TIPE - LA COMPRESSION DE DONNÉES

Et son application aux images

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Introduction

• **Compression**: maximum d'informations avec une taille minimal

- Deux types : avec (lossy) ou sans perte (lossless).
- Dans le domaine urbain : beaucoup d'informations → compression ⇒ stocker et traiter efficacement

Procédés de Compression

Compression sans perte : entropique et algorithmique

• Une réorganisation des données

Application de transformées mathématiques

Implémentation de l'algorithme JPEG

Entropie et Codage Optimal (1)

Théorie de l'Information de Shannon

Théorie probabiliste quantifiant l'information d'un ensemble de messages.

Définition - Entropie

Pour une source X comportant n symboles, un symbole x_i ayant une probabilité $p_i=\mathbb{P}(X=x_i)$ d'apparaître, l'entropie H est définie par :

$$H(X) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

Entropie et Codage Optimal (2)

Définition - Code de Source

Un **code de source C** pour une variable aléatoire X de distribution de probabilité p, est une application de Ω (ensemble des symboles sources) vers A^* (où A est l'alphabet du code).

Définition - Code Uniquement Décodable

Un code est dit uniquement décodable si

$$\forall x, y \in \Omega^+, \ x \neq y \implies C^+(x) \neq C^+(y)$$

Entropie et Codage Optimal (3)

Définition - Code Préfixe

Un code est dit préfixe si aucun mot de code n'est le préfixe d'un autre mot de code

Rq. : Code préfixe ⇒ code uniquement décodable

Un code non-préfixe

$$\begin{array}{c|cc}
a & 0 \\
b & 1 \\
c & 01
\end{array}$$

$$ab = 01 = c \rightarrow$$
Non uniquement décodable

Un code **préfixe**

| a | 0 |
|---|----|
| b | 10 |
| c | 11 |

Chaque code est unique

Entropie et Codage Optimal (4)

Inégalité de Kraft

Pour un code défini sur un alphabet de taille D, et un alphabet de source Ω de taille $|\Omega|$, alors il est **préfixé** si et seulement si

$$\sum_{i=1}^{|\Omega|} D^{-l_i} \le 1$$

 $l_i =$ longueur des mots du codes

Théorème du Codage de Source - Shannon 1948

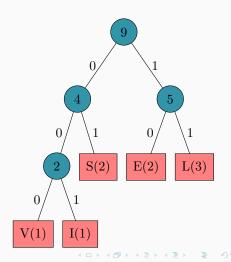
Lorsque l'efficacité de la compression augmente, la longueur moyenne du code tend vers l'entropie ${\cal H}.$

Codage De Huffman (1)

- Codage optimal au niveau symbole, à longueur variable
- Impose un nombre entier de bits pour un symbole
- Exemple de codage de "LES VILLES" :

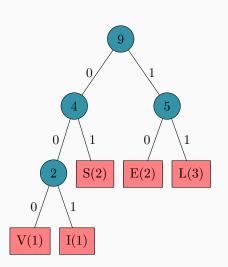
| symbole source | fréquence |
|----------------|-----------|
| L | 3 |
| Е | 2 |
| S | 2 |
| V | 1 |
| 1 | 1 |

Arbre de Huffman de "LES VILLES"



Codage De Huffman (2)

Arbre de Huffman de "LES VILLES"



• Table de code de Huffman

| symbole source | code |
|----------------|------|
| L | 11 |
| E | 10 |
| S | 01 |
| V | 000 |
| I | 001 |

• Code de Huffman : 111001000001111111001

Codage Arithmétique (1)

- Codage optimal au niveau bit, à longueur variable
- Principe : codage par morceaux et non par symbole (Huffman)
- Exemple de codage de "VILLE" :

| symbole source | probabilité | intervalle |
|----------------|-------------|--------------|
| V | 1/5 | [0; 0, 2[|
| I | 1/5 | [0, 2; 0, 4[|
| L | 2/5 | [0, 4; 0, 8[|
| E | 1/5 | [0, 8; 1[|

Ajout d'un symbole s:

$$BB = BS - BI$$

$$BS \leftarrow BI + BB \times BS_s$$

$$BI \leftarrow BI + BB \times BI_s$$

Codage Arithmétique (2)

| symbole source | probabilité | intervalle |
|----------------|-------------|------------|
| V | 1/5 | [0; 0.2[|
| ļ | 1/5 | [0.2; 0.4[|
| L | 2/5 | [0.4; 0.8[|
| E | 1/5 | [0.8; 1[|

Ajout de
$$s = V$$
:

$$BS = BB = 1$$
, $BI = 0$, $BS_s = 0.2$, $BI_s = 0$

$$BS \leftarrow 0 + 1 \times 0.2 = 0.2$$

$$BI \leftarrow 0 + 1 \times 0 = 0$$

Ajout, ensuite, de s' = I:

$$BS = BB = 0.2$$
, $BI = 0$, $BS_{s'} = 0.4$, $BI_{s'} = 0.2$

$$BS \leftarrow 0 + 0.2 \times 0.4 = 0.08$$

$$BI \leftarrow 0 + 0.2 \times 0.2 = 0.04$$

. . .

Codage Arithmétique (3)

| symbole source | probabilité | intervalle |
|----------------|-------------|------------|
| V | 1/5 | [0; 0.2[|
| 1 | 1/5 | [0.2; 0.4[|
| L | 2/5 | [0.4; 0.8[|
| E | 1/5 | [0.8; 1[|

Code de "VILLE" : $x \in [0.06752; 0.0688]$ Par exemple x = 0.068 fonctionne.

Décompression :

$$x \in [0; 0.2] \to V$$

$$2 x \leftarrow \frac{x - BI_V}{p_V} = 0.34$$

$$x \in [0.2; 0.4] \to VI$$

$$x \leftarrow \frac{x - BI_I}{p_I} = 0.7$$

. . .

Mot décodé : VILLE

La Représentation d'Image

Image =
$$\begin{pmatrix} (r, g, b)_{1,1} & \dots & (r, g, b)_{1,p} \\ \vdots & \ddots & \vdots \\ (r, g, b)_{n,1} & \dots & (r, g, b)_{n,p} \end{pmatrix}$$





Référence Image: laou.fr

La Représentation YCbCr

Transformation YCbCr

$$\varphi \colon [0, 255]^3 \to [0, 255] \times [-128, 127]^2$$

 $X = (r, g, b) \longmapsto TX = (Y, Cb, Cr)$

$$T = 255 \begin{pmatrix} K_r & K_r & K_b \\ -\frac{1}{2} \cdot \frac{K_r}{1 - K_b} & -\frac{1}{2} \cdot \frac{K_g}{1 - K_b} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \cdot \frac{K_g}{1 - K_r} & -\frac{1}{2} \cdot \frac{K_b}{1 - K_r} \end{pmatrix} \text{ et } K_r + K_g + K_b = 1$$

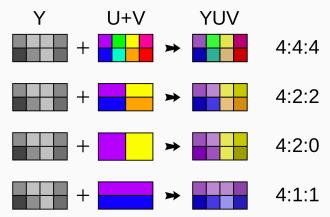
Rq.: En général $K_r = 0.299, K_q = 0.587, K_b = 0.114$

La Représentation YCbCr (2)



Sous-Échantillonage

- Principe : Cb, Cr moins importants -> moyenner ces valeurs sur plusieurs pixels
- Exemples de sous-échantillonnages :



Référence Schéma: wikipedia.org

DCT (transformée en cosinus discrète) (1)

Transformation DCT

$$\psi \colon \mathbb{R}^N \to \mathbb{R}^N$$

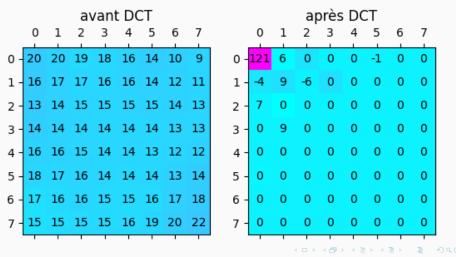
$$(x_0, \dots, x_{N-1}) \longmapsto \left(\sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}(n+\frac{1}{2})k\right]\right)_{k \in [0,N-1]}$$

On peut rendre la matrice associée à ψ orthogonale en multipliant le terme X_0 par $\frac{1}{\sqrt{N}}$ et toute la matrice par $\sqrt{2/N}$.

2D-DCT $\rightarrow \psi$ sur chaque ligne puis chaque colonne

DCT (2)

- "Continuité" des images → peu de variations des hautes fréquences
- Compactage de l'énergie vers les basses fréquences



Quantification

- Seule étape avec perte de la compression JPEG
- Réduction des coefficients
- Différence entre Y, Cb et Cr
- Fonction de quantification :

$$\varepsilon \colon \mathcal{M}_{8,8}(\mathbb{Z})^2 \to \mathcal{M}_{8,8}(\mathbb{Z})$$

$$Q, B \longmapsto \lfloor B/Q \rfloor$$

• Souvent Q dépend d'un facteur de qualité q (dans notre cas : $q \in [\![1,100]\!])$

Codage par plages (Run-Length Encoding)

- Tire son avantage des **répétitions** de symboles
- Exemple :

$$\underbrace{AAAA}_{4\times A}\underbrace{B}_{1\times B}\underbrace{CCC}_{3\times C}\underbrace{BBBBBBB}_{7\times B}\xrightarrow{RLE}A4B1C3B7$$

15 caractères \rightarrow 8 caractères

 Pour faire apparaître ces répétitions dans les images, lecture de la matrice en zigzag :



Image Originale



Taille: 10,3 Mo

Image Compressée (q = 100, 4:4:4) (presque lossless)

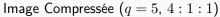


$$\begin{aligned} & \text{Taille: 2,2 Mo} \\ & \text{Ratio: } \eta = \frac{T_{init}}{T_{compres}} = 4.68 \end{aligned}$$

Image Compressée (q = 50, 4:2:2)



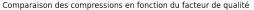
$$\begin{aligned} & \text{Taille}: 519 \text{ Ko} \\ & \text{Ratio}: \eta = \frac{T_{init}}{T_{compres}} = 19.8 \end{aligned}$$

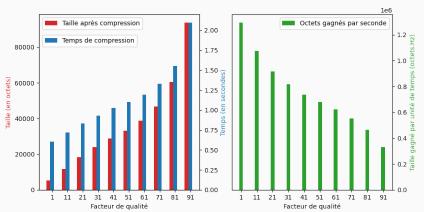




$$\begin{aligned} & \text{Taille}: 77 \text{ Ko} \\ & \text{Ratio}: \eta = \frac{T_{init}}{T_{compres}} = 134 \end{aligned}$$

Comparaison (1)

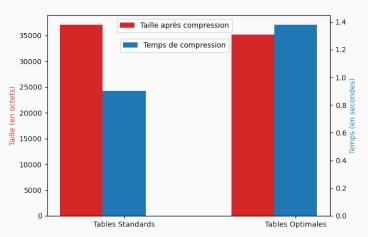




Conclusion : JPEG plus efficace lorsque la qualité baisse

Comparaison (2)

Comparaison des compressions en fonction des tables de Huffman utilisées



Conclusion : Le gain en espace est proportionnellement inférieure au gain en temps

Effectué

- √ Étude théorique des moyens de compression
- ✓ Implémentation des méthodes de compression en python
- ✓ Implémentation d'un encoder JPEG utilisant le codage de Huffman en python
- Implémentation d'un encoder JPEG utilisant le codage arithmétique en python
- $\sim\,$ Comparaison des méthodes de compression sur des images
 - √ Comparaison en fonction du facteur de qualité
 - ✓ Comparaison en fonction du codage de Huffman
 - Comparaison en fonction du codage entropique (Huffman ou Arithmétique)

Annexe - Démonstration Inégalité de Kraft (1)

Pour toute la démonstration, on considère un codage h ayant la propriété du préfixe. Soit s_1,\ldots,s_k et l_1,\ldots,l_k les mots codes et leur longueur.

On considère le cas où la taille de l'alphabet D=2 (binaire), l'inégalité de Kraft s'écrit donc :

$$\sum_{i=1}^{k} 2^{-l_i} \leqslant 1$$

On note $m=\max l_i$ la plus grande longueur des mots codes. Comme le code a la propriété du préfixe, on peut placer, de manière unique, ces mots codes sur un arbre $\mathcal T$ binaire équilibré de profondeur m chaque mot code étant une feuille de cet arbre.

On peut donc compter le nombre de feuilles issues du mot de code s_i dans l'arbre complet binaire $\mathcal T$ de profondeur m Comme le mot code est à la profondeur l_i , le nœud correspondant est le père de 2^{m-l_i} feuilles à la profondeur m.

Annexe - Démonstration Inégalité de Kraft (2)

Comme le codage a la propriété du préfixe les sous arbres issus des mots codes sont disjoints et donc le nombre total de feuilles issues des mots codes à la profondeur m est égal à

$$\sum_{i=1}^{k} 2^{m-l_i}$$

Ce nombre de feuilles est inférieur ou égal au nombre total de feuilles à la profondeur m, c'est à dire à 2^m . En écrivant cette inégalité on obtient :

$$\sum_{i=1}^{k} 2^{m-l_i} \le 2^m \implies \sum_{i=1}^{k} 2^{-l_i} \le 1$$

La réciproque se montre en partant de l'arbre complet de profondeur m et en construisant les mots codes à partir des l_i .

Annexe - Exemple Matrice de Quantification

| 10 | 7 | 6 | 10 | 14 | 24 | 31 | 37 | 10 | 11 | 14 | 28 | 59 | 59 | 59 | 59 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 7 | 7 | 8 | 11 | 16 | 35 | 36 | 33 | 11 | 13 | 16 | 40 | 59 | 59 | 59 | 59 |
| 8 | 8 | 10 | 14 | 24 | 34 | 41 | 34 | 14 | 16 | 34 | 59 | 59 | 59 | 59 | 59 |
| 8 | 10 | 13 | 17 | 31 | 52 | 48 | 37 | 28 | 40 | 59 | 59 | 59 | 59 | 59 | 59 |
| 11 | 13 | 22 | 34 | 41 | 65 | 62 | 46 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 |
| 14 | 21 | 33 | 38 | 49 | 62 | 68 | 55 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 |
| 29 | 38 | 47 | 52 | 62 | 73 | 72 | 61 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 |
| 43 | 55 | 57 | 59 | 67 | 60 | 62 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 | 59 |

Quantization table 2 Référence Image: researchgate.net

Quantization table 1

Annexe - Listing Programmation

- ycbcr.py \rightarrow slides 32-33
- ullet huffman.py o slides 33-35
- fft.py \rightarrow slides 36-39
- complex.py → slides 39-44
- arithmetic_coding.py → slides 44-47
- utils.py \rightarrow slides 47-59
- huffman_jpeg.py → slides 59-73
- encoder.py \rightarrow slides 73-80
- $\bullet \ \ \text{comparator.py} \to \text{slides} \ 80\text{-}89$

Selected files 9 printable files

```
ycbcr.py
huffman.py
fft.py
complex.py
arithmetic_coding.py
encoder/tutils.py
encoder/huffman_ipeg.py
encoderencoder.py
encoder/comparator.py
```

ycbcr.py

```
1 # Module transforming RGB images into YCbCr
 2 import numpy as np
 3 import numpy.linalg as alg
 5
   mat = np.array(
6
        [[65.481, 128.553, 24.966], [-37.797, -74.203, 112.0], [112.0, -93.786, -18.214]]
 7
 8
   col = np.arrav([[16, 128, 128]])
10
12
   def rab to vcbcr(rab: tuple) -> tuple:
13
        a = np.asarray(rgb)
       b = mat.dot(a)
14
15
        return tuple(b + col)
16
18 def ycbcr_to_rgb(t: tuple) -> tuple:
19
        a = np.asarray(t)
```

```
20 | b = alg.inv(mat)
21 | c = a - col
22 | d = b.dot(c[0])
23 | return tuple(d)
```

huffman.py

```
# Module computing Huffman compression
3
   from collections import Counter, namedtuple
   from heapq import heapify, heappop, heappush
6
7
   # Node in a Huffman Tree
   Node = namedtuple("Node", ["char", "freq"])
10
   class HuffmanCompressor:
       """Huffman compression implementation"""
12
13
       def init (self):
14
           self.encoding table = {}
15
           self.decoding table = {}
16
17
       def build tables(self, s: str):
18
            """create both the encodingn and decoding tables of a given string
19
20
       parameters
21
22
           -s : string used to build the tables
23
24
       return
25
26
           - fill both the encoding and decoding table of the given class instance"""
27
```

```
28
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45
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59
60
61
62
63
64
65
```

```
freq table = Counter(s)
# create a heap of the nodes in the tree
heap = []
for char, freq in freq table.items():
    heap.append(Node(char, freg))
heapify(heap)
# create the Huffman tree
while len(heap) > 1:
    left node = heappop(heap)
    right node = heappop(heap)
    combined node = Node(None, left node.freg + right node.freg)
    heappush(heap, combined node)
def build encoding table(node, code=''):
    if node.char is not None:
        # if the node is a leaf, add it to the encoding table
        self.encoding_table[node.char] = code
        return
   # if the node is not a leaf, recursively build the encoding table
    build_encoding_table(node.left, code + '0')
    build encoding table(node.right, code + '1')
build encoding_table(heap[0])
def build decoding table(node, code=''):
    if node.char is not None:
        # if the node is a leaf, add it to the decoding table
        self.decoding table[code] = node.char
        return
    # if the node is not a leaf, recursively build the decoding table
    build decoding table(node.left, code + "0")
    build decoding table(node.right, code + "1")
build decoding table(heap[0])
```

```
66
         def compress(self, s: str) -> str:
 67
             """compress the inputed string
 68
 69
         parameters
 70
 71
             -s : string to be compressed
 72
 73
         return
 74
 75
             - compressed string"""
 76
             compressed = ""
 77
             for char in s:
 78
                 compressed += self.encoding table[char]
 79
             return compressed
 80
 81
         def decompress(self, compressed: str) -> str:
 82
             """decompress the inputed string
 83
 84
         parameters
 85
 86
             -s : string to be compressed
 87
 88
         return
 89
             - decompressed string"""
 90
 91
             decompressed = ""
 92
             i = 0
 93
             while i < len(compressed):</pre>
                 for j in range(i+1, len(compressed)+1):
 94
 95
                     if compressed[i:i] in self.decoding table:
 96
                         decompressed += self.decoding_table[compressed[i:j]]
 97
                         i = j
 98
                         break
 99
100
             return decompressed
101
102
```

103

fft.py

```
# fast-fourier transforms
   import complex as cpx
   from numpy import log2
   from cmath import pi, exp, cos
   from scipy.fftpack import dct, idct
8
   def FFT(vector:list) -> list:
10
        """calculate the fast fourier tranform of a vector
11
12
       parameters
13
14
           -vector : list of Complex object
15
16
       return
17
            - 1-D fast fourier transform of the vector"""
18
19
       n = len(vector)
20
       assert log2(n).is_integer(), "make sure that the length of the arguement is a power of 2"
       if n == 1:
21
22
            return vector
23
       polv even, polv odd = vector[::2] , vector[1::2]
       res even, res_odd = FFT(poly_even), FFT(poly_odd)
24
25
        res = [cpx.Complex(0)] * n
26
       for i in range(n//2):
27
            w i = cpx.exp to literal(-2*pi*i/n)
28
           product = w j * res odd[j]
29
           res[i] = res even[i] + product
30
            res[i + n//2] = res even[i] - product
31
        return res
```

```
33
   def IFFT aux(vector:list) -> list:
34
       """auxiliary function that makes the recursive steps of the IFFT algorithm
35
       parameters
36
37
           -vector : list of Complex object
38
39
       return
40
41
            - partial inverse of the 1-D fast fourier transform of the vector (lack the division by n)"""
42
       n = len(vector)
43
       assert log2(n).is integer(), "make sure that the length of the arguement is a power of 2"
44
        if n == 1:
45
            return vector
46
       poly even, poly odd = vector[::2] , vector[1::2]
47
       res even, res odd = IFFT aux(poly even), IFFT aux(poly odd)
48
        res = \lceil cpx.Complex(0) \rceil * n
49
       for j in range(n//2):
            w_j = cpx.exp_{to_literal((2 * pi * j) / n)}
50
51
           product = w i * res odd[i]
52
           res[j] = res even[j] + product
53
            res[i + n//2] = res even[i] - product
54
        return res
55
56
   def IFFT(vector:list) -> list:
57
        """caclulate the inverse of the fast fourier tranform of a vector (in order to have ifft(fft(polv)) ==
   polv)
58
59
       parameters
60
61
            -vector : list of Complex object
62
63
       return
64
65
            - inverse of the 1-D fast fourier transform of the vector"""
66
       n = len(vector)
67
       res = IFFT aux(vector)
68
       for i in range(n):
```

```
69
            res[i] = res[i] / cpx.Complex(n)
70
        return res
71
    def DCT(vector:list, orthogonalize:bool =False, norm="forward"):
73
         """calculate the one-dimensional type-II discrete cosine tranform of a matrix (MAKHOUL) (using the FFT
    function previously defined)
74
75
        parameters
76
77
            - vector: list of Numerical Object
78
79
        return
80
81
            - discrete cosine tranform of the input"""
82
        N = len(vector)
83
        temp = vector[::2] + vector[-1 - N % 2 ::-2]
84
        temp = FFT(temp)
85
        factor = - pi / (N * 2)
86
        result = [2 * (val * (cpx.exp to literal(i * factor))).re for (i, val) in enumerate(temp)]
87
        if orthogonalize:
88
            result[0] *= 2 ** (-1 / 2)
         if norm == "ortho":
89
90
            result[0] *= (N) **(-1 / 2)
91
            result[1::] = [(2 / N) ** (1 / 2) * result[i] for i in range(1, len(result))]
92
        return result
93
94
    def IDCT(vector:list):
95
         """calculate the one-dimensional "inverse" type-III discrete cosine tranform of a matrix (MAKHOUL) (using
    the FFT function previously defined)
96
97
        parameters
98
99
            - vector: list of Numerical Object
100
101
        return
102
103
            - type-III discrete cosine tranform of the input"""
104
        N = len(vector)
```

◆□▶ ◆□▶ ◆□▶ ◆□▶ □ り<0</p>

```
105
        factor = - pi / (N * 2)
106
        temp = [(cpx.Complex(val) if i > 0 else (cpx.Complex(val) / cpx.Complex(2))) * cpx.exp to literal(i *
     factor) for (i, val) in enumerate(vector)]
107
        temp = FFT(temp)
108
        temp = [val.re for val in temp]
109
        result = [None] * N
110
        result[ : : 2] = temp[ : (N + 1) // 2]
111
        result[-1 - N \% 2 : : -2] = temp[(N + 1) // 2 : ]
112
        return result
113
114 if name == " main ":
115
        vectorCpx= [cpx.Complex(5), cpx.Complex(2), cpx.Complex(4), cpx.Complex(8)]
        vector = [5, 2, 4, 8]
116
117
        print("DCT : ", DCT(vectorCpx))
118
        print("inverse + DCT : ", IDCT((DCT(vectorCpx))))
119
        print("scipy dct :", dct(vector))
120
        print("scipy + inverse dct: ", dct(idct(vector)))
121
        print("scipy dct (ortho) : ", dct(vector, norm = "ortho"))
122
        print("scipv inverse + dct (ortho) : ", idct(dct(vector, norm="ortho"), norm="ortho"))
```

complex.py

3 4

8

11

```
1 | # Module computing complex numbers
   # disclaimer : this class is not made to deal with less than 1e-10 values
   from numpy import arctan2, cos, pi, sin, sqrt
  from math import isclose
   from typing import Union, List
9
10
   class Complex:
       """Computing complex numbers"""
       def __init__(self, real=0., imaginary=0.):
13
                self.re = real # round(real, 15)
```

```
14
                self.im = imaginary # round(imaginary,15)
15
        def str (self) -> str:
16
            if self.im == 0.:
17
                string = f"{self.re}"
18
            elif self.re == 0:
19
                string = f"i({self.im})"
20
            else:
21
                string = f"{self.re} + i({self.im})"
22
            return string
23
        repr = str
24
        def eq (self, other) -> bool:
25
            return bool(isclose(self.re, other.re) and isclose(self.im, other.im))
26
        def is null(self):
27
            return isclose(self.re. 0) and isclose(self.im. 0)
28
        def is real(self):
29
            return isclose(self.im, 0)
30
        def is imaginary(self):
31
            return isclose(self.re, 0)
32
        def arg(self):
33
            """return the argument of the complex number
34
            return None if 0"""
35
           if self.is null():
36
                arg = None
37
            elif isclose(self.re, 0) and self.im > 0:
38
                arg = pi / 2
39
            elif isclose(self.re, 0) and self.im < 0:</pre>
40
                arg = - pi / 2
41
            else:
42
                arg = round(arctan2(self.im, self.re), 15)
43
            return ard
44
        def module(self):
45
            """return the module of the complex number"""
46
            return round(sart(self.re**2 + self.im**2), 15)
47
        def conjuagate(self):
48
            return (Complex(self.re, -self.im))
49
        #arithmetic
50
        def add__(self, other):
51
            return Complex(self.re + other.re, self.im + other.im)
```

```
52
       def sub (self, other):
53
           return Complex(self.re - other.re, self.im - other.im)
54
       def mul (self. other):
55
           real = (self.re * other.re) - (self.im * other.im)
56
            imaginary = (self.re * other.im) + (self.im * other.re)
57
           return Complex(real, imaginary)
58
       def truediv (self, other):
59
           if other.is null():
60
                raise ValueError("Error : dividing by 0")
61
           elif other.is real():
62
                return Complex(self.re / other.re, self.im / other.re)
63
            else:
64
                denominator = (other.re ** 2) + (other.im ** 2)
65
                real = ((self.re * other.re) + (self.im * other.im)) / denominator
66
                imaginary = ((self.im * other.re) - (self.re * other.im)) / denominator
67
                return Complex(real, imaginary)
68
   Num = Union[int, float]
70
   def addition(*complexes:Complex) -> Complex: #partially depreciated (can still be usefull for more iterable
   arguments)
72
        """calculate the sum of complex numbers
74
       parameters
75
76
           - *complexes : iterable type of Complex
77
78
        return
79
80
           - sum of the complex numbers"""
81
82
       res = Complex(0)
83
       for number in complexes:
84
           res re += number re
85
           res.im += number.im
86
        return res
87
```

```
88 def difference(cpx1:Complex, cpx2:Complex = Complex(0)): #fully depreciated (replaced by sub Complex
     methods)
 89
         """calculate the difference of two complex numbers
 90
 91
         parameters
 92
 93
            - cpx1 : Complex number
 94
            - cpx2 : Complex number to subtract to cpx1 (=Complex(0) by default)
 95
 96
         return
 97
 98
                 difference of the two complex numbers"""
 99
         res = Complex()
         res.re = cpx1.re - cpx2.re
100
101
         res.im = cpx1.im - cpx2.im
102
         return res
103
104
    def product(*complexes:Complex) -> Complex: #partially depreciated (can still be usefull for more iterable
     arguments)
         """calculate the product of complex numbers
105
106
107
         parameters
108
109
             - *complexes : iterable type of Complex
110
111
         return
112
113
             - product of the complex numbers"""
114
         res = Complex(1)
115
         for number in complexes:
116
             re = res.re * number.re - res.im * number.im
117
             im = res.re * number.im + res.im * number.re
118
             res.re = re
119
            res.im = im
120
         return res
121
122 def exp to literal(arg:float, module:float = 1.0) -> Complex:
123
         """ return the literal expression of a complex number defined by its argument and module
```

```
125
        parameters
126
127
            - arg : type(float) (should be between 0 and 2pi)
128
            - module : type(float) (must have a positive value)(=1 by default)
129
130
         return
131
132
            - Complex number associated"""
133
        assert(module >= 0), "second-argument(module) must have a positive value"
        return Complex(module*cos(arg), module*sin(arg))
134
135
136
    def nth root(n:int, cpx:Complex = Complex(1)) -> Complex:
137
         """calculate the nth root of a complex number
138
139
        parameters
140
141
            - n : type(int)
142
            - complex : type(Complex) (=Complex(1) by default) (must not be Complex(0))
143
144
        return
145
146
            - list of the nth roots"""
147
        assert(cpx.re != 0 or cpx.im != 0), "second argument must be a non-zero complex number"
148
        module = cpx.module()
        arg = cpx.arg()
149
150
        if arg is not None:
151
            return exp to literal((arg/n), module**(1/n))
152
        else:
153
            return Complex(1) #Not used case but just here to ensure nth root cannot return None
154
155
156
    def nth roots unitv(n:int) -> list:
157
         """ calculate the n roots of unity
158
159
        parameter
160
161
            - n : type(int) : must be a positive integer
                                                                       ←□→ ←□→ ←□→ □
```

```
163
         return
164
165
             - a list of Complex containing the n roots of unity"""
166
         roots = [Complex(1) for i in range(n)]
167
         for k in range(0,n):
168
             roots[k] = exp to literal((2*k*pi/n), 1.0)
169
         return roots
170
171
    def inverse nth roots unity(n:int) -> list:
172
         """ calculate the inversed n roots of unity
173
174
         parameter
175
176
         - n : type(int) : must be a positive integer
177
178
         return
179
180
         - a list of Complex containing the inversed n roots of unity"""
         roots = [Complex(1) for i in range(n)]
181
182
         for k in range(0.n):
183
             roots[k] = exp to literal((-2*k*pi/n), 1.0)
         return roots
184
185
186
    def make complex(values:List[Num]) -> List[Complex]:
         res = []
187
188
         for value in values:
189
             res.extend([Complex(value)])
190
         return res
191
192
193 if name == " main ":
194
         pass
195
```

${ t arithmetic_coding.py}$

```
def proba(data):
2
 3
       Créer le dictionnaire de probabilités d'apparition des différents caractères
 4
       assert len(data) != 0
       d = \{\}
 6
 7
       for x in data:
 8
            d[x] = d.get(x, 0) + (1 / len(data))
9
       return d
10
11
12
   def create_int(data):
13
14
       Créer le dictionnaire des intervalles des différents caractères connaissant les données
15
16
       p = proba(data)
17
       d = \{\}
18
       n = 0.0
19
       for c, v in p.items():
20
           d[c] = (n, n + v)
21
           n += v
       return d
23
24
25
   def create int2(p):
26
       Créer le dictionnaire des intervalles des différents caractères connaissant les probas des différents
   caractères
28
29
       d = \{\}
30
       n = 0.0
31
       for c, v in p.items():
32
           d[c] = (n, n + v)
33
           n += v
```

35

return d

```
36
37
   def encode(data):
38
39
        effectue l'encodage des données
40
41
        int = create int(data)
42
        value = (0.0, 1.0)
43
        for x in data:
44
            d = value[1] - value[0]
45
            sup = value[0] + d * int[x][1]
46
            inf = value[0] + d * int[x][0]
47
            value = (inf, sup)
48
        return (value[0] + value[1]) / 2
49
50
51
   def appartient(x, int):
52
53
        teste l'appartenance de x à un intervalle fermé à gauche et ouvert à droite
54
55
        assert len(int) == 2
56
        return x >= int[0] and x < int[1]
57
58
59
   def inverse(dic):
60
61
        renvoie le dictionnaire où les clés et valeurs sont inversées
62
63
        d = \{\}
64
        for c, v in dic.items():
65
            d\Gamma v1 = c
66
        return d
67
68
69
   def decode(n, p, nbr carac):
70
        d = inverse(create int2(p))
71
        res = []
72
        i = n
73
        while len(res) < nbr carac:</pre>
```

```
76
                     res.append(v)
 77
                     i = (i - c[0]) / (c[1] - c[0])
 78
                     break
 79
         return res
 80
 81
 82
     # Examples
 83
     if name == " main ":
 85
         print(encode("WIKI"))
 86
         print(decode(0.171875, {"W": 0.25, "I": 0.5, "K": 0.25}, 4))
 87
         print(encode("AAABBCCCCC"))
 88
         print(decode(0.010783125000000005, {"A": 0.3, "B": 0.2, "C": 0.5}, 10))
 89
         print(encode([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]))
 90
         print(
 91
             decode(
 92
                 encode([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]),
 93
 94
                     1: 0.1,
 95
                     2: 0.1.
 96
                     3: 0.1.
 97
                     4: 0.1,
 98
                     5: 0.1,
 99
                     6: 0.1.
100
                     7: 0.1.
101
                     8: 0.1
                     9: 0.1
102
103
                     10: 0.1
104
                 },
105
                 10
106
107
108
encoder/utils.pv
```

75

for c, v in d.items():

if appartient(i, c):

```
1 | import numpy as np
   import math
   import cv2
   from io import BytesIO
5
6
   # DCT block size
   BH, BW = 8.8
9
10
11
   class MARKER:
12
       SOI = b"\xff\xd8"
13
       APP0 = b"\xff\xe0"
14
       APPn = (b"\xff\xe1", b"\xff\xef") # n=1~15
15
       DOT = b"\xff\xdb"
16
       SOF0 = b"\xff\xc0"
17
       DHT = b"\xff\xc4"
18
       DRI = b"\xff\xdd"
19
       SOS = b"\xff\xda"
20
       EOI = b"\xff\xd9"
21
22
   class ComponentInfo:
24
       def init (self, id , horizontal, vertical, qt id, dc ht id, ac ht id):
25
           self.id = id
26
           self.horizontal = horizontal
27
           self.vertical = vertical
28
           self.qt id = qt id
29
           self.dc ht id = dc ht id
30
           self.ac ht id = ac ht id
31
32
       @classmethod
33
       def default(cls):
34
           return cls. init (*[0 for in range(6)])
35
36
       def encode SOS info(self):
```

```
37
            return int2bytes(self.id , 1) + int2bytes(
38
                (self.dc ht id << 4) + self.ac ht id, 1
39
40
41
       def encode SOFO info(self):
42
            return (
43
                int2bytes(self.id , 1)
44
               + int2bytes((self.horizontal << 4) + self.vertical, 1)
45
               + int2bytes(self.qt id, 1)
46
47
48
       def repr (self):
49
            return (
50
                f"{self.id_}: qt-{self.qt_id}, ht-(dc-{self.dc_ht_id}, "
51
                f"ac-{self.ac ht id}), sample-{self.vertical, self.horizontal} "
52
53
54
55
   class BitStreamReader:
        """simulate hitwise read"""
56
57
58
       def __init__(self, bytes_: bytes):
59
            self.bits = np.unpackbits(np.frombuffer(bytes , dtype=np.uint8))
60
            self.index = 0
61
62
       def read bit(self):
63
            if self.index >= self.bits.size:
64
                raise EOFError("Ran out of element")
            self.index += 1
65
66
            return self.bits[self.index - 1]
67
68
       def read int(self, length):
           result = 0
69
           for in range(length):
70
71
                result = result * 2 + self.read bit()
72
           return result
73
74
       def repr (self):
```

```
75
             return f"[{self.index}, {self.bits.size}]"
 76
 77
 78
     class BitStreamWriter:
 79
         """simulate bitwise write"""
 80
 81
         def init (self, length=10000):
 82
             self.index = 0
 83
             self.bits = np.zeros(length, dtype=np.uint8)
 84
 85
         def write bitstring(self, bitstring):
 86
             length = len(bitstring)
 87
             if length + self.index > self.bits.size * 8:
 88
                 arr = np.zeros((length + self.index) // 8 * 2, dtype=np.uint8)
 89
                 arr[: self.bits.size] = self.bits
 90
                 self.bits = arr
 91
             for bit in bitstring:
 92
                 self.bits[self.index // 8] |= int(bit) << (7 - self.index % 8)</pre>
 93
                 self.index += 1
 94
 95
         def to bytes(self):
 96
             return self.bits[: math.ceil(self.index / 8)].tobvtes()
 97
 98
         def to hex(self):
             length = math.ceil(self.index / 8) * 8
 99
100
             for i in range(self.index. length):
101
                 self.bits[i] = 1
102
             bvtes = np.packbits(self.bits[:length])
             return " ".join(f"{b:2x}" for b in bytes )
103
104
105
106
    class BytesWriter(BytesI0):
107
         def init (self, *args, **kwargs):
108
             super(BytesWriter, self). init (*args, **kwargs)
109
110
         def add bytes(self, *args):
111
             self.write(b"".join(args))
112
```

```
114 def bytes2int(bytes , byteorder="big"):
115
         return int.from bytes(bytes , byteorder)
116
117
118
    def int2bytes(int : int, length):
119
         return int .to bytes(length, byteorder="big")
120
121
122
    def decode 2s complement(complement, length) -> int:
123
         if length == 0:
124
             return 0
125
         if complement >> (length - 1) == 1: # sign bit equal to one
126
             number = complement
127
         else: # sign bit equal to zero
128
             number = 1 - 2**length + complement
129
         return number
130
131
132
    def encode 2s complement(number) -> str:
133
         """return the 2's complement representation as string"""
134
         if number == 0:
             return ""
135
136
         if number > 0:
137
             complement = bin(number)[2:1]
138
         else:
139
             length = int(np.log2(-number)) + 1
             complement = bin(number - (1 - 2**length))[2:].zfill(length)
140
         return complement
141
142
143
144
    def load quantization table(quality, component):
         # the below two tables was processed by zigzag encoding
145
         # in JPEG bit stream, the table is also stored in this order
146
147
         if component == "lum":
148
             q = np.array(
149
150
                     16,
```

| 101 | |
|-----|--|
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11, 12, 14, 12, 10, 16, 14, 13, 14, 18, 17, 16, 19, 24, 40, 26, 24, 22, 22, 24, 49, 35, 37, 29, 40, 58, 51, 61, 60, 57, 51, 56, 55, 64, 72, 92, 78, 64,

```
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                      87,
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                      113,
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                      121,
207
                      112,
208
                      100,
209
                      120,
210
                      92,
211
                      101,
                      103,
213
                      99,
214
215
                  dtype=np.int32,
216
217
         elif component == "chr":
218
              q = np.array(
219
220
                      17,
221
                      18,
222
                      18,
                      24,
224
                      21,
225
                      24,
226
                      47,
```

68,

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99,
            99,
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            99,
            99,
            99,
            99,
            99,
            99.
        ],
        dtype=np.int32,
else:
    raise ValueError(
            f"component should be either 'lum' or 'chr', "
            f"but '{component}' was found."
if 0 < quality < 50:
    q = np.minimum(np.floor(50 / quality * q + 0.5), 255)
elif 50 <= quality <= 100:
    q = np.maximum(np.floor((2 - quality / 50) * q + 0.5), 1)
else:
    raise ValueError("quality should belong to (0, 100].")
return q.astype(np.int32)
```

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292 293 294

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297

298

299

300

```
303 | def zigzag points(rows, cols):
        # constants for directions
        UP, DOWN, RIGHT, LEFT, UP RIGHT, DOWN LEFT = range(6)
        move func = {
            UP: lambda p: (p[0] - 1, p[1]),
            DOWN: lambda p: (p[0] + 1, p[1]),
            LEFT: lambda p: (p[0], p[1] - 1),
             RIGHT: lambda p: (p[0], p[1] + 1),
            UP RIGHT: lambda p: move(UP, move(RIGHT, p)),
             DOWN LEFT: lambda p: move(DOWN, move(LEFT, p)),
        # move the point in different directions
        def move(direction, point):
             return move func[direction](point)
        # return true if point is inside the block bounds
        def inbounds(p):
             return 0 \le p[0] < rows and 0 \le p[1] < cols
        # start in the top-left cell
        now = (0, 0)
        # True when moving up-right, False when moving down-left
        move up = True
        trace = []
        for i in range(rows * cols):
             trace.append(now)
             if move up:
                 if inbounds(move(UP RIGHT, now)):
                     now = move(UP RIGHT, now)
                 else:
                     move up = False
                     if inbounds(move(RIGHT, now)):
                         now = move(RIGHT, now)
                     else:
```

305

306 307

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312

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314 315 316

317

318

319 320

321

322

323 324

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326 327

328

329

330 331

332

333

334

335

336

338

339

```
341
                         now = move(DOWN, now)
342
             else:
343
                 if inbounds(move(DOWN LEFT, now)):
344
                     now = move(DOWN LEFT, now)
345
                 else:
346
                     move up = True
347
                     if inbounds(move(DOWN, now)):
348
                         now = move(DOWN, now)
349
                     else:
350
                         now = move(RIGHT, now)
351
352
         for rows = cols = 8, the actual 1-D index:
353
             0, 1, 8, 16, 9, 2, 3, 10, 17, 24, 32, 25, 18, 11, 4, 5,
354
             12, 19, 26, 33, 40, 48, 41, 34, 27, 20, 13, 6, 7, 14, 21, 28,
355
             35, 42, 49, 56, 57, 50, 43, 36, 29, 22, 15, 23, 30, 37, 44, 51,
356
             58, 59, 52, 45, 38, 31, 39, 46, 53, 60, 61, 54, 47, 55, 62, 63
357
358
         return trace
359
360
361
    def RGB2YCbCr(im):
362
         im = im.astvpe(np.float32)
363
         im = cv2.cvtColor(im, cv2.COLOR RGB2YCrCb)
364
         RGB [0, 255]
365
366
         opency uses the following equations to conduct color conversion in float32
367
             Y = 0.299 * R + 0.587 * G + 0.114 * B
368
             Cb = (B - Y) * 0.564 + 0.5
             Cr = (R - Y) * 0.713 + 0.5
369
370
         Y [0, 255], Cb, Cr [-128, 127]
371
372
         # convert YCrCb to YCbCr
         Y, Cr, Cb = np.split(im, 3, axis=-1)
373
374
         im = np.concatenate([Y, Cb, Cr], axis=-1)
375
         return im
376
377
378
    def YCbCr2RGB(im):
```

```
379
         im = im.astype(np.float32)
380
        Y, Cb, Cr = np.split(im, 3, axis=-1)
381
        im = np.concatenate([Y, Cr, Cb], axis=-1)
382
         im = cv2.cvtColor(im, cv2.COLOR YCrCb2RGB)
383
384
        Y [0, 255], Cb, Cr [-128, 127]
385
        conversion equation (float32):
386
             B = (Cb - 0.5) / 0.564 + Y
387
            R = (Cr - 0.5) / 0.713 + Y
388
            G = (Y - 0.299 * R - 0.114 * B) / 0.587
389
        RGB [0, 255]
390
391
        return im
392
393
394
    def bits required(n):
395
        n = abs(n)
396
        result = 0
397
        while n > 0:
398
             n >>= 1
399
            result += 1
400
        return result
401
402
403
    def divide blocks(im. mh. mw):
404
        h, w = im.shape[:2]
405
        return im.reshape(h // mh, mh, w // mw, mw).swapaxes(1, 2).reshape(-1, mh, mw)
406
407
408
    def restore image(block, nh, nw):
409
        bh, bw = block.shape[1:]
410
        return block.reshape(nh, nw, bh, bw).swapaxes(1, 2).reshape(nh * bh, nw * bw)
411
412
    def flatten(lst):
413
414
        return [item for sublist in lst for item in sublist]
415
416
```

```
417 | def averageMatrix(
418
        arrayMatrix,
419
    ): # given an array of 2D-array, return the average (coef by coef) 2D array
420
        avgMatrix = np.zeros like(arrayMatrix[0])
421
        for i in range(avgMatrix.shape[0]):
422
            for j in range(avgMatrix.shape[1]):
423
                 avgMatrix[i, j] = np.average(arrayMatrix[:, i, j])
424
        return avgMatrix
425
426
427 if name == " main ":
        arrMatrix = np.array([[[1, 2], [3, 4]], [[5, 2], [3, 4]]))
428
429
        print(averageMatrix(arrMatrix))
430
```

encoder/huffman_jpeg.py

import numpy as np

```
MAX CLEN = 32 # assumed maximum initial code length
4
5
 6
   def getFreg(data):
7
       freq = [0] * 257
8
       for elem in data:
9
            frea[elem] += 1
       freq[256] = 1
10
11
       return freq
12
13
14
   def jpegGenerateOptimalTable(freq):
15
       bits = [0] * (MAX CLEN + 1)
16
       bitPos = [0] * (MAX_CLEN + 1)
17
       codesize = [0] * 257
```

```
18
        nzIndex = [0] * 257
19
20
        others = \lceil -1 \rceil \times 257
21
22
        numNzSymbols = 0
23
        for i in range(257):
24
            if freq[i]:
25
                 nzIndex[numNzSymbols] = i
26
                 freq[numNzSymbols] = freq[i]
27
                 numNzSymbols += 1
28
29
        huffval = [0] * (numNzSymbols - 1)
30
31
        while True:
32
            c1 = -1
33
            c2 = -1
34
            v = 10000000000
35
            v^2 = 10000000000
36
            for i in range(numNzSymbols):
37
                 if freq[i] <= v2:
38
                     if freq[i] <= v:</pre>
39
                         c2 = c1
40
                         v2 = v
41
                         v = freq[i]
42
                         c1 = i
43
                     else:
44
                         v2 = frea[i]
45
                         c2 = i
46
47
            if c2 < 0:
48
                 break
49
50
            frea[c1] += frea[c2]
51
            freq[c2] = 1000000001
52
53
            codesize[c1] += 1
54
            while others[c1] >= 0:
55
                 c1 = others[c1]
```

```
codesize[c1] += 1
    others[c1] = c2
   codesize[c2] += 1
    while others[c2] >= 0:
        c2 = others[c2]
        codesize[c2] += 1
for i in range(numNzSymbols):
    bits[codesize[i]] += 1
p = 0
for i in range(1, MAX_CLEN + 1):
    bitPos[i] = p
    p += bits[i]
for i in range(MAX CLEN, 16, -1):
    while bits[i] > 0:
        i = i - 2
        while bits[j] == 0:
           i -= 1
        bits[i] -= 2
        bits[i - 1] += 1
        bits[i + 1] += 2
        bits[j] -= 1
i = MAX CLEN
while bits[i] == 0:
   i -= 1
bits[i] -= 1
for i in range(numNzSymbols - 1):
    huffval[bitPos[codesize[i]]] = nzIndex[i]
    bitPos[codesize[i]] += 1
return bits, huffval
```

57 58

59 60

61

62

63

64 65

66

67 68

69

70

71

72 73

74

75

76

77

78

79

80

81

82 83

84

85

86 87 88

89

90

91 92

```
94
 95
     def jpegGenerateHuffmanTable(bits, huffval):
 96
         huffsize = [0] * 257
 97
         huffcode = [0] * 257
 98
 99
         p = 0
100
         for l in range(1, 17):
101
             i = bits[l]
102
             while i:
103
                 huffsize[p] = l
104
                  p += 1
105
                  i -= 1
106
107
         huffsize[p] = 0
108
         lastp = p
109
110
         code = 0
111
         si = huffsize[0]
         0 = q
112
113
         while huffsize[p]:
114
             while huffsize[p] == si:
115
                 huffcode[p] = code
116
                 code += 1
117
                 p += 1
118
             code <<= 1
119
             si += 1
120
121
         ehufco = [0] * 257
122
         ehufsi = \lceil 0 \rceil \times 257
123
         for p in range(lastp):
124
125
             i = huffval[p]
126
             ehufco[i] = huffcode[p]
             ehufsi[i] = huffsize[p]
127
128
129
         return ehufsi, ehufco
130
131
```

```
132
    def jpegTransformTable(ehufsi, ehufco):
133
         table = {}
134
         for i in range(len(ehufco)):
135
             if ehufsi[i] != 0:
136
                 endCode = bin(ehufco[i])[2:]
137
                 nbZeros = ehufsi[i] - len(endCode)
138
                 table[i] = "0" * nbZeros + endCode
139
         return table
140
141
142
    def jpegCreateHuffmanTable(arr):
143
         freq = getFreg(arr)
144
         bits, huffval = jpegGenerateOptimalTable(freg)
145
         ehufsi, ehufco = jpegGenerateHuffmanTable(bits, huffval)
146
         table = jpegTransformTable(ehufsi, ehufco)
147
         return table
148
149
150
    def convert huffman table(table):
151
         """convert huffman table to count and weigh"""
152
         # table[int] = string
153
         pairs = sorted(table.items(), key=lambda x: (len(x[1]), x[1]))
154
         weigh, codes = zip(*pairs)
155
         weigh = np.array(weigh, dtype=np.uint8)
         # count[i]: there are count[i] codes of length i+1
156
157
         count = np.zeros(16, dtvpe=np.uint8)
158
         for c in codes:
159
             count[len(c) - 1] += 1
160
         return count, weigh
161
162
163
     def read huffman code(table, stream):
         prefix = ""
164
         while prefix not in table:
165
             prefix += str(stream.read_bit())
166
167
         return table[prefix]
168
169
```

```
170 | def reverse(table):
171
         return {v: k for k, v in table.items()}
172
173
174
    # 4 recommended huffman tables in JPEG standard
175 # luminance DC
176 RM Y DC = {
177
         "00": 0,
178
         "010": 1,
179
         "011": 2,
180
         "100": 3,
181
         "101": 4,
182
         "110": 5,
183
         "1110": 6,
        "11110": 7,
184
185
         "111110": 8,
186
         "1111110": 9,
187
         "11111110": 10,
         "111111110": 11,
188
189 }
190
191 # luminance AC
192 RM Y AC = {
         "00": 1,
193
194
         "01": 2.
195
         "100": 3,
196
         "1010": 0,
        "1011": 4,
197
198
         "1100": 17.
         "11010": 5,
199
         "11011": 18,
200
201
         "11100": 33,
         "111010": 49,
202
203
         "111011": 65.
204
         "1111000": 6,
205
         "1111001": 19,
206
         "1111010": 81,
207
         "1111011": 97,
```

```
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209
         "11111001": 34,
210
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         "111111000": 129,
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216
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223
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224
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225
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227
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239
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240
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241
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```
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325
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350
351
         "11111111111111011": 247.
352
         "1111111111111100": 248,
353
         "1111111111111101": 249.
354
         "1111111111111110": 250.
355 }
356
357 # chroma DC
358 RM C DC = {
         "00": 0,
359
```

```
360
         "01": 1,
361
         "10": 2,
362
         "110": 3,
363
         "1110": 4,
364
         "11110": 5,
365
         "111110": 6,
         "1111110": 7,
366
367
         "11111110": 8,
368
         "111111110": 9,
369
         "1111111110": 10,
370
         "11111111110": 11,
371 }
372
373 # chroma AC
374 RM C AC = {
375
         "00": 0,
376
         "01": 1,
377
         "100": 2,
378
         "1010": 3,
379
         "1011": 17,
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```
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533
         "1111111111111011": 247,
534
         "1111111111111100": 248,
535
         "1111111111111101": 249.
536
         "1111111111111110": 250,
537 }
538
539 if __name__ == "__main__":
         arr = np.array([np.random.randint(-127, 128) for in range(64)])
540
541
         table = ipeqCreateHuffmanTable(arr)
542
         print(table)
543
```

encoder/encoder.py

```
1 from math import ceil
   import cv2
   import numpy as np
4 from PIL import Image
   from pathlib import Path
 6
 7
   from utils import *
   from huffman jpeq import *
9
10
11
   def padding(im, mh, mw):
12
13
       pad use boundary pixels so that its height and width are
14
       the multiple of the height and width of MCUs, respectively
15
16
       h, w, d = im.shape
       if h % mh == 0 and w % mw == 0:
17
18
            return im
19
       hh, ww = ceil(h / mh) * mh, ceil(w / mw) * mw
20
       im_ex = np.zeros_like(im, shape=(hh, ww, d))
21
       im ex[:h, :w] = im
22
       im \ ex[:, w:] = im \ ex[:, w - 1 : w]
23
        im_ex[h:, :] = im_ex[h - 1 : h, :]
24
       return im ex
25
26
27
   mcu sizes = {
28
       "4:2:0": (BH * 2, BW * 2),
29
       "4:1:1": (BH * 2, BW * 2),
30
       "4:2:2": (BH, BW * 2),
31
       "4:4:4": (BH, BW),
32
33
34
35
   def scan_blocks(mcu, mh, mw):
36
37
       scan MCU to blocks for DPCM, for 4:2:0, the scan order is as follows:
```

```
38
39
40
41
42
43
44
       blocks = (
45
            mcu.reshape(-1, mh // BH, BH, mw // BW, BW).swapaxes(2, 3).reshape(-1, BH, BW)
46
47
       return blocks
48
49
50
   def DCT(blocks):
51
       dct = np.zeros_like(blocks)
52
       for i in range(blocks.shape[0]):
53
            dct[i] = cv2.dct(blocks[i])
54
       return dct
55
56
57
   def zigzag_encode(dct):
58
       trace = zigzag points(BH, BW)
59
       zz = np.zeros_like(dct).reshape(-1, BH * BW)
       for i, p in enumerate(trace):
60
            zz[:, i] = dct[:, p[0], p[1]]
61
62
       return zz
63
64
65
   def quantization(dct, table):
66
       ret = dct / table[None]
67
       return np.round(ret).astvpe(np.int32)
68
69
70
   def DPCM(dct):
71
72
       encode the DC differences
73
74
       dc pred = dct.copy()
75
       dc pred[1:, 0] = dct[1:, 0] - dct[:-1, 0]
```

```
76
         return dc pred
 77
 78
 79
     def run length encode(arr):
 80
         # determine where the sequence is ending prematurely
 81
         last nonzero = -1
 82
         for i, elem in enumerate(arr):
 83
             if elem != 0:
 84
                last nonzero = i
 85
         rss, values = [], []
 86
         run lenath = 0
 87
         for i, elem in enumerate(arr):
 88
             if i > last nonzero:
 89
                 rss.append(0)
 90
                 values.append(0)
 91
                 break
 92
             elif elem == 0 and run length < 15:
 93
                 run length += 1
 94
             else:
 95
                 size = bits required(elem)
 96
                 rss.append((run length << 4) + size)
 97
                 values.append(elem)
 98
                 run length = 0
 99
         return rss. values
100
101
102
    def encode_header(qts, hts, cop_infos, height, width):
103
         writer = BvtesWriter()
104
         add bytes = writer.add bytes
105
         add bytes(
106
             MARKER.SOI.
107
             MARKER.APP0,
108
             b"\x00\x10", # length = 16
109
             b"JFIF\x00", # identifier = JFIF0
             b"\x01\x01", # version
110
111
             b"\x00", # unit
112
             b"\x00\x01", # x density
113
             b"\x00\x01", # y density
```

```
b"\x00\x00", # thumbnail data
114
115
116
        for id , gt in enumerate(gts):
117
             add bytes(
118
                 MARKER.DQT,
119
                 b"\x00C", # length = 67
120
                 # precision (8 bits), table id, = 0, id_
121
                 int2bytes(id , 1),
122
                 gt.astype(np.uint8).tobytes(),
123
124
        cop_num = len(cop_infos)
125
        add bytes(
126
            MARKER.SOFO.
127
             int2bytes(8 + 3 * cop_num, 2), # length
128
             int2bytes(8, 1), # 8 bit precision
129
             int2bytes(height, 2),
130
             int2bvtes(width, 2),
             int2bytes(cop num, 1),
133
        add bytes(*[info.encode SOF0 info() for info in cop infos])
134
135
        # type << 4 + id, (type 0: DC, 1 : AC)
136
        type ids = [b"\x00", b"\x10", b"\x01", b"\x11"]
137
        for type id, ht in zip(type ids, hts):
138
             count, weigh = convert huffman table(ht)
139
             ht bytes = count.tobytes() + weigh.tobytes()
140
             add bytes(
141
                 MARKER.DHT.
142
                 int2bvtes(len(ht bvtes) + 3, 2), # length
143
                 type id.
                 ht bytes.
145
146
147
        add bytes(
148
            MARKER.SOS,
149
             int2bytes(6 + cop num * 2, 2), # length
150
            int2bytes(cop_num, 1),
151
```

131

132

144

```
add bytes(*[info.encode SOS info() for info in cop infos])
153
         add bytes(b"\x00\x3f\x00")
154
         return writer
155
156
157
    def encode mcu(mcu, hts):
158
         bit stream = BitStreamWriter()
159
         for cur in mcu:
160
             for dct, (dc ht, ac ht) in zip(cur, hts):
161
                 dc code = encode 2s complement(dct[0])
                 container = [dc ht[len(dc code)], dc code]
162
163
                 rss, values = run length encode(dct[1:])
164
                 for rs, v in zip(rss, values):
165
                     container.append(ac ht[rs])
                     container.append(encode 2s complement(v))
166
167
                 bitstring = "".join(container)
168
                 bit stream.write bitstring(bitstring)
169
         return bit stream.to bytes()
170
171
172
    def encode jpeg(im, quality=95, subsample="4:2:0", use rm ht=True):
173
         im = np.expand dims(im, axis=-1) if im.ndim == 2 else im
174
         height, width, depth = im.shape
175
176
         mh. mw = mcu sizes[subsample] if depth == 3 else (BH, BW)
177
         im = padding(im, mh, mw)
178
         im = RGB2YCbCr(im) if depth == 3 else im
179
180
         # DC level shift for luminance.
         # the shift of chroma was completed by color conversion
181
182
         Y im = im\Gamma:, :, 0\Gamma - 128
183
         # divide image into MCUs
184
         mcu = divide blocks(Y im, mh, mw)
         # MCU to blocks, for luminance there are more than one blocks in each MCU
185
         Y = scan blocks(mcu, mh, mw)
186
187
         Y dct = DCT(Y)
188
         # the quantization table was already processed by zigzag scan,
         # so we apply zigzag encoding to DCT block first
189
```

```
190
        Y z = zigzag encode(Y dct)
191
        gt y = load quantization table(quality, "lum")
192
        Y q = quantization(Y z, qt v)
193
        Y p = DPCM(Y q)
194
        # whether to use recommended huffman table
195
        if use rm ht:
196
             Y dc ht, Y ac ht = reverse(RM Y DC), reverse(RM Y AC)
197
         else:
198
             Y dc ht = jpeqCreateHuffmanTable(np.vectorize(bits required)(Y p[:, 0]))
199
             Y ac ht = jpeqCreateHuffmanTable(
200
                 flatten(run length encode(Y p[i, 1:])[0] for i in range(Y p.shape[0]))
201
202
        qts, hts = [qt y], [Y dc ht, Y ac ht]
203
        cop_infos = [ComponentInfo(1, mw // BW, mh // BH, 0, 0, 0)]
        # the number of Y DCT blocks in an MCU
204
205
        num = (mw // BW) * (mh // BH)
206
        mcu hts = [(Y dc ht, Y ac ht) for in range(num)]
207
        # assign DCT blocks to MCUs
208
        mcu = Y p.reshape(-1, num, BH * BW)
209
210
        if depth == 3:
211
             # chroma subsample
212
             ch = im[:: mh // BH, :: mw // BW, 1:]
213
             Cb = divide blocks(ch[:, :, 0], BH, BW)
214
             Cr = divide blocks(ch[:, :, 1], BH, BW)
215
             Cb \ dct, Cr \ dct = DCT(Cb), DCT(Cr)
216
             Cb_z, Cr_z = zigzag_encode(Cb_dct), zigzag_encode(Cr_dct)
217
             gt c = load guantization table(guality, "chr")
218
             Cb g, Cr g = quantization(Cb z, gt c), quantization(Cr z, gt c)
219
             Cb p, Cr p = DPCM(Cb q), DPCM(Cr q)
220
             if use rm ht:
221
                 C dc ht, C ac ht = reverse(RM C DC), reverse(RM C AC)
222
             else:
223
                 ch = np.concatenate(ΓCb p, Cr pl, axis=0)
224
                 C dc ht = jpegCreateHuffmanTable(np.vectorize(bits required)(ch [:, 0]))
225
                 C ac ht = jpeqCreateHuffmanTable(
226
                     flatten(run length encode(ch [i, 1:])[0] for i in range(ch .shape[0]))
227
                                                                       4 D F A A B F A B F
```

```
228
            qts.append(qt c), hts.extend([C dc ht, C ac ht])
229
            cop infos.extend(
230
                [ComponentInfo(2, 1, 1, 1, 1, 1), ComponentInfo(3, 1, 1, 1, 1, 1)]
231
232
            mcu hts.extend((C dc ht, C ac ht) for in range(2))
233
            mcu = np.concatenate([mcu , Cb p[:, None], Cr p[:, None]], axis=1)
234
235
        writer = encode header(gts, hts, cop infos, height, width)
236
        bytes = encode mcu(mcu , mcu hts)
237
        writer.write(bytes .replace(b"\xff", b"\xff\x00"))
238
        writer.write(MARKER.E0I)
239
        return writer.getvalue()
240
241
242 def write jpeq(filename, im, quality=95, subsample="4:2:0", use rm ht=True):
243
        bytes = encode jpeq(im, quality, subsample, use rm ht)
244
        Path(filename).write bytes(bytes )
245
246
247 def main():
248
        im = Image.open("./data/villeLyon.jpg")
249
        write_jpeg("data/villeLyonLow.jpg", np.array(im), 5, "4:1:1", False)
250
251
252 if name == " main ":
253
        main()
254
```

encoder/comparator.py

```
import numpy as np
import encoder
import sys, os
from pathlib import Path
```

```
5 | from PIL import Image
6 import cv2
   import pandas as pd
   import time as t
   import shutil
10 import utils
11 import matplotlib.pyplot as plt
12 from encoder import DCT, padding
13 from scipy.fftpack import dct
14
   import random as rd
15
16
   LIM = 2 # number of files to test to
17
18
19
   def compare(
20
       qualities=None,
21
       dataDirectory=None,
       outputDirectory=None,
23
       subsamples=None,
24
       useStdHuffmanTable=None.
25
       DeleteFilesAfterward=True.
26
   ):
27
       if qualities is None:
28
           qualities = [np.random.randint(0, 101)]
29
       if subsamples is None:
30
           subsamples = \Gamma"4:2:2"1
31
       if dataDirectory is None:
32
           dataDirectorv = "./data/datasetBmp"
33
       if useStdHuffmanTable is None:
34
           useStdHuffmanTable = [False]
35
       stat = np.zeros(
36
           (LTM * len(qualities) * len(subsamples) * len(useStdHuffmanTable), 6).
37
           dtvpe=object.
38
          # dim 0 : quality factor, dim 1 : subsample method, dim 2 : usage of std Hf Tables, dim 3 : size
   before compression, dim 4: size after compression, dim 5: time to compress
39
       i = 0
       i max = LIM * len(qualities) * len(subsamples) * len(useStdHuffmanTable)
40
41
       filesTreated = rd.choices(os.listdir(dataDirectory), k=LTM)
                                                                      ◆□▶ ◆圖▶ ◆臺▶ - 臺
```

```
42
        for quality in qualities:
43
            for subsample in subsamples:
44
                for hfTables in useStdHuffmanTable:
45
                    outputDirectory = f"./data/treated/quality{quality}-subsample{subsample}-stdHf{hfTables}"
46
                    for filename in filesTreated:
47
                        f = os.path.join(dataDirectory, filename)
48
                        if not os.path.exists(outputDirectory):
49
                            os.makedirs(outputDirectory)
50
                        f out = os.path.join(outputDirectory, filename + ".jpq")
51
                        if os.path.isfile(f):
52
                            previousSize = os.stat(f).st size
53
                            image = Image.open(f)
54
                            time = t.time()
55
                            encoder.write ipea(
56
                                f out, np.array(image), quality, subsample, hfTables
57
58
                            time = t.time() - time
59
                            newSize = os.stat(f out).st size
60
                            stat[i][0] = quality
61
                            stat[i][1] = subsample
62
                            stat[i][2] = hfTables
63
                            stat[i][3] = previousSize
                            stat[i][4] = newSize
64
65
                            stat[i][5] = time
66
                        i += 1
67
                        print(f"{i}/{i max}", end="\r")
68
                    if DeleteFilesAfterward:
69
                        shutil.rmtree(outputDirectory)
70
        return stat
71
72
   def write stat(statFile, stat, quality, subsample, standHuffTables):
74
       with open(statFile, "a+") as f:
75
           f.write("\n" * 2)
76
           f.write("New sample \n")
77
           f.write(f"Size of sample : {LIM} images \n")
78
           f.write(
```

```
79
                 f"Parameters of compression : (quality) {quality}, (subsample) {subsample}, (usage of standard
    HuffTables) {'Yes' if standHuffTables else 'No'} \n"
80
81
             avgPreviousSize = np.average(stat[:, 0])
82
             avgNewSize = np.average(stat[:, 1])
83
             f.write(
84
                 f"Average size of image before compression : {avqPreviousSize} bytes \n"
85
86
             f.write(f"Average size of images after compression : {avgNewSize} bytes \n")
87
             f.write(f"Ratio is {avgPreviousSize / avgNewSize:.2f}")
88
89
90
    def write stat csv(output, stat):
91
        if os.path.isfile(output):
92
             pd.DataFrame(stat).to csv(output, mode="a", index=False, header=False)
93
        else:
94
             pd.DataFrame(stat).to csv(
95
                 output,
96
                 index=False.
97
                 header=[
98
                     "quality",
99
                     "subsample",
100
                     "stdHuffmanTables",
101
                     "oldSize".
102
                     "newSize".
103
                     "time".
104
                 ],
105
106
107
108
    def csv to stat(csvFile):
109
        stat = pd.read csv(csvFile)
110
         return stat
111
112
113
    def dataInterpreation(dataFrame):
114
        df = dataFrame
115
        qualities = df["quality"].unique()
```

```
qualitySize = {}
116
117
         qualityTime = {}
118
         for quality in qualities:
119
             qualitySize[quality] = int(df[df["quality"] == quality]["newSize"].mean())
120
             qualityTime[quality] = round(df[df["quality"] == quality]["time"].mean(), 3)
121
         stdSize = int(df[df["stdHuffmanTables"] == True]["newSize"].mean())
122
         stdTime = round(df[df["stdHuffmanTables"] == True]["time"].mean(), 3)
123
         nonStdSize = int(df[df["stdHuffmanTables"] == False]["newSize"].mean())
124
         nonStdTime = round(df[df["stdHuffmanTables"] == False]["time"].mean(), 3)
125
126
         plt.rcParams["figure.figsize"] = [10, 5]
127
128
         fig, (ax1, ax3) = plt.subplots(1, 2)
129
         ax2 = ax1.twinx()
130
131
         fig.suptitle("Comparaison des compressions en fonction du facteur de qualité")
132
133
         width = 0.25
134
135
         initialSize = 786486
136
         xaxis = list(qualitySize.kevs())
137
         yaxisSize = np.array(list(qualitySize.values()))
138
         vaxisTime = np.arrav(list(qualityTime.values()))
139
         yaxisRatio = (initialSize - yaxisSize) / yaxisTime
140
141
         color1 = "tab:red"
142
         color2 = "tab:blue"
143
         color3 = "tab:green"
144
145
         ax1.bar(
146
             np.arange(len(qualitySize)) - width.
147
            yaxisSize,
148
            width.
149
            tick label=xaxis.
150
            color=color1,
151
            label="Taille après compression",
152
153
         ax2.bar(
```

```
154
             np.arange(len(qualityTime)),
155
            yaxisTime,
156
            width.
157
             tick label=xaxis,
158
             color=color2,
159
             label="Temps de compression",
160
161
         ax3.bar(
162
             np.arange(len(qualitySize)),
163
            yaxisRatio,
            width.
164
165
             tick label=xaxis,
166
             color=color3,
167
             label="Octets gagnés par seconde",
168
169
170
         ax1.legend(loc="upper left")
171
         ax2.legend(loc="upper left", bbox to anchor=(0, 0.9))
172
         ax3.legend(loc="upper right")
173
174
         ax3.yaxis.tick right()
175
176
         ax1.set xlabel("Facteur de qualité")
177
         ax1.set ylabel("Taille (en octets)", color=color1)
         ax2.set vlabel("Temps (en secondes)", color=color2)
178
179
         ax3.set xlabel("Facteur de qualité")
180
         ax3.set ylabel("Taille gagné par unité de temps (octets.Hz)", color=color3)
         ax3.vaxis.set label position("right")
181
182
183
         plt.savefig("./data/treated/compressionComparaison", transparent=True)
184
185
         plt.rcParams["figure.figsize"] = [7, 5]
         plt.clf()
186
187
188
         fig = plt.figure()
189
         ax1 = fig.add subplot(111)
190
         ax2 = ax1.twinx()
191
```

```
192
         fig.suptitle(
193
             "Comparaison des compressions en fonction des tables de Huffman utilisées"
194
195
196
         yaxisSize = [stdSize, nonStdSize]
197
         yaxisTime = [stdTime, nonStdTime]
198
199
         labels = ["Tables Standards", "Tables Optimales"]
200
         ax1.bar(
201
             np.arange(2) - width / 2,
202
            vaxisSize.
203
            width,
204
            tick label=labels,
205
            color=color1.
            label="Taille après compression",
206
207
208
         ax2.bar(
209
             np.arange(2) + width / 2,
210
            vaxisTime.
211
            width.
212
            tick label=labels,
213
            color=color2,
214
            label="Temps de compression".
215
216
217
         ax1.legend(loc="upper center", bbox to anchor=(0.45, 1))
218
         ax2.legend(loc="upper center", bbox to anchor=(0.45, 0.9))
219
220
         ax1.set vlabel("Taille (en octets)", color=color1)
221
         ax2.set vlabel("Temps (en secondes)", color=color2)
222
         plt.savefig("./data/treated/compressionComparaison2", transparent=True)
224
225
226 def energyCompaction(imgPath):
227
         img = cv2.imread(imgPath)
228
229
         imgYCrCB = cv2.cvtColor(
```

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```
230
            img, cv2.COLOR RGB2YCrCb
231
         ) # Convert RGB to YCrCb (Cb applies V, and Cr applies U).
232
233
        Y, Cr, Cb = cv2.split(padding(imgYCrCB, 8, 8))
234
        Y = Y.astype("int") - 128
235
        blocks Y = utils.divide blocks(Y, 8, 8)
236
        dctBlocks Y = np.zeros like(blocks Y)
237
        for i in range(len(blocks Y)):
238
            dctBlocks Y[i] = dct(
239
                dct(blocks Y[i], axis=0, norm="ortho"), axis=1, norm="ortho"
240
241
        avg Y = utils.averageMatrix(blocks Y)
242
        avgDct Y = utils.averageMatrix(dctBlocks Y)
243
244
        x = np.random.randint(blocks Y.shape[0])
245
        arr1 = blocks Y[x]
246
        arr2 = dctBlocks Y[x]
247
248
        fig. (ax1, ax2) = plt.subplots(1, 2)
249
250
        valueMax, valueMin = max(np.max(arr1), np.max(arr2)), min(
251
            np.min(arr1), np.min(arr2)
252
253
        # fiq.suptitle('Matrice de la luminance de "villeLyon.jpg"')
254
255
        ax1.matshow(arr1, cmap="cool", vmin=valueMin, vmax=valueMax)
256
        ax1.set title("avant DCT")
257
258
        ax2.matshow(arr2, cmap="cool", vmin=valueMin, vmax=valueMax)
259
        ax2.set title("après DCT")
260
261
        for i in range(arr1.shape[0]):
            for i in range(arr1.shape[1]):
262
263
                cNormal = int(arr1[i, i])
264
                cDct = int(arr2[i, j])
265
                ax1.text(i, j, str(cNormal), va="center", ha="center")
266
                ax2.text(i, i, str(cDct), va="center", ha="center")
267
        plt.savefig("./data/energyCompaction.png", transparent=True)
                                                                       ◆□▶ ◆□▶ ◆□▶ ◆□▶
```

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```
269
270
    def rqbToYCbCr channel bis():
271
         img = cv2.imread("./data/villeLyon.jpg") # Read input image in BGR format
273
         imgYCrCB = cv2.cvtColor(
274
            ima. cv2.COLOR BGR2YCrCb
275
         ) # Convert RGB to YCrCb (Cb applies V, and Cr applies U).
276
277
        Y, Cr, Cb = cv2.split(imgYCrCB)
278
279
        # Fill Y and Cb with 128 (Y level is middle gray, and Cb is "neutralized").
280
        onlyCr = imgYCrCB.copy()
281
        onlyCr[:, :, 0] = 128
282
        onlyCr[:, :, 2] = 128
283
        onlyCr as bgr = cv2.cvtColor(
284
            onlvCr, cv2.COLOR YCrCb2BGR
285
         ) # Convert to BGR - used for display as false color
286
287
        # Fill Y and Cr with 128 (Y level is middle gray, and Cr is "neutralized").
288
        onlvCb = imgYCrCB.copv()
289
        onlyCb[:, :, 0] = 128
        onlvCb[:, :, 1] = 128
290
291
        onlyCb as bgr = cv2.cvtColor(
            onlvCb, cv2.C0L0R YCrCb2BGR
292
293
         ) # Convert to BGR - used for display as false color
294
295
        cv2.imshow("ima", ima)
296
        cv2.imshow("Y", Y)
297
        cv2.imshow("onlvCb as bgr", onlvCb as bgr)
298
        cv2.imshow("onlvCr as bgr", onlvCr as bgr)
299
        cv2.waitKey()
        cv2.destrovAllWindows()
300
301
        cv2.imwrite("./data/treated/villeLyon Y.jpg", Y)
302
303
        cv2.imwrite("./data/treated/villeLyon Cb.jpg", onlyCb as bgr)
304
        cv2.imwrite("./data/treated/villeLvon Cr.ipg", onlvCr as bgr)
305
```

268

```
306
307
    if name == " main ":
308
         # compare()
309
         # rgbToYCbCr channel bis()
310
         # energyCompaction("./data/villeLyon.jpg")
311
         \# test = np.array([[93, 90, 83, 68, 61, 61, 46, 21],
312
                           [102, 92, 95, 77, 65, 60, 49, 32],
313
                           [69, 55, 47, 57, 65, 60, 72, 65],
314
                           [55, 55, 40, 42, 23, 1, 11, 38],
315
                           [55, 57, 47, 53, 35, 59, -2, 26],
                           [64, 41, 42, 55, 60, 57, 25, -8],
316
317
                           [77, 87, 58, -2, -5, 14, -10, -35],
318
                           [38, 14, 33, 33, -21, -23, -43, -34]])
319
         # print(dct(dct(test, axis=0, norm="ortho"), axis=1, norm='ortho'))
320
321
         # stat = compare(qualities = list(range(1, 101, 10)), subsamples=['4:4:4', '4:2:0', '4:1:1', '4:2:2'],
     useStdHuffmanTable=[True, False], DelèteFilesAfterward=True)
322
         # write stat csv("./data/treated/stat.csv", stat)
323
         stat = csv to stat("./data/treated/stat.csv")
324
         dataInterpreation(stat)
325
```