Basic Definitions

Alexander Golovnev

Outline

Definitions

An isolated vertex forms a component

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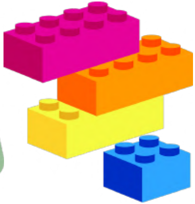

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The Degree of a Vertex

The number of friends

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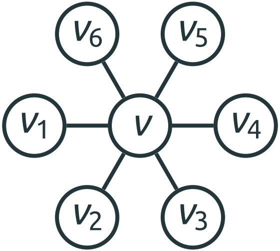
- I.e., the Degree of a vertex is the number of its neighbors
- The degree of a vertex v is denoted by $\deg(v)$
- The degree of a graph is the maximum degree of its vertices

The Degree of a Vertex: Examples

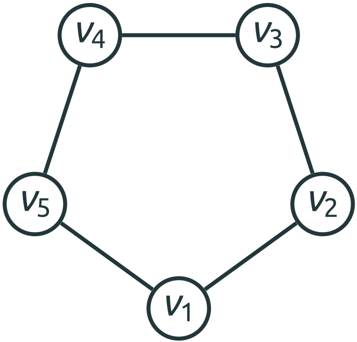
The degree of v is 6: $\deg(v) = 6$

The degree of v_6 is 1: $\deg(v_6) = 1$

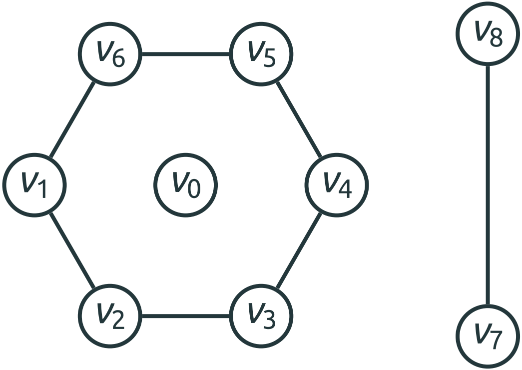


The Degree of a Vertex: Examples

The degree of every vertex is 2: $\forall i, \deg(v_i) = 2$

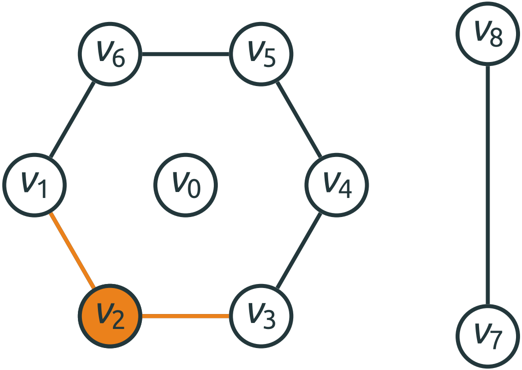


Isolated Vertices



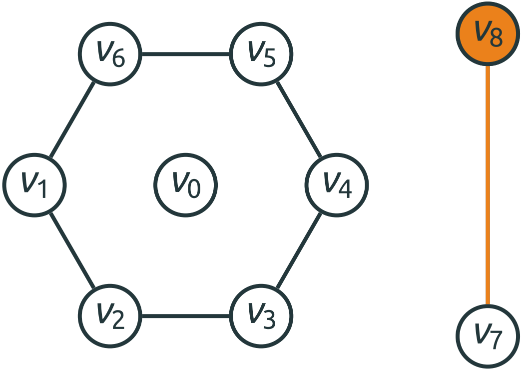
Isolated Vertices

$\deg(v_2) = 2$



Isolated Vertices

$$\deg(v_2) = 2$$
$$\deg(v_8) = 1$$

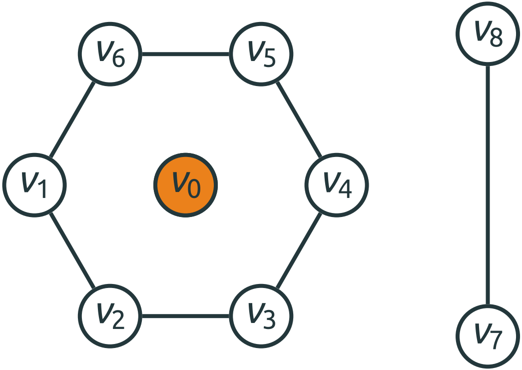


Isolated Vertices

$\deg(v_2) = 2$

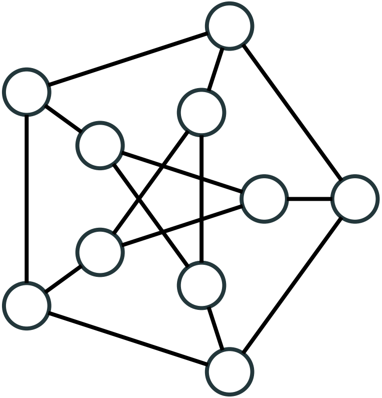
$\deg(v_8) = 1$

$\deg(v_0) = 0$. v_0 is an Isolated Vertex



Regular Graphs

A Regular graph is a graph where each vertex has the same degree



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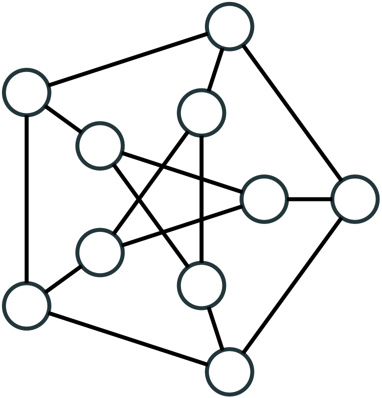
A regular graph of degree k is also called k-Regular



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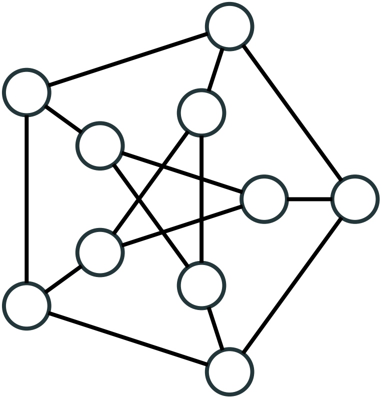
A regular graph of degree *k* is also called *k*-Regular

E.g., this graph is 3-Regular



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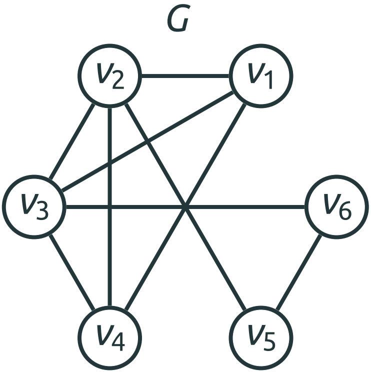
- I.e., $(u, v) \in \bar{E}$ if and only if $(u, v) \notin E$

Complement Graph



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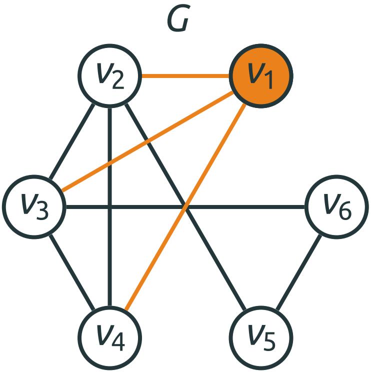

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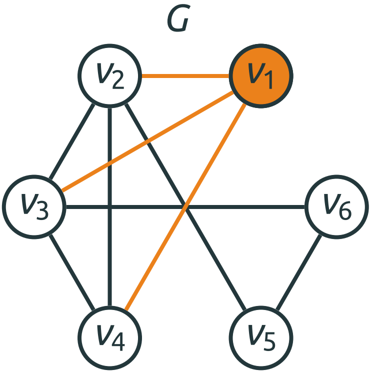

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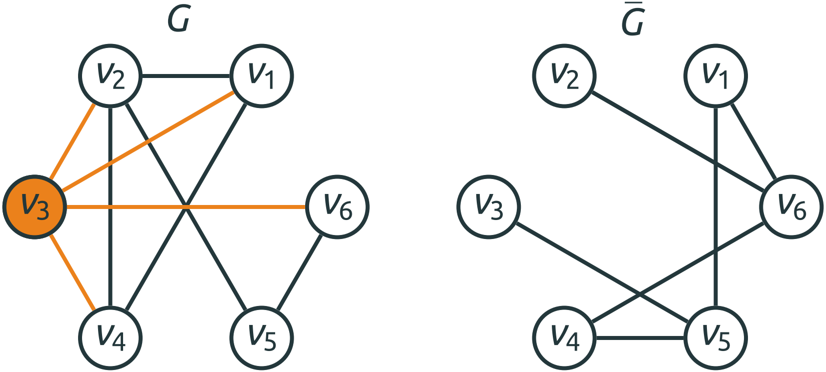

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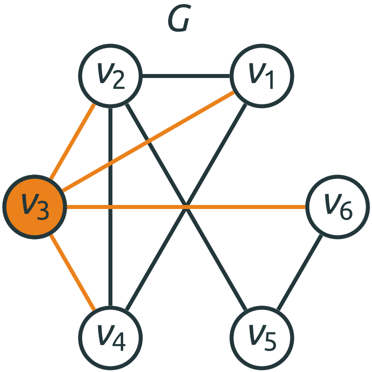

Outline

The Degree of a Vertex

Paths

Connectivity

Directed Graphs

Weighted Graphs

Paths

Is there a path from one point to another?

Walks

- A Walk in a graph is a sequence of edges, such that each edge (except for the first one) starts with a vertex where the previous edge ended

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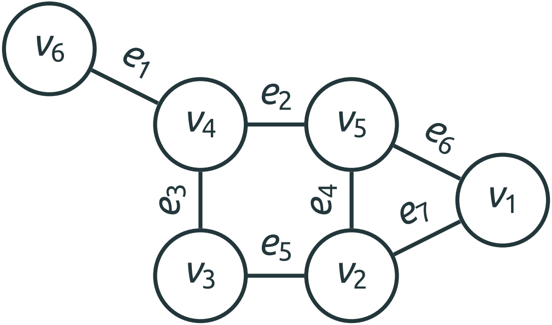
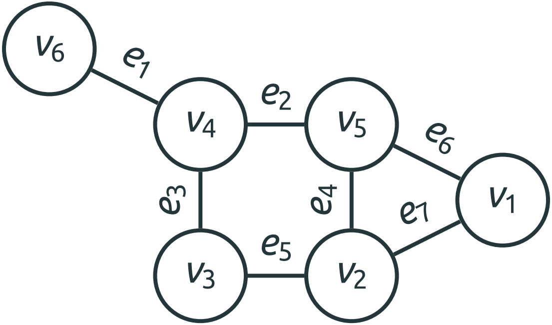
- The Length of a walk is the number of edges in it
- A Path is a walk where all edges are distinct
- A Simple Path is a walk where all vertices are distinct

Walks: Examples



Walks: Examples

A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$



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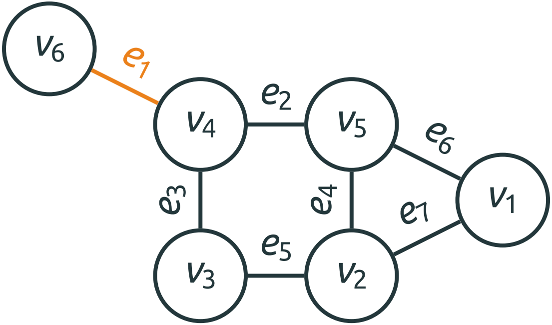

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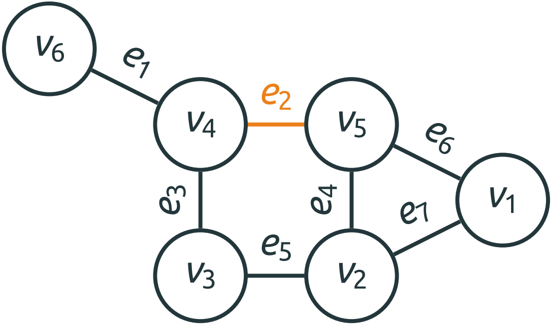

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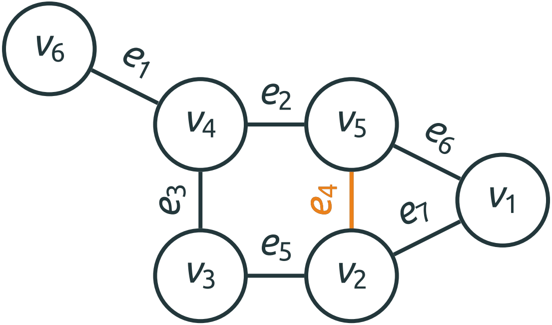

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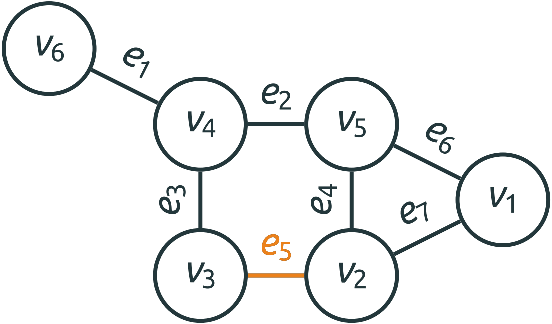

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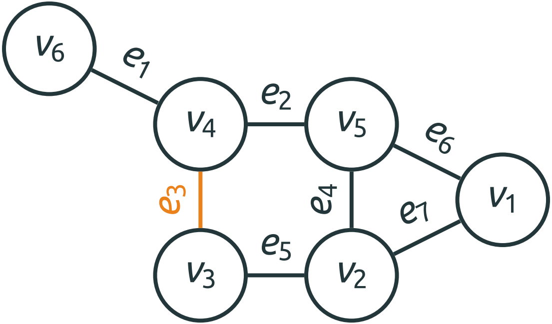

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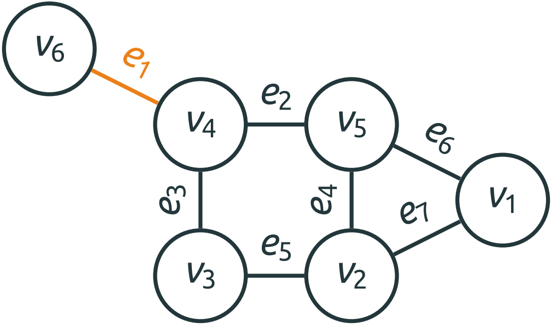
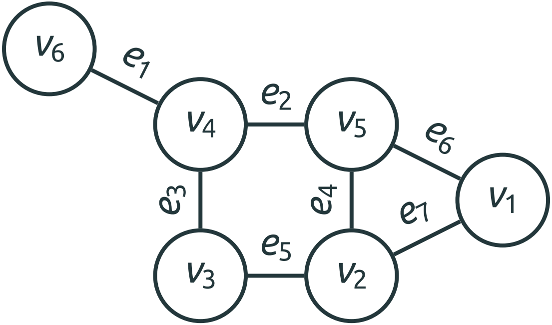


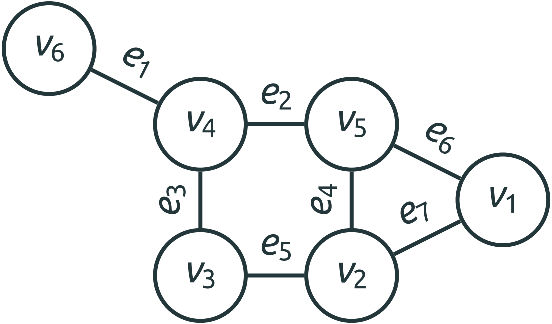
Walks: Examples

A walk of length 6: $(e_1, e_2, e_4, e_5, e_3, e_1)$
Not a path: uses e_1 twice



Walks: Examples

A path of length 4: (e_7, e_6, e_4, e_5)



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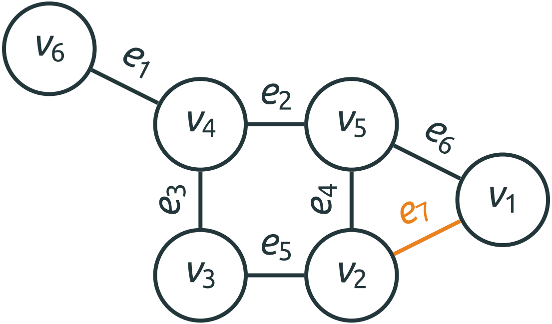

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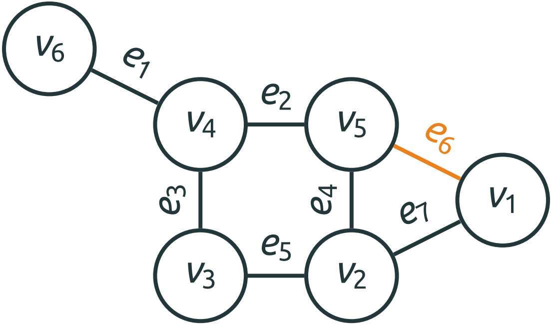

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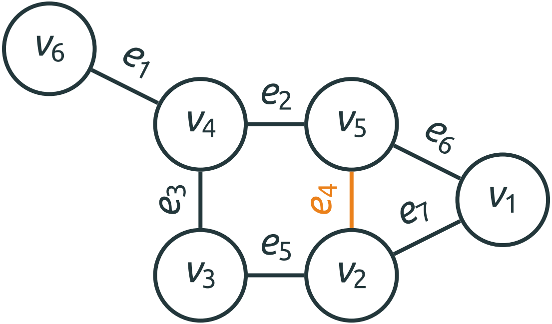

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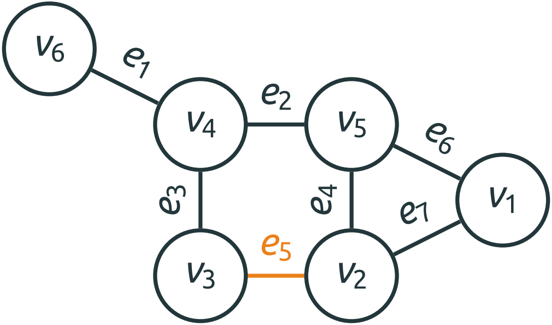

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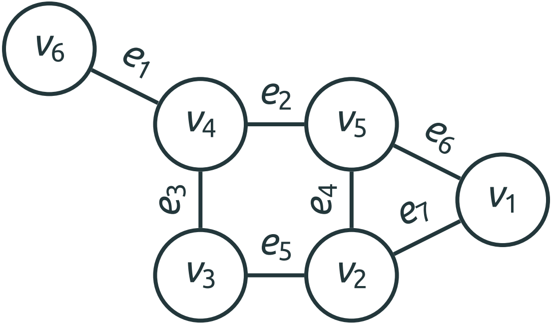
Not a simple path: visits v_2 twice



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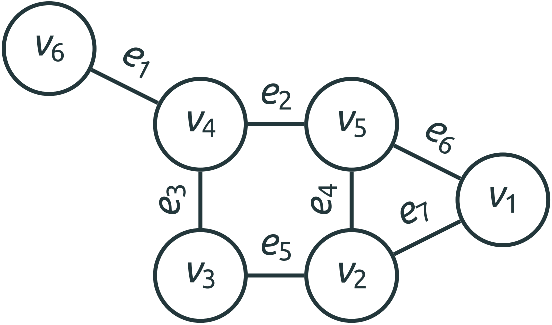

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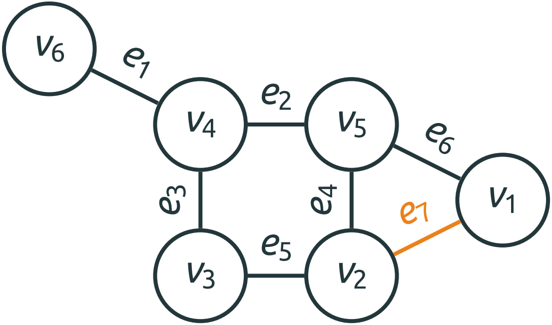

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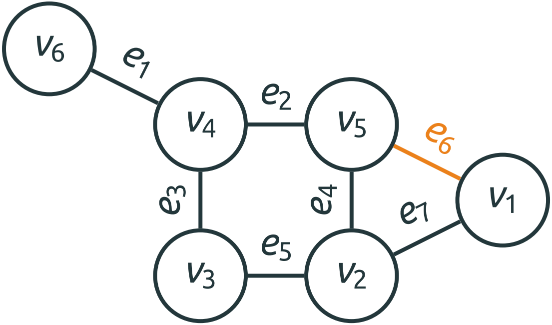

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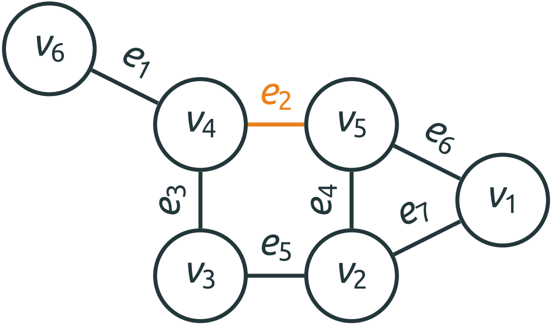

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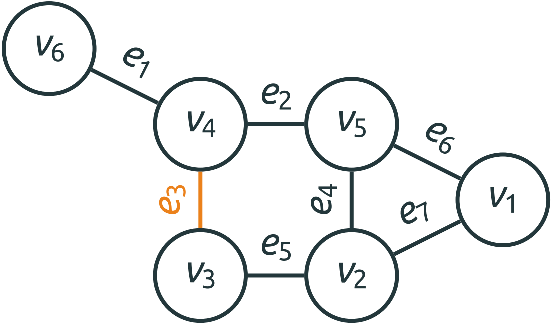
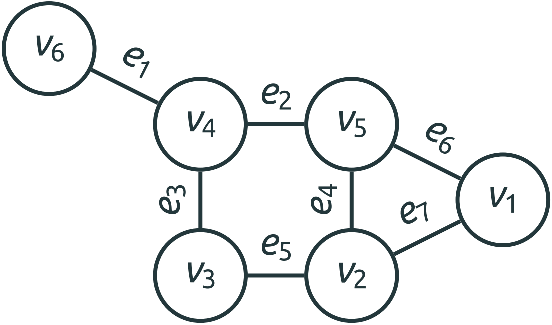


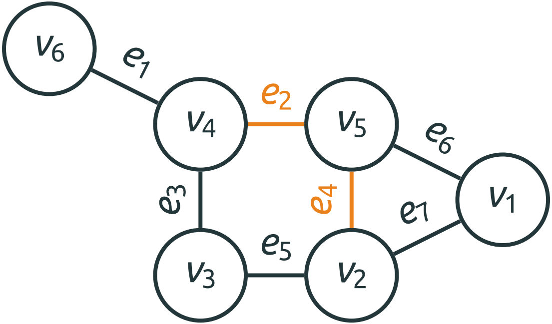
Walks: Examples

It is sometimes convenient to specify a path (walk) by a list of its vertices



Walks: Examples

(v_4, v_5, v_2) is a path of length 2



Cycles

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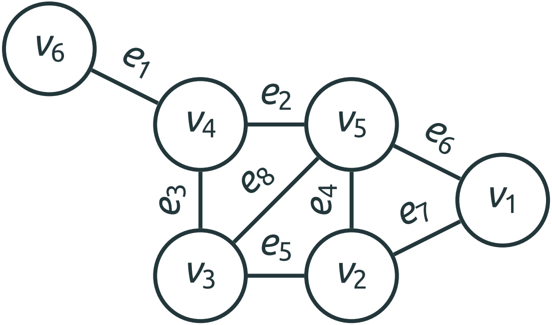
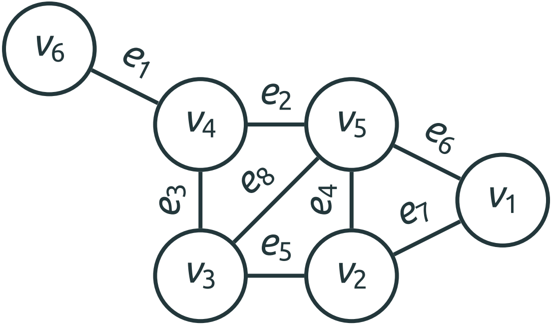
- In particular, all the edges in a Cycle are distinct
- A Simple Cycle is a cycle where all vertices except for the first one are distinct. (And there first vertex is taken twice)

Cycles: Examples



Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$



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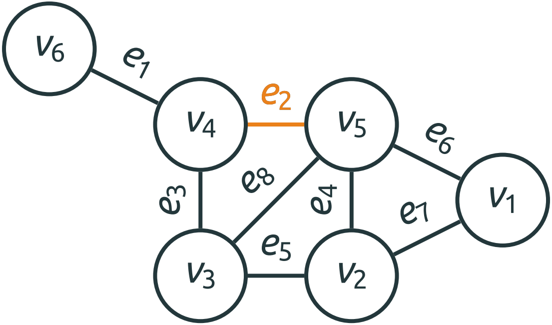

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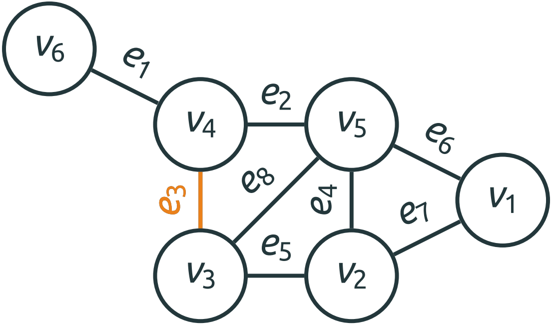

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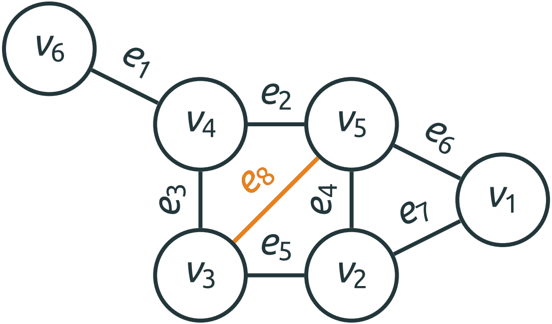

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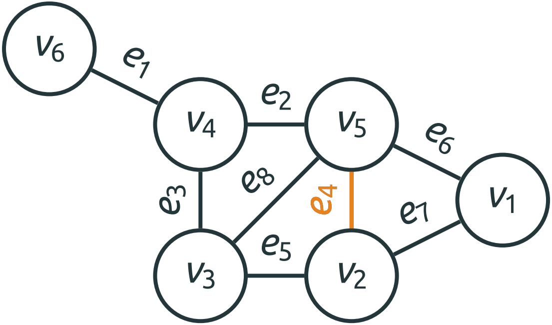

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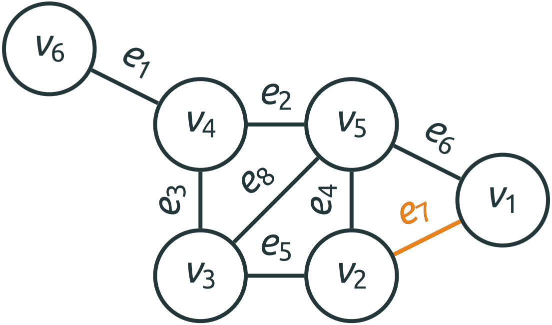

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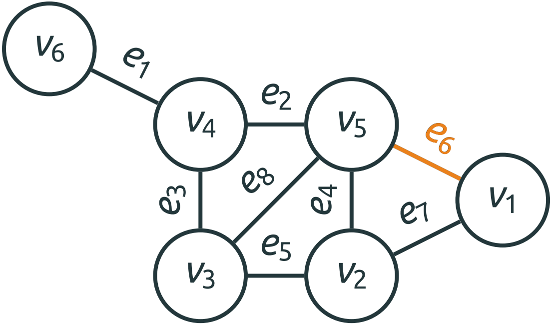
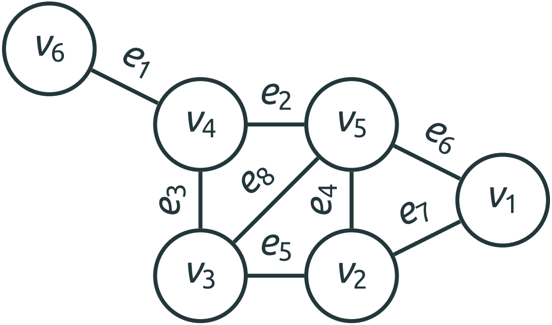


Cycles: Examples

A cycle of length 6: $(e_2, e_3, e_8, e_4, e_7, e_6)$

Not a simple cycle: visits v_5 three times



Cycles: Examples

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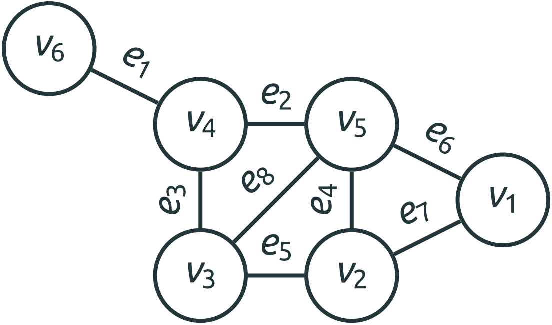

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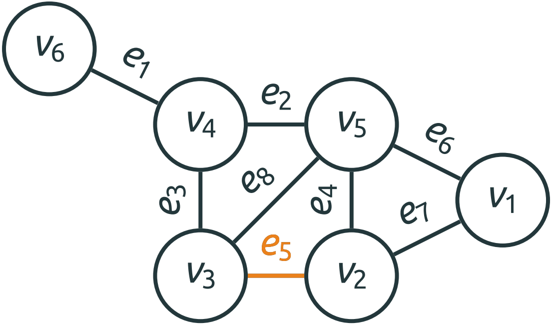

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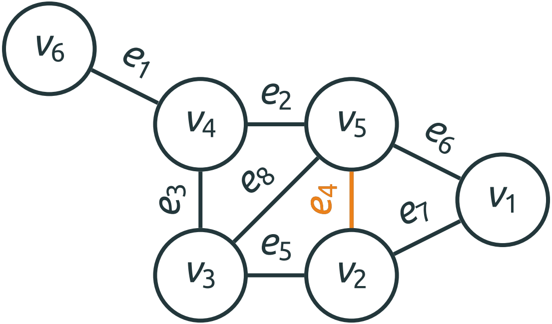

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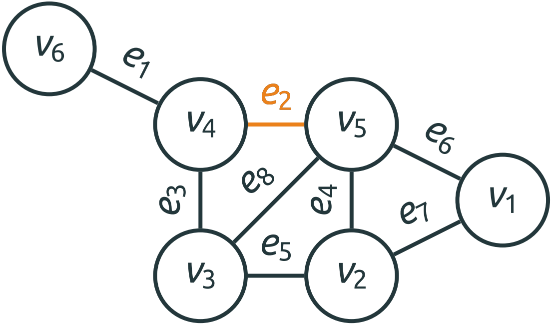

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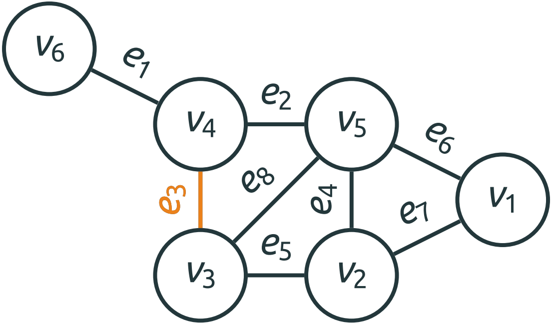
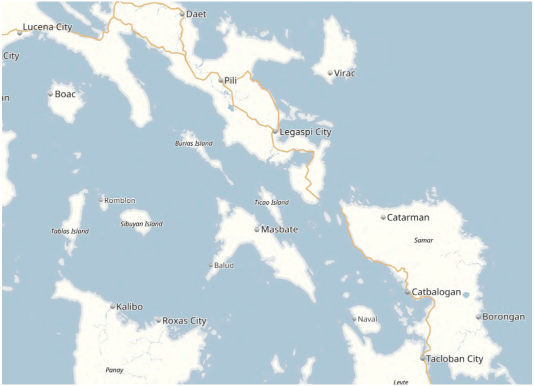


Outline

Connected Components

The number of islands



Connectivity

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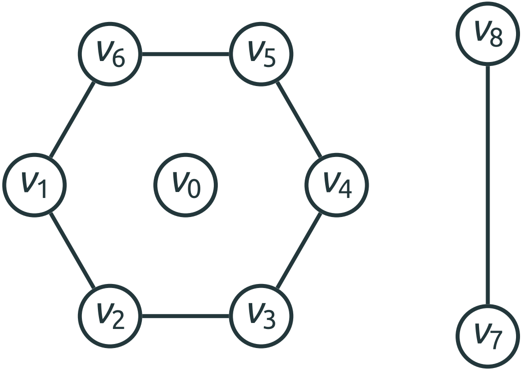
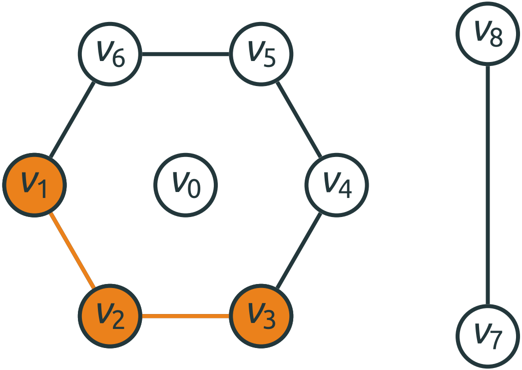
- I.e., a connected subgraph of G which is not contained in a larger connected subgraph of G

Connected Components: Examples



Connected Components: Examples

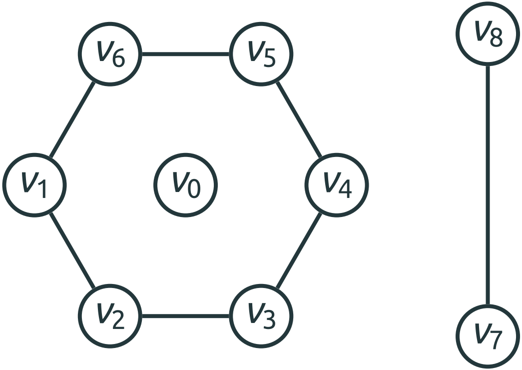
v_1, v_2, v_3 form a connected subgraph



Connected Components: Examples

v_1, v_2, v_3 form a connected subgraph

But not a connected component



Connected Components: Examples

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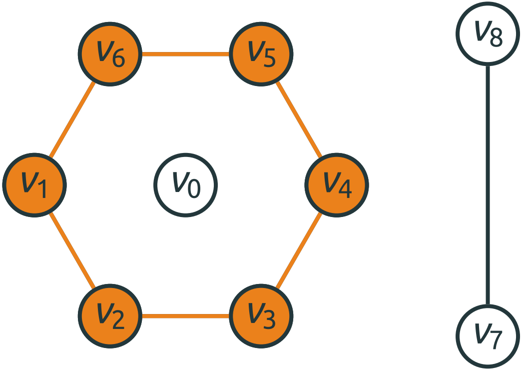
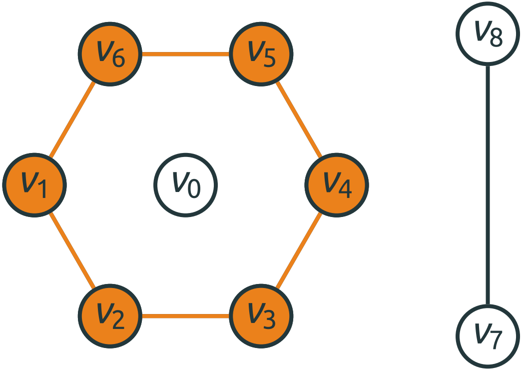


Connected Components: Examples

v_1, v_2, v_3 form a connected subgraph

But not a connected component

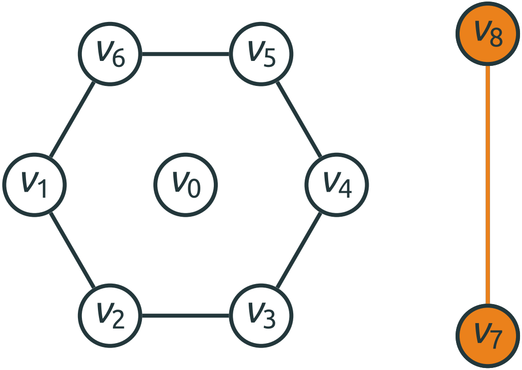
$v_1, v_2, v_3, v_4, v_5, v_6$ form a Connected Component



Connected Components: Examples

v_7, v_8 form a Connected Component

$v_1, v_2, v_3, v_4, v_5, v_6$ form a Connected Component

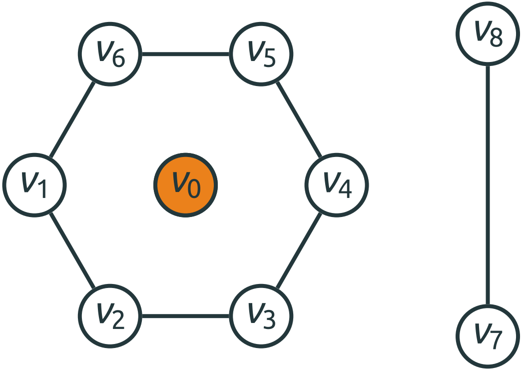


Connected Components: Examples

v_0 forms a Connected Component

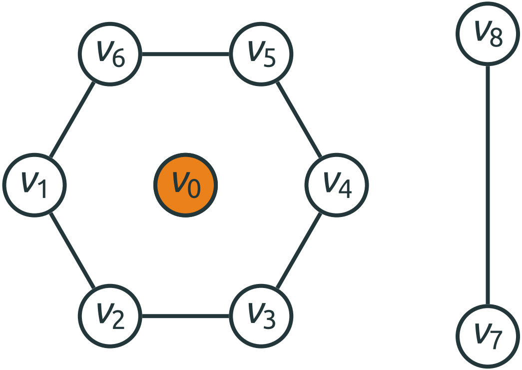
v_7, v_8 form a Connected Component

$v_1, v_2, v_3, v_4, v_5, v_6$ form a Connected Component



Connected Components: Examples

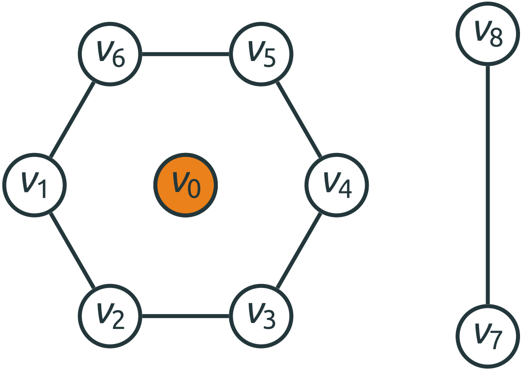
Each isolated vertex forms a Connected Component



Connected Components: Examples



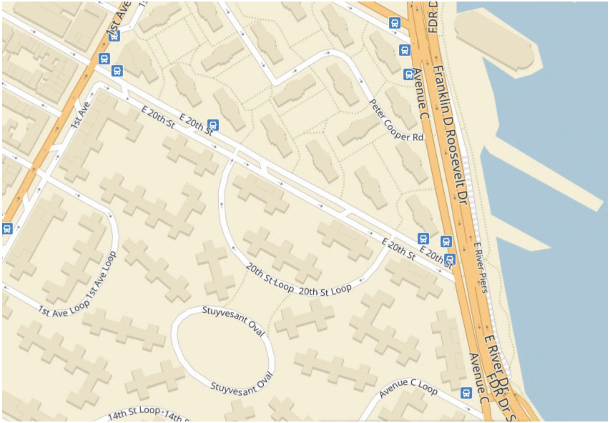

Each isolated vertex forms a Connected Component



Outline

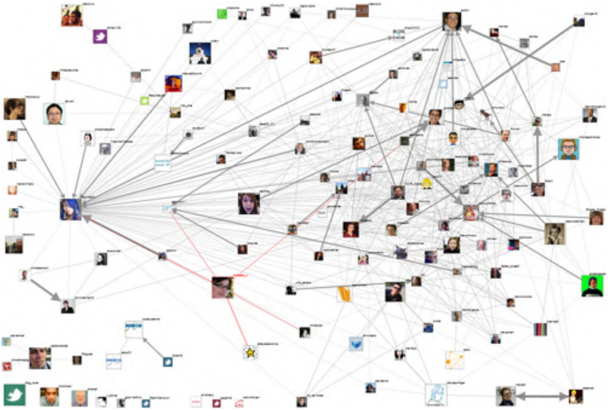
Directed Graphs

One-way Streets



Directed Graphs

Followers



Undirected Edge (Edge)

Edge $\{u, v\}$

Directed Edge (Arc)

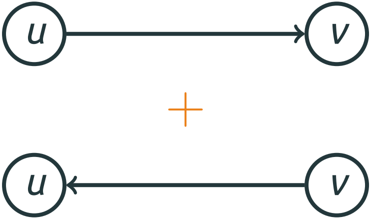
Arc (u, v)

Directed Edge (Arc)

Arc (u, v)

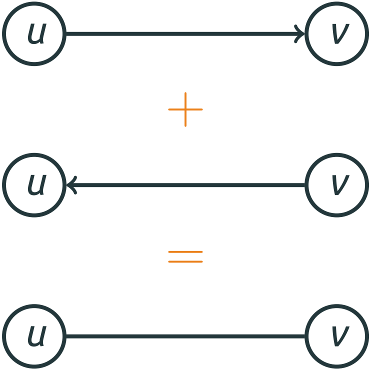
Directed Edge (Arc)

Arc (u, v)



Directed Edge (Arc)

Arc (u, v)



The Degree of a Vertex

- The Indegree of a vertex v is the number of edges ending at v

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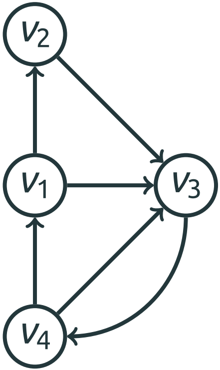
- The Indegree of a vertex *v* is the number of edges ending at *v*

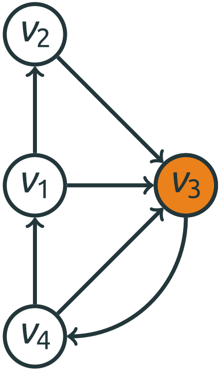
- The Outdegree of a vertex *v* is the number of edges leaving *v*

The Degree of a Vertex: Examples



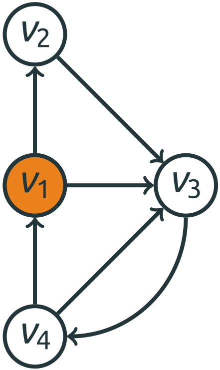
The Degree of a Vertex: Examples

The Indegree of v_3 is 3,
the Outdegree of v_3 is 1

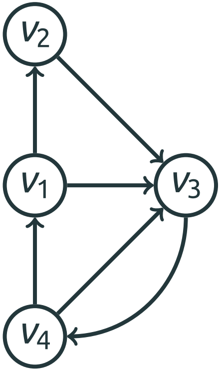


The Degree of a Vertex: Examples

The Indegree of v_1 is 1,
the Outdegree of v_1 is 2

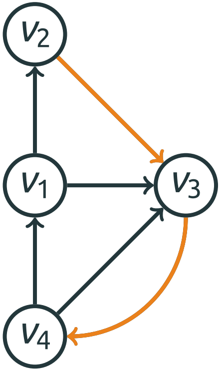


Directed Paths



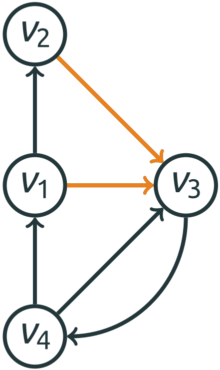
Directed Paths

(v_2, v_3, v_4) is a
Path of length 2



Directed Paths

(v_1, v_3, v_2) is not a Path



Outline

The Degree of a Vertex

Paths

Connectivity

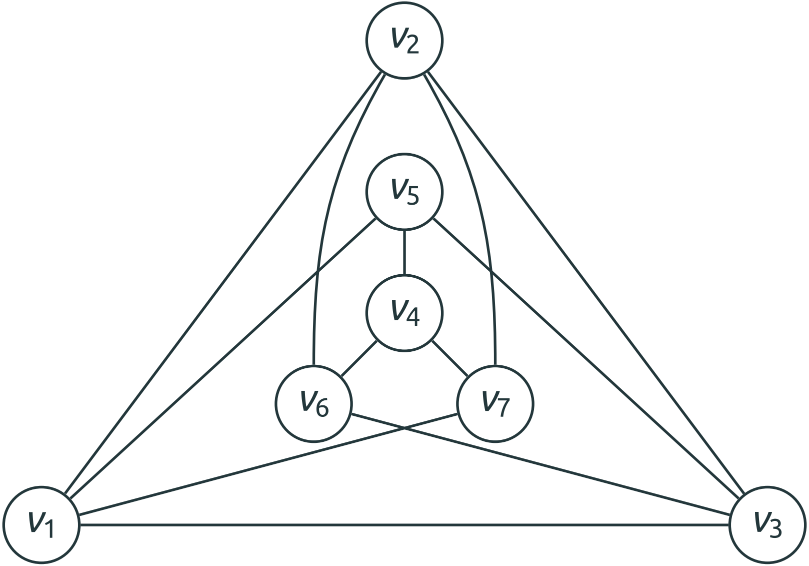
Directed Graphs

Weighted Graphs

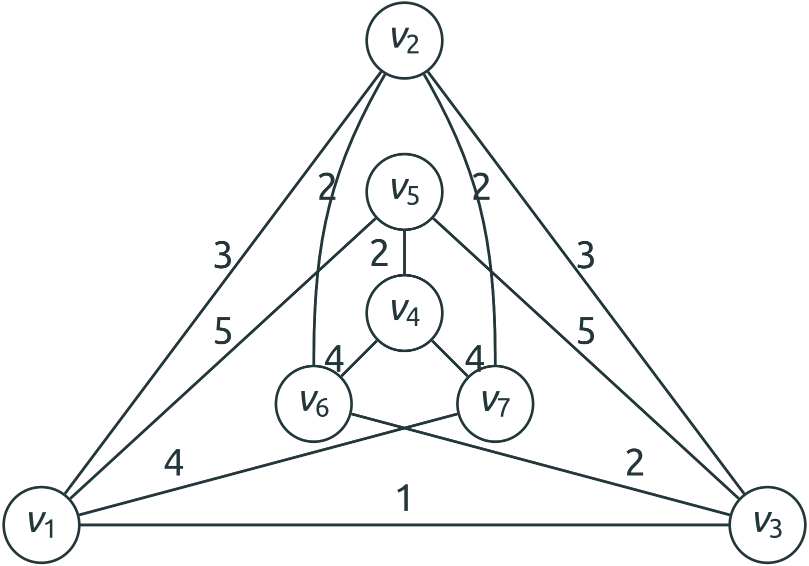
Weighted Graphs

Distance, Driving Time, etc.

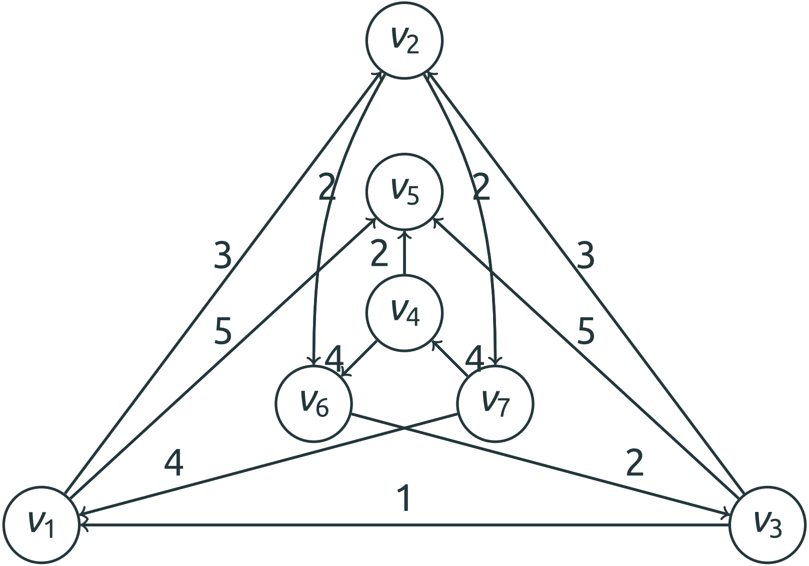
Weighted Graphs: Examples



Weighted Graphs: Examples



Weighted Graphs: Examples



Weighted Paths

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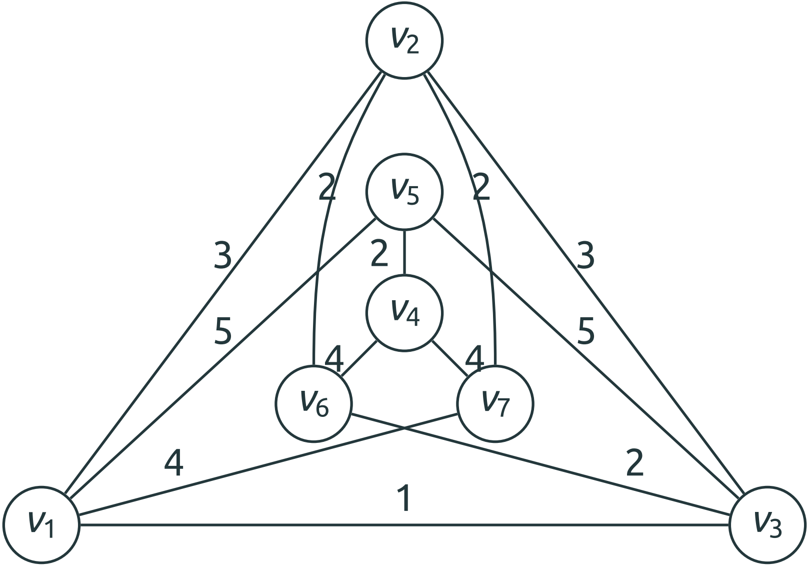
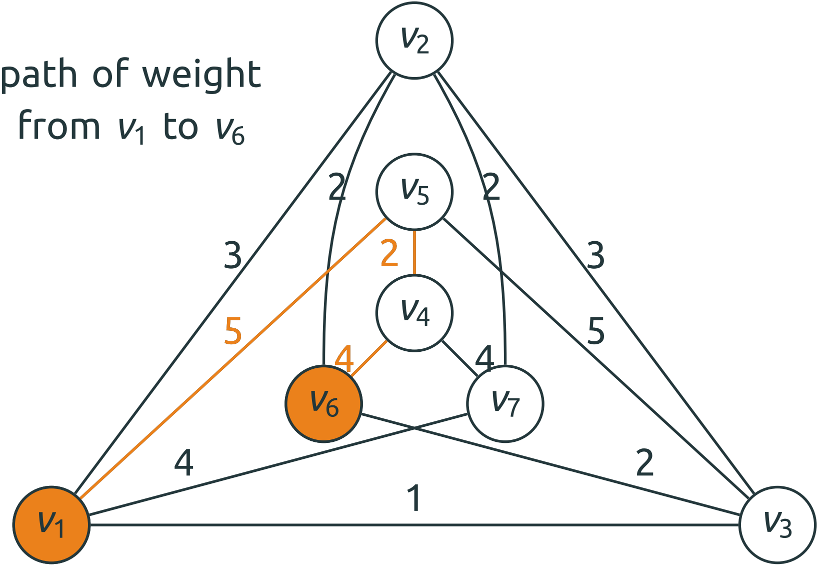
- The Weight of a path is the sum of the weights of its edges
- A Shortest Path between two vertices is a path of the minimum weight
- The Distance between two vertices is the length of a shortest path between them

Weighted Paths: Examples



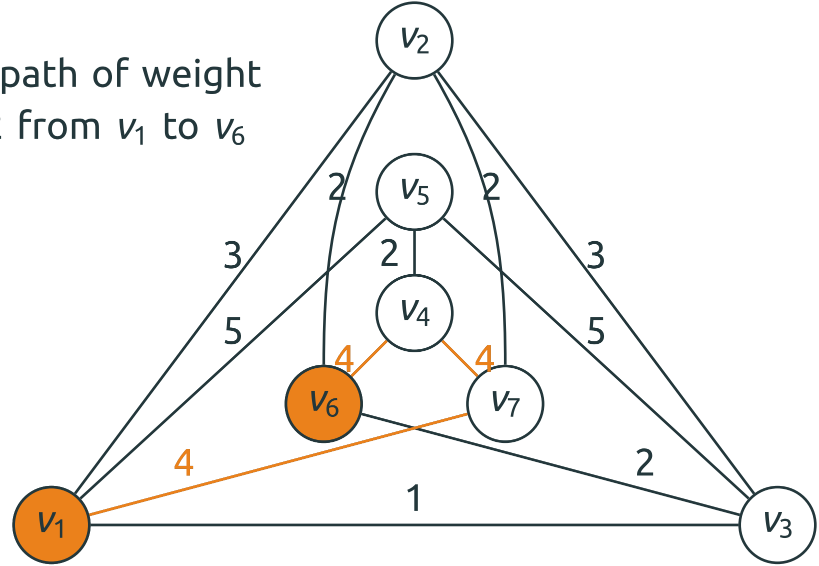
Weighted Paths: Examples

A path of weight 11 from v_1 to v_6



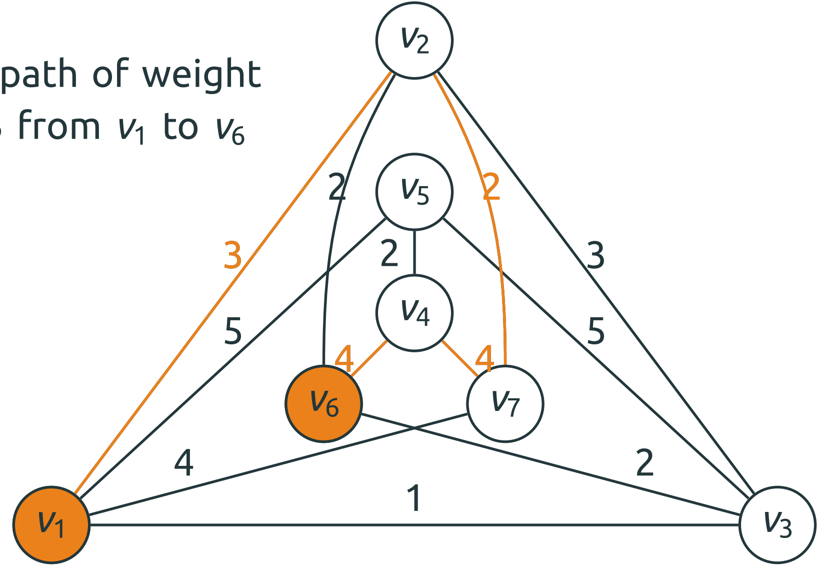
Weighted Paths: Examples

A path of weight 12 from v_1 to v_6



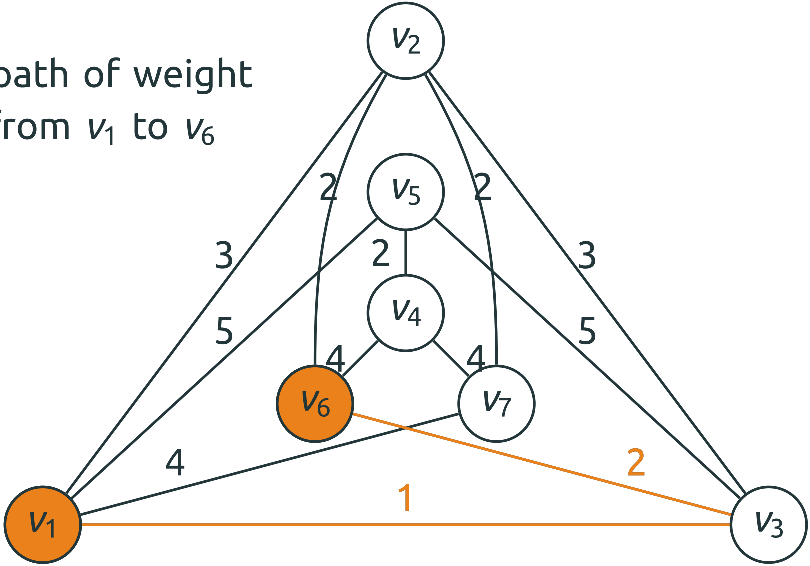
Weighted Paths: Examples

A path of weight 13 from v_1 to v_6



Weighted Paths: Examples

A path of weight 3 from v_1 to v_6



Weighted Paths: Examples

The distance between v_1 and v_6 is 3