

Introduction to Graph Theory

Why Graphs?

Alexander Golovnev

Outline

What is a Graph?

Graph Examples

Graph Applications

Graphs

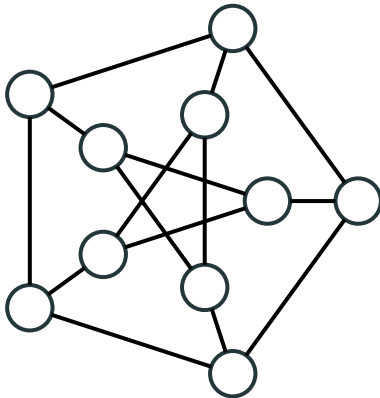
- A set of **Objects**

Graphs

- A set of **Objects**
- **Relations** between pairs of objects

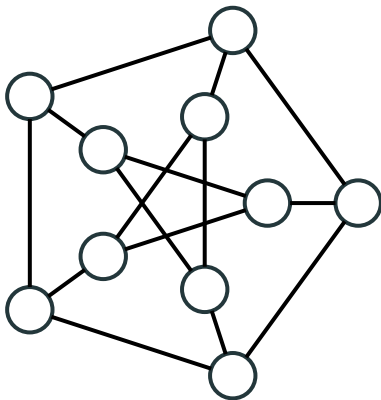
Graphs

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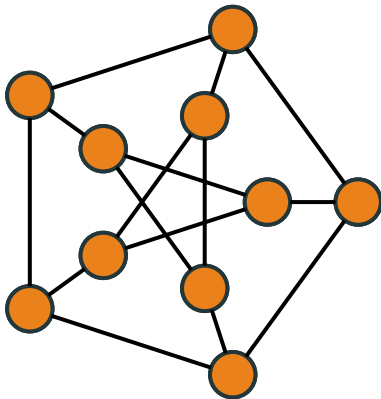
Definition

- A **Graph** $G = (V, E)$



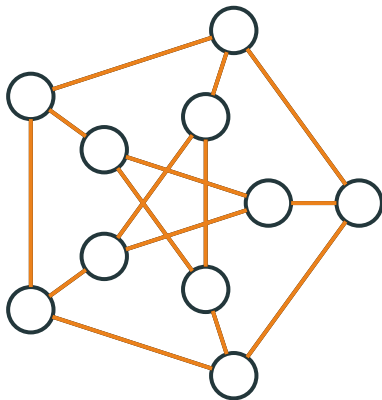
Definition

- A **Graph** $G = (V, E)$
- A set V of **Vertices/Nodes**



Definition

- A **Graph** $G = (V, E)$
- A set V of **Vertices/Nodes**
- A set E of **Edges**



Vocabulary



Vocabulary



- We can name individual vertices and edges

Vocabulary



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Vocabulary



- We can name individual vertices and edges
- e Connects u and v

Vocabulary



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- u and v are **End Points** of e

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- u and e are **Incident**

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Vocabulary



- We can name individual vertices and edges
- e **Connects** u and v
- u and v are **End Points** of e
- u and e are **Incident**
- u and v are **Adjacent**
- u and v are **Neighbors**

Drawing a Graph

Objects: $\{A, B, C, D\}$

Relations: $\{\{A, C\}, \{D, A\}, \{B, D\}, \{C, B\}\}$

Drawing a Graph

Objects: {A,B,C,D}

Relations: {{A,C},{D,A},{B,D},{C,B}}



Drawing a Graph

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ⓑ

Ⓐ

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ⓑ

ⓒ

Ⓐ

Drawing a Graph

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(B)

(C)

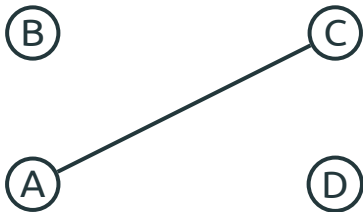
(A)

(D)

Drawing a Graph

Objects: {A,B,C,D}

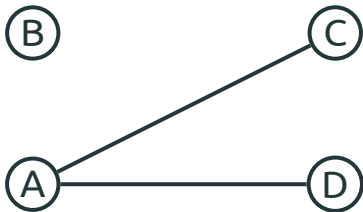
Relations: {{A,C},{D,A},{B,D},{C,B}}



Drawing a Graph

Objects: {A,B,C,D}

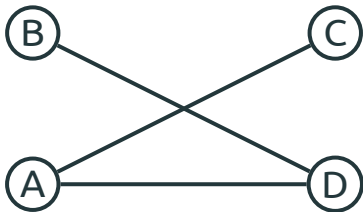
Relations: {{A,C},{D,A},{B,D},{C,B}}



Drawing a Graph

Objects: $\{A, B, C, D\}$

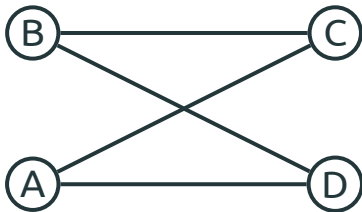
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Drawing a Graph

Objects: $\{A, B, C, D\}$

Relations: $\{\{C, A\}, \{D, A\}, \{B, D\}, \{C, B\}\}$



Directed Graph



- It is often convenient to consider **Directed Edges (Arcs)**

Directed Graph



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Directed Graph



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Directed Graph



- It is often convenient to consider **Directed Edges (Arcs)**
- They describe **asymmetric** relations
- There is a flight from A to B, but not the other way around
- Such a graph is called **Directed**

Drawing a Directed Graph

Objects: $\{A, B, C, D\}$

Relations: $\{(A, C), (D, A), (B, D), (C, B)\}$



Drawing a Directed Graph

Objects: $\{A, B, C, D\}$

Relations: $\{(A, C), (D, A), (B, D), (C, B)\}$



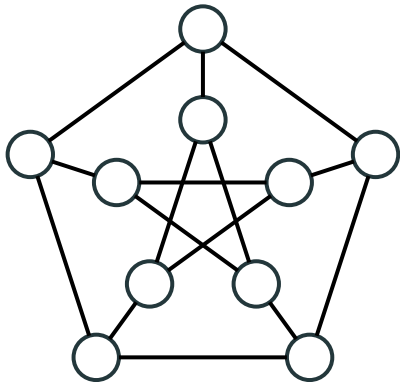
Drawing a Directed Graph

Objects: {A,B,C,D}

Relations: {(C,A),(D,A),(B,D),(C,B)}

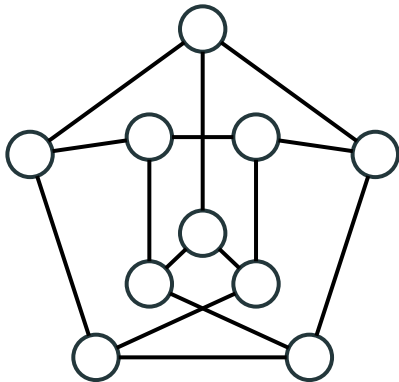
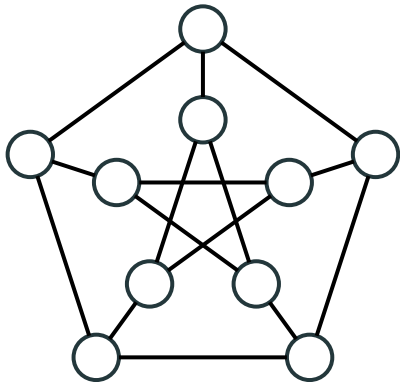


Many Ways to Draw a Graph



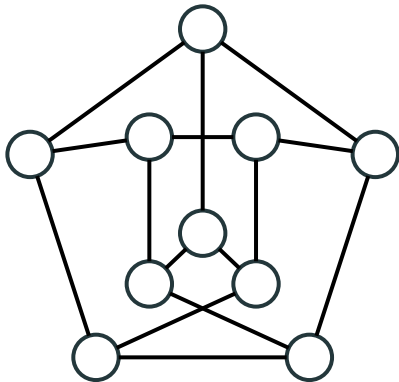
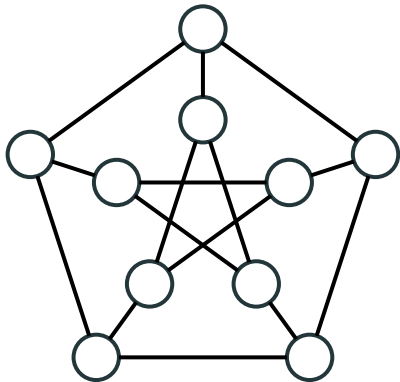
Many Ways to Draw a Graph

Are these graphs the same?



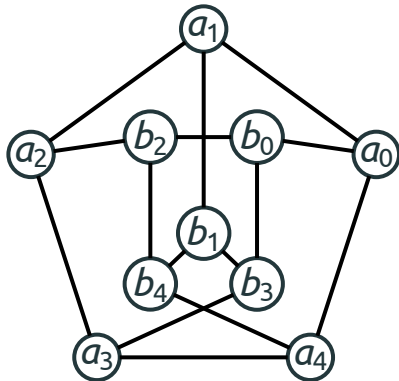
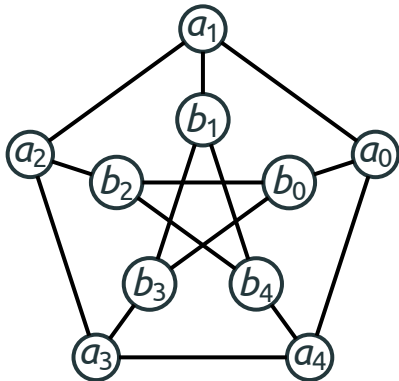
Many Ways to Draw a Graph

10 vertices and 15 edges?



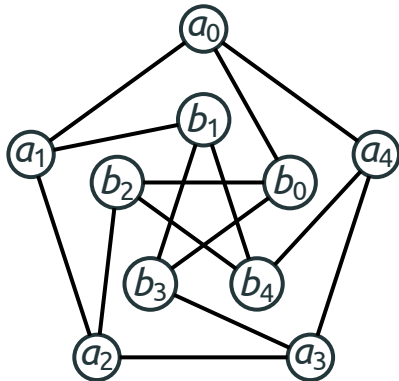
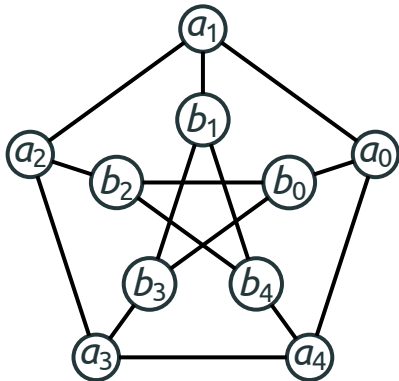
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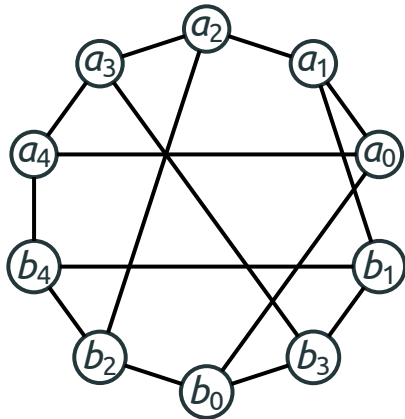
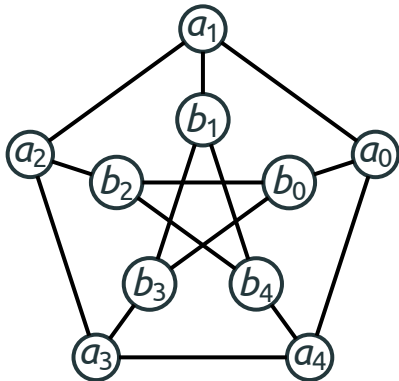
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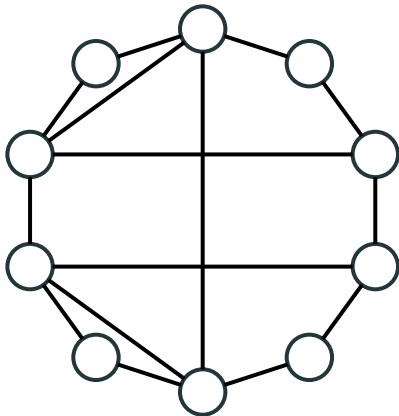
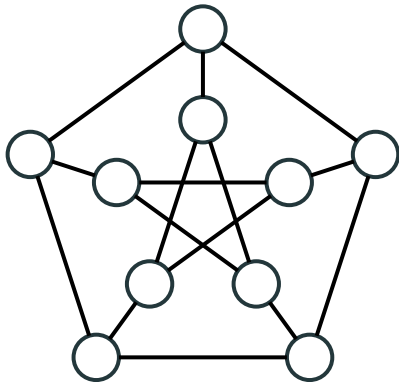


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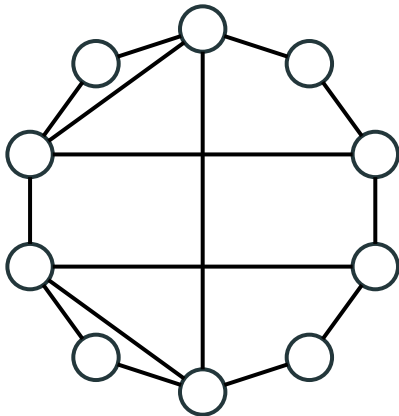
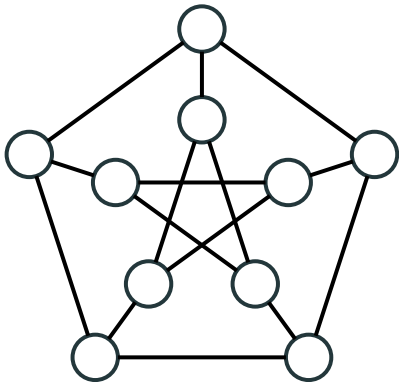


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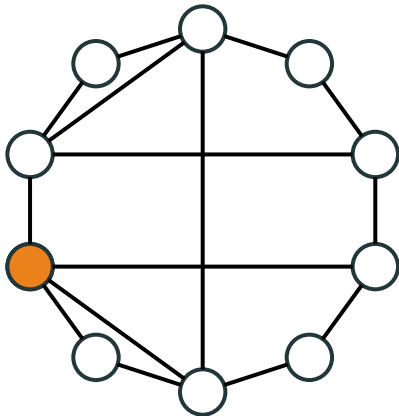
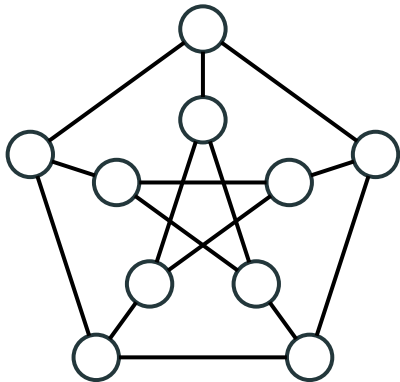
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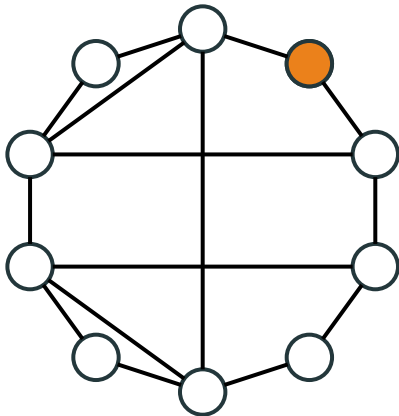
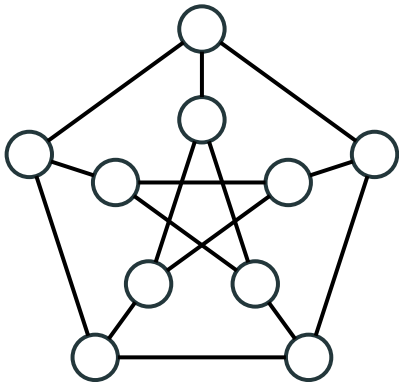
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Are These Graphs the Same?

10 vertices and 15 edges?



Graph Drawing is Beautiful!



Donald E. Knuth

Graph drawing is the best possible field I can think of: It merges aesthetics, mathematical beauty and wonderful algorithms.

It therefore provides a harmonic balance between the left and right brain parts.

Outline

What is a Graph?

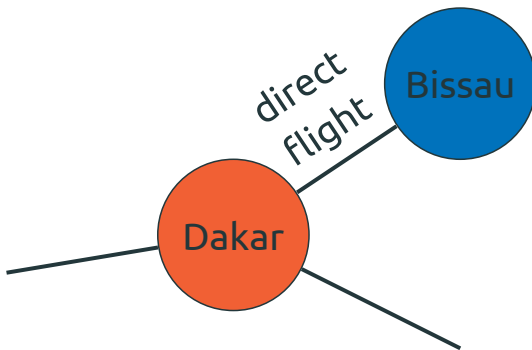
Graph Examples

Graph Applications

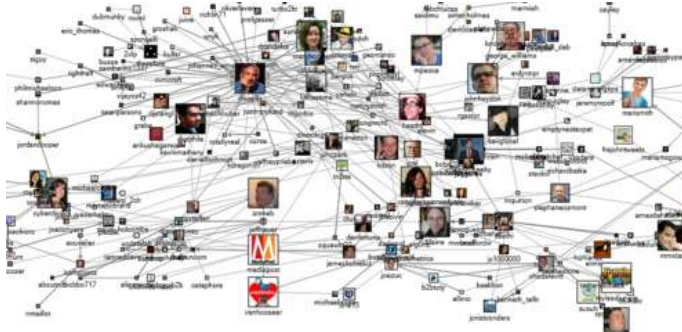
Airlines Graph



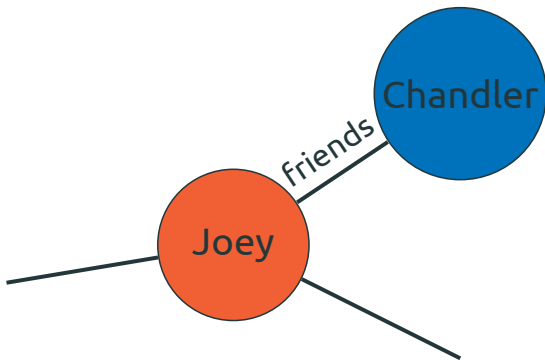
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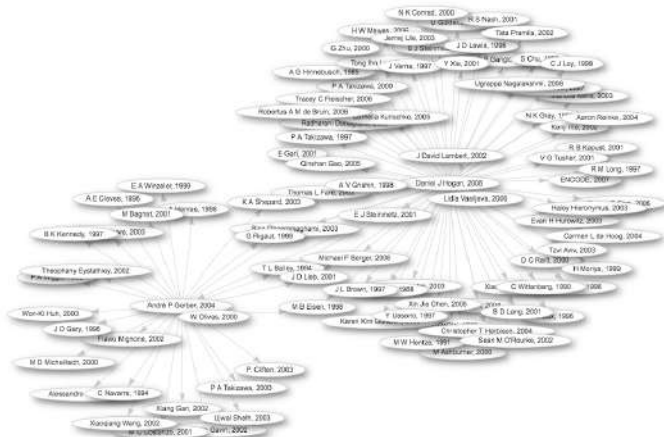
Facebook Graph



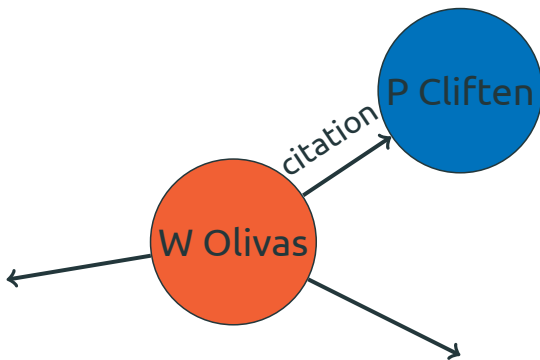
Facebook Graph



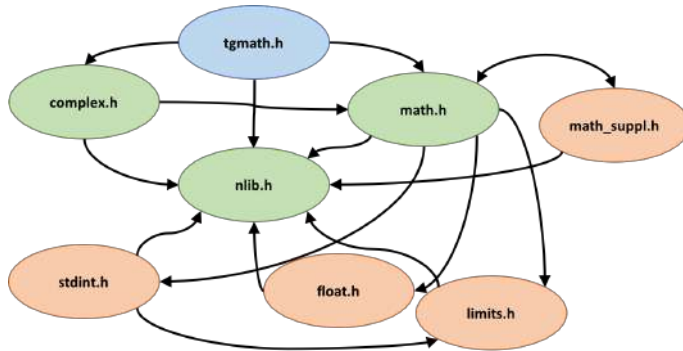
Citation Graph For a Paper



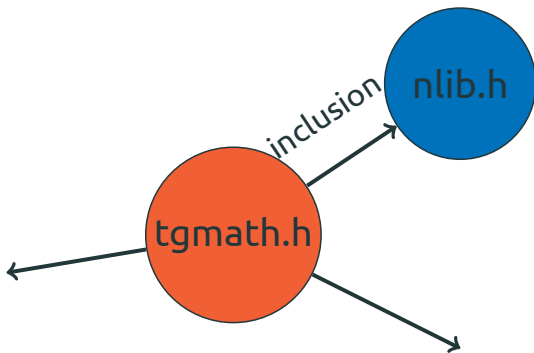
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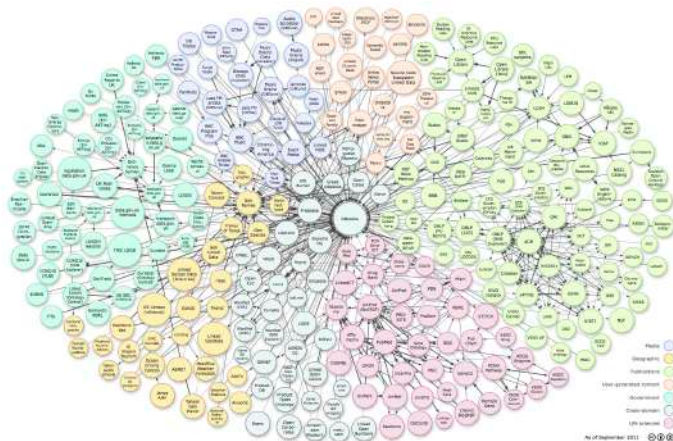
Dependency Graph



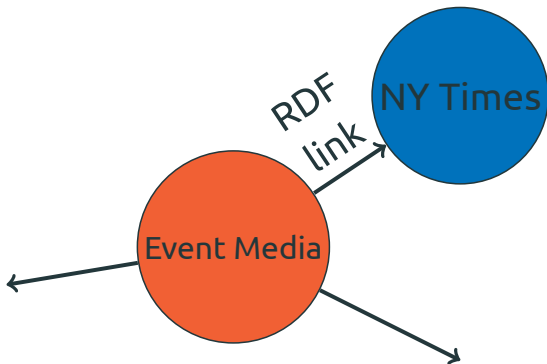
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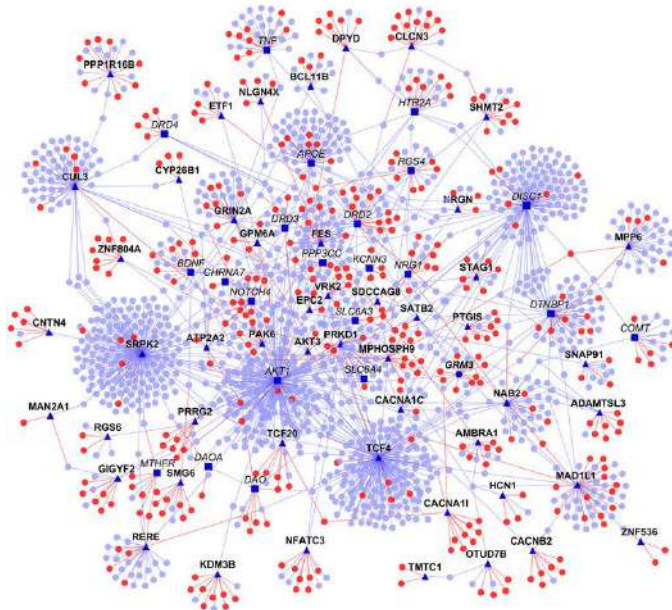
Linked Open Data Diagram



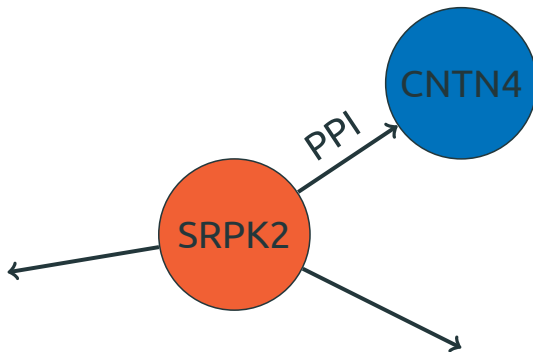
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Schizophrenia Protein-Protein Interaction



Schizophrenia Protein-Protein Interaction (PPI)



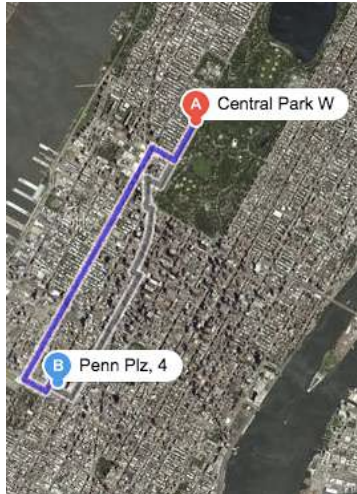
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Graph Examples

Graph Applications

Navigation



Navigation



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graph¹


/ɡrɑːf/ 

noun

- a diagram showing the relation between variable quantities, typically of two variables, each measured along one of a pair of axes at right angles.
synonyms: [chart](#), [diagram](#), [More](#)

verb

- plot or trace on a graph.
synonyms: [plot](#), [trace](#), [draw up](#), [delineate](#)
"we graphed the new prices"

 Translations, word origin, and more definitions

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Graph - Wikipedia

<https://en.wikipedia.org/wiki/Graph> ▼

Graph (topology), a topological space resembling a graph in the sense of discrete mathematics. Graph of a function. Chart, a means of representing data (also called a graph).

Graph of a function - Wikipedia

https://en.wikipedia.org/wiki/Graph_of_a_function ▼

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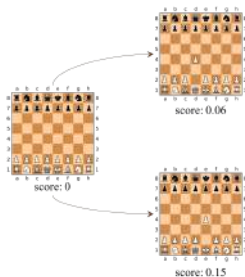
Game Strategies



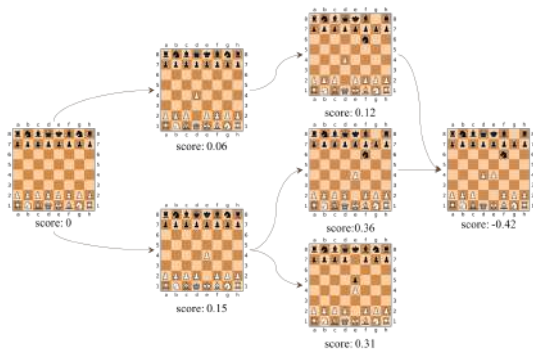
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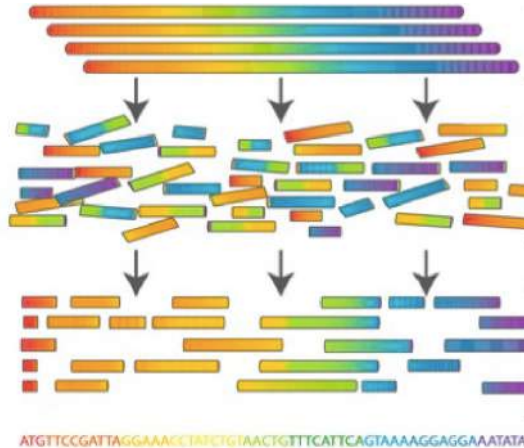
Game Strategies



Game Strategies



Genome Assembly



GSM



GSM



GSM

4 Frequency
Ranges

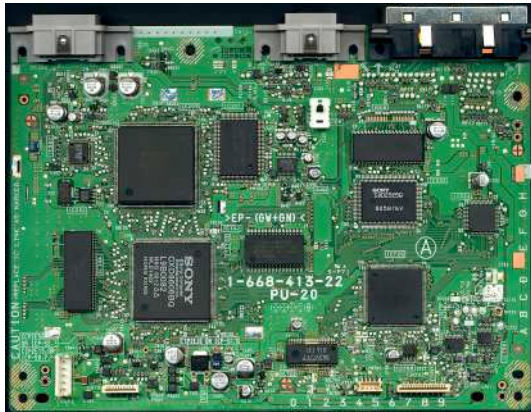


GSM

4 Frequency
Ranges



Computer Chips



Computer Chips

