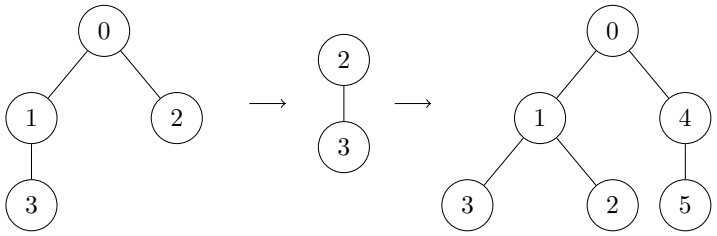
TD - Fil de priorité et Tas

I Un tas de question

1. 

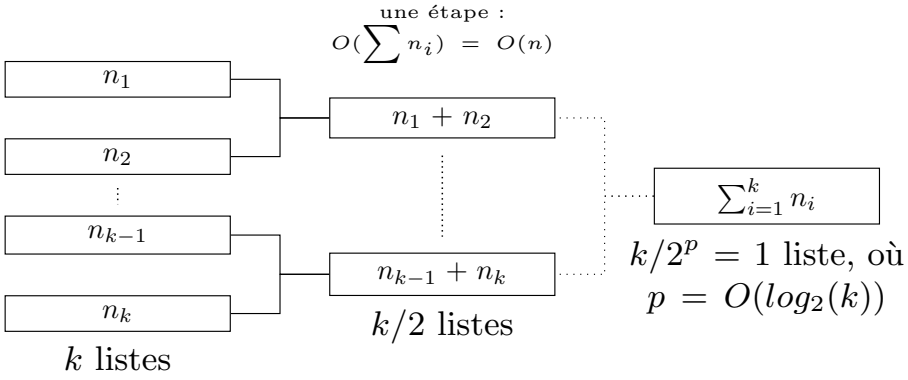
2.

3.
```
let rec is_tas_max heap =
    test = ref true in
    for i = 0 to heap.n - 1 do
        if a.(i) > a.((i-1)/2) then
        test := false
    done;
    !test ;;
```

4.
```
type 'a heap = {a : 'a array ; mutable n : int}
type 'a tree = E | N of 'a * 'a tree * 'a tree

let heap_to_tree heap=
    let rec aux i = (*renvoie le sous-arbre enraciné en heap.a *)
    if i >= heap.n
        then E
        else N(heap.a.(i), aux(2*i + 1), aux(2 * (i + 1))) in
    aux 0 ;;
```

5.
```
let rec fusion l1, l2 = match l1, l2 with
    |[], _ -> l2
    |_, [] -> l1
    |e1::q1, e2::q2 -> if e1 < e2 then
                            e1::fusion q1 l2
                        else
                            e2::fusion l1 q2 ;;
```

La complexité est $O(n+m)$



d'où une complexité en $O(nlog(k))$

```
let rec etape ll = match ll with
    |l1::l2::q -> (fusion l1 l2)::etape q
```

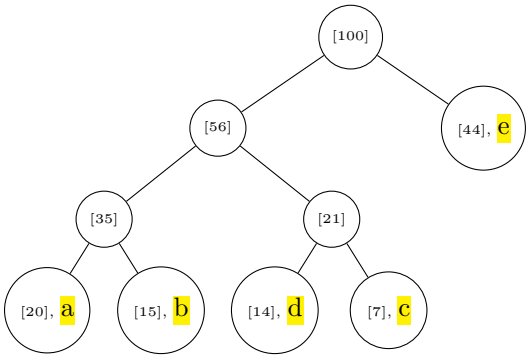
```
    |_ -> ll ;;

let rec kfusion ll = match ll with
    |[] -> []
    |[l] -> l
    |_ -> kfusion (etape ll) ;;
```

II Compression de Huffman

1.
```
let rec read arb lst =
match arb with
| F(chr) -> chr, lst
| N(g, d) -> match lst with
                |0::t -> read g t
                |1::t -> read d t
                |_ -> failwith "liste non valide"
```

2.
```
let rec decode arb lst =
    match (read arb lst) with
        |chr, [] -> chr::[]
        |chr, lst_end -> chr::(decode arb lst_end) ;;
```

3.



4.
```
let to_huffman freq =
    let fp = create () in
    let rec fill freq=
    match freq with
    |[] -> ()
    |(f, chr)::t -> add fp (f, F(chr)) ; fill t in
    fill freq;
    while len_over_2 fp do
    let (f1, a1), (f2, a2) = extract_min fp, extract_min fp in
        add fp (f1 + f2, N(a1, a2))
    done;
    snd (extract_min fp) ;;
```

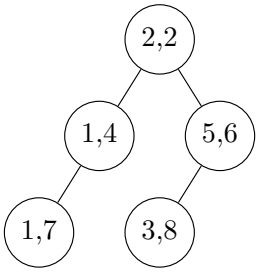
5.
```
let to_dict arb =
    let dict = Hashtbl.create 100 in
    let rec aux arb dict code =
        match arb with
        |F(chr) -> Hashtbl.add dict chr (List.rev code)
        |N(g, d) -> aux g dict (0::code) ; aux d dict (1::code)
    in
    aux arb dict [] ; dict ;;
```

6.
```
let rec code lst dict =
    match lst with
    |[] -> []
    |h::t -> (Hashtbl.find dict h) @ code t dict
```

III Arbretas

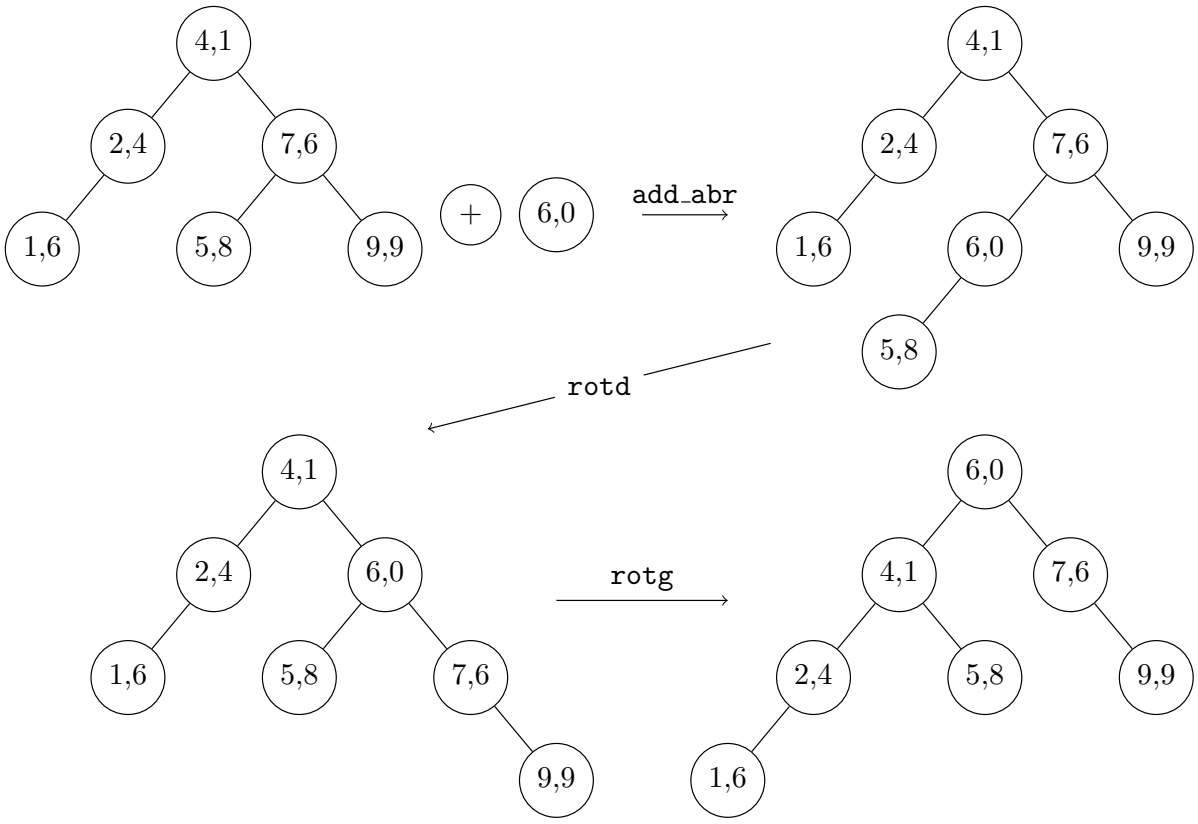
1.
```
let swap t i j =
    let tmp = t.(j) in
    t.(j) <- t.(i); t.(i) <- tmp
;;
```

2. La fonction `shuffle t` permutte tous les éléments du tableau `t` avec un autre éléments du tableau choisi au hasard dans les indices inférieurs. Après avoir terminé la boucle `for`, le tableau a donc subit une permutation aléatoires.

3.


4. Soit $n \in \mathbb{N}$, le nombre d'éléments de l'abretas, on note $e_1, ..., e_n \in \mathbb{N}^2$ ces éléments. Montrons par récurrence forte sur n l'unicité de l'arbretas.
On note $P(n)$ l'assertion "Un arbretas à n éléments distincts est unique"
Initialisation : Si $n = 1$, alors l'abretas est évidemment unique.
Hérédité : Soit $n \in \mathbb{N}$, supposons $P(n), P(n-1), ..., P(1)$, montrons $P(n+1)$: On choisi i tel que $sie_i = (x, p)$, alors p est la priorité minimum, alors par propriété de l'abretas, e_i est la racine, on peut alors creer les sous-arbres gauche et droite, de taille au plus n qui sont donc uniques par *hypothèse de récurrence*.
D'où l'unicité.

5.
```
let rotd treap = match treap with
|N(r, N(gr, gg, gd), d) -> N(gr, N(r, gd, d), gg)
|_ -> treap
```

6.


7.
```
let prio tree = match tree with
|V -> max_int
|N((_, p), _, _) -> p
;;
```

8.
```
let rec add treap e =
  let elem, _ = e in
  match treap with
  |V -> N(e, V, V)
  |N((x, p), g, d) -> if elem >= x then
                          let d_upt = add d e in
                              if (prio d_upt) < p then
                                rotg (N((x,p), g, d_upt))
                              else N((x,p), g, d_upt)
                      else
                          let g_upt = add g e in
                          if (prio g_upt) < p then
                              rotd (N((x,p), g_upt, d))
                            else N((x,p), g_upt, d)
                            ;;
```

9.
```
let rec del treap e = match treap with
|V -> V
|N((x,p), g, d) -> if e > x then
                     N((x,p), g, (del d e))
                   else if e < x then
                     N((x,p), (del g e), d)
                   else match g,d with
                     |V, V -> V
                     |V, f |f, V -> f
                     |_ -> if prio g < prio d then
                           let treap_rot = rotd(treap) in
                             del treap_rot e
                           else let treap_rot = rotg(treap) in
                             del treap_rot e
                        ;;
```