# Physics 155 HW # 3

1.) The logarithmic derivative of a function $f(x)$ is given by:

$$\frac{\frac{df(x)}{dx}}{f(x)} = \frac{d}{dx} \ln f(x) \tag{1}$$

a.) Compute the logarithmic derivative of $ax^n$ and $a(x-c)^n$.

b.) Prove that the logarithmic derivative of a product of two functions $W(x) = U(x)V(x)$ is the sum of the logarithmic derivatives of $U$ and $V$.

c.) Find the logarithmic derivative of

$$(x-a_1)^{n_1}(x-a_2)^{n_2}(x-a_3)^{n_3}...(x-a_p)^{n_p}$$

for integers $n_1$, $n_2$,..., $n_p$.

d.) Find a function whose logarithmic derivative is 0, 1, or $2x$.

2.) Use the appropriate trig substitution to integrate (*i.e.* compute the antiderivative)

$$\int dx \sqrt{\frac{1+x}{1-x}} \tag{2}$$

*Hint:* manipulate the equation to get a $\sqrt{1-x^2}$ in the denominator before using a trig substitution.

3.) Use induction to show that

$$\frac{d^n}{dx^n}(fg) = \frac{d^n f}{dx^n} g + \binom{n}{1} \frac{d^{n-1} f}{dx^{n-1}} \frac{dg}{dx} + \binom{n}{2} \frac{d^{n-2} f}{dx^{n-2}} \frac{d^2 g}{dx^2} + \ldots + \binom{n}{n-1} \frac{df}{dx} \frac{d^{n-1} g}{dx^{n-1}} + f \frac{d^n g}{dx^n} \qquad (3)$$

where $\binom{n}{m} = \dfrac{n!}{m!(n-m)!}$ is the binomial coefficient.

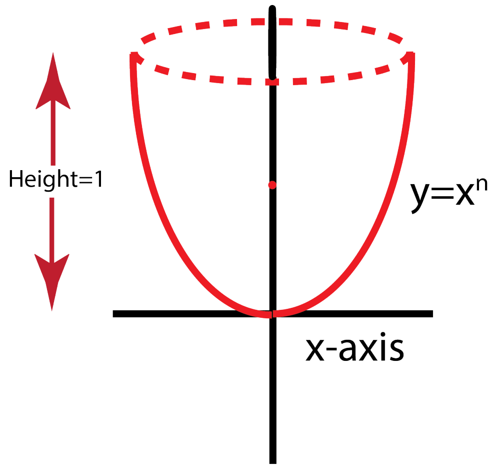
4.) Evaluate the following integrals analytically:

a.) $$\int_0^a \frac{dx}{x^2 + a^2} \qquad (4)$$

b.) $$\int_{-1}^1 dx \sqrt{e^x} \qquad (5)$$

c.) $$\int_0^1 dx \cos^{-1}\left(\sqrt{1 - x^2}\right) \qquad (6)$$

5.) Evaluate the surface area and the volume of the surface of revolution given by a function $y = x^n$ with a height $h = 1$.




For the integrals, work them out to a one-dimensional integral. You won't be able to integrate the general case, so do only $n = 1$ and $n = 2$, which can be integrated. Also determine these values numerically (give a decimal answer).