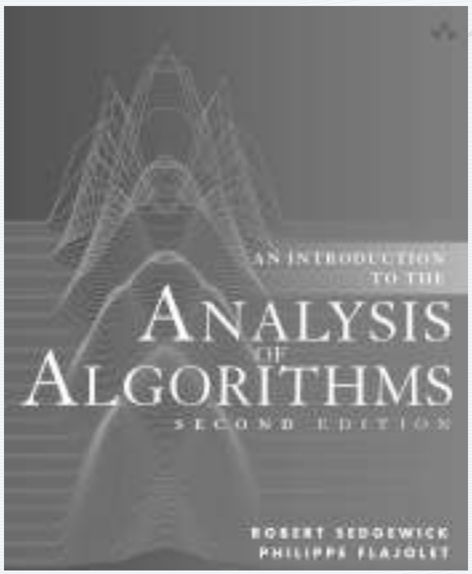

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6. Trees

- Trees and forests
- Binary search trees
- Path length
- **Other types of trees**

Other types of trees in combinatorics

Classic tree structures:

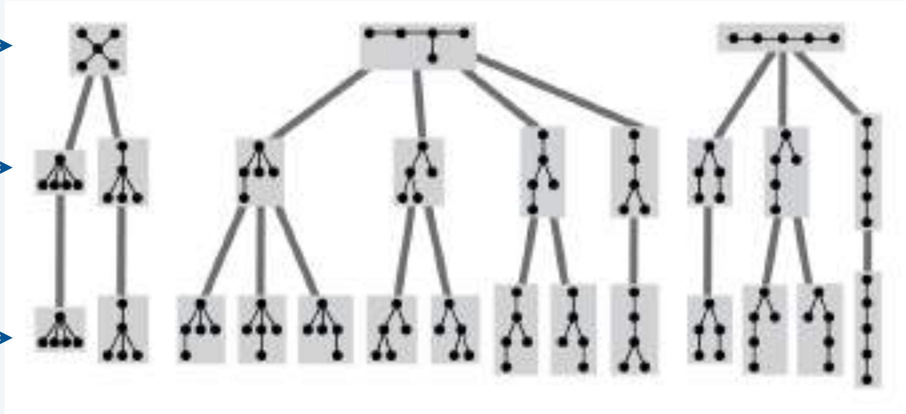
- The free tree, an acyclic connected graph.
- The rooted tree, a free tree with a distinguished root node.
- The ordered tree, a rooted tree where the order of the subtrees is significant.

Ex. 5-node trees:

3 free trees →

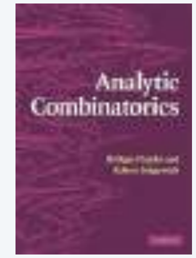
9 rooted trees →

14 ordered trees →



Enumeration? Path length? Stay tuned for *Analytic Combinatorics*

Other types of trees in algorithmics



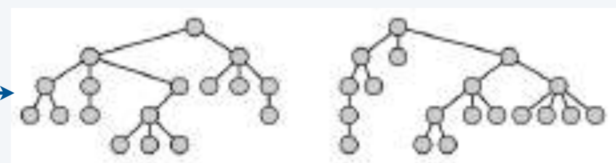
Variations on binary trees:

- The *t*-ary tree, where each node has *exactly t* children.
- The *t*-restricted tree, where each node has *at most t* children.
- The 2-3 tree, the method of choice in symbol-table implementations.

3-ary tree →

← 4-ary tree

3-restricted tree →



← 4-restricted tree

2-3 tree →

← 2-3-4 tree

Enumeration? Path length? Stay tuned for *Analytic Combinatorics*

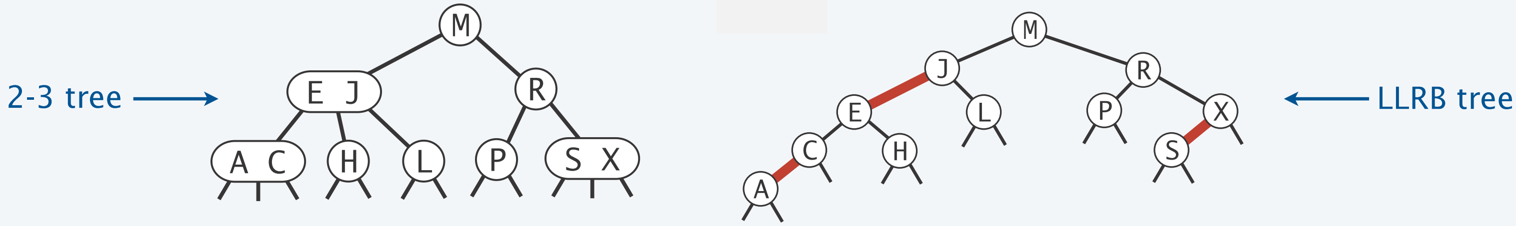
An unsolved problem

Balanced trees are the method of choice for symbol tables

- Same search code as BSTs.
- Slight overhead for insertion.
- Guaranteed height < $2\lg N$.
- Most algorithms use 2-3 or 2-3-4 tree representations.

Ex. LLRB (left-leaning red-black) trees.

Section 3.3

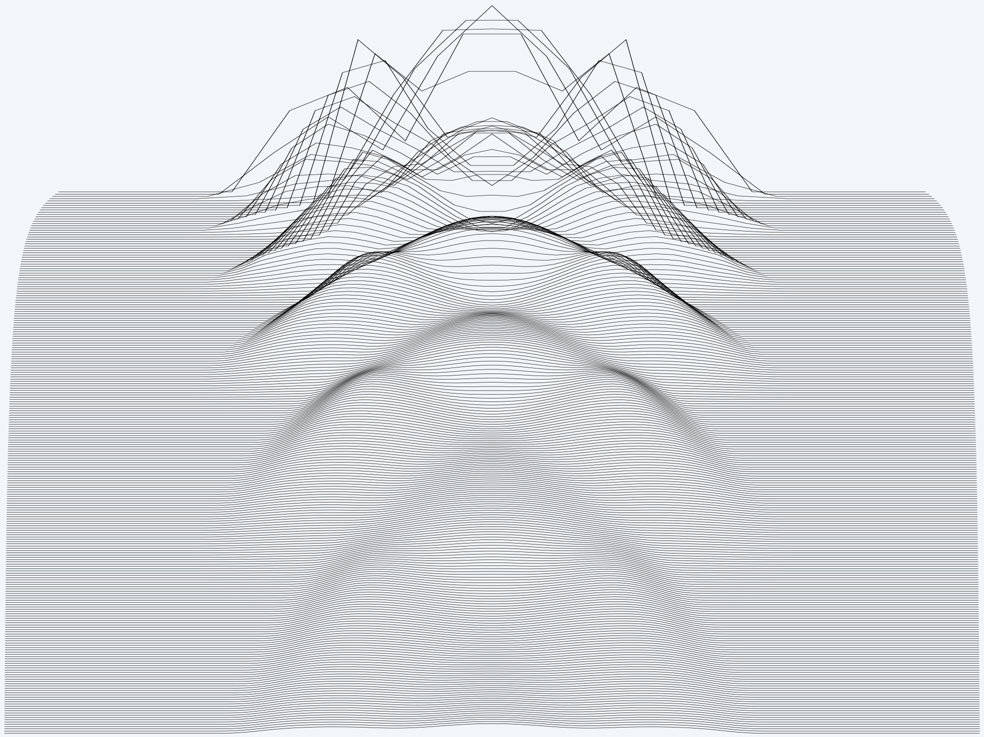


Q. Path length of balanced tree built from a random permutation?

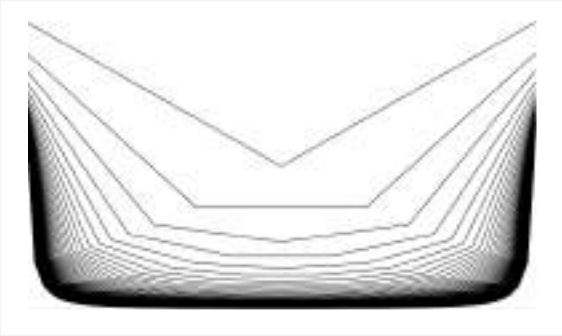
a property of permutations, not trees

Balanced tree distribution

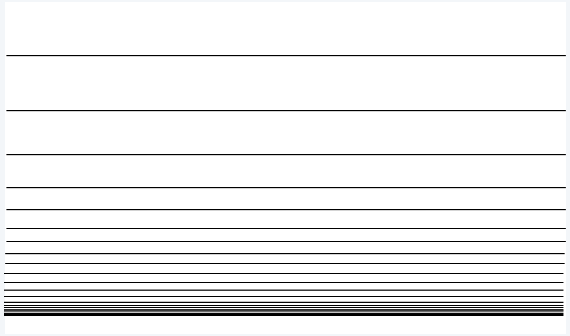
Probability that the root is of rank k in a randomly-chosen AVL tree.



Random binary tree

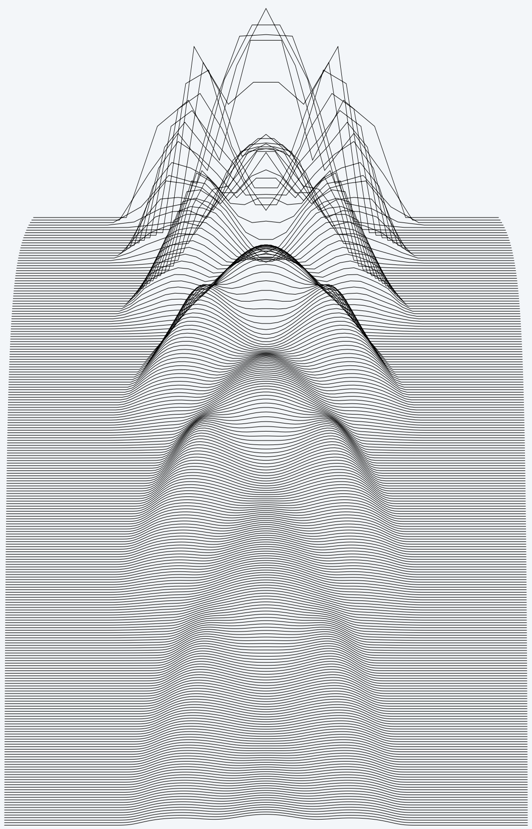


BST built from a random permutation

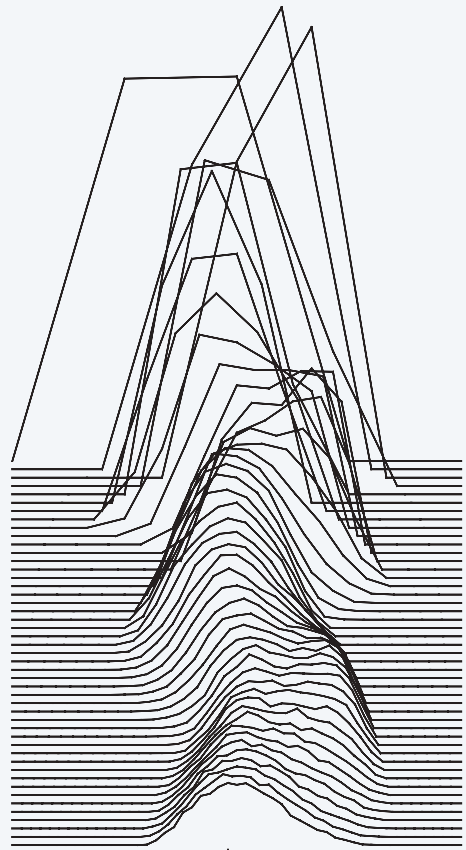


An unsolved problem

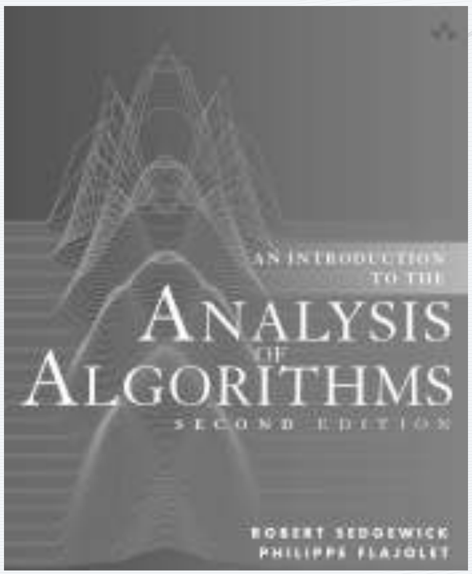
Q. Path length of balanced tree built from a random permutation?



random AVL tree



LLRB tree built from random perm (empirical)



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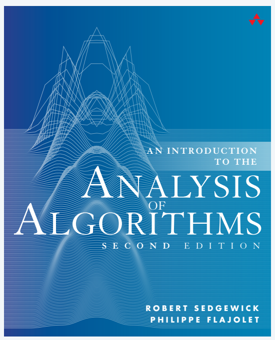
6. Trees

- Trees and forests
- Binary search trees
- Path length
- Other types of trees
- **Exercises**

Exercise 6.6

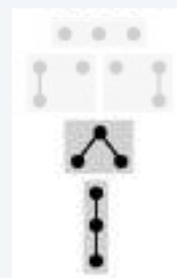
Tree enumeration via the symbolic method.



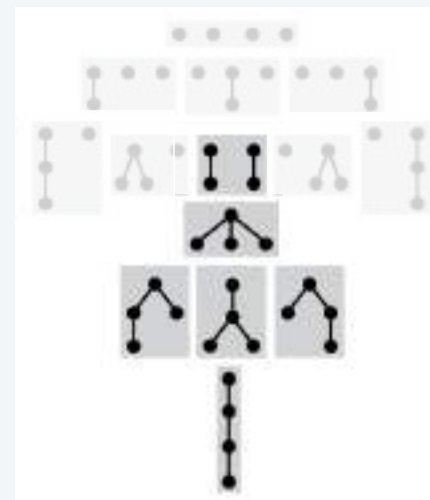
Exercise 6.6 What proportion of the forests with N nodes have no trees consisting of a single node? For $N = 1, 2, 3$, and 4, the answer is 0, $1/2$, $2/5$, and $3/7$, respectively.

1/1

1/2



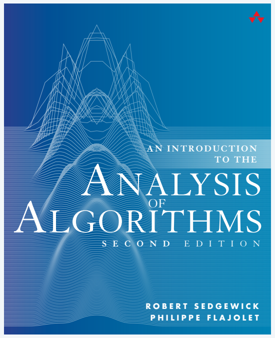
2/5



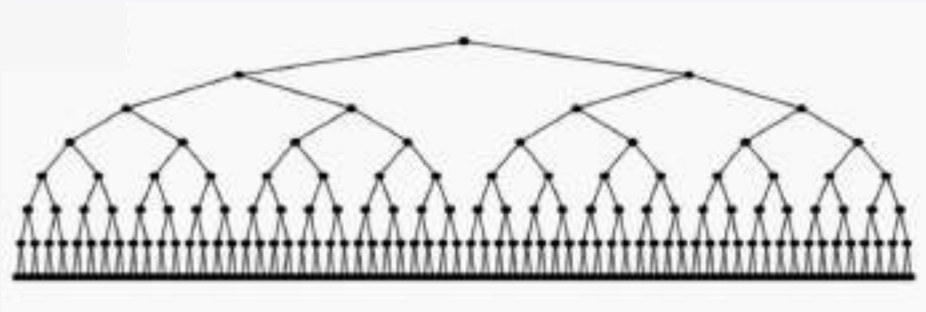
6/14

Exercise 6.27

Compute the probability that a BST is perfectly balanced.

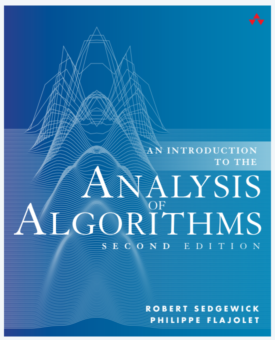


Exercise 6.27 For $N = 2^n - 1$, what is the probability that a perfectly balanced tree structure (all 2^n external nodes on level n) will be built, if all $N!$ key insertion sequences are equally likely?



Exercises 6.43

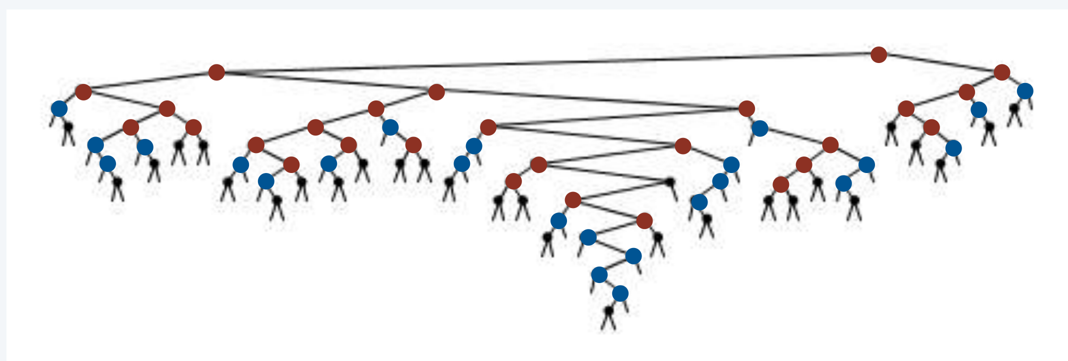
Parameters for BSTs built from a random permutation.



Answer these questions for BSTs built from a random permutation.

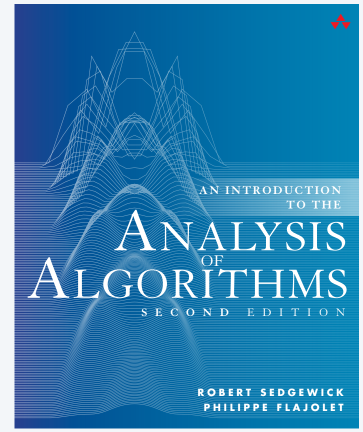
Exercise 5.15 Find the average number of internal nodes in a binary tree of size n with both children internal.

Exercise 5.16 Find the average number of internal nodes in a binary tree of size n with one child internal and one child external.



Assignments for next lecture

1. Read pages 257-344 in text.



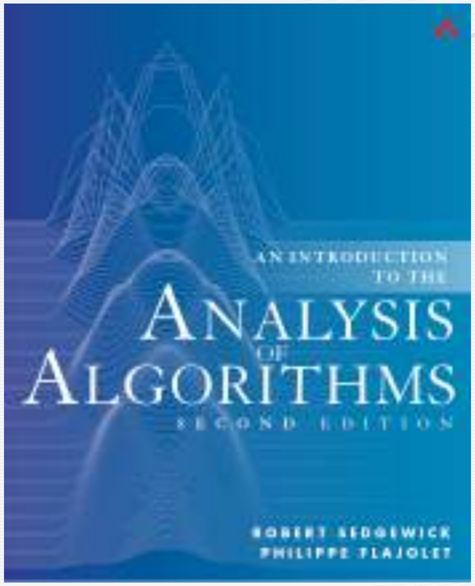
2. Run experiments to validate mathematical results.

Experiment 1. Generate 1000 random permutations for $N = 100$, 1000, and 10,000 and compare the average path length and height of the generated trees with the values predicted by analysis.

Experiment 2. *Extra credit.* Do the same for random binary trees.

3. Write up solutions to Exercises 6.6, 6.27, and 6.43.

ANALYTIC COMBINATORICS
PART ONE



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6. Trees