Physics 155 HW # 2

1.) Using the methods developed in class, find the general formula for

$$\sum_{j=1}^{N} j^3 \tag{1}$$

2.) Evaluate $\int_0^\infty dx\, e^{-ax}$ for a real number $a > 0$. Use the following procedure:

- a.) expand e^{-ax} in a MacLaurin series in x
- b.) interchange the limit of integration and summing the series
- c.) integrate each term in the series, but do not evaluate at the two limits yet
- d.) sum the series to express the antiderivative as a function of e^{-ax}
- e.) evaluate at the limits to get the final answer

Note that step b.) is only valid for an absolutely convergent series.

3.) a.) Repeat the above procedure for $\int_0^1 \sin(ax)\, dx$ for $a > 0$.

b.) Find a mathematical argument for why the above integral in a.) is less than or equal to $\frac{a}{2}$ by considering an appropiate bound for $\sin(ax)$.

4.) In lecture we stated, without proof, that

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \tag{2}$$

Provide a proof for this result and numerically evaluate for a range of different n values. How large does n need to be to get 4 digits of accuracy for e?

5.) Describe why a logarithm table to base 10 is more convenient to calculate with than a table to base e.