every undirected graph is a digraph (with edges in both directions).
- DFS is a digraph algorithm.

DFS (to visit a vertex v)

Mark v as visited.
Recursively visit all unmarked
vertices w pointing from v.

Depth-first search demo

To visit a vertex v :
- Mark vertex v as visited.
- Recursively visit all unmarked vertices pointing from v.



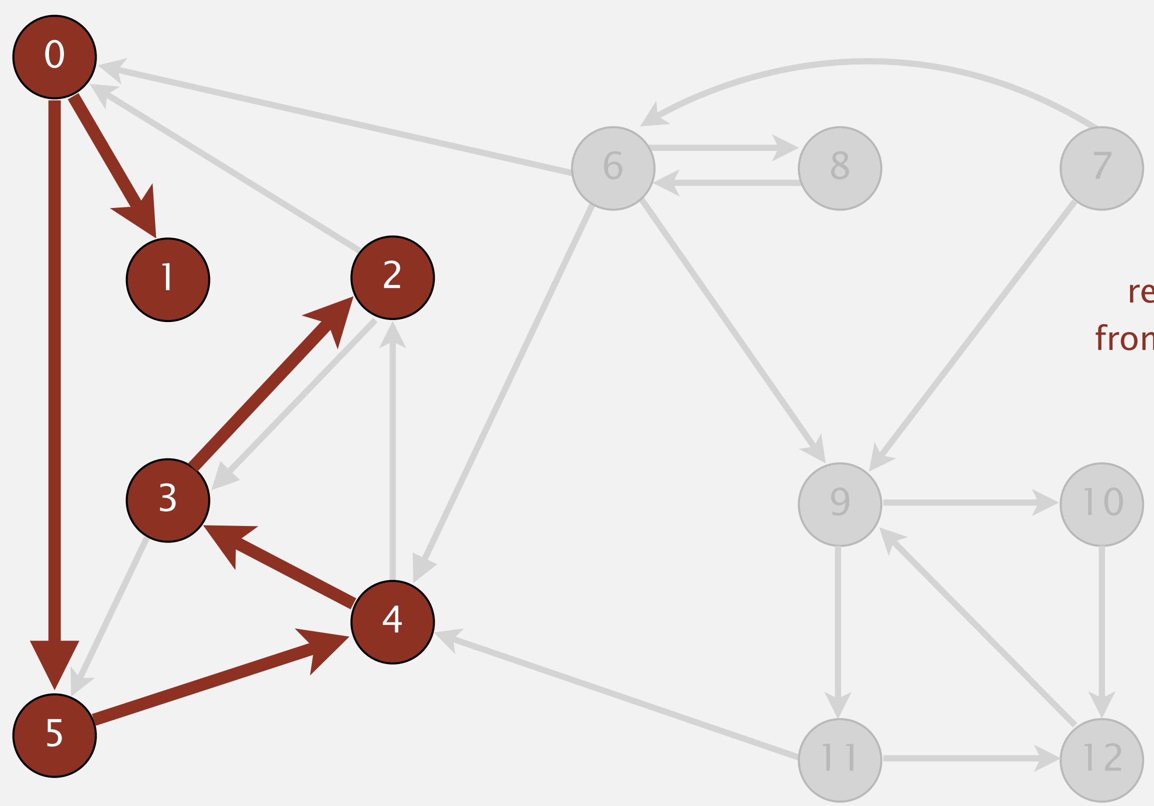
a directed graph

4→2
2→3
3→2
6→0
0→1
2→0
11→12
12→9
9→10
9→11
8→9
10→12
11→4
4→3
3→5
6→8
8→6
5→4
0→5
6→4
6→9
7→6

Depth-first search demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices pointing from v.



reachable from vertex 0

v	marked[]	edgeTo[]
0	T	–
1	T	0
2	T	3
3	T	4
4	T	5
5	T	0
6	F	–
7	F	–
8	F	–
9	F	–
10	F	–
11	F	–
12	F	–

Depth-first search (in undirected graphs)

Recall code for undirected graphs.

```
public class DepthFirstSearch
{
   private boolean[] marked;

   public DepthFirstSearch(Graph G, int s)
   {
      marked = new boolean[G.V()];
      dfs(G, s);
   }

   private void dfs(Graph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w]) dfs(G, w);
   }

   public boolean visited(int v)
   {  return marked[v];  }
}
```

true if path to s

constructor marks vertices connected to s

recursive DFS does the work

client can ask whether any vertex is connected to s

Depth-first search (in directed graphs)

Code for directed graphs identical to undirected one.

[substitute `Digraph` for `Graph`]

```
public class DirectedDFS
{
   private boolean[] marked;

   public DirectedDFS(Digraph G, int s)
   {
      marked = new boolean[G.V()];
      dfs(G, s);
   }

   private void dfs(Digraph G, int v)
   {
      marked[v] = true;
      for (int w : G.adj(v))
         if (!marked[w]) dfs(G, w);
   }

   public boolean visited(int v)
   {  return marked[v];  }
}
```

true if path from s

constructor marks vertices reachable from s

recursive DFS does the work

client can ask whether any vertex is reachable from s

Reachability application: program control-flow analysis

Every program is a digraph.

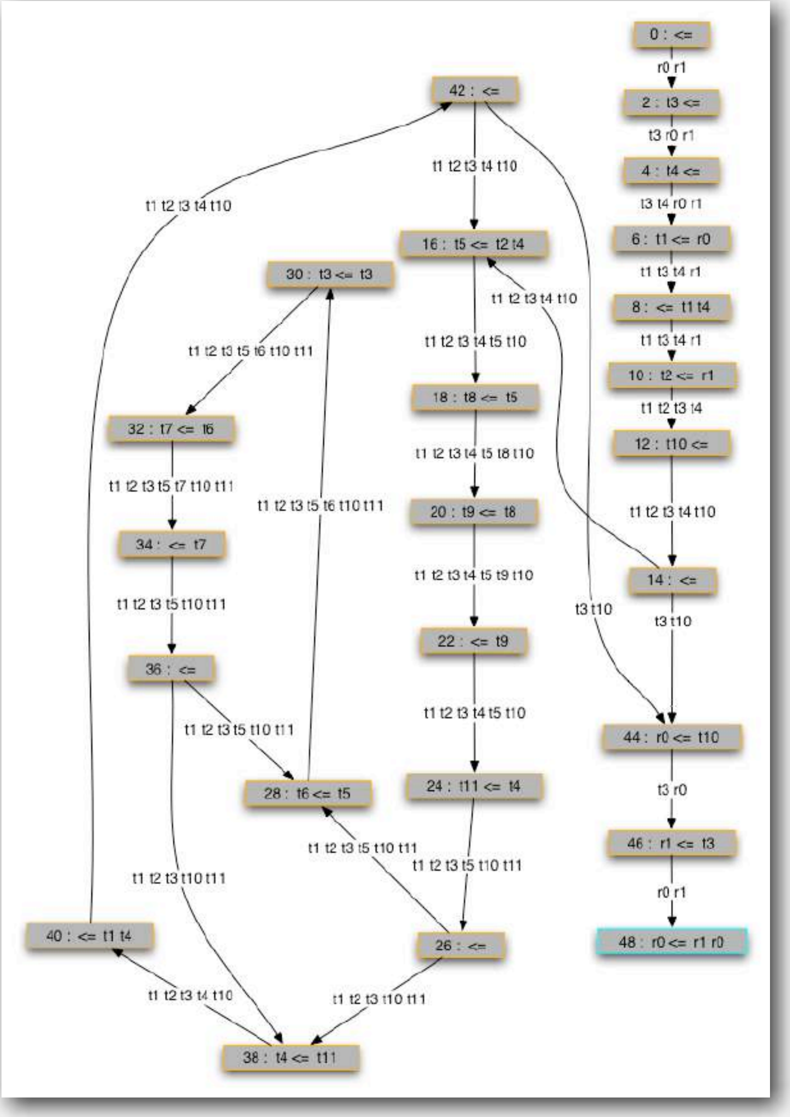
- Vertex = basic block of instructions (straight-line program).
- Edge = jump.

Dead-code elimination.

Find (and remove) unreachable code.

Infinite-loop detection.

Determine whether exit is unreachable.



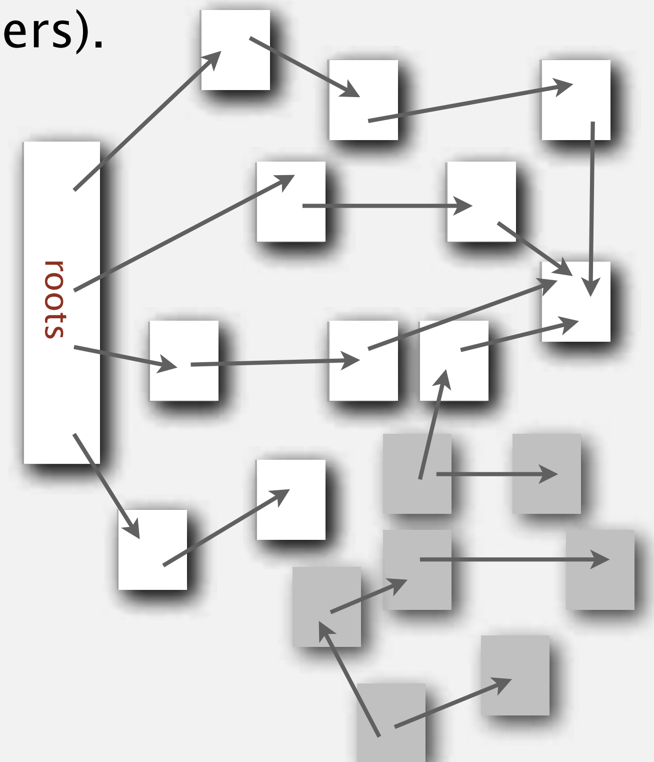
Reachability application: mark-sweep garbage collector

Every data structure is a digraph.

- Vertex = object.
- Edge = reference.

Roots. Objects known to be directly accessible by program (e.g., stack).

Reachable objects. Objects indirectly accessible by program (starting at a root and following a chain of pointers).

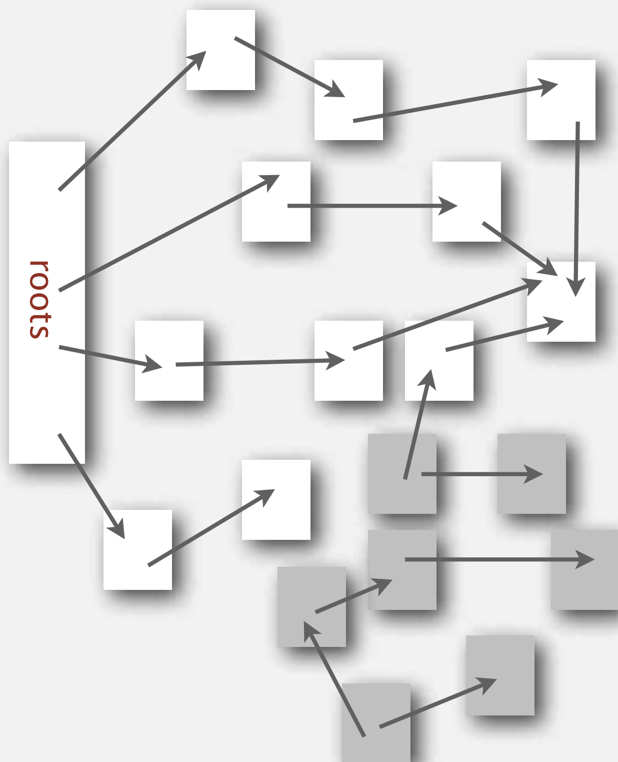


Reachability application: mark-sweep garbage collector

Mark-sweep algorithm. [McCarthy, 1960]

- Mark: mark all reachable objects.
- Sweep: if object is unmarked, it is garbage (so add to free list).

Memory cost. Uses 1 extra mark bit per object (plus DFS stack).



Depth-first search in digraphs summary

DFS enables direct solution of simple digraph problems.

- ✓ Reachability.
- Path finding.
- Topological sort.
- Directed cycle detection.

Basis for solving difficult digraph problems.

- 2-satisfiability.
- Directed Euler path.
- Strongly-connected components.

SIAM J. COMPUT.
Vol. 1, No. 2, June 1972

DEPTH-FIRST SEARCH AND LINEAR GRAPH ALGORITHMS*

ROBERT TARJAN†

Abstract. The value of depth-first search or "backtracking" as a technique for solving problems is illustrated by two examples. An improved version of an algorithm for finding the strongly connected components of a directed graph and an algorithm for finding the biconnected components of an undirect graph are presented. The space and time requirements of both algorithms are bounded by $k_1V + k_2E + k_3$ for some constants k_1, k_2, and k_3, where V is the number of vertices and E is the number of edges of the graph being examined.

Breadth-first search in digraphs

Same method as for undirected graphs.

- Every undirected graph is a digraph (with edges in both directions).
- BFS is a digraph algorithm.

BFS (from source vertex s)

Put s onto a FIFO queue, and mark s as visited.
Repeat until the queue is empty:

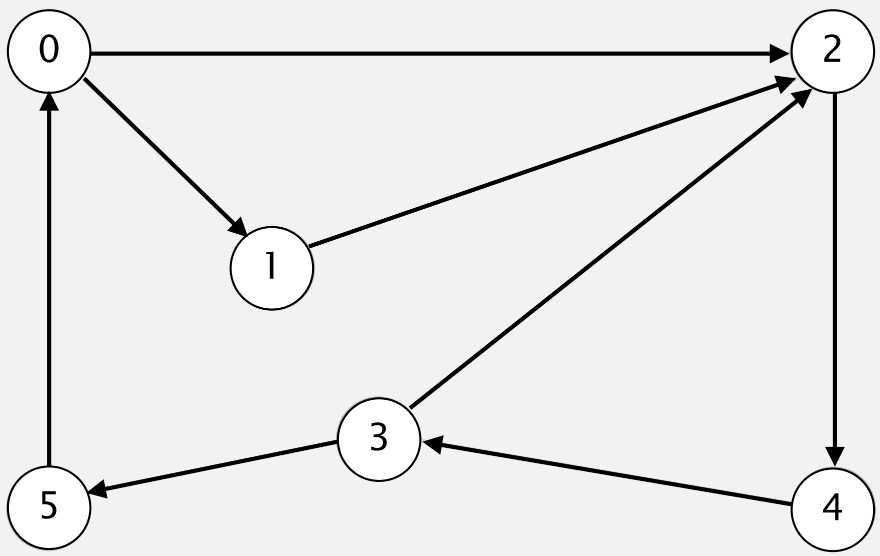
- **remove the least recently added vertex v**
- **for each unmarked vertex pointing from v:**
  add to queue and mark as visited.

Proposition. BFS computes shortest paths (fewest number of edges) from s to all other vertices in a digraph in time proportional to $E + V$.

Directed breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices pointing from v and mark them.



graph G

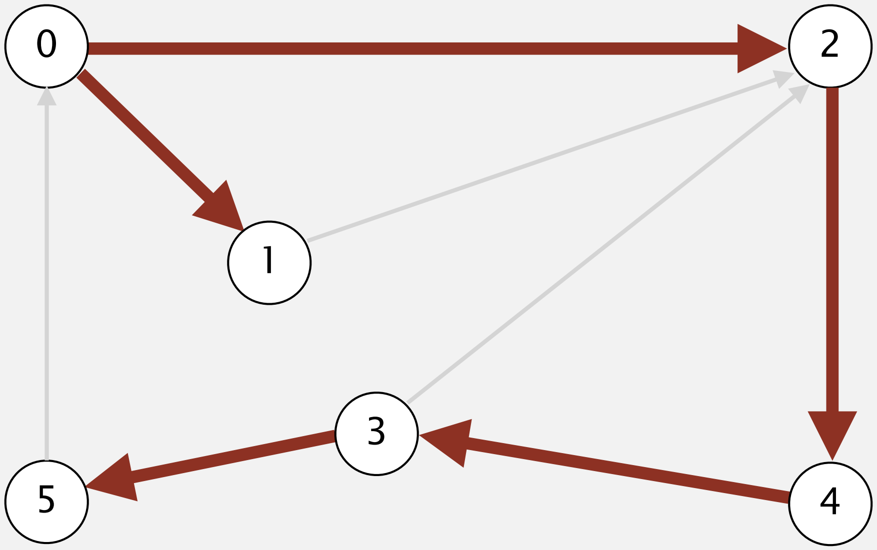
tinyDG2.txt

```
V → 6
    8 ← E
    5 0
    2 4
    3 2
    1 2
    0 1
    4 3
    3 5
    0 2
```

Directed breadth-first search demo

Repeat until queue is empty:

- Remove vertex v from queue.
- Add to queue all unmarked vertices pointing from v and mark them.



v	edgeTo[]	distTo[]
0	–	0
1	0	1
2	0	1
3	4	3
4	2	2
5	3	4

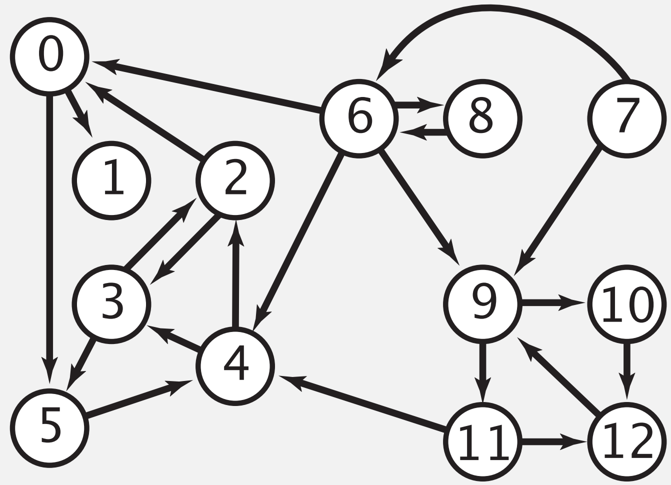
done

Multiple-source shortest paths

Multiple-source shortest paths. Given a digraph and a set of source vertices, find shortest path from any vertex in the set to each other vertex.

Ex. $S = \{ 1, 7, 10 \}$.

- Shortest path to 4 is $7 \to 6 \to 4$.
- Shortest path to 5 is $7 \to 6 \to 0 \to 5$.
- Shortest path to 12 is $10 \to 12$.
- ...



Q. How to implement multi-source shortest paths algorithm?

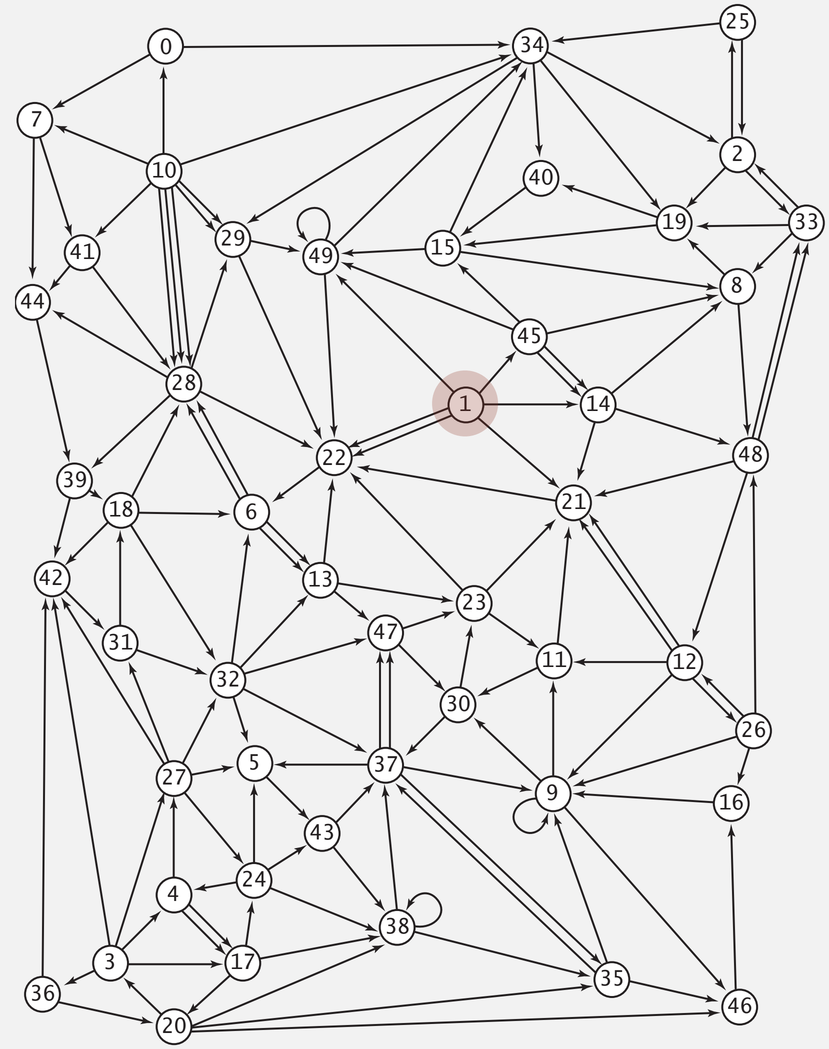
A. Use BFS, but initialize by enqueuing all source vertices.

Breadth-first search in digraphs application: web crawler

Goal. Crawl web, starting from some root web page, say `www.princeton.edu`.

Solution. [BFS with implicit digraph]

- Choose root web page as source s.
- Maintain a `Queue` of websites to explore.
- Maintain a `SET` of discovered websites.
- Dequeue the next website and enqueue websites to which it links (provided you haven't done so before).



Q. Why not use DFS?

Bare-bones web crawler: Java implementation

```
Queue<String> queue = new Queue<String>();
SET<String> marked = new SET<String>();

String root = "http://www.princeton.edu";
queue.enqueue(root);
marked.add(root);

while (!queue.isEmpty())
{
   String v = queue.dequeue();
   StdOut.println(v);
   In in = new In(v);
   String input = in.readAll();

   String regexp = "http://(\\w+\\.)*(\\w+)";
   Pattern pattern = Pattern.compile(regexp);
   Matcher matcher = pattern.matcher(input);
   while (matcher.find())
   {
      String w = matcher.group();
      if (!marked.contains(w))
      {
         marked.add(w);
         queue.enqueue(w);
      }
   }
}
```

queue of websites to crawl

set of marked websites

start crawling from root website

read in raw html from next website in queue

use regular expression to find all URLs in website of form `http://xxx.yyy.zzz` [crude pattern misses relative URLs]

if unmarked, mark it and put on the queue

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