Design and Analysis of Algorithms I

QuickSort

Analysis III: Final Calculations

Average Running Time of QuickSort

<u>QuickSort Theorem</u> : for every input array of length n, the average running time of QuickSort (with random pivots) is O(nlog(n))

<u>Note</u> : holds for every input. [no assumptions on the data]

- recall our guiding principles !
- "average" is over random choices made by the algorithm (i.e., pivot choices)

The Story So Far

$$E[C] = 2\underbrace{\sum_{i=1}^{n-1}\sum_{j=1}^{n}\overbrace{\frac{1}{j-i+1}}}_{\theta(n^2)\ \ terms}$$

How big can this be ?

(*)

<= n choices for i

<u>Note</u> : for each fixed i, the inner sum is

$$\sum_{j=i+1}^{n}\frac{1}{j-i+1} = 1/2 + 1/3 + \ldots$$

$$So\ \ E[C] \leq 2 \cdot n \cdot \sum_{k=2}^{n}\frac{1}{k}$$

<u>Claim</u> : this is <= ln(n)

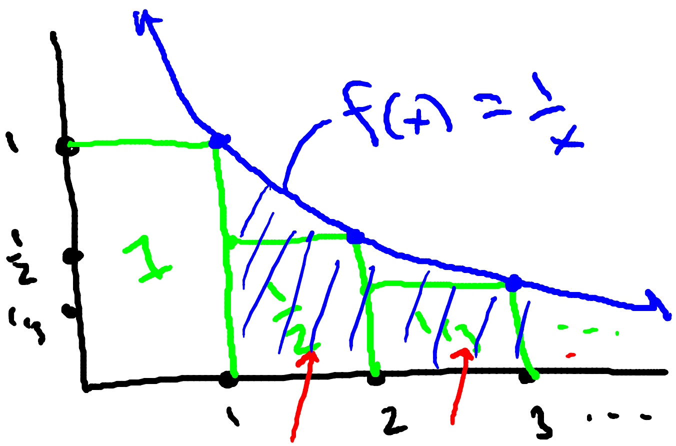
Tim Roughgarden

Completing the Proof

$$E[C] \leq 2 \cdot n \cdot \sum_{k=2}^{n} \frac{1}{k}$$

$$Claim \quad \sum_{k=2}^{n} \frac{1}{k} \leq \ln n$$

<u>Proof of Claim</u>



$$So \quad \sum_{k=2}^{n} \frac{1}{n} \leq \int_{1}^{n} \frac{1}{x} dx$$

$$= \ln x \;\Big|_{1}^{n}$$

$$= \ln n - \ln 1$$

$$= \ln n$$

Q.E.D. (CLAIM)

<u>So</u> :
E[C] <=
2n ln n

Q.E.D.