Physics 155 HW # 9

1.) a.) Find all solutions to

$$\begin{array}{rrrrl} x_1 & -2x_2 & +3x_3 & -x_4 & = 0 \\ -x_1 & & +2x_3 & +x_4 & = 0 \\ 2x_1 & +x_2 & & -2x_4 & = 0 \end{array} \tag{1}$$

b.)

$$\begin{array}{rrrl} x_1 & +x_2 & -x_3 & = 1 \\ 2x_1 & +x_2 & +3x_3 & = 2 \\ & x_2 & -5x_3 & = 1 \end{array} \tag{2}$$

c.)

$$\begin{array}{rrrrrl} x_1 & -2x_2 & +x_3 & -x_4 & +2x_5 & = -7 \\ & x_2 & +x_3 & +2x_4 & -x_5 & = 5 \\ x_1 & -x_2 & +2x_3 & +2x_4 & +2x_5 & = -1 \end{array} \tag{3}$$

For part (c), also find the solution of the homogeneous system and use it to find the general solution by adding the homogeneous solution to the particular solution of the non-homogeneous system.

2.) Find the determinants

a.)

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \qquad (4)$$

b.)

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \qquad (5)$$

c.)

$$\begin{vmatrix} 1 & 3 & -1 & 2 \\ 2 & 1 & 3 & 1 \\ -1 & 2 & -1 & 3 \\ -2 & 1 & 2 & -3 \end{vmatrix} \qquad (6)$$

Use either the formula for the determinant, the expansion in cofactors, or the row reduction technique to solve each case.

d.)

$$\begin{vmatrix} 1 & 2 & -1 & 3 & -2 \\ 2 & 0 & 4 & -5 & 1 \\ -3 & 1 & 6 & 0 & -7 \\ 0 & 3 & 1 & -5 & 2 \\ -2 & 6 & 3 & -1 & 2 \end{vmatrix} \qquad (7)$$

3.) Determine if the following matrices are non-singular. If so, find their inverse.

a.)

$$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \tag{8}$$

b.)

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{9}$$

c.)

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \tag{10}$$

d.)

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \tag{11}$$

e.)

$$\begin{pmatrix} 1 & 3 & -1 & 2 \\ 2 & 1 & 3 & 1 \\ -1 & 2 & -4 & 1 \\ -2 & 1 & 2 & -3 \end{pmatrix} \tag{12}$$

4.) Calculate the determinant.

$$P(x) = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{pmatrix} \tag{13}$$

and plot the determinant as a function of x. You will find $P(x)$ vanishes at 4 points in x. Explain why it vanishes at these values.

5.) Calculate the inverse of H_3 in two ways

$$H_3 = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix} \tag{14}$$

One via row reduction and one via the cofactor formula.