Warm-up

Alexander Golovnev

Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

Airlines Graph

Consider a small country with five cities: A, B, C, D, E.

There are six flights:

$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D?
With one stop?
With two stops?

Airlines Graph

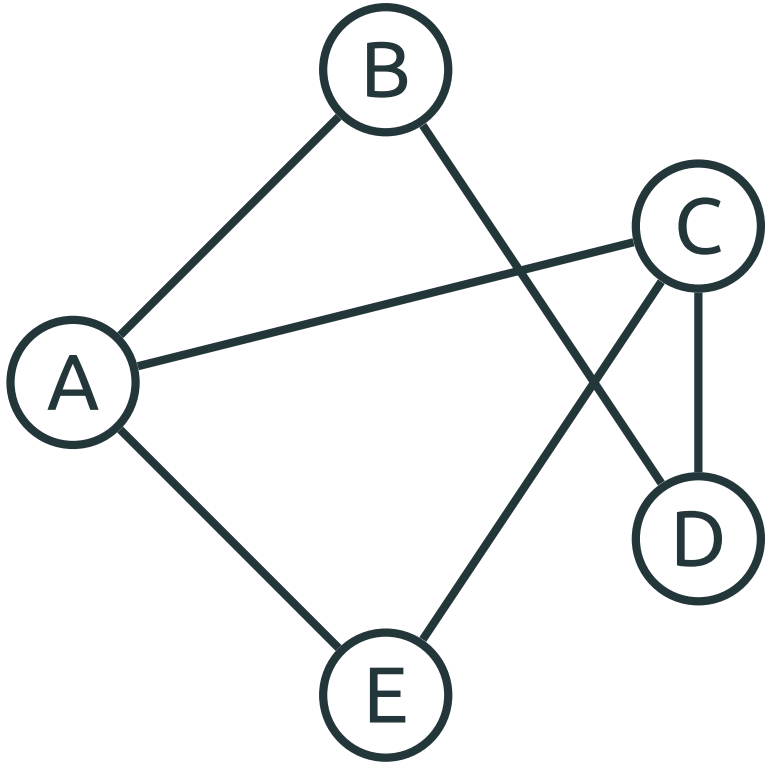
Consider a small country with five cities: A, B, C, D, E.

There are six flights:

$A - B, A - C, A - E,$

$B - D, C - D, C - E.$

Is there a direct flight from A to D?
With one stop?
With two stops?



Airlines Graph

Consider a small country with five cities: A, B, C, D, E.

There are six flights:

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Is there a direct flight from A to D?
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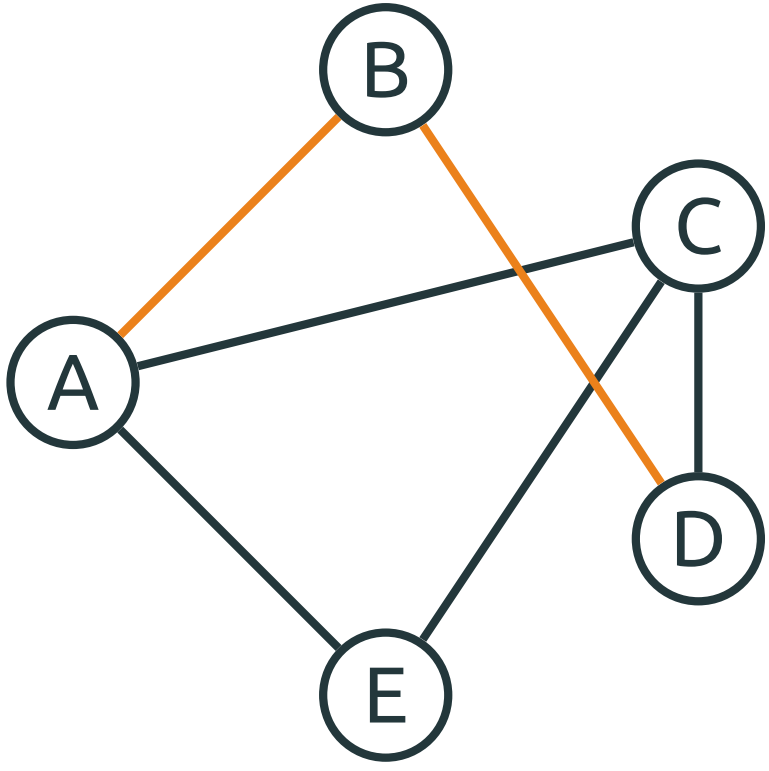

Airlines Graph

Consider a small country with five cities: A, B, C, D, E.

There are six flights:

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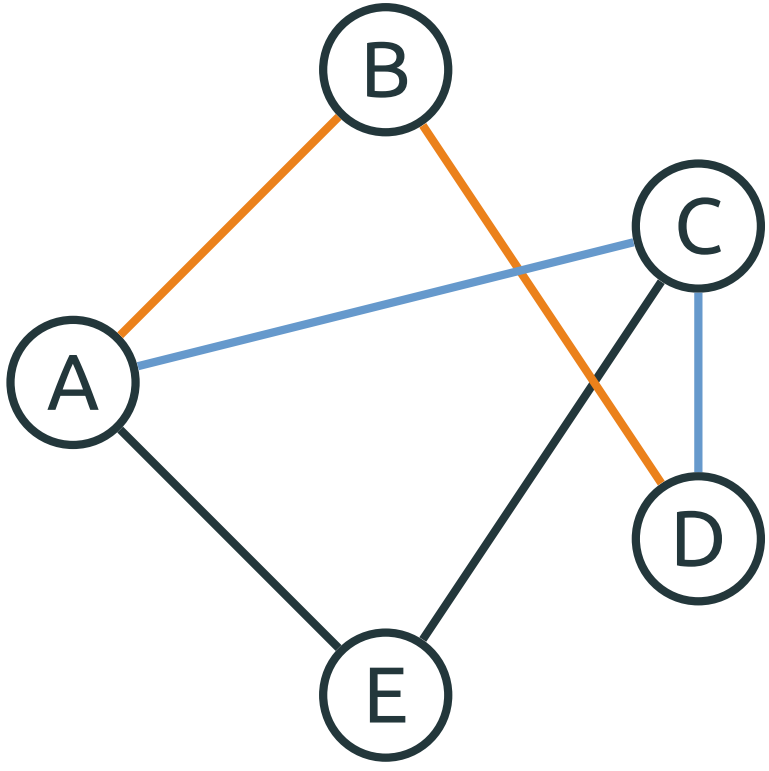


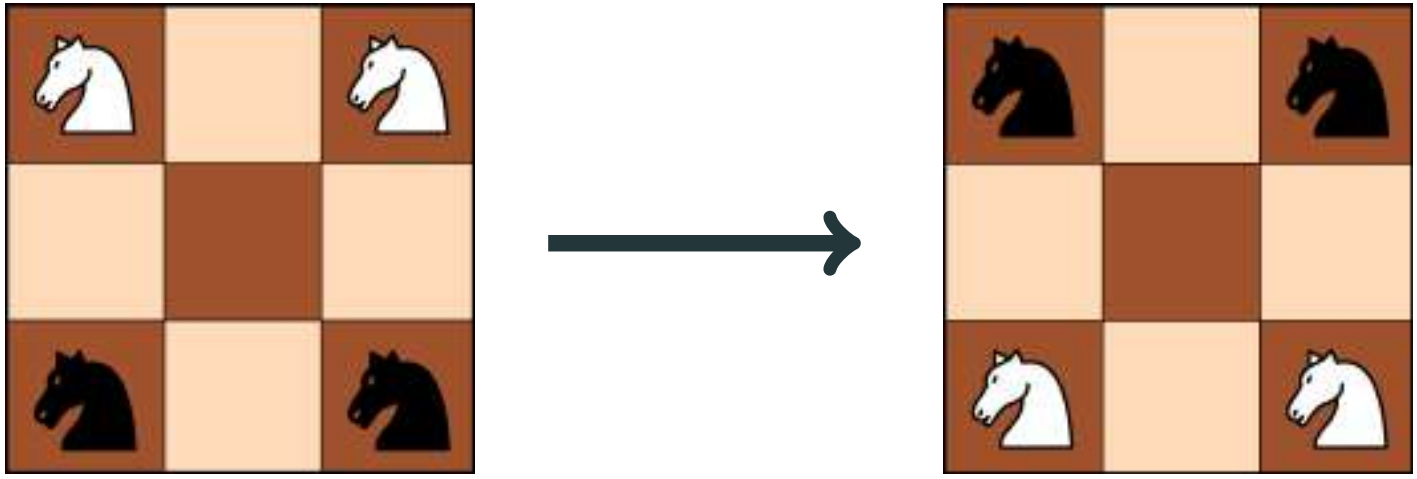
Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

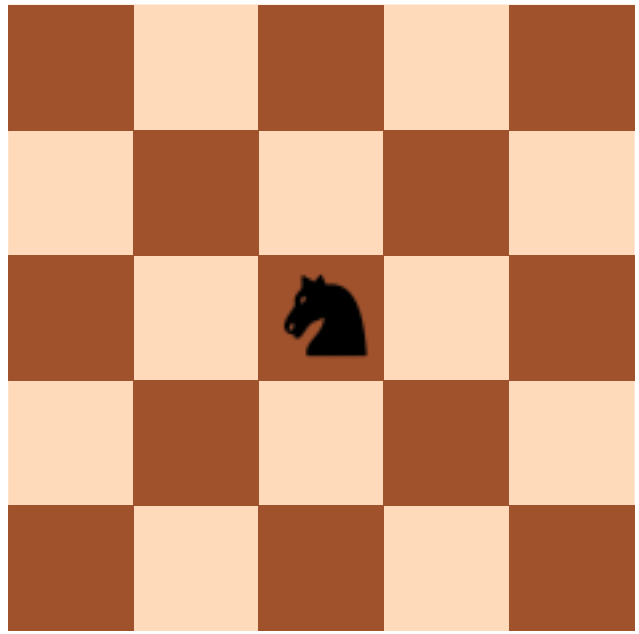
Guarini's Puzzle



Exchange the places of the white knights and the black knights.

Chess Knight

A chess knight can move in an **L** shape in any direction



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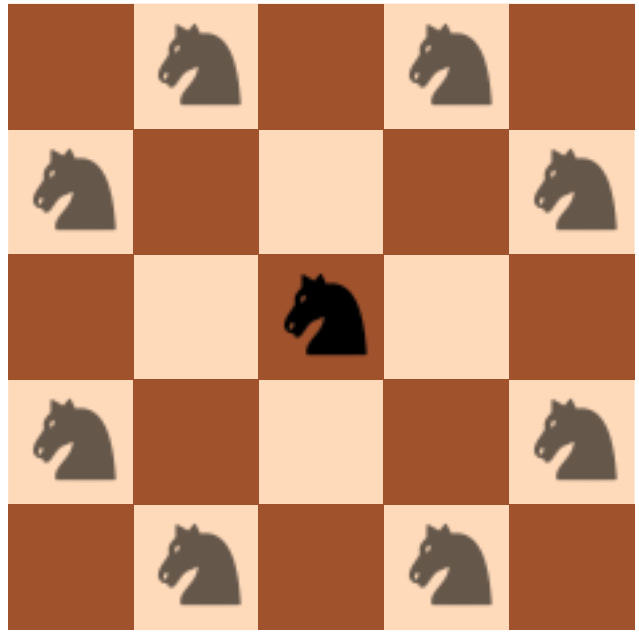
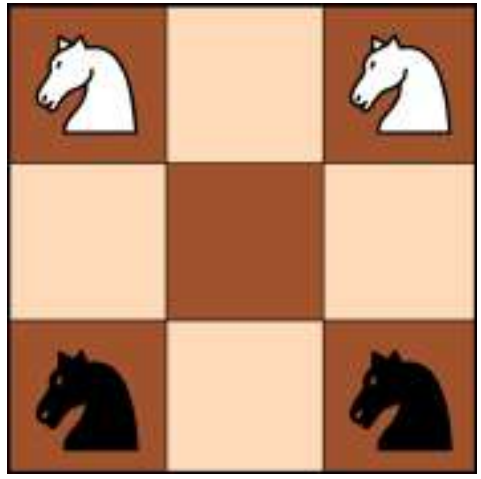


Guarini's Puzzle. Solution



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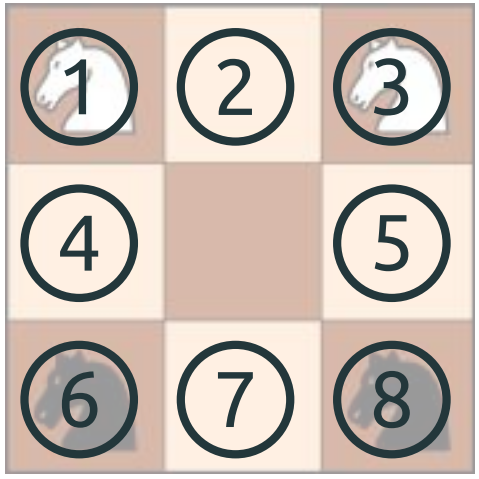

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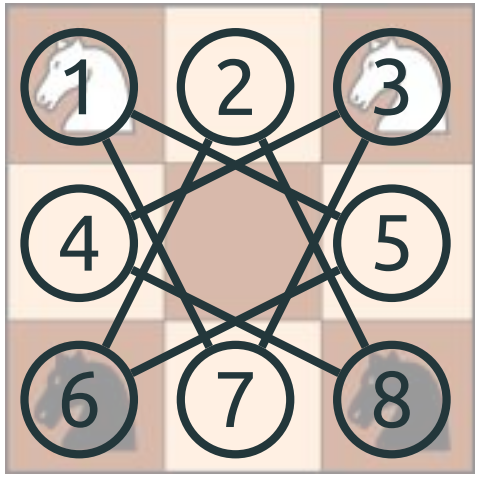

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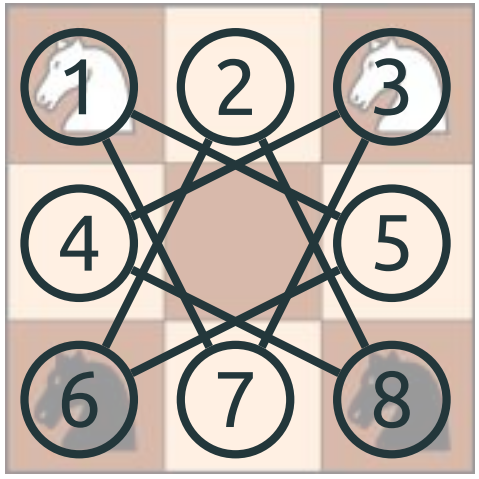

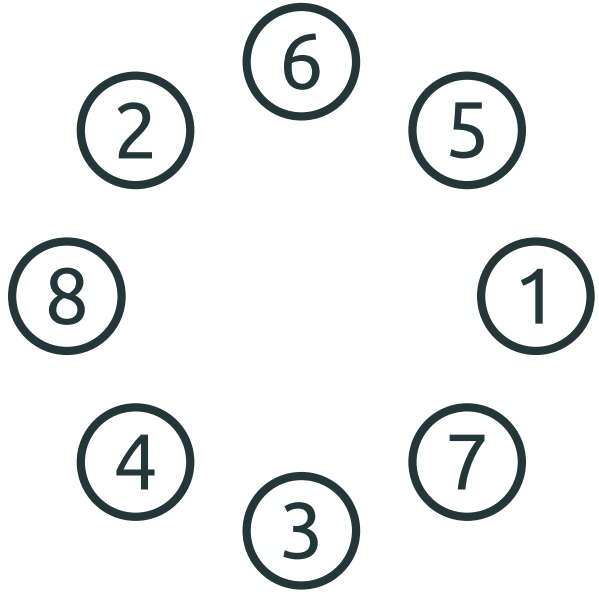

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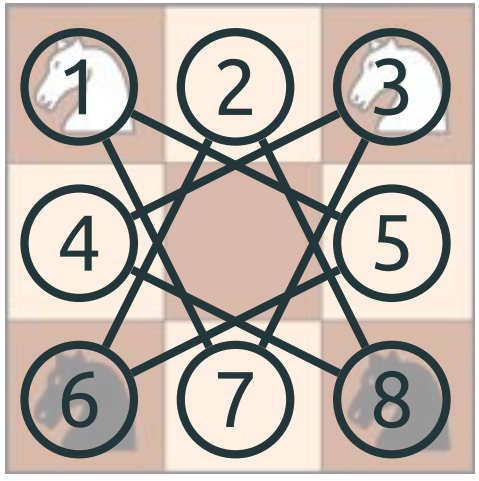

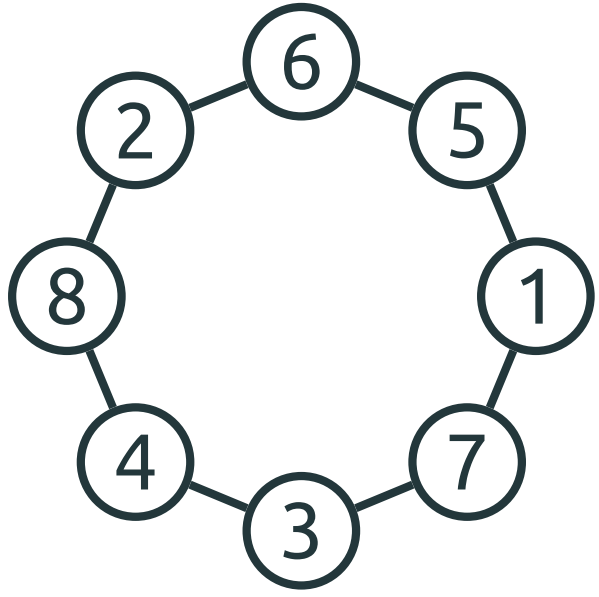

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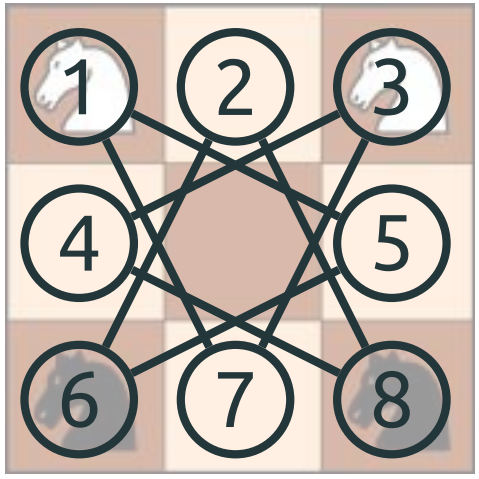

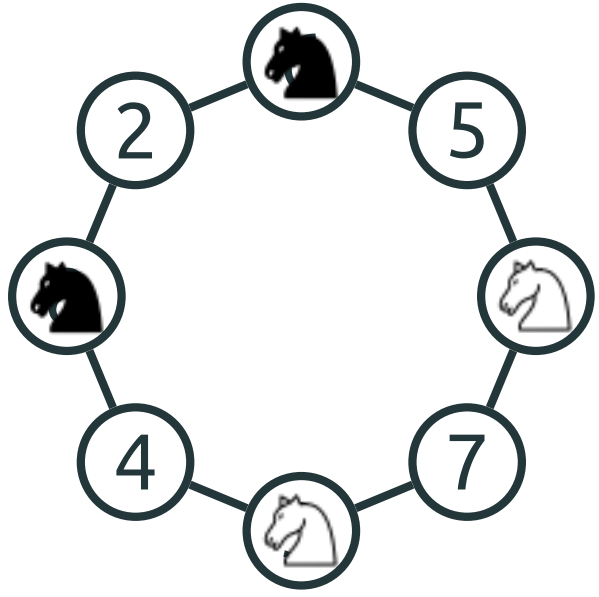

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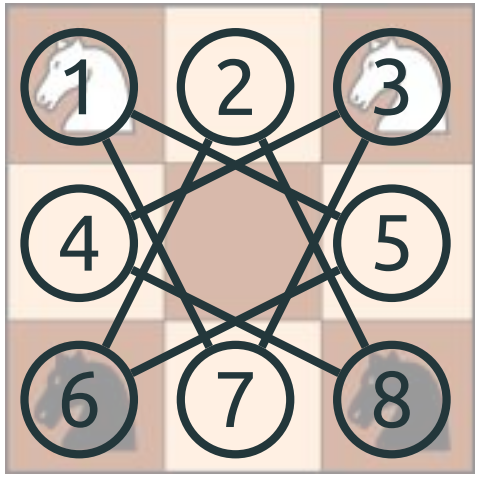

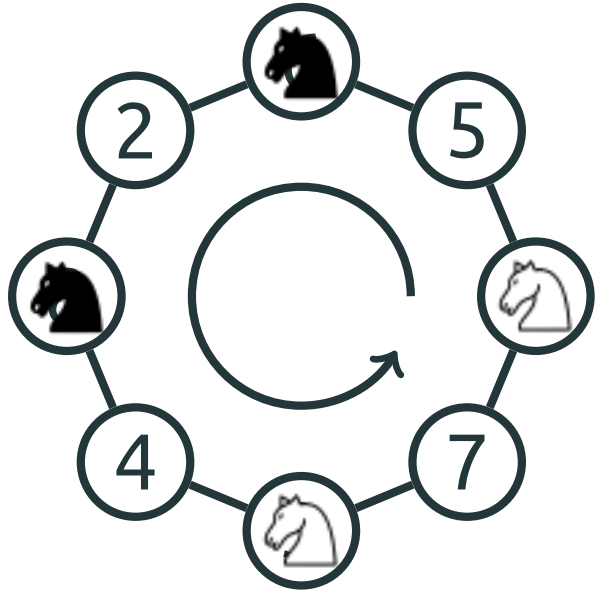

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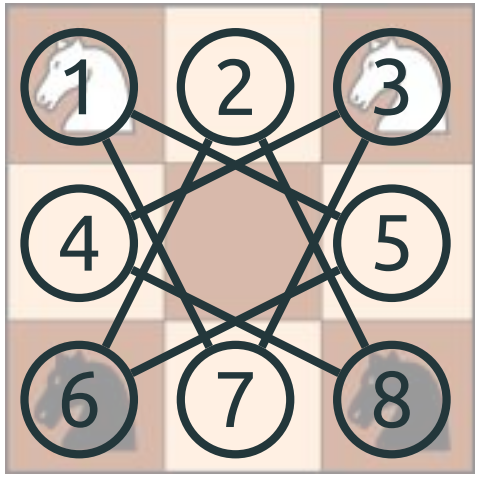

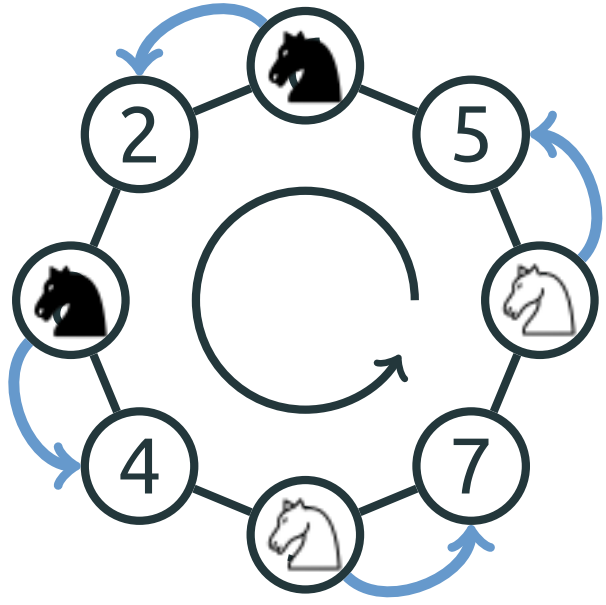

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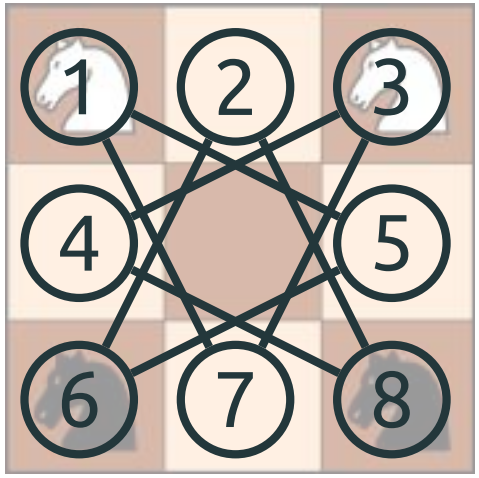

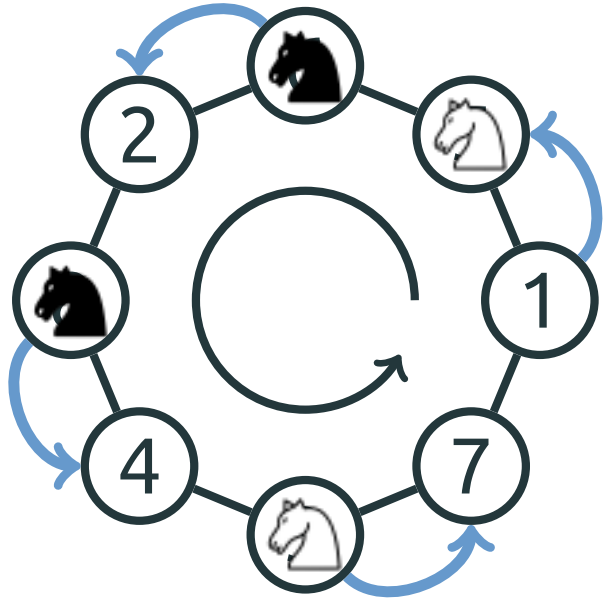

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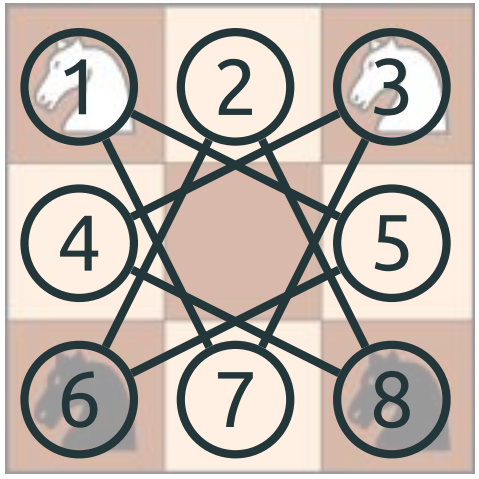

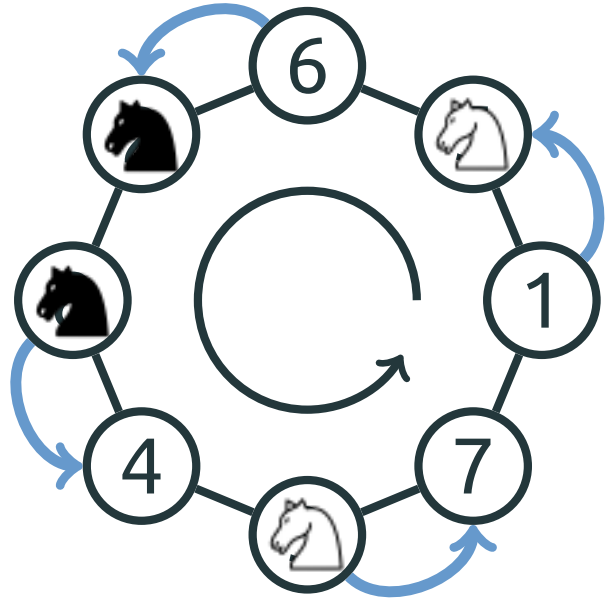

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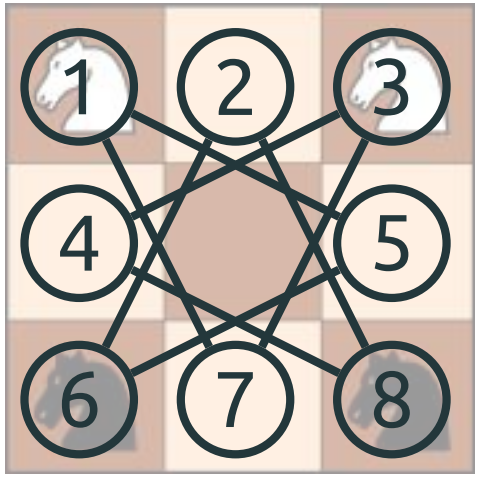

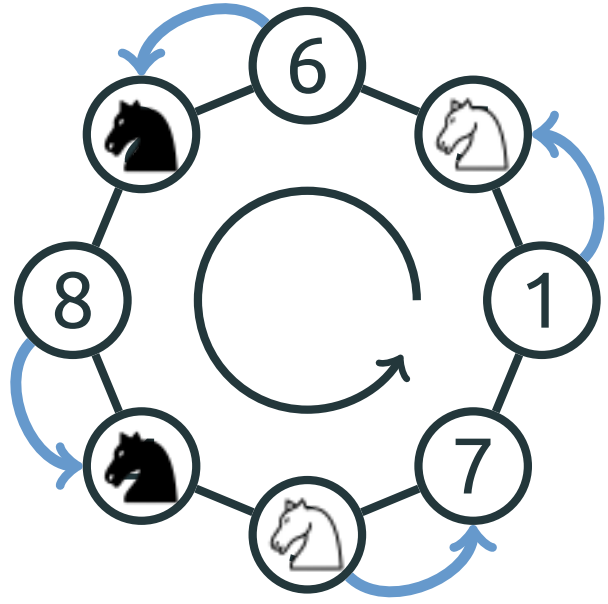

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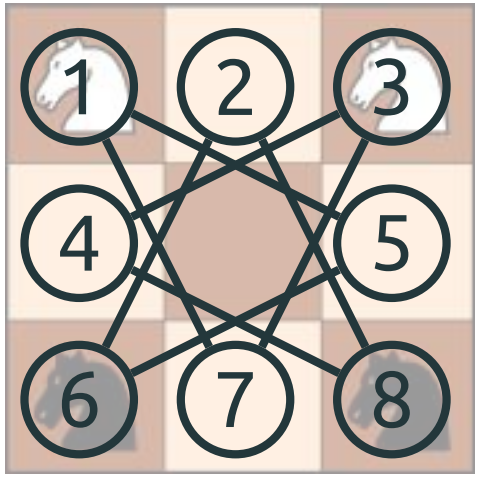

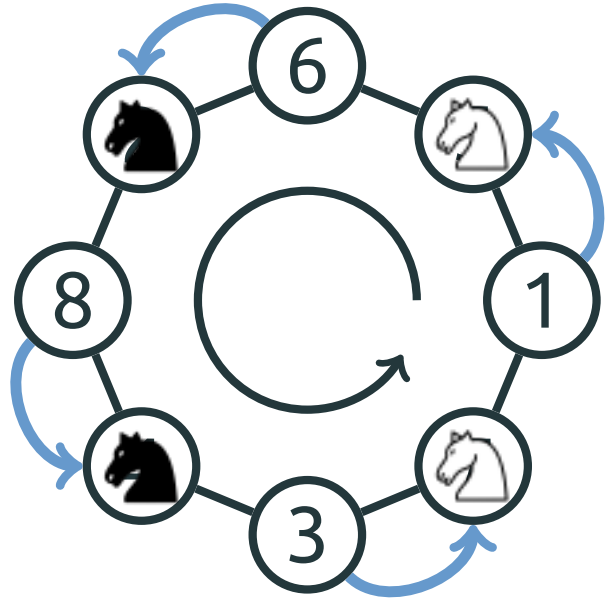

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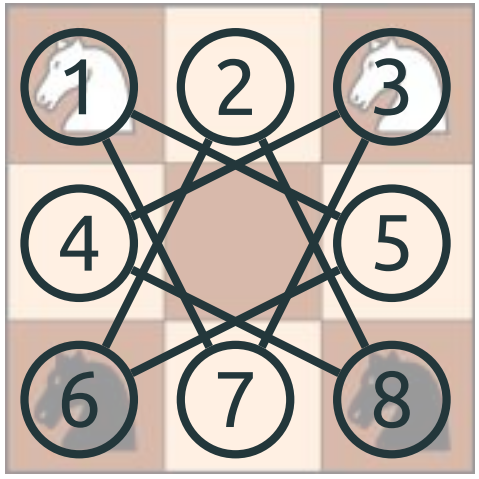

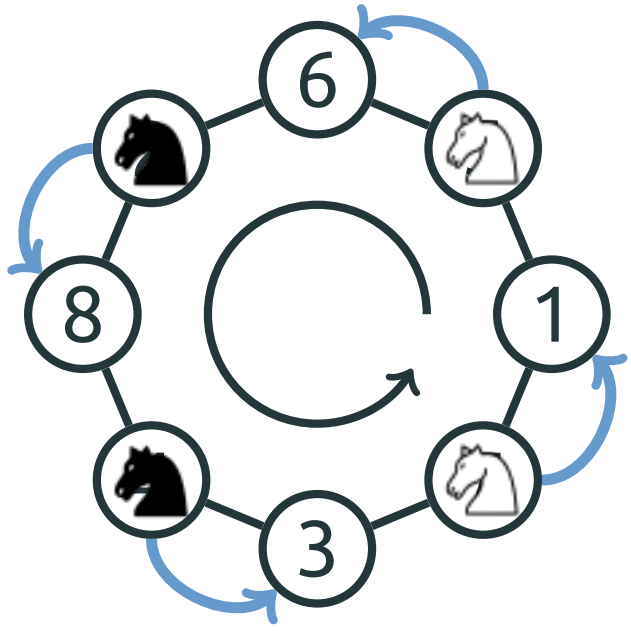

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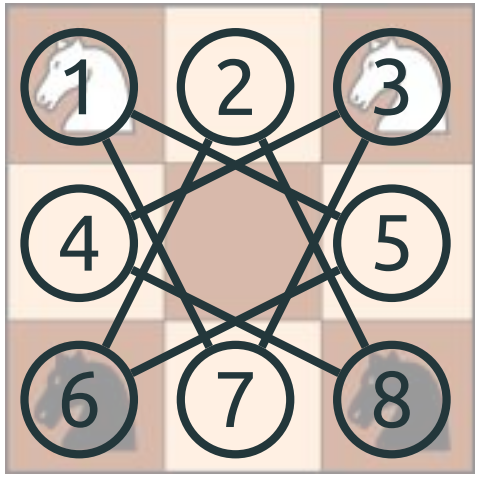

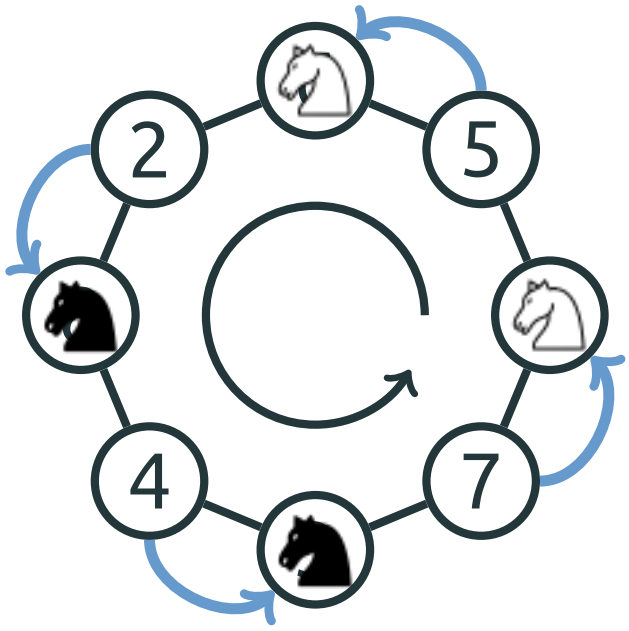

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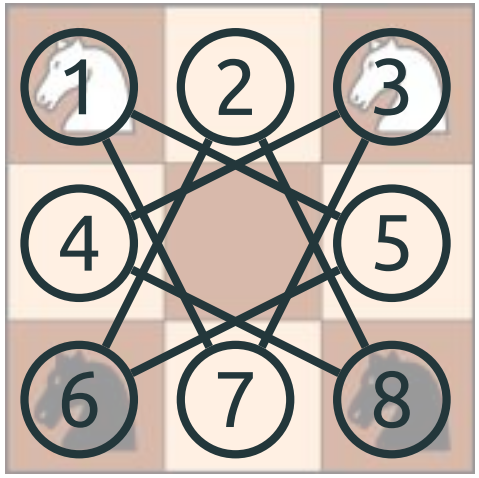

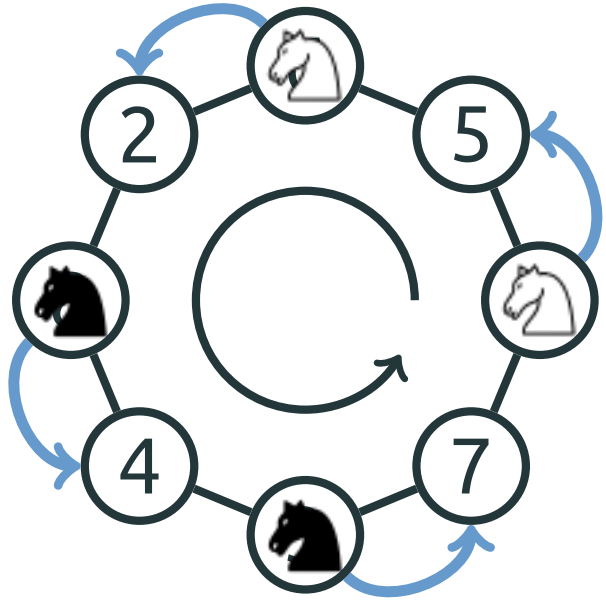

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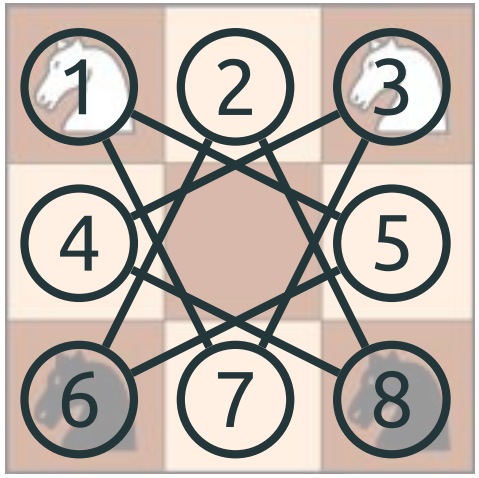

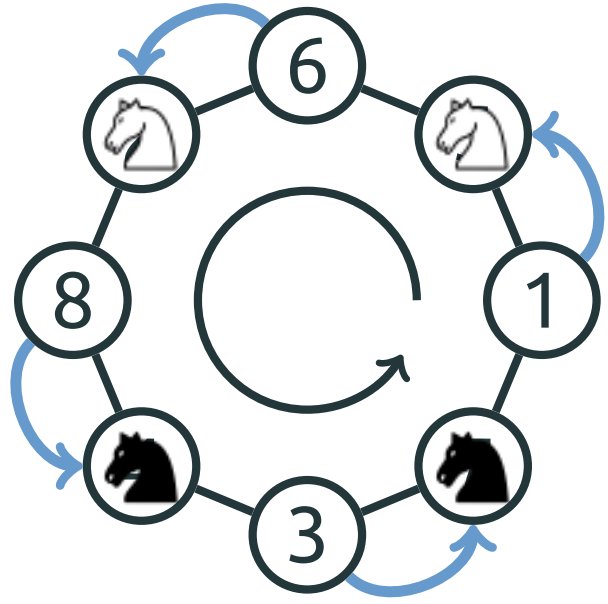

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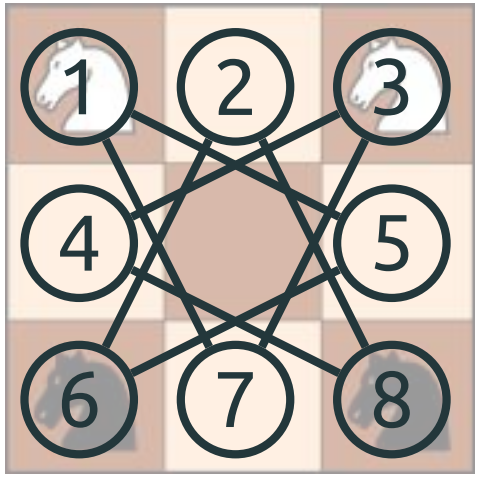

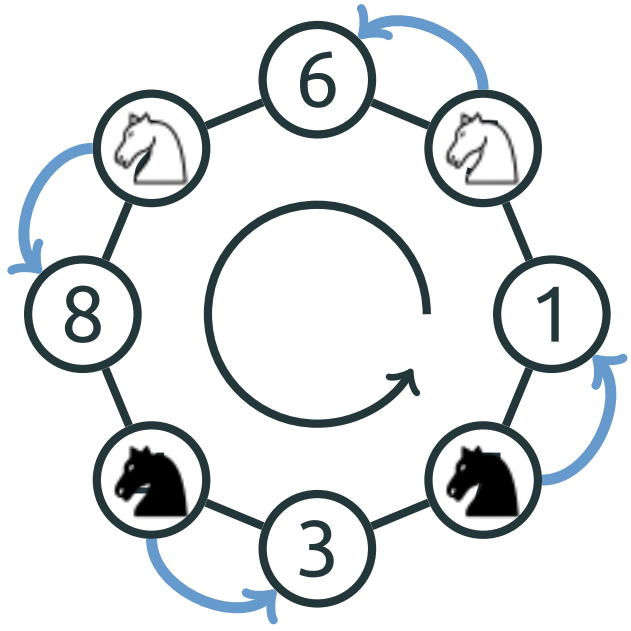

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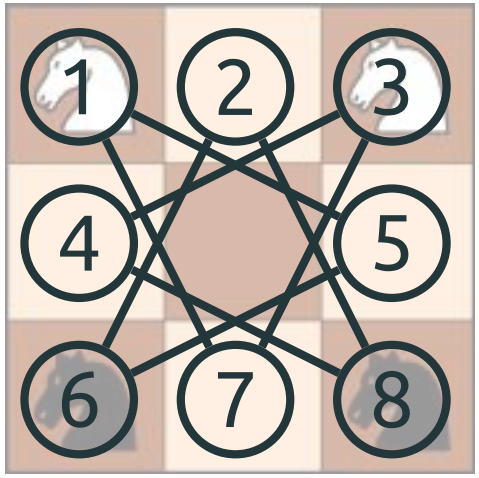

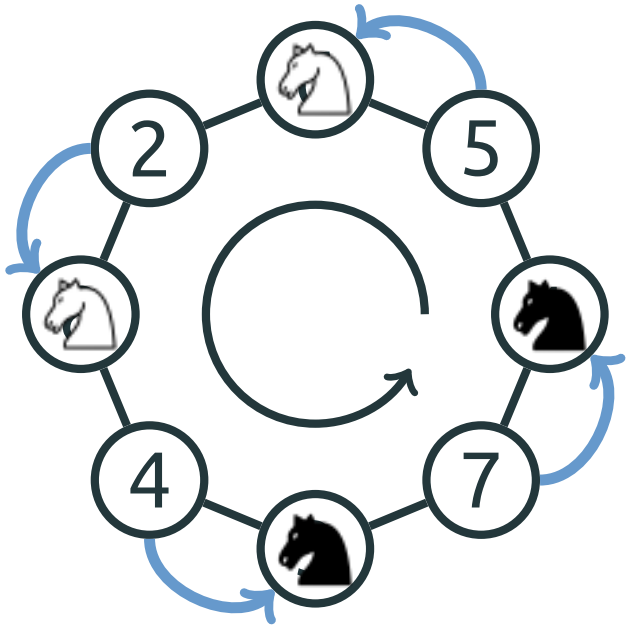

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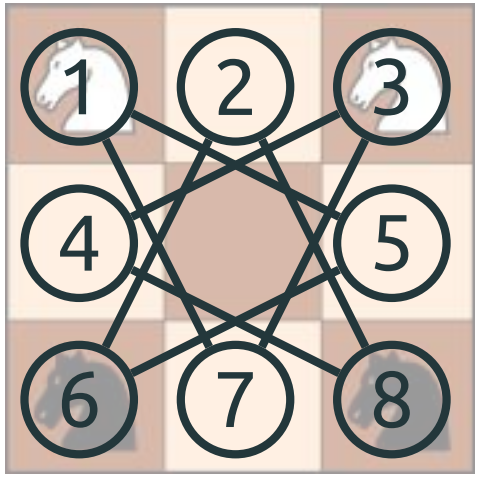

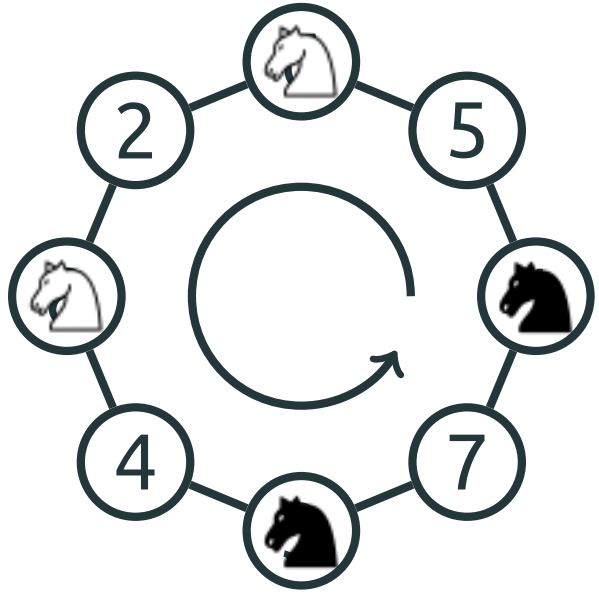

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Outline

Airlines Graph

Knight Transposition

Seven Bridges of Königsberg

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Königsberg, Prussia, 1735

Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

Cross each bridge
exactly once

Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

Cross each bridge
exactly once

Leonhard Euler

Seven Bridges of Königsberg

Königsberg, Prussia, 1735

Walk through Königsberg

Cross each bridge
exactly once

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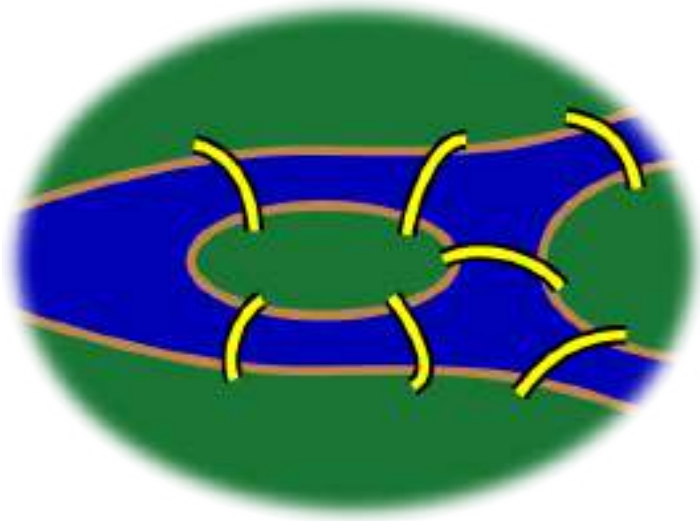
Impossible!

Bridges of Königsberg. Graph

Bridges of Königsberg. Graph



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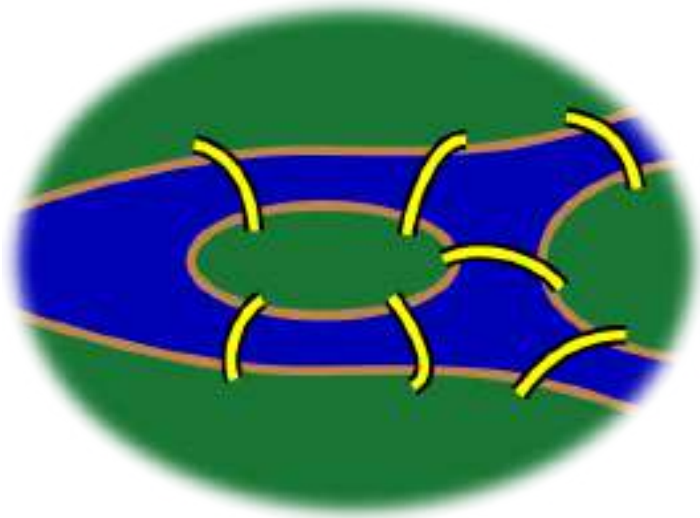

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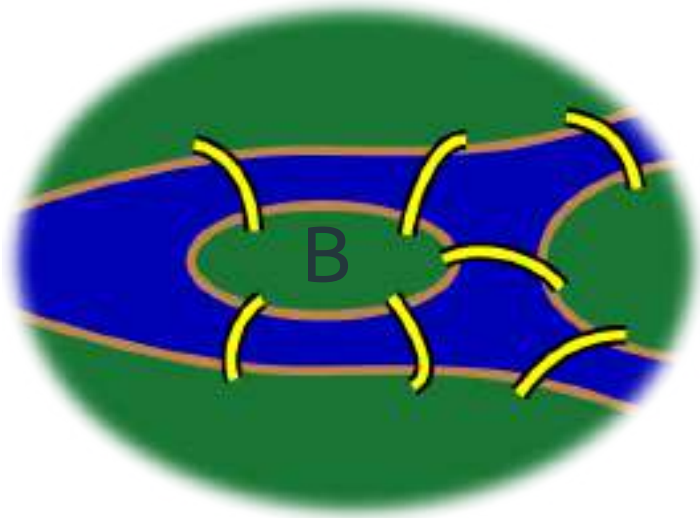

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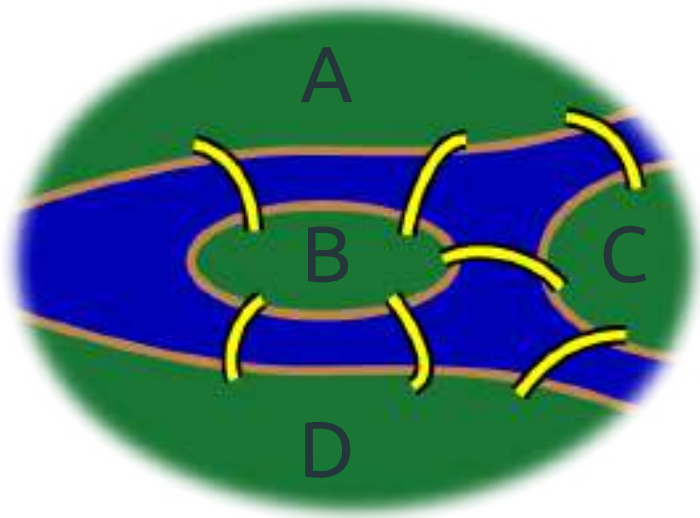
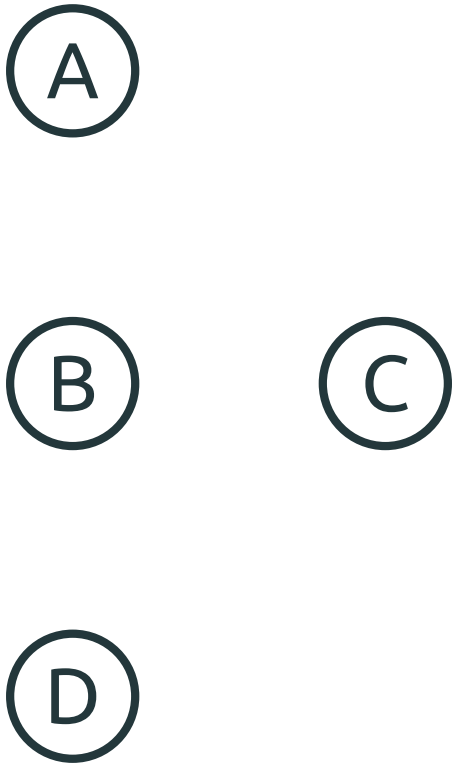
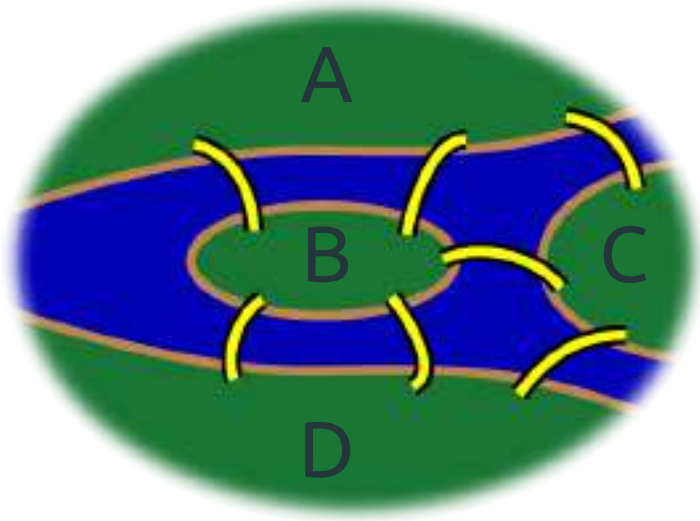
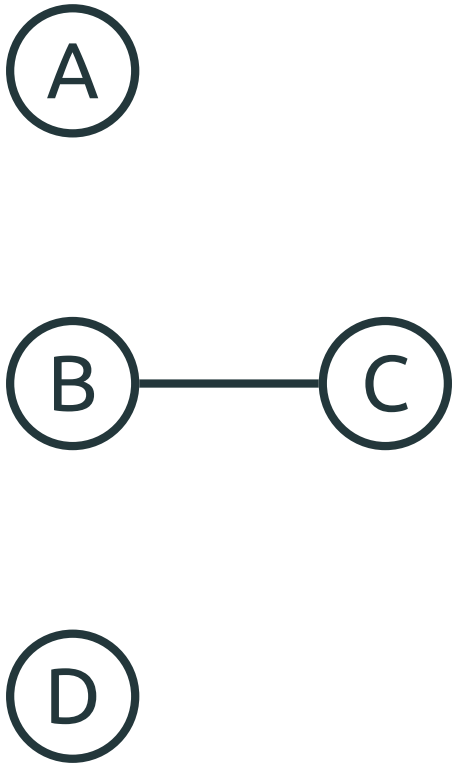


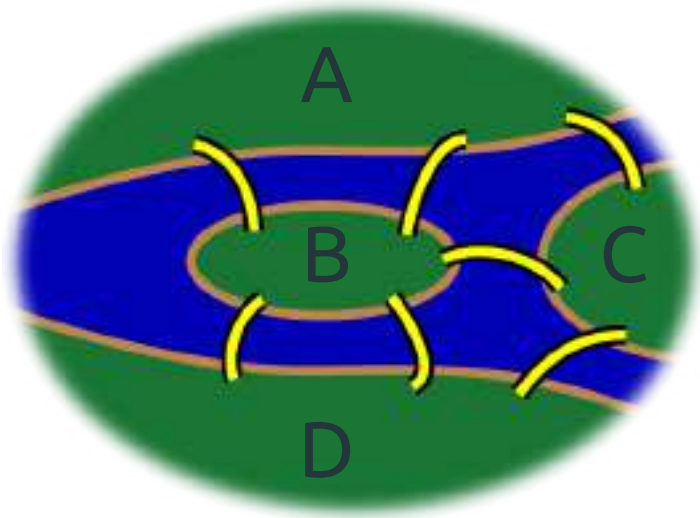
Bridges of Königsberg. Graph



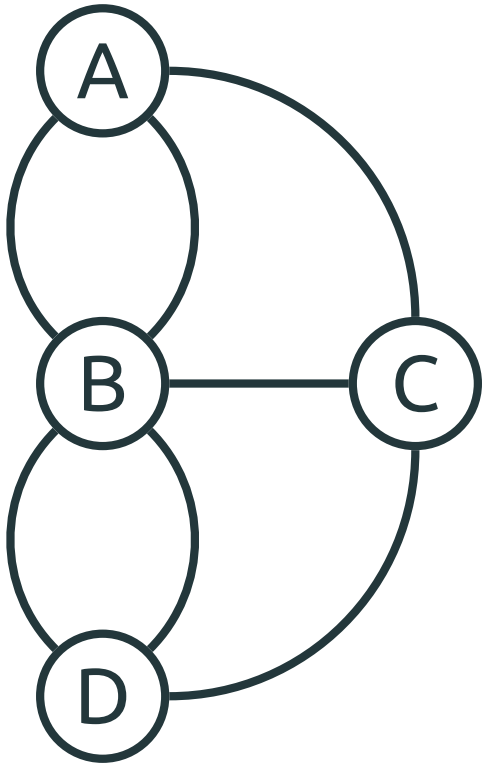


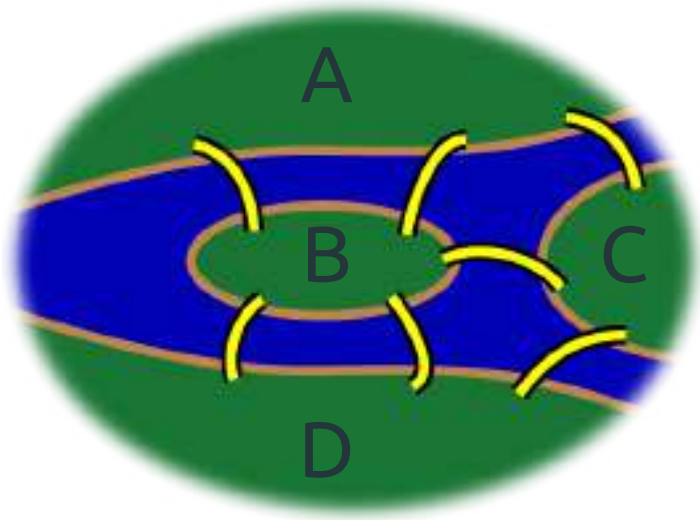
Bridges of Königsberg. Graph



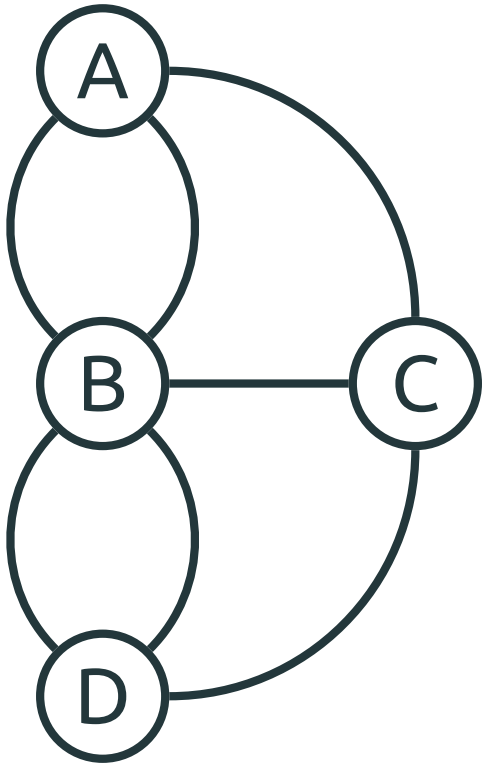


Bridges of Königsberg. Graph





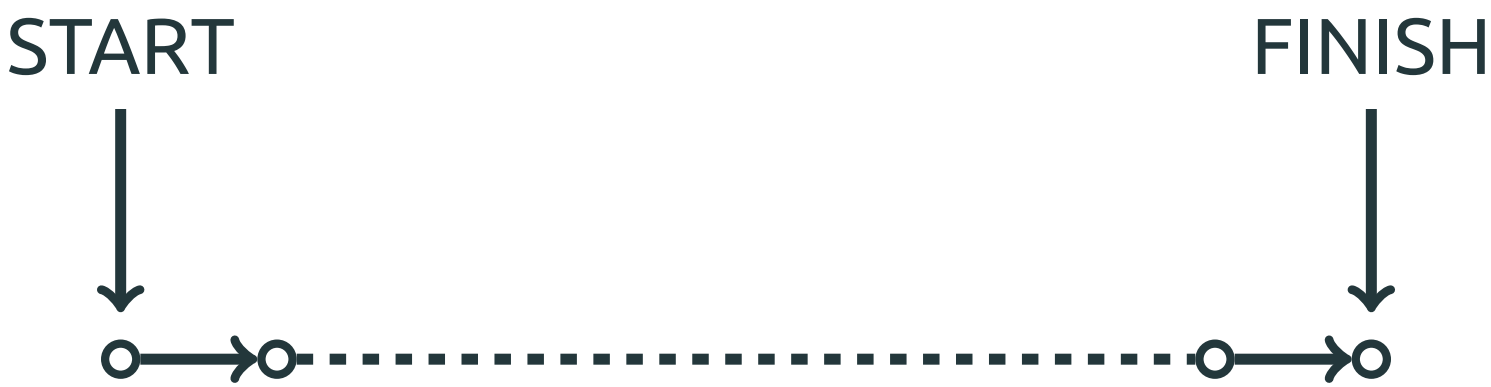
Bridges of Königsberg. Graph



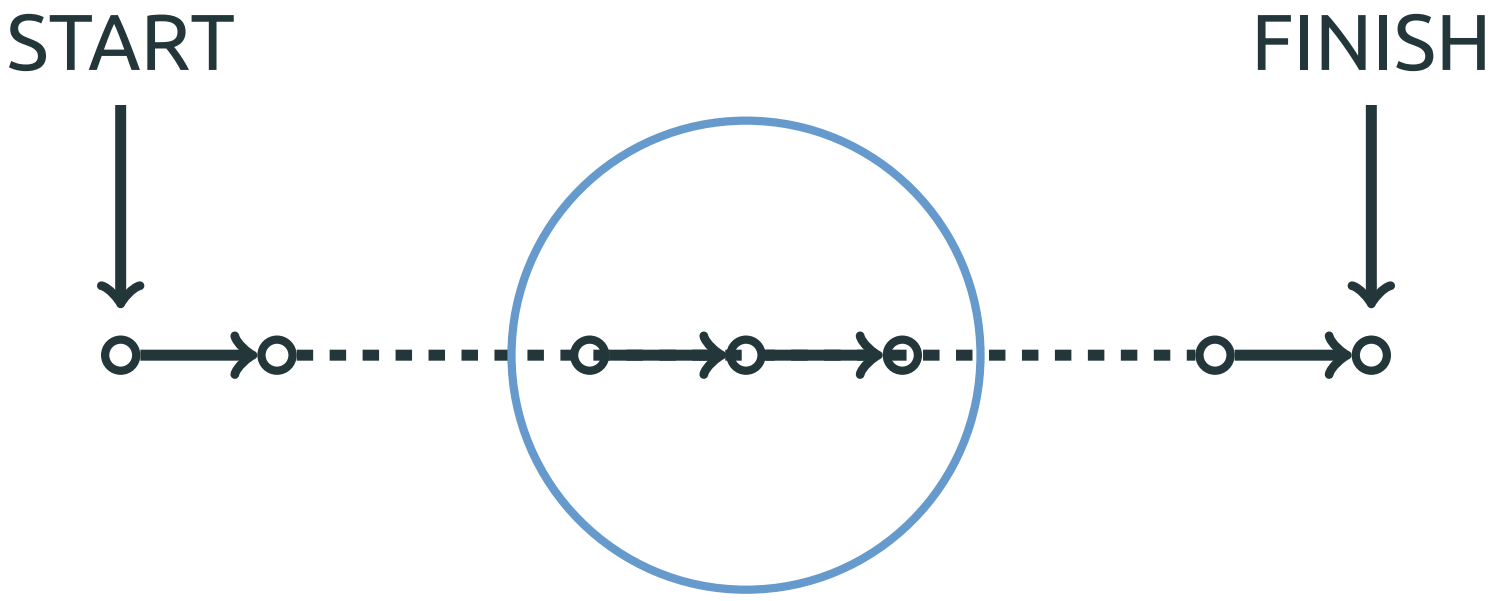
Is there a path which visits every edge exactly once?

Eulerian Path

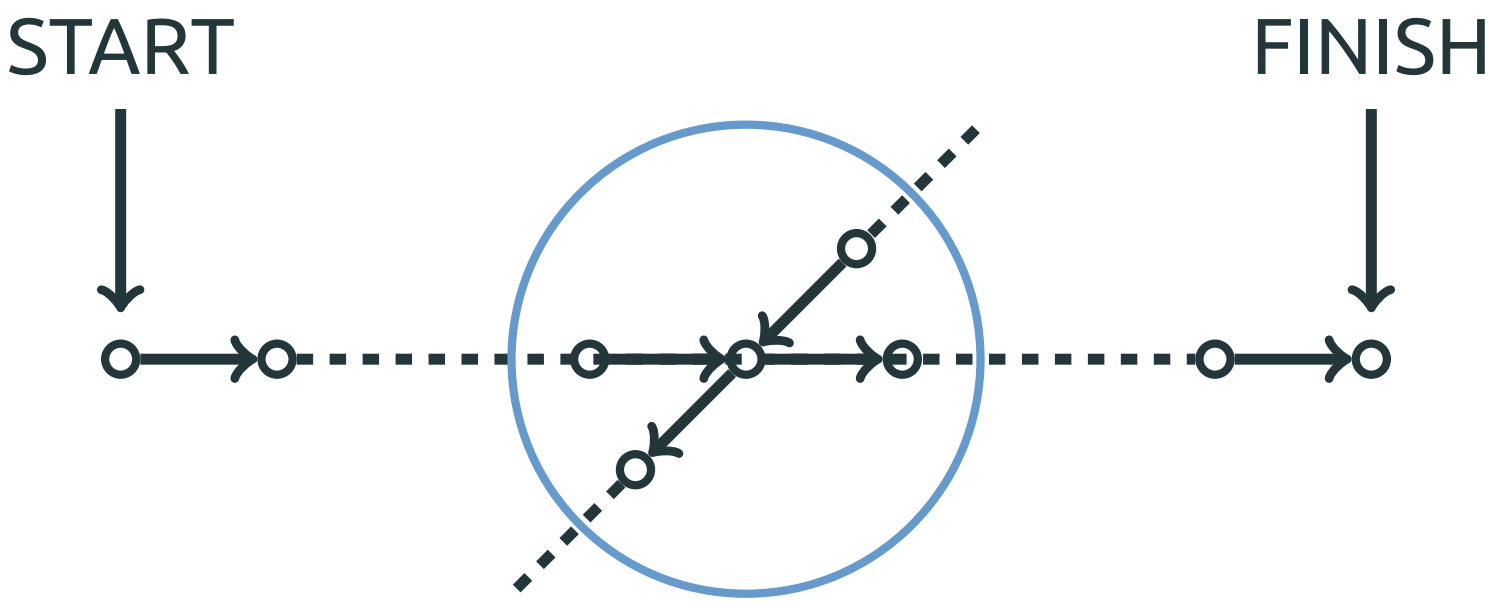
Eulerian Path



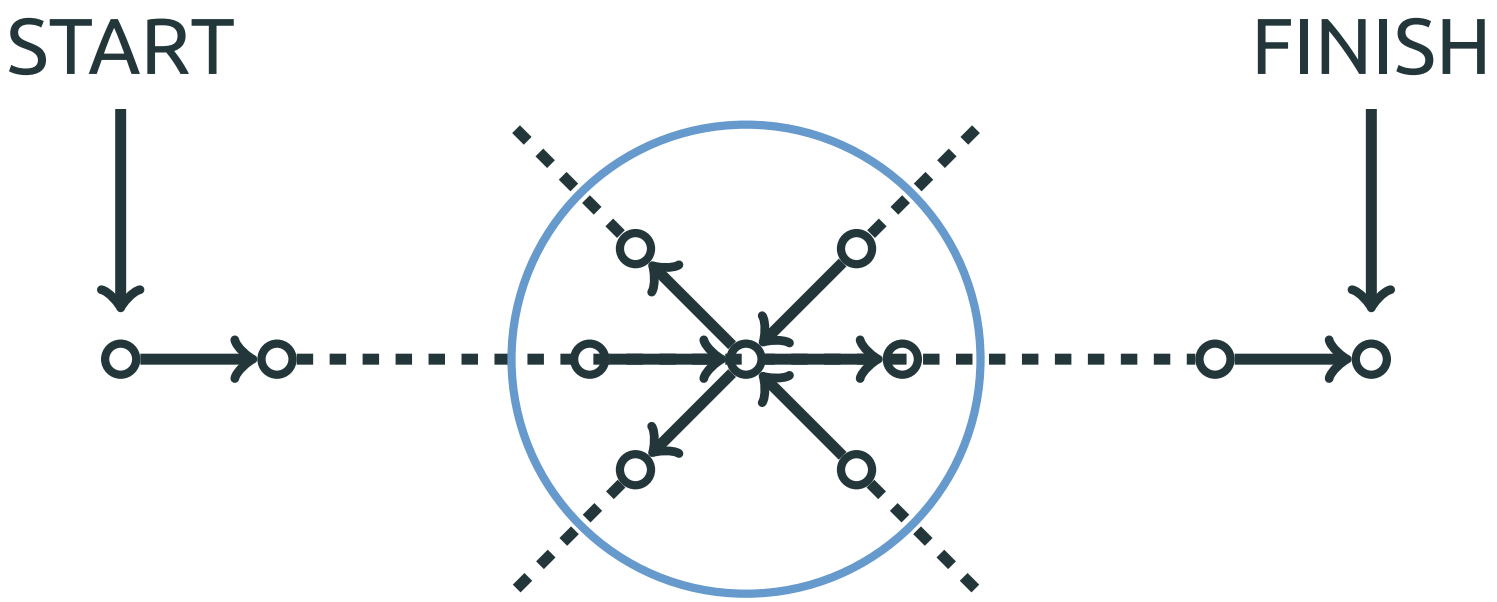
Eulerian Path



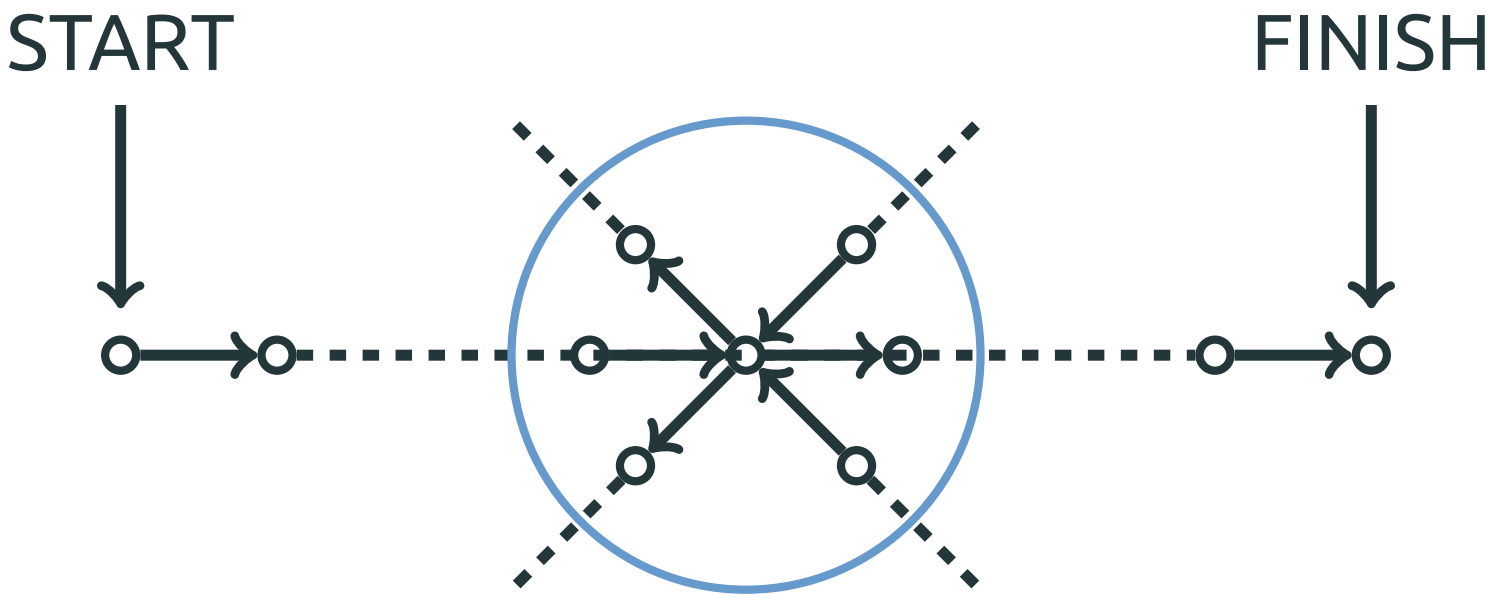
Eulerian Path



Eulerian Path

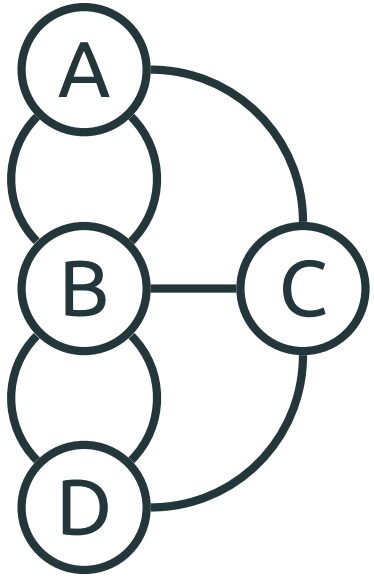


Eulerian Path



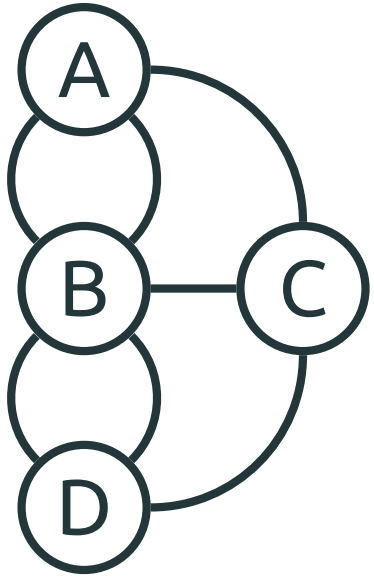
All but START and FINISH vertices
have even number of neighbors

Eulerian Path



All but START and FINISH vertices have even number of neighbors

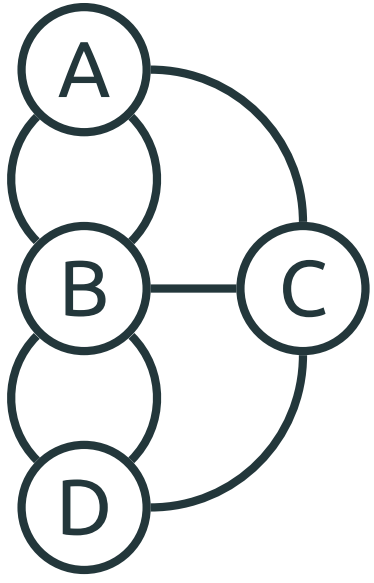
Eulerian Path



All four vertices have
odd number of neighbors

All but START and FINISH vertices
have even number of neighbors

Eulerian Path

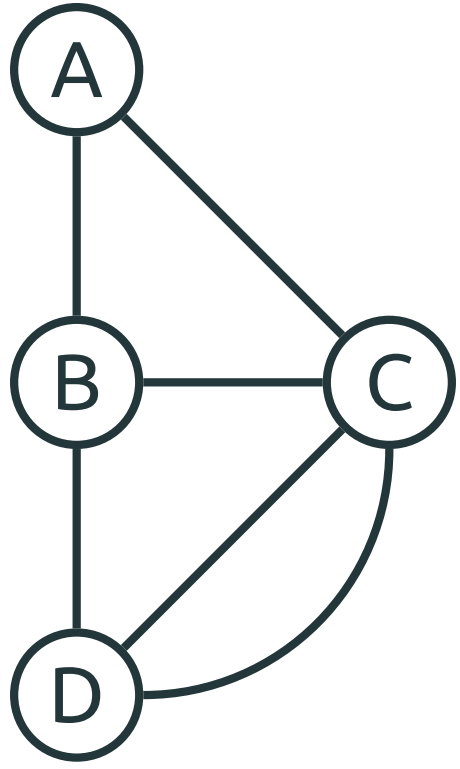


All four vertices have
odd number of neighbors
Impossible!

All but START and FINISH vertices
have even number of neighbors

Six Bridges of Kaliningrad

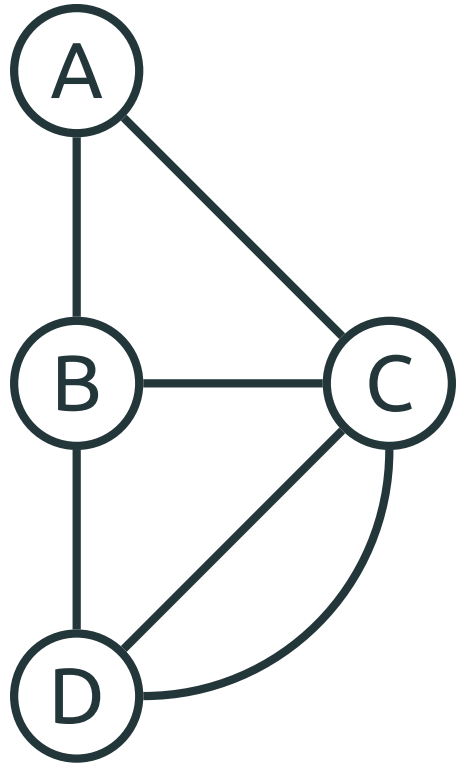
Six Bridges of Kaliningrad



Six Bridges of Kaliningrad

B and D have odd number of neighbors

If there exists an Eulerian path, B and D must be START and FINISH



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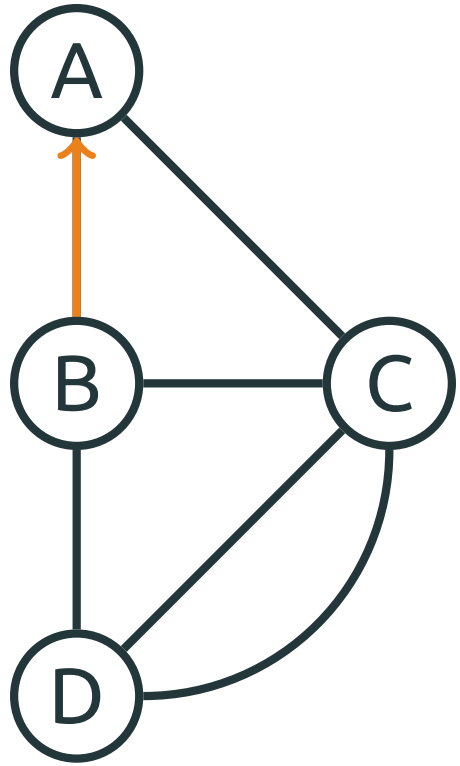

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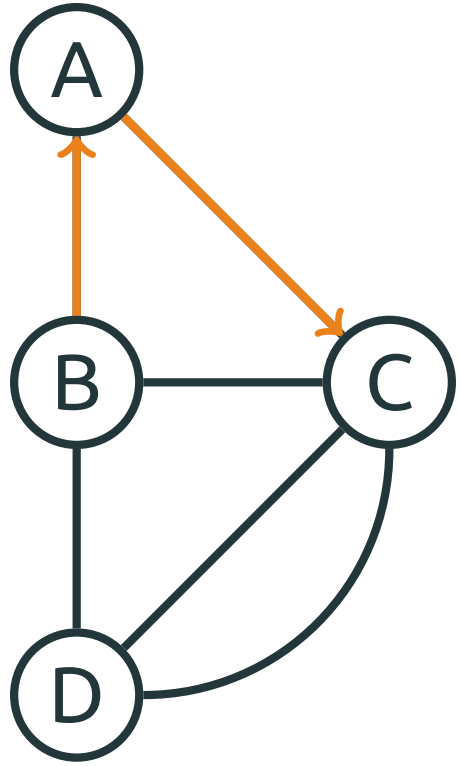

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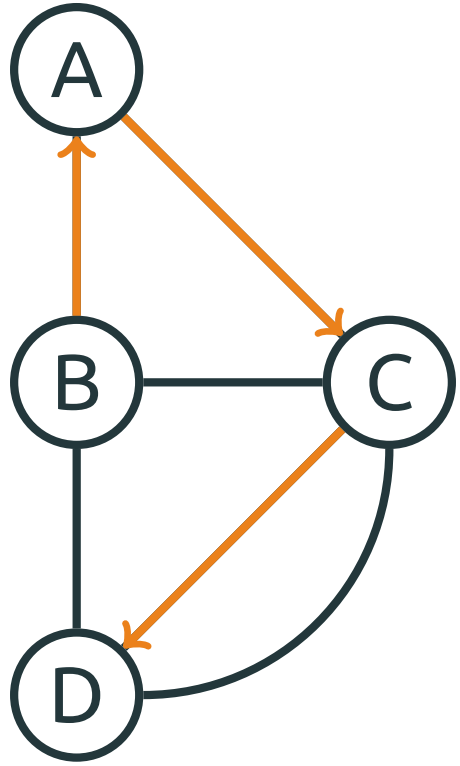

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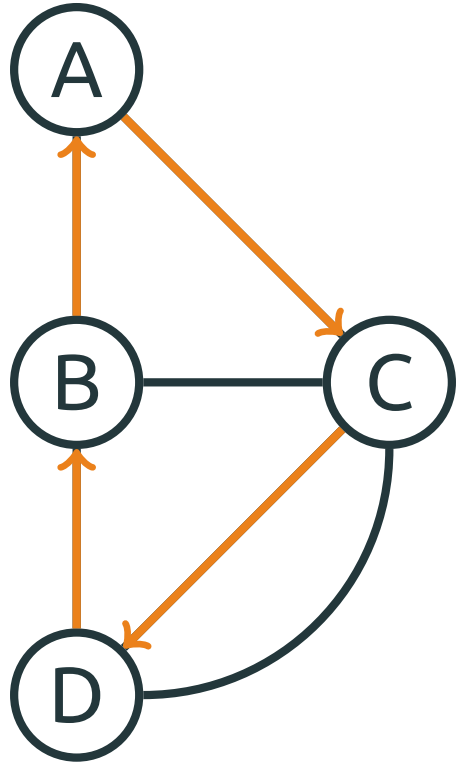

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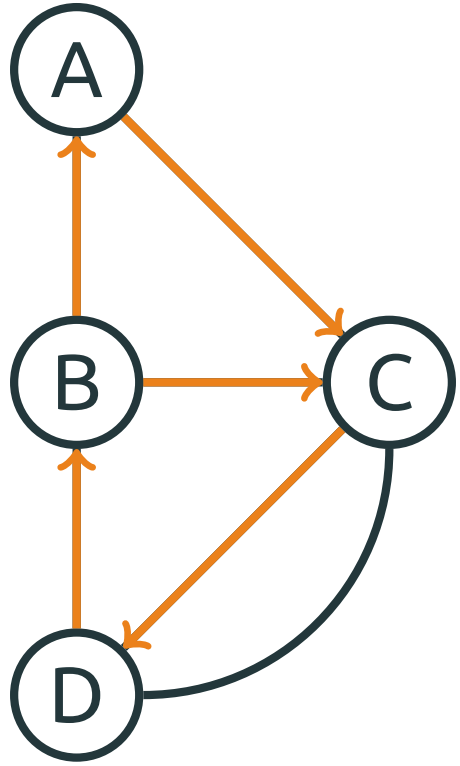

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