Physics 155 HW # 7

1.) Calculate $\vec{F} = \vec{\nabla} f$ for each of the following scalar functions. Also show $\vec{\nabla} \times \vec{F} = \vec{0}$ for each of them.

a.) $f = xyz$
b.) $f = x^2 + y^2 + z^2$
c.) $f = xy + yz + zx$
d.) $f = 3x^2 - 4y^2$
e.) $f = e^{-x}\sin(y)$

2.) Verify the following identities:

a.) $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$
b.) $\vec{\nabla}(\vec{F}\cdot\vec{G}) = (\vec{G}\cdot\vec{\nabla})\vec{F} + (\vec{F}\cdot\vec{\nabla})\vec{G} + \vec{F}\times(\vec{\nabla}\times\vec{G}) + \vec{G}\times(\vec{\nabla}\times\vec{F})$
c.) $\vec{\nabla}\times(\vec{F}\times\vec{G}) = (\vec{G}\cdot\vec{\nabla})\vec{F} - (\vec{F}\cdot\vec{\nabla})\vec{G} + \vec{F}(\vec{\nabla}\cdot\vec{G}) - \vec{G}(\vec{\nabla}\cdot\vec{F})$
d.) $\vec{\nabla}\times(\vec{\nabla}\times\vec{F}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{F}) - \nabla^2\vec{F}$

3. The total heat Q in an object with volume V is given by

$$Q = C \iiint\limits_V T(x,y,z,t)\rho(x,y,z)\,dV \tag{1}$$

where C is a constant called the specific heat, which is a property of the material, $T(x,y,z,t)$ and $\rho(x,y,z)$ are the temperature and density of the object at those locations and time (the object is a solid so the density is independent of time).

The rate at which heat flows out of a surface S, that bounds the object, is

$$\frac{dQ}{dt} = \kappa \iint\limits_S \hat{n}\cdot\vec{\nabla}T\,dS \tag{2}$$

where κ is the (constant) thermal conductivity of the object.

Use these facts to derive the heat flow equation.

$$\nabla^2 T = \alpha \frac{dT}{dt} \tag{3}$$

and find what α is in terms of C, ρ, and κ.
Hint: the divergence theorem will need to be used.

4.) a.) Find the charge density $\rho(x, y, z)$ that produces the electric field

$$\vec{E} = g(\hat{\imath}x + \hat{\jmath}y + \hat{k}z) \tag{4}$$

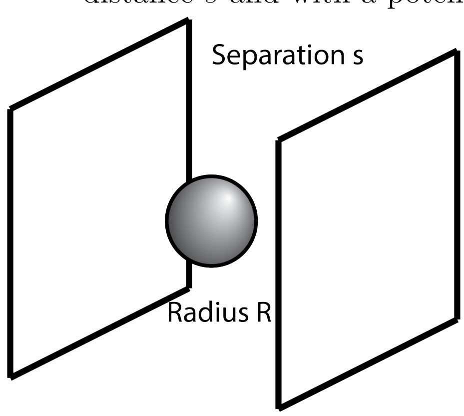
for constant g.

b.) Find the potential Φ such that $\vec{E} = -\vec{\nabla}\Phi$

c.) Verify Poisson's equation: $\nabla^2\Phi = -\dfrac{\rho}{\epsilon_0}$

Hint: Use Gauss' law to solve for part (a) and make an appropriate guess to get part (b).

5.) a.) Similar to the problem we worked out in lecture 16, we have a sphere of radius $R \ll s$ sitting between two plates separated by a distance s and with a potential difference between them.



The sphere is at $\Phi = 0$, while the plates have a voltage between them that produces a field $\vec{E} = E_0\hat{\imath}$ far from the sphere. Find the potential outside the sphere and in between the plates. It is best to use a spherical coordinate system.

b.) Show there is no net charge on the sphere.

c.) Repeat part (a) if $\Phi = V_0 \neq 0$ on the sphere.