TIPE - LA COMPRESSION DE DONNÉES

Et son application aux images

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Introduction

- **Compression** : maximum d'informations avec une taille minimal
- Deux types : **avec** (lossy) ou **sans** perte (lossless).
- Dans le domaine urbain : beaucoup d'informations $\rightarrow$ compression $\Rightarrow$ **stocker et traiter efficacement**

Procédés de Compression

- Compression sans perte : **entropique** et **algorithmique**
- Une **réorganisation** des données
- Application de **transformées mathématiques**
- Implémentation de l'**algorithme JPEG**

Entropie et Codage Optimal (1)

Théorie de l'Information de Shannon

Théorie probabiliste **quantifiant l'information** d'un ensemble de messages.

Définition - Entropie

Pour une source X comportant n symboles, un symbole x_i ayant une probabilité $p_i = \mathbb{P}(X = x_i)$ d'apparaître, l'entropie H est définie par :

$$H(X) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

Entropie et Codage Optimal (2)

Définition - Code de Source

Un **code de source C** pour une variable aléatoire X de distribution de probabilité p, est une application de Ω (ensemble des symboles sources) vers A^* (où A est l'alphabet du code).

Définition - Code Uniquement Décodable

Un code est dit **uniquement décodable** si

$$\forall x, y \in \Omega^+,\ x \neq y \implies C^+(x) \neq C^+(y)$$

Définition - Code Préfixe

Un code est dit préfixe si **aucun mot de code n'est le préfixe d'un autre mot** de code

Rq. : Code **préfixe** $\implies$ code **uniquement décodable**

Un code **non-préfixe**

a	0
b	1
c	01

$ab = 01 = c \rightarrow$ **Non uniquement décodable**

Un code **préfixe**

a	0
b	10
c	11

Chaque code est **unique**

Entropie et Codage Optimal (4)

Inégalité de Kraft

Pour un code défini sur un alphabet de taille D, et un alphabet de source Ω de taille $|\Omega|$, alors il est **préfixé** si et seulement si

$$\sum_{i=1}^{|\Omega|} D^{-l_i} \leq 1$$

$l_i =$ longueur des mots du codes

Théorème du Codage de Source - Shannon 1948

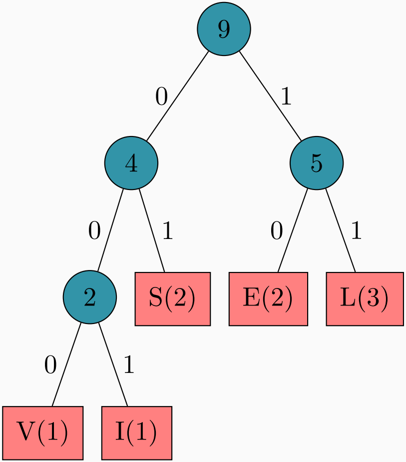
Lorsque l'efficacité de la compression augmente, la longueur moyenne du code tend vers l'entropie H.

Codage De Huffman (1)

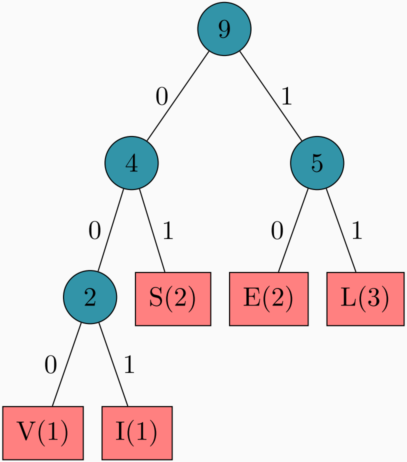
- Codage **optimal** au niveau **symbole**, à longueur **variable**
- Impose un **nombre entier de bits** pour un symbole
- Exemple de codage de "LES VILLES" :

symbole source	fréquence
L	3
E	2
S	2
V	1
I	1

Arbre de Huffman de "LES VILLES"



Arbre de Huffman de "LES VILLES"



- **Table de code** de Huffman

symbole source	code
L	11
E	10
S	01
V	000
I	001

- **Code** de Huffman : 1110010000011111001

Codage Arithmétique (1)

- Codage **optimal** au niveau **bit**, à longueur **variable**
- Principe : codage **par morceaux** et non par symbole (Huffman)
- Exemple de codage de "VILLE" :

symbole source	probabilité	intervalle
V	$1/5$	$[0; 0,2[$
I	$1/5$	$[0,2; 0,4[$
L	$2/5$	$[0,4; 0,8[$
E	$1/5$	$[0,8; 1[$

Ajout d'un symbole s :

1. $BB = BS - BI$
2. $BS \leftarrow BI + BB \times BS_s$
3. $BI \leftarrow BI + BB \times BI_s$

Codage Arithmétique (2)

symbole source	probabilité	intervalle
V	$1/5$	$[0; 0.2[$
I	$1/5$	$[0.2; 0.4[$
L	$2/5$	$[0.4; 0.8[$
E	$1/5$	$[0.8; 1[$

Ajout de $s =$ V :
$BS = BB = 1,\ BI = 0,\ BS_s = 0.2,\ \ BI_s = 0$
$BS \leftarrow 0 + 1 \times 0.2 = 0.2$
$BI \leftarrow 0 + 1 \times 0 = 0$

Ajout, ensuite, de $s' =$ I :
$BS = BB = 0.2,\ BI = 0,\ BS_{s'} = 0.4,\ \ BI_{s'} = 0.2$
$BS \leftarrow 0 + 0.2 \times 0.4 = 0.08$
$BI \leftarrow 0 + 0.2 \times 0.2 = 0.04$

...

Codage Arithmétique (3)

symbole source	probabilité	intervalle
V	$1/5$	$[0; 0.2[$
I	$1/5$	$[0.2; 0.4[$
L	$2/5$	$[0.4; 0.8[$
E	$1/5$	$[0.8; 1[$

Code de "VILLE" : $x \in [0.06752; 0.0688]$
Par exemple $x = 0.068$ fonctionne.

Décompression :

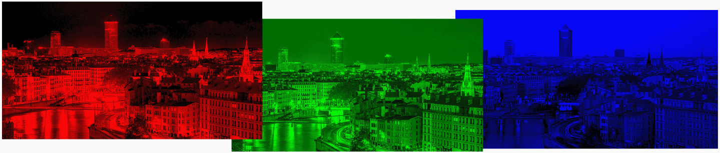
1. $x \in [0; 0.2] \rightarrow$ V
2. $x \leftarrow \frac{x - BI_V}{p_V} = 0.34$
3. $x \in [0.2; 0.4] \rightarrow$ VI
4. $x \leftarrow \frac{x - BI_I}{p_I} = 0.7$

...

5. Mot **décodé** : VILLE

La Représentation d'Image

$$\text{Image} = \begin{pmatrix} (r,g,b)_{1,1} & \cdots & (r,g,b)_{1,p} \\ \vdots & \ddots & \vdots \\ (r,g,b)_{n,1} & \cdots & (r,g,b)_{n,p} \end{pmatrix}$$



Référence Image: *laou.fr*

La Représentation YCbCr

Transformation YCbCr

$$\varphi\colon [\![0,255]\!]^3 \to [\![0,255]\!] \times [\![-128,127]\!]^2$$
$$X = (r,g,b) \longmapsto TX = (Y, Cb, Cr)$$

$$T = 255 \begin{pmatrix} K_r & K_r & K_b \\ -\frac{1}{2}\cdot\frac{K_r}{1-K_b} & -\frac{1}{2}\cdot\frac{K_g}{1-K_b} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2}\cdot\frac{K_g}{1-K_r} & -\frac{1}{2}\cdot\frac{K_b}{1-K_r} \end{pmatrix} \text{ et } K_r + K_g + K_b = 1$$

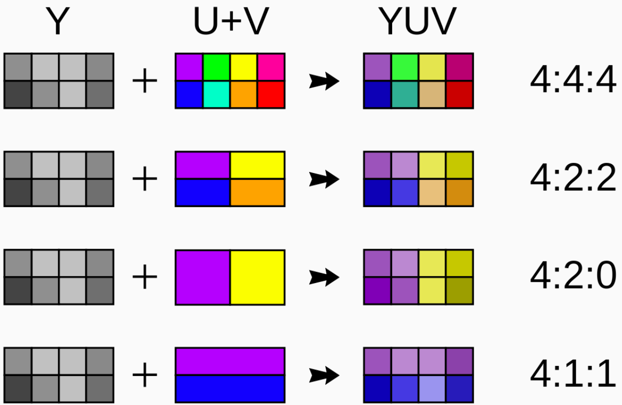
Rq. : En général $K_r = 0.299,\ K_g = 0.587,\ K_b = 0.114$

La Représentation YCbCr (2)

Œil humain → Y **prédomine**, Cb et Cr moins importants

Sous-Échantillonage

- **Principe** : Cb, Cr moins importants -> **moyenner** ces valeurs sur **plusieurs pixels**
- Exemples de **sous-échantillonnages** :



Référence Schéma: *wikipedia.org*

DCT (transformée en cosinus discrète) (1)

Transformation DCT

$$\psi \colon \mathbb{R}^N \to \mathbb{R}^N$$
$$(x_0, \ldots, x_{N-1}) \longmapsto \left(\sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}(n + \frac{1}{2})k \right] \right)_{k \in [\![0, N-1]\!]}$$

On peut rendre la matrice associée à ψ **orthogonale** en multipliant le terme X_0 par $\frac{1}{\sqrt{N}}$ et toute la matrice par $\sqrt{2/N}$.

2D-DCT $\to$ ψ sur chaque ligne puis chaque colonne

DCT (2)

- **"Continuité"** des images → peu de variations des **hautes fréquences**
- **Compactage** de l'énergie vers les **basses fréquences**

avant DCT

	0	1	2	3	4	5	6	7
0	20	20	19	18	16	14	10	9
1	16	17	17	16	16	14	12	11
2	13	14	15	15	15	15	14	13
3	14	14	14	14	14	14	13	13
4	16	16	15	14	14	13	12	12
5	18	17	16	14	14	14	13	14
6	17	16	16	15	15	16	17	18
7	15	15	15	15	16	19	20	22

après DCT

	0	1	2	3	4	5	6	7
0	121	6	0	0	0	-1	0	0
1	-4	9	-6	0	0	0	0	0
2	7	0	0	0	0	0	0	0
3	0	9	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0

Quantification

- Seule étape **avec perte** de la compression **JPEG**
- **Réduction** des coefficients
- **Différence** entre Y, Cb et Cr
- Fonction de quantification :

$$\begin{aligned} \varepsilon \colon \mathcal{M}_{8,8}(\mathbb{Z})^2 &\to \mathcal{M}_{8,8}(\mathbb{Z}) \\ Q, B &\longmapsto \lfloor B/Q \rfloor \end{aligned}$$

- Souvent Q dépend d'un **facteur de qualité** q (dans notre cas : $q \in [\![1, 100]\!]$)

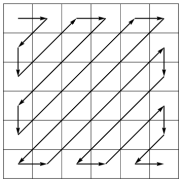
Codage par plages (Run-Length Encoding)

- Tire son avantage des **répétitions** de symboles
- Exemple :

$$\underbrace{AAAA}_{4\times A}\underbrace{B}_{1\times B}\underbrace{CCC}_{3\times C}\underbrace{BBBBBBB}_{7\times B} \xrightarrow[RLE]{} A4B1C3B7$$

  15 caractères $\rightarrow$ 8 caractères

- Pour faire apparaître ces répétitions dans les **images**, lecture de la matrice en **zigzag** :



Exemple de Compression

Image Originale

Taille : 10,3 Mo

Exemple de Compression

Image Compressée ($q = 100$, $4:4:4$) (presque **lossless**)

Taille : 2,2 Mo

Ratio : $\eta = \frac{T_{init}}{T_{compres}} = 4.68$

Exemple de Compression

Image Compressée ($q = 50$, $4:2:2$)

Taille : 519 Ko

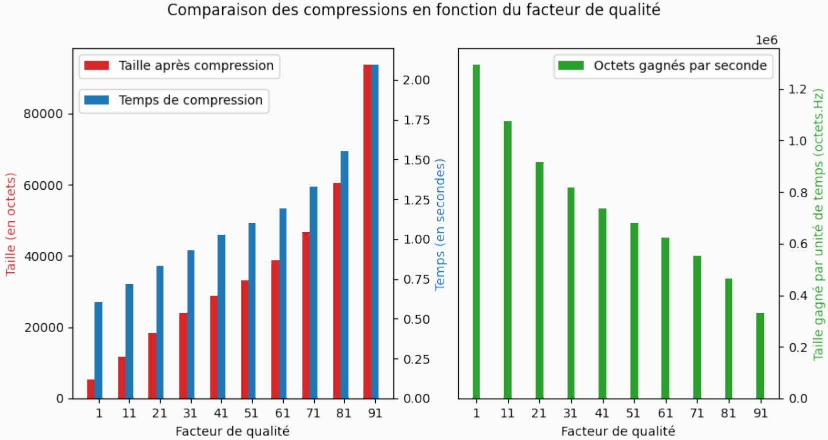
Ratio : $\eta = \frac{T_{init}}{T_{compres}} = 19.8$

Image Compressée ($q = 5$, $4 : 1 : 1$)

Taille : 77 Ko

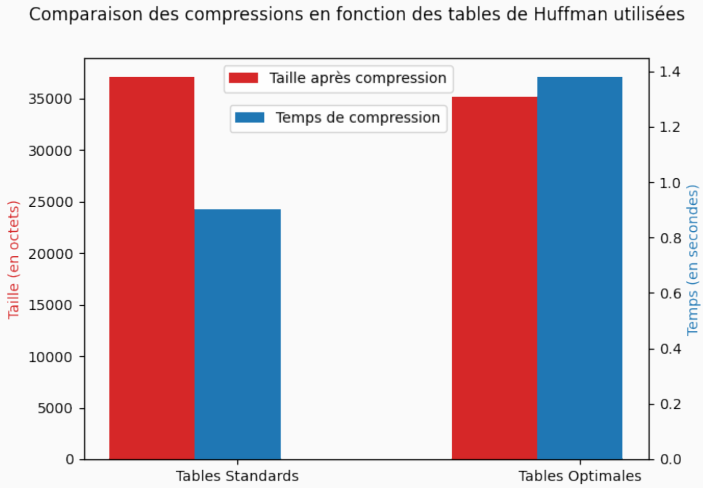
Ratio : $\eta = \frac{T_{init}}{T_{compres}} = 134$

Comparaison (1)



Conclusion : JPEG **plus efficace** lorsque la **qualité baisse**

Comparaison (2)



Conclusion : Le **gain en espace** est proportionnellement **inférieure** au **gain en temps**

Effectué

- ✓ **Étude théorique** des moyens de compression
- ✓ **Implémentation** des méthodes de compression en **python**
- ✓ **Implémentation** d'un encoder JPEG utilisant le **codage de Huffman** en **python**
- × **Implémentation** d'un encoder JPEG utilisant le **codage arithmétique** en **python**
- ∼ Comparaison des méthodes de compression sur des images
  - ✓ Comparaison en fonction du **facteur de qualité**
  - ✓ Comparaison en fonction du **codage de Huffman**
  - × Comparaison en fonction du **codage entropique** (Huffman ou Arithmétique)

Annexe - Démonstration Inégalité de Kraft (1)

Pour toute la démonstration, on considère un codage h ayant la propriété du préfixe. Soit $s_1, \dots, s_k$ et $l_1, \dots, l_k$ les mots codes et leur longueur.
On considère le cas où la taille de l'alphabet $D = 2$ (binaire), l'inégalité de Kraft s'écrit donc :

$$\sum_{i=1}^{k} 2^{-l_i} \leqslant 1$$

On note $m = \max l_i$ la plus grande longueur des mots codes. Comme le code a la propriété du préfixe, on peut placer, de manière unique, ces mots codes sur un arbre $\mathcal{T}$ binaire équilibré de profondeur m chaque mot code étant une feuille de cet arbre.
On peut donc compter le nombre de feuilles issues du mot de code s_i dans l'arbre complet binaire $\mathcal{T}$ de profondeur m Comme le mot code est à la profondeur l_i, le nœud correspondant est le père de 2^{m-l_i} feuilles à la profondeur m.

Annexe - Démonstration Inégalité de Kraft (2)

Comme le codage a la propriété du préfixe les sous arbres issus des mots codes sont disjoints et donc le nombre total de feuilles issues des mots codes à la profondeur m est égal à

$$\sum_{i=1}^{k} 2^{m-l_i}$$

Ce nombre de feuilles est inférieur ou égal au nombre total de feuilles à la profondeur m, c'est à dire à 2^m. En écrivant cette inégalité on obtient :

$$\sum_{i=1}^{k} 2^{m-l_i} \leqslant 2^m \implies \sum_{i=1}^{k} 2^{-l_i} \leqslant 1$$

La réciproque se montre en partant de l'arbre complet de profondeur m et en construisant les mots codes à partir des l_i.

Annexe - Démonstration Theorème du Codage de Source (1)

Pour cette démonstration, **on se limitera aux cas du codage par symboles** (type Huffman).
Montrons d'abord qu'étant donné un code uniquement décodable opérant sur une source X qui peut prendre ses valeurs dans l'alphabet fini A avec la distribution de probabilité p alors la longueur moyenne de ce code vérifie : $H(X) \leqslant \bar{l}$.

Si un code est uniquement décodable, il vérifie l'inégalité de Kraft, donc $\sum 2^{-l_i} \leqslant 1$.
On définie donc une nouvelle loi de probabilité : $q_i = a2^{-l_i}$ avec $a \geqslant 1$ tel que $\sum q_i = 1$.
L'inégalité de Gibbs (admise) donne :

$$\begin{aligned}
\underbrace{-\sum p_i \log_2(p_i)}_{=H(X)} &\leqslant -\sum p_i \log_2(q_i) \\
&= -\sum p_i \log_2(a2^{-l_i}) \\
&= \underbrace{-\sum p_i \log_2(\underbrace{a}_{\geqslant 1})}_{\leqslant 0} + \underbrace{\sum p_i l_i}_{=\bar{l}}
\end{aligned}$$

D'où $H(X) \leqslant \bar{l}$

Annexe - Démonstration Theorème du Codage de Source (2)

Montrons dorénavant qu'il existe un code uniquement décodable pour la source X de loi p tel que la longueur moyenne du code vérifie $\bar{l} < H(X) + 1$.

Pour tout i, notons $l_i \in \mathbb{N}^*$ tel que $2^{-l_i} \leqslant p_i < 2^{-l_i+1}$.
On a alors

$$\begin{aligned} & - l_i \leqslant \log_2 p_i < -l_i + 1 \\ \iff & - \log_2 p_i \leqslant l_i < - \log_2 p_k + 1 \end{aligned}$$

On alors directement : $H(X) \leqslant \bar{l} < H(X) + 1$ De plus $\sum 2^{l_k} \leqslant \sum 2^{\log_2 p_k} = 1$
L'inégalité de Kraft est donc vérifiée, on peut donc effectivement construire ce code, **d'où le résultat**.

Annexe - Exemple Matrice de Quantification

10	7	6	10	14	24	31	37
7	7	8	11	16	35	36	33
8	8	10	14	24	34	41	34
8	10	13	17	31	52	48	37
11	13	22	34	41	65	62	46
14	21	33	38	49	62	68	55
29	38	47	52	62	73	72	61
43	55	57	59	67	60	62	59

Quantization table 1

10	11	14	28	59	59	59	59
11	13	16	40	59	59	59	59
14	16	34	59	59	59	59	59
28	40	59	59	59	59	59	59
59	59	59	59	59	59	59	59
59	59	59	59	59	59	59	59
59	59	59	59	59	59	59	59
59	59	59	59	59	59	59	59

Quantization table 2

Référence Image: *researchgate.net*

Annexe - Listing Programmation

- ycbcr.py → slides 34-35
- huffman.py → slides 35-37
- fft.py → slides 38-41
- complex.py → slides 41-46
- arithmetic_coding.py → slides 46-49
- utils.py → slides 49-61
- huffman_jpeg.py → slides 61-75
- encoder.py → slides 75-82
- comparator.py → slides 82-91

Selected files

9 printable files

ycbcr.py
huffman.py
fft.py
complex.py
arithmetic_coding.py
encoder/utils.py
encoder/huffman_jpeg.py
encoder/encoder.py
encoder/comparator.py

ycbcr.py

```python
# Module transforming RGB images into YCbCr
import numpy as np
import numpy.linalg as alg

mat = np.array(
    [[65.481, 128.553, 24.966], [-37.797, -74.203, 112.0], [112.0, -93.786, -18.214]]
)

col = np.array([[16, 128, 128]])


def rgb_to_ycbcr(rgb: tuple) -> tuple:
    a = np.asarray(rgb)
    b = mat.dot(a)
    return tuple(b + col)


def ycbcr_to_rgb(t: tuple) -> tuple:
    a = np.asarray(t)
```

```
    b = alg.inv(mat)
    c = a - col
    d = b.dot(c[0])
    return tuple(d)

```

huffman.py

```
# Module computing Huffman compression


from collections import Counter, namedtuple
from heapq import heapify, heappop, heappush


# Node in a Huffman Tree
Node = namedtuple("Node", ["char", "freq"])

class HuffmanCompressor:
    """Huffman compression implementation"""
    def __init__(self):
        self.encoding_table = {}
        self.decoding_table = {}

    def build_tables(self, s: str):
        """create both the encodingn and decoding tables of a given string

    parameters
    ----------
        -s : string used to build the tables

    return
    ------
        - fill both the encoding and decoding table of the given class instance"""

```

```python
        freq_table = Counter(s)

        # create a heap of the nodes in the tree
        heap = []
        for char, freq in freq_table.items():
            heap.append(Node(char, freq))
        heapify(heap)

        # create the Huffman tree
        while len(heap) > 1:
            left_node = heappop(heap)
            right_node = heappop(heap)
            combined_node = Node(None, left_node.freq + right_node.freq)
            heappush(heap, combined_node)

        def build_encoding_table(node, code=''):
            if node.char is not None:
                # if the node is a leaf, add it to the encoding table
                self.encoding_table[node.char] = code
                return
            # if the node is not a leaf, recursively build the encoding table
            build_encoding_table(node.left, code + '0')
            build_encoding_table(node.right, code + '1')

        build_encoding_table(heap[0])


        def build_decoding_table(node, code=''):
            if node.char is not None:
                # if the node is a leaf, add it to the decoding table
                self.decoding_table[code] = node.char
                return
            # if the node is not a leaf, recursively build the decoding table
            build_decoding_table(node.left, code + "0")
            build_decoding_table(node.right, code + "1")

        build_decoding_table(heap[0])

```

```python
    def compress(self, s: str) -> str:
        """compress the inputed string

    parameters
    ----------
        -s : string to be compressed

    return
    ------
        - compressed string"""
        compressed = ""
        for char in s:
            compressed += self.encoding_table[char]
        return compressed

    def decompress(self, compressed: str) -> str:
        """decompress the inputed string

    parameters
    ----------
        -s : string to be compressed

    return
    ------
        - decompressed string"""
        decompressed = ""
        i = 0
        while i < len(compressed):
            for j in range(i+1, len(compressed)+1):
                if compressed[i:j] in self.decoding_table:
                    decompressed += self.decoding_table[compressed[i:j]]
                    i = j
                    break

        return decompressed
```

fft.py

```python
# fast-fourier transforms

import complex as cpx
from numpy import log2
from cmath import pi, exp, cos
from scipy.fftpack import dct, idct


def FFT(vector:list) -> list:
    """calculate the fast fourier tranform of a vector

    parameters
    ----------
        -vector : list of Complex object

    return
    ------
        - 1-D fast fourier transform of the vector"""
    n = len(vector)
    assert log2(n).is_integer(), "make sure that the length of the arguement is a power of 2"
    if n == 1:
        return vector
    poly_even, poly_odd = vector[::2] , vector[1::2]
    res_even, res_odd = FFT(poly_even), FFT(poly_odd)
    res = [cpx.Complex(0)] * n
    for j in range(n//2):
        w_j = cpx.exp_to_literal(-2*pi*j/n)
        product = w_j * res_odd[j]
        res[j] = res_even[j] + product
        res[j + n//2] = res_even[j] - product
    return res
```

```python

def IFFT_aux(vector:list) -> list:
    """auxiliary function that makes the recursive steps of the IFFT algorithm
    parameters
    ----------
        -vector : list of Complex object

    return
    ------
        - partial inverse of the 1-D fast fourier transform of the vector (lack the division by n)"""
    n = len(vector)
    assert log2(n).is_integer(), "make sure that the length of the arguement is a power of 2"
    if n == 1:
        return vector
    poly_even, poly_odd = vector[::2] , vector[1::2]
    res_even, res_odd = IFFT_aux(poly_even), IFFT_aux(poly_odd)
    res = [cpx.Complex(0)] * n
    for j in range(n//2):
        w_j = cpx.exp_to_literal((2 * pi * j) / n)
        product = w_j * res_odd[j]
        res[j] = res_even[j] + product
        res[j + n//2] = res_even[j] - product
    return res

def IFFT(vector:list) -> list:
    """caclulate the inverse of the fast fourier tranform of a vector (in order to have ifft(fft(poly)) ==
poly)

    parameters
    ----------
        -vector : list of Complex object

    return
    ------
        - inverse of the 1-D fast fourier transform of the vector"""
    n = len(vector)
    res = IFFT_aux(vector)
    for i in range(n):
```

```python
        res[i] = res[i] / cpx.Complex(n)
    return res

def DCT(vector:list, orthogonalize:bool =False, norm="forward"):
    """calculate the one-dimensional type-II discrete cosine tranform of a matrix (MAKHOUL) (using the FFT function previously defined)

    parameters
    ----------
        - vector: list of Numerical Object

    return
    ------
        - discrete cosine tranform of the input"""
    N = len(vector)
    temp = vector[ : : 2] + vector[-1 - N % 2 : : -2]
    temp = FFT(temp)
    factor = - pi / (N * 2)
    result = [2 * (val * (cpx.exp_to_literal(i * factor))).re for (i, val) in enumerate(temp)]
    if orthogonalize:
        result[0] *= 2 ** (-1 / 2)
    if norm == "ortho":
        result[0] *= (N) **(-1 / 2)
        result[1::] = [(2 / N) ** (1 / 2) * result[i] for i in range(1, len(result))]
    return result

def IDCT(vector:list):
    """calculate the one-dimensional "inverse" type-III discrete cosine tranform of a matrix (MAKHOUL) (using the FFT function previously defined)

    parameters
    --------
        - vector: list of Numerical Object

    return
    ------
        - type-III discrete cosine tranform of the input"""
    N = len(vector)
```

```python
    factor = - pi / (N * 2)
    temp = [(cpx.Complex(val) if i > 0 else (cpx.Complex(val) / cpx.Complex(2))) * cpx.exp_to_literal(i * factor) for (i, val) in enumerate(vector)]
    temp = FFT(temp)
    temp = [val.re for val in temp]
    result = [None] * N
    result[ : : 2] = temp[ : (N + 1) // 2]
    result[-1 - N % 2 : : -2] = temp[(N + 1) // 2 : ]
    return result

if __name__ == "__main__":
    vectorCpx= [cpx.Complex(5), cpx.Complex(2), cpx.Complex(4), cpx.Complex(8)]
    vector = [5, 2, 4, 8]
    print("DCT : ", DCT(vectorCpx))
    print("inverse + DCT : ", IDCT((DCT(vectorCpx))))
    print("scipy dct :", dct(vector))
    print("scipy + inverse dct: ", dct(idct(vector)))
    print("scipy dct (ortho) : ", dct(vector, norm = "ortho"))
    print("scipy inverse + dct (ortho) : ", idct(dct(vector, norm="ortho"), norm="ortho"))
```

complex.py

```python
# Module computing complex numbers
# disclaimer : this class is not made to deal with less than 1e-10 values


from numpy import arctan2, cos, pi, sin, sqrt
from math import isclose
from typing import Union, List


class Complex:
    """Computing complex numbers"""
    def __init__(self, real=0., imaginary=0.):
            self.re = real # round(real, 15)
```

```python
            self.im = imaginary # round(imaginary,15)
    def __str__(self) -> str:
        if self.im == 0.:
            string = f"{self.re}"
        elif self.re == 0:
            string = f"i({self.im})"
        else:
            string = f"{self.re} + i({self.im})"
        return string
    __repr__ = __str__
    def __eq__(self, other) -> bool:
        return bool(isclose(self.re, other.re) and isclose(self.im, other.im))
    def is_null(self):
        return isclose(self.re, 0) and isclose(self.im, 0)
    def is_real(self):
        return isclose(self.im, 0)
    def is_imaginary(self):
        return isclose(self.re, 0)
    def arg(self):
        """return the argument of the complex number
        return None if 0"""
        if self.is_null():
            arg = None
        elif isclose(self.re, 0) and self.im > 0:
            arg = pi / 2
        elif isclose(self.re, 0) and self.im < 0:
            arg = - pi / 2
        else:
            arg = round(arctan2(self.im, self.re), 15)
        return arg
    def module(self):
        """return the module of the complex number"""
        return round(sqrt(self.re**2 + self.im**2), 15)
    def conjuagate(self):
        return (Complex(self.re, -self.im))
    #arithmetic
    def __add__(self, other):
        return Complex(self.re + other.re, self.im + other.im)
```

```python
    def __sub__(self, other):
        return Complex(self.re - other.re, self.im - other.im)
    def __mul__(self, other):
        real = (self.re * other.re) - (self.im * other.im)
        imaginary = (self.re * other.im) + (self.im * other.re)
        return Complex(real, imaginary)
    def __truediv__(self, other):
        if other.is_null():
            raise ValueError("Error : dividing by 0")
        elif other.is_real():
            return Complex(self.re / other.re, self.im / other.re)
        else:
            denominator = (other.re ** 2) + (other.im ** 2)
            real = ((self.re * other.re) + (self.im * other.im)) / denominator
            imaginary = ((self.im * other.re) - (self.re * other.im)) / denominator
            return Complex(real, imaginary)

Num = Union[int, float]

def addition(*complexes:Complex) -> Complex: #partially depreciated (can still be usefull for more iterable arguments)
    """calculate the sum of complex numbers

    parameters
    ----------
        - *complexes : iterable type of Complex

    return
    ------
        - sum of the complex numbers"""

    res = Complex(0)
    for number in complexes:
        res.re += number.re
        res.im += number.im
    return res

```

```python
def difference(cpx1:Complex, cpx2:Complex = Complex(0)): #fully depreciated (replaced by __sub__ Complex methods)
    """calculate the difference of two complex numbers

    parameters
    ----------
        - cpx1 : Complex number
        - cpx2 : Complex number to subtract to cpx1 (=Complex(0) by default)

    return
    ------
        -   difference of the two complex numbers"""
    res = Complex()
    res.re = cpx1.re - cpx2.re
    res.im = cpx1.im - cpx2.im
    return res

def product(*complexes:Complex) -> Complex: #partially depreciated (can still be usefull for more iterable arguments)
    """calculate the product of complex numbers

    parameters
    ----------
        - *complexes : iterable type of Complex

    return
    ------
        - product of the complex numbers"""
    res = Complex(1)
    for number in complexes:
        re = res.re * number.re - res.im * number.im
        im = res.re * number.im + res.im * number.re
        res.re = re
        res.im = im
    return res

def exp_to_literal(arg:float, module:float = 1.0) -> Complex:
    """ return the literal expression of a complex number defined by its argument and module
```

```python

    parameters
    ----------
        - arg : type(float) (should be between 0 and 2pi)
        - module : type(float) (must have a positive value)(=1 by default)

    return
    ------
        - Complex number associated"""
    assert(module >= 0), "second-argument(module) must have a positive value"
    return Complex(module*cos(arg), module*sin(arg))

def nth_root(n:int, cpx:Complex = Complex(1)) -> Complex:
    """calculate the nth root of a complex number

    parameters
    ----------
        - n : type(int)
        - complex : type(Complex) (=Complex(1) by default) (must not be Complex(0))

    return
    ------
        - list of the nth roots"""
    assert(cpx.re != 0 or cpx.im != 0), "second argument must be a non-zero complex number"
    module = cpx.module()
    arg = cpx.arg()
    if arg is not None:
        return exp_to_literal((arg/n), module**(1/n))
    else:
        return Complex(1) #Not used case but just here to ensure nth_root cannot return None


def nth_roots_unity(n:int) -> list:
    """ calculate the n roots of unity

    parameter
    ---------
        - n : type(int) : must be a positive integer
```

```

    return
    ------
        - a list of Complex containing the n roots of unity"""
    roots = [Complex(1) for i in range(n)]
    for k in range(0,n):
        roots[k] = exp_to_literal((2*k*pi/n), 1.0)
    return roots

def inverse_nth_roots_unity(n:int) -> list:
    """ calculate the inversed n roots of unity

    parameter
    ---------
    - n : type(int) : must be a positive integer

    return
    ------
    - a list of Complex containing the inversed n roots of unity"""
    roots = [Complex(1) for i in range(n)]
    for k in range(0,n):
        roots[k] = exp_to_literal((-2*k*pi/n), 1.0)
    return roots

def make_complex(values:List[Num]) -> List[Complex]:
    res = []
    for value in values:
        res.extend([Complex(value)])
    return res


if __name__ == "__main__":
    pass

```

arithmetic_coding.py

```python
def proba(data):
    """
    Créer le dictionnaire de probabilités d'apparition des différents caractères
    """
    assert len(data) != 0
    d = {}
    for x in data:
        d[x] = d.get(x, 0) + (1 / len(data))
    return d


def create_int(data):
    """
    Créer le dictionnaire des intervalles des différents caractères connaissant les données
    """
    p = proba(data)
    d = {}
    n = 0.0
    for c, v in p.items():
        d[c] = (n, n + v)
        n += v
    return d


def create_int2(p):
    """
    Créer le dictionnaire des intervalles des différents caractères connaissant les probas des différents caractères
    """
    d = {}
    n = 0.0
    for c, v in p.items():
        d[c] = (n, n + v)
        n += v
    return d

```

```python

def encode(data):
    """
    effectue l'encodage des données
    """
    int = create_int(data)
    value = (0.0, 1.0)
    for x in data:
        d = value[1] - value[0]
        sup = value[0] + d * int[x][1]
        inf = value[0] + d * int[x][0]
        value = (inf, sup)
    return (value[0] + value[1]) / 2


def appartient(x, int):
    """
    teste l'appartenance de x à un intervalle fermé à gauche et ouvert à droite
    """
    assert len(int) == 2
    return x >= int[0] and x < int[1]


def inverse(dic):
    """
    renvoie le dictionnaire où les clés et valeurs sont inversées
    """
    d = {}
    for c, v in dic.items():
        d[v] = c
    return d


def decode(n, p, nbr_carac):
    d = inverse(create_int2(p))
    res = []
    i = n
    while len(res) < nbr_carac:
```

```python
        for c, v in d.items():
            if appartient(i, c):
                res.append(v)
                i = (i - c[0]) / (c[1] - c[0])
                break
    return res


# Examples

if __name__ == "__main__":
    print(encode("WIKI"))
    print(decode(0.171875, {"W": 0.25, "I": 0.5, "K": 0.25}, 4))
    print(encode("AAABBCCCCC"))
    print(decode(0.010783125000000005, {"A": 0.3, "B": 0.2, "C": 0.5}, 10))
    print(encode([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]))
    print(
        decode(
            encode([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]),
            {
                1: 0.1,
                2: 0.1,
                3: 0.1,
                4: 0.1,
                5: 0.1,
                6: 0.1,
                7: 0.1,
                8: 0.1,
                9: 0.1,
                10: 0.1,
            },
            10,
        )
    )

```

encoder/utils.py

```python
import numpy as np
import math
import cv2
from io import BytesIO


# DCT block size
BH, BW = 8, 8


class MARKER:
    SOI = b"\xff\xd8"
    APP0 = b"\xff\xe0"
    APPn = (b"\xff\xe1", b"\xff\xef")  # n=1~15
    DQT = b"\xff\xdb"
    SOF0 = b"\xff\xc0"
    DHT = b"\xff\xc4"
    DRI = b"\xff\xdd"
    SOS = b"\xff\xda"
    EOI = b"\xff\xd9"


class ComponentInfo:
    def __init__(self, id_, horizontal, vertical, qt_id, dc_ht_id, ac_ht_id):
        self.id_ = id_
        self.horizontal = horizontal
        self.vertical = vertical
        self.qt_id = qt_id
        self.dc_ht_id = dc_ht_id
        self.ac_ht_id = ac_ht_id

    @classmethod
    def default(cls):
        return cls.__init__(*[0 for _ in range(6)])

    def encode_SOS_info(self):
```

```python
        return int2bytes(self.id_, 1) + int2bytes(
            (self.dc_ht_id << 4) + self.ac_ht_id, 1
        )

    def encode_SOF0_info(self):
        return (
            int2bytes(self.id_, 1)
            + int2bytes((self.horizontal << 4) + self.vertical, 1)
            + int2bytes(self.qt_id, 1)
        )

    def __repr__(self):
        return (
            f"{self.id_}: qt-{self.qt_id}, ht-(dc-{self.dc_ht_id}, "
            f"ac-{self.ac_ht_id}), sample-{self.vertical, self.horizontal} "
        )


class BitStreamReader:
    """simulate bitwise read"""

    def __init__(self, bytes_: bytes):
        self.bits = np.unpackbits(np.frombuffer(bytes_, dtype=np.uint8))
        self.index = 0

    def read_bit(self):
        if self.index >= self.bits.size:
            raise EOFError("Ran out of element")
        self.index += 1
        return self.bits[self.index - 1]

    def read_int(self, length):
        result = 0
        for _ in range(length):
            result = result * 2 + self.read_bit()
        return result

    def __repr__(self):
```

```
        return f"[{self.index}, {self.bits.size}]"


class BitStreamWriter:
    """simulate bitwise write"""

    def __init__(self, length=10000):
        self.index = 0
        self.bits = np.zeros(length, dtype=np.uint8)

    def write_bitstring(self, bitstring):
        length = len(bitstring)
        if length + self.index > self.bits.size * 8:
            arr = np.zeros((length + self.index) // 8 * 2, dtype=np.uint8)
            arr[: self.bits.size] = self.bits
            self.bits = arr
        for bit in bitstring:
            self.bits[self.index // 8] |= int(bit) << (7 - self.index % 8)
            self.index += 1

    def to_bytes(self):
        return self.bits[: math.ceil(self.index / 8)].tobytes()

    def to_hex(self):
        length = math.ceil(self.index / 8) * 8
        for i in range(self.index, length):
            self.bits[i] = 1
        bytes_ = np.packbits(self.bits[:length])
        return " ".join(f"{b:2x}" for b in bytes_)


class BytesWriter(BytesIO):
    def __init__(self, *args, **kwargs):
        super(BytesWriter, self).__init__(*args, **kwargs)

    def add_bytes(self, *args):
        self.write(b"".join(args))

```

```python

def bytes2int(bytes_, byteorder="big"):
    return int.from_bytes(bytes_, byteorder)


def int2bytes(int_: int, length):
    return int_.to_bytes(length, byteorder="big")


def decode_2s_complement(complement, length) -> int:
    if length == 0:
        return 0
    if complement >> (length - 1) == 1:  # sign bit equal to one
        number = complement
    else:  # sign bit equal to zero
        number = 1 - 2**length + complement
    return number


def encode_2s_complement(number) -> str:
    """return the 2's complement representation as string"""
    if number == 0:
        return ""
    if number > 0:
        complement = bin(number)[2:]
    else:
        length = int(np.log2(-number)) + 1
        complement = bin(number - (1 - 2**length))[2:].zfill(length)
    return complement


def load_quantization_table(quality, component):
    # the below two tables was processed by zigzag encoding
    # in JPEG bit stream, the table is also stored in this order
    if component == "lum":
        q = np.array(
            [
                16,
```

```
                    11,
                    12,
                    14,
                    12,
                    10,
                    16,
                    14,
                    13,
                    14,
                    18,
                    17,
                    16,
                    19,
                    24,
                    40,
                    26,
                    24,
                    22,
                    22,
                    24,
                    49,
                    35,
                    37,
                    29,
                    40,
                    58,
                    51,
                    61,
                    60,
                    57,
                    51,
                    56,
                    55,
                    64,
                    72,
                    92,
                    78,
                    64,
```

```
                68,
                87,
                69,
                55,
                56,
                80,
                109,
                81,
                87,
                95,
                98,
                103,
                104,
                103,
                62,
                77,
                113,
                121,
                112,
                100,
                120,
                92,
                101,
                103,
                99,
            ],
            dtype=np.int32,
        )
    elif component == "chr":
        q = np.array(
            [
                17,
                18,
                18,
                24,
                21,
                24,
                47,
```

```
                    26,
                    26,
                    47,
                    99,
                    66,
                    56,
                    66,
                    99,
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                99,
            ],
            dtype=np.int32,
        )
    else:
        raise ValueError(
            (
                f"component should be either 'lum' or 'chr', "
                f"but '{component}' was found."
            )
        )
    if 0 < quality < 50:
        q = np.minimum(np.floor(50 / quality * q + 0.5), 255)
    elif 50 <= quality <= 100:
        q = np.maximum(np.floor((2 - quality / 50) * q + 0.5), 1)
    else:
        raise ValueError("quality should belong to (0, 100].")
    return q.astype(np.int32)
```

```python
def zigzag_points(rows, cols):
    # constants for directions
    UP, DOWN, RIGHT, LEFT, UP_RIGHT, DOWN_LEFT = range(6)

    move_func = {
        UP: lambda p: (p[0] - 1, p[1]),
        DOWN: lambda p: (p[0] + 1, p[1]),
        LEFT: lambda p: (p[0], p[1] - 1),
        RIGHT: lambda p: (p[0], p[1] + 1),
        UP_RIGHT: lambda p: move(UP, move(RIGHT, p)),
        DOWN_LEFT: lambda p: move(DOWN, move(LEFT, p)),
    }

    # move the point in different directions
    def move(direction, point):
        return move_func[direction](point)

    # return true if point is inside the block bounds
    def inbounds(p):
        return 0 <= p[0] < rows and 0 <= p[1] < cols

    # start in the top-left cell
    now = (0, 0)

    # True when moving up-right, False when moving down-left
    move_up = True
    trace = []

    for i in range(rows * cols):
        trace.append(now)
        if move_up:
            if inbounds(move(UP_RIGHT, now)):
                now = move(UP_RIGHT, now)
            else:
                move_up = False
                if inbounds(move(RIGHT, now)):
                    now = move(RIGHT, now)
                else:
```

```python
                    now = move(DOWN, now)
        else:
            if inbounds(move(DOWN_LEFT, now)):
                now = move(DOWN_LEFT, now)
            else:
                move_up = True
                if inbounds(move(DOWN, now)):
                    now = move(DOWN, now)
                else:
                    now = move(RIGHT, now)
    """
    for rows = cols = 8, the actual 1-D index:
        0, 1, 8, 16, 9, 2, 3, 10, 17, 24, 32, 25, 18, 11, 4, 5,
        12, 19, 26, 33, 40, 48, 41, 34, 27, 20, 13, 6, 7, 14, 21, 28,
        35, 42, 49, 56, 57, 50, 43, 36, 29, 22, 15, 23, 30, 37, 44, 51,
        58, 59, 52, 45, 38, 31, 39, 46, 53, 60, 61, 54, 47, 55, 62, 63
    """
    return trace


def RGB2YCbCr(im):
    im = im.astype(np.float32)
    im = cv2.cvtColor(im, cv2.COLOR_RGB2YCrCb)
    """
    RGB [0, 255]
    opencv uses the following equations to conduct color conversion in float32
        Y = 0.299 * R + 0.587 * G + 0.114 * B
        Cb = (B - Y) * 0.564 + 0.5
        Cr = (R - Y) * 0.713 + 0.5
    Y [0, 255], Cb, Cr [-128, 127]
    """
    # convert YCrCb to YCbCr
    Y, Cr, Cb = np.split(im, 3, axis=-1)
    im = np.concatenate([Y, Cb, Cr], axis=-1)
    return im


def YCbCr2RGB(im):
```

```python
    im = im.astype(np.float32)
    Y, Cb, Cr = np.split(im, 3, axis=-1)
    im = np.concatenate([Y, Cr, Cb], axis=-1)
    im = cv2.cvtColor(im, cv2.COLOR_YCrCb2RGB)
    """
    Y [0, 255], Cb, Cr [-128, 127]
    conversion equation (float32):
        B = (Cb - 0.5) / 0.564 + Y
        R = (Cr - 0.5) / 0.713 + Y
        G = (Y - 0.299 * R - 0.114 * B) / 0.587
    RGB [0, 255]
    """
    return im


def bits_required(n):
    n = abs(n)
    result = 0
    while n > 0:
        n >>= 1
        result += 1
    return result


def divide_blocks(im, mh, mw):
    h, w = im.shape[:2]
    return im.reshape(h // mh, mh, w // mw, mw).swapaxes(1, 2).reshape(-1, mh, mw)


def restore_image(block, nh, nw):
    bh, bw = block.shape[1:]
    return block.reshape(nh, nw, bh, bw).swapaxes(1, 2).reshape(nh * bh, nw * bw)


def flatten(lst):
    return [item for sublist in lst for item in sublist]
```

```python
def averageMatrix(
    arrayMatrix,
):  # given an array of 2D-array, return the average (coef by coef) 2D array
    avgMatrix = np.zeros_like(arrayMatrix[0])
    for i in range(avgMatrix.shape[0]):
        for j in range(avgMatrix.shape[1]):
            avgMatrix[i, j] = np.average(arrayMatrix[:, i, j])
    return avgMatrix


if __name__ == "__main__":
    arrMatrix = np.array([[[1, 2], [3, 4]], [[5, 2], [3, 4]]])
    print(averageMatrix(arrMatrix))

```

encoder/huffman_jpeg.py

```python
import numpy as np

MAX_CLEN = 32  # assumed maximum initial code length


def getFreq(data):
    freq = [0] * 257
    for elem in data:
        freq[elem] += 1
    freq[256] = 1
    return freq


def jpegGenerateOptimalTable(freq):
    bits = [0] * (MAX_CLEN + 1)
    bitPos = [0] * (MAX_CLEN + 1)
    codesize = [0] * 257
```

```
    nzIndex = [0] * 257

    others = [-1] * 257

    numNzSymbols = 0
    for i in range(257):
        if freq[i]:
            nzIndex[numNzSymbols] = i
            freq[numNzSymbols] = freq[i]
            numNzSymbols += 1

    huffval = [0] * (numNzSymbols - 1)

    while True:
        c1 = -1
        c2 = -1
        v = 1000000000
        v2 = 1000000000
        for i in range(numNzSymbols):
            if freq[i] <= v2:
                if freq[i] <= v:
                    c2 = c1
                    v2 = v
                    v = freq[i]
                    c1 = i
                else:
                    v2 = freq[i]
                    c2 = i

        if c2 < 0:
            break

        freq[c1] += freq[c2]
        freq[c2] = 1000000001

        codesize[c1] += 1
        while others[c1] >= 0:
            c1 = others[c1]
```

```
            codesize[c1] += 1

        others[c1] = c2

        codesize[c2] += 1
        while others[c2] >= 0:
            c2 = others[c2]
            codesize[c2] += 1

    for i in range(numNzSymbols):
        bits[codesize[i]] += 1

    p = 0
    for i in range(1, MAX_CLEN + 1):
        bitPos[i] = p
        p += bits[i]

    for i in range(MAX_CLEN, 16, -1):
        while bits[i] > 0:
            j = i - 2
            while bits[j] == 0:
                j -= 1
            bits[i] -= 2
            bits[i - 1] += 1
            bits[j + 1] += 2
            bits[j] -= 1

    i = MAX_CLEN
    while bits[i] == 0:
        i -= 1
    bits[i] -= 1

    for i in range(numNzSymbols - 1):
        huffval[bitPos[codesize[i]]] = nzIndex[i]
        bitPos[codesize[i]] += 1

    return bits, huffval
```

```python
def jpegGenerateHuffmanTable(bits, huffval):
    huffsize = [0] * 257
    huffcode = [0] * 257

    p = 0
    for l in range(1, 17):
        i = bits[l]
        while i:
            huffsize[p] = l
            p += 1
            i -= 1

    huffsize[p] = 0
    lastp = p

    code = 0
    si = huffsize[0]
    p = 0
    while huffsize[p]:
        while huffsize[p] == si:
            huffcode[p] = code
            code += 1
            p += 1
        code <<= 1
        si += 1

    ehufco = [0] * 257
    ehufsi = [0] * 257

    for p in range(lastp):
        i = huffval[p]
        ehufco[i] = huffcode[p]
        ehufsi[i] = huffsize[p]

    return ehufsi, ehufco
```

```python
def jpegTransformTable(ehufsi, ehufco):
    table = {}
    for i in range(len(ehufco)):
        if ehufsi[i] != 0:
            endCode = bin(ehufco[i])[2:]
            nbZeros = ehufsi[i] - len(endCode)
            table[i] = "0" * nbZeros + endCode
    return table


def jpegCreateHuffmanTable(arr):
    freq = getFreq(arr)
    bits, huffval = jpegGenerateOptimalTable(freq)
    ehufsi, ehufco = jpegGenerateHuffmanTable(bits, huffval)
    table = jpegTransformTable(ehufsi, ehufco)
    return table


def convert_huffman_table(table):
    """convert huffman table to count and weigh"""
    # table[int] = string
    pairs = sorted(table.items(), key=lambda x: (len(x[1]), x[1]))
    weigh, codes = zip(*pairs)
    weigh = np.array(weigh, dtype=np.uint8)
    # count[i]: there are count[i] codes of length i+1
    count = np.zeros(16, dtype=np.uint8)
    for c in codes:
        count[len(c) - 1] += 1
    return count, weigh


def read_huffman_code(table, stream):
    prefix = ""
    while prefix not in table:
        prefix += str(stream.read_bit())
    return table[prefix]
```

```python
def reverse(table):
    return {v: k for k, v in table.items()}


# 4 recommended huffman tables in JPEG standard
# luminance DC
RM_Y_DC = {
    "00": 0,
    "010": 1,
    "011": 2,
    "100": 3,
    "101": 4,
    "110": 5,
    "1110": 6,
    "11110": 7,
    "111110": 8,
    "1111110": 9,
    "11111110": 10,
    "111111110": 11,
}

# luminance AC
RM_Y_AC = {
    "00": 1,
    "01": 2,
    "100": 3,
    "1010": 0,
    "1011": 4,
    "1100": 17,
    "11010": 5,
    "11011": 18,
    "11100": 33,
    "111010": 49,
    "111011": 65,
    "1111000": 6,
    "1111001": 19,
    "1111010": 81,
    "1111011": 97,
```

```
    "11111000": 7,
    "11111001": 34,
    "11111010": 113,
    "111110110": 20,
    "111110111": 50,
    "111111000": 129,
    "111111001": 145,
    "111111010": 161,
    "1111110110": 8,
    "1111110111": 35,
    "1111111000": 66,
    "1111111001": 177,
    "1111111010": 193,
    "11111110110": 21,
    "11111110111": 82,
    "11111111000": 209,
    "11111111001": 240,
    "111111110100": 36,
    "111111110101": 51,
    "111111110110": 98,
    "111111110111": 114,
    "111111111000000": 130,
    "1111111110000010": 9,
    "1111111110000011": 10,
    "1111111110000100": 22,
    "1111111110000101": 23,
    "1111111110000110": 24,
    "1111111110000111": 25,
    "1111111110001000": 26,
    "1111111110001001": 37,
    "1111111110001010": 38,
    "1111111110001011": 39,
    "1111111110001100": 40,
    "1111111110001101": 41,
    "1111111110001110": 42,
    "1111111110001111": 52,
    "1111111110010000": 53,
    "1111111110010001": 54,
```

```
    "1111111110010010": 55,
    "1111111110010011": 56,
    "1111111110010100": 57,
    "1111111110010101": 58,
    "1111111110010110": 67,
    "1111111110010111": 68,
    "1111111110011000": 69,
    "1111111110011001": 70,
    "1111111110011010": 71,
    "1111111110011011": 72,
    "1111111110011100": 73,
    "1111111110011101": 74,
    "1111111110011110": 83,
    "1111111110011111": 84,
    "1111111110100000": 85,
    "1111111110100001": 86,
    "1111111110100010": 87,
    "1111111110100011": 88,
    "1111111110100100": 89,
    "1111111110100101": 90,
    "1111111110100110": 99,
    "1111111110100111": 100,
    "1111111110101000": 101,
    "1111111110101001": 102,
    "1111111110101010": 103,
    "1111111110101011": 104,
    "1111111110101100": 105,
    "1111111110101101": 106,
    "1111111110101110": 115,
    "1111111110101111": 116,
    "1111111110110000": 117,
    "1111111110110001": 118,
    "1111111110110010": 119,
    "1111111110110011": 120,
    "1111111110110100": 121,
    "1111111110110101": 122,
    "1111111110110110": 131,
    "1111111110110111": 132,
```

```
    "1111111110111000": 133,
    "1111111110111001": 134,
    "1111111110111010": 135,
    "1111111110111011": 136,
    "1111111110111100": 137,
    "1111111110111101": 138,
    "1111111110111110": 146,
    "1111111110111111": 147,
    "1111111111000000": 148,
    "1111111111000001": 149,
    "1111111111000010": 150,
    "1111111111000011": 151,
    "1111111111000100": 152,
    "1111111111000101": 153,
    "1111111111000110": 154,
    "1111111111000111": 162,
    "1111111111001000": 163,
    "1111111111001001": 164,
    "1111111111001010": 165,
    "1111111111001011": 166,
    "1111111111001100": 167,
    "1111111111001101": 168,
    "1111111111001110": 169,
    "1111111111001111": 170,
    "1111111111010000": 178,
    "1111111111010001": 179,
    "1111111111010010": 180,
    "1111111111010011": 181,
    "1111111111010100": 182,
    "1111111111010101": 183,
    "1111111111010110": 184,
    "1111111111010111": 185,
    "1111111111011000": 186,
    "1111111111011001": 194,
    "1111111111011010": 195,
    "1111111111011011": 196,
    "1111111111011100": 197,
    "1111111111011101": 198,
```

```
    "1111111111011110": 199,
    "1111111111011111": 200,
    "1111111111100000": 201,
    "1111111111100001": 202,
    "1111111111100010": 210,
    "1111111111100011": 211,
    "1111111111100100": 212,
    "1111111111100101": 213,
    "1111111111100110": 214,
    "1111111111100111": 215,
    "1111111111101000": 216,
    "1111111111101001": 217,
    "1111111111101010": 218,
    "1111111111101011": 225,
    "1111111111101100": 226,
    "1111111111101101": 227,
    "1111111111101110": 228,
    "1111111111101111": 229,
    "1111111111110000": 230,
    "1111111111110001": 231,
    "1111111111110010": 232,
    "1111111111110011": 233,
    "1111111111110100": 234,
    "1111111111110101": 241,
    "1111111111110110": 242,
    "1111111111110111": 243,
    "1111111111111000": 244,
    "1111111111111001": 245,
    "1111111111111010": 246,
    "1111111111111011": 247,
    "1111111111111100": 248,
    "1111111111111101": 249,
    "1111111111111110": 250,
}

# chroma DC
RM_C_DC = {
    "00": 0,
```

```
    "01": 1,
    "10": 2,
    "110": 3,
    "1110": 4,
    "11110": 5,
    "111110": 6,
    "1111110": 7,
    "11111110": 8,
    "111111110": 9,
    "1111111110": 10,
    "11111111110": 11,
}

# chroma AC
RM_C_AC = {
    "00": 0,
    "01": 1,
    "100": 2,
    "1010": 3,
    "1011": 17,
    "11000": 4,
    "11001": 5,
    "11010": 33,
    "11011": 49,
    "111000": 6,
    "111001": 18,
    "111010": 65,
    "111011": 81,
    "1111000": 7,
    "1111001": 97,
    "1111010": 113,
    "11110110": 19,
    "11110111": 34,
    "11111000": 50,
    "11111001": 129,
    "111110100": 8,
    "111110101": 20,
    "111110110": 66,
```

```
    "111110111": 145,
    "111111000": 161,
    "111111001": 177,
    "111111010": 193,
    "1111110110": 9,
    "1111110111": 35,
    "1111111000": 51,
    "1111111001": 82,
    "1111111010": 240,
    "11111110110": 21,
    "11111110111": 98,
    "11111111000": 114,
    "11111111001": 209,
    "111111110100": 10,
    "111111110101": 22,
    "111111110110": 36,
    "111111110111": 52,
    "11111111100000": 225,
    "111111111000010": 37,
    "111111111000011": 241,
    "1111111110001000": 23,
    "1111111110001001": 24,
    "1111111110001010": 25,
    "1111111110001011": 26,
    "1111111110001100": 38,
    "1111111110001101": 39,
    "1111111110001110": 40,
    "1111111110001111": 41,
    "1111111110010000": 42,
    "1111111110010001": 53,
    "1111111110010010": 54,
    "1111111110010011": 55,
    "1111111110010100": 56,
    "1111111110010101": 57,
    "1111111110010110": 58,
    "1111111110010111": 67,
    "1111111110011000": 68,
    "1111111110011001": 69,
```

```
    "1111111110011010": 70,
    "1111111110011011": 71,
    "1111111110011100": 72,
    "1111111110011101": 73,
    "1111111110011110": 74,
    "1111111110011111": 83,
    "1111111110100000": 84,
    "1111111110100001": 85,
    "1111111110100010": 86,
    "1111111110100011": 87,
    "1111111110100100": 88,
    "1111111110100101": 89,
    "1111111110100110": 90,
    "1111111110100111": 99,
    "1111111110101000": 100,
    "1111111110101001": 101,
    "1111111110101010": 102,
    "1111111110101011": 103,
    "1111111110101100": 104,
    "1111111110101101": 105,
    "1111111110101110": 106,
    "1111111110101111": 115,
    "1111111110110000": 116,
    "1111111110110001": 117,
    "1111111110110010": 118,
    "1111111110110011": 119,
    "1111111110110100": 120,
    "1111111110110101": 121,
    "1111111110110110": 122,
    "1111111110110111": 130,
    "1111111110111000": 131,
    "1111111110111001": 132,
    "1111111110111010": 133,
    "1111111110111011": 134,
    "1111111110111100": 135,
    "1111111110111101": 136,
    "1111111110111110": 137,
    "1111111110111111": 138,
```

```
    "1111111111000000": 146,
    "1111111111000001": 147,
    "1111111111000010": 148,
    "1111111111000011": 149,
    "1111111111000100": 150,
    "1111111111000101": 151,
    "1111111111000110": 152,
    "1111111111000111": 153,
    "1111111111001000": 154,
    "1111111111001001": 162,
    "1111111111001010": 163,
    "1111111111001011": 164,
    "1111111111001100": 165,
    "1111111111001101": 166,
    "1111111111001110": 167,
    "1111111111001111": 168,
    "1111111111010000": 169,
    "1111111111010001": 170,
    "1111111111010010": 178,
    "1111111111010011": 179,
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    "1111111111011001": 185,
    "1111111111011010": 186,
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    "1111111111011101": 196,
    "1111111111011110": 197,
    "1111111111011111": 198,
    "1111111111100000": 199,
    "1111111111100001": 200,
    "1111111111100010": 201,
    "1111111111100011": 202,
    "1111111111100100": 210,
    "1111111111100101": 211,
```

```
    "1111111111100110": 212,
    "1111111111100111": 213,
    "1111111111101000": 214,
    "1111111111101001": 215,
    "1111111111101010": 216,
    "1111111111101011": 217,
    "1111111111101100": 218,
    "1111111111101101": 226,
    "1111111111101110": 227,
    "1111111111101111": 228,
    "1111111111110000": 229,
    "1111111111110001": 230,
    "1111111111110010": 231,
    "1111111111110011": 232,
    "1111111111110100": 233,
    "1111111111110101": 234,
    "1111111111110110": 242,
    "1111111111110111": 243,
    "1111111111111000": 244,
    "1111111111111001": 245,
    "1111111111111010": 246,
    "1111111111111011": 247,
    "1111111111111100": 248,
    "1111111111111101": 249,
    "1111111111111110": 250,
}

if __name__ == "__main__":
    arr = np.array([np.random.randint(-127, 128) for _ in range(64)])
    table = jpegCreateHuffmanTable(arr)
    print(table)
```

encoder/encoder.py

```python
from math import ceil
import cv2
import numpy as np
from PIL import Image
from pathlib import Path

from utils import *
from huffman_jpeg import *


def padding(im, mh, mw):
    """
    pad use boundary pixels so that its height and width are
    the multiple of the height and width of MCUs, respectively
    """
    h, w, d = im.shape
    if h % mh == 0 and w % mw == 0:
        return im
    hh, ww = ceil(h / mh) * mh, ceil(w / mw) * mw
    im_ex = np.zeros_like(im, shape=(hh, ww, d))
    im_ex[:h, :w] = im
    im_ex[:, w:] = im_ex[:, w - 1 : w]
    im_ex[h:, :] = im_ex[h - 1 : h, :]
    return im_ex


mcu_sizes = {
    "4:2:0": (BH * 2, BW * 2),
    "4:1:1": (BH * 2, BW * 2),
    "4:2:2": (BH, BW * 2),
    "4:4:4": (BH, BW),
}


def scan_blocks(mcu, mh, mw):
    """
    scan MCU to blocks for DPCM, for 4:2:0, the scan order is as follows:
```

```
    --------- | ---------
    | 0 | 1 | | | 4 | 5 |
    --------- | ---------
    | 2 | 3 | | | 6 | 7 |
    --------- | ---------
    """
    blocks = (
        mcu.reshape(-1, mh // BH, BH, mw // BW, BW).swapaxes(2, 3).reshape(-1, BH, BW)
    )
    return blocks


def DCT(blocks):
    dct = np.zeros_like(blocks)
    for i in range(blocks.shape[0]):
        dct[i] = cv2.dct(blocks[i])
    return dct


def zigzag_encode(dct):
    trace = zigzag_points(BH, BW)
    zz = np.zeros_like(dct).reshape(-1, BH * BW)
    for i, p in enumerate(trace):
        zz[:, i] = dct[:, p[0], p[1]]
    return zz


def quantization(dct, table):
    ret = dct / table[None]
    return np.round(ret).astype(np.int32)


def DPCM(dct):
    """
    encode the DC differences
    """
    dc_pred = dct.copy()
    dc_pred[1:, 0] = dct[1:, 0] - dct[:-1, 0]
```

```python
    return dc_pred


def run_length_encode(arr):
    # determine where the sequence is ending prematurely
    last_nonzero = -1
    for i, elem in enumerate(arr):
        if elem != 0:
            last_nonzero = i
    rss, values = [], []
    run_length = 0
    for i, elem in enumerate(arr):
        if i > last_nonzero:
            rss.append(0)
            values.append(0)
            break
        elif elem == 0 and run_length < 15:
            run_length += 1
        else:
            size = bits_required(elem)
            rss.append((run_length << 4) + size)
            values.append(elem)
            run_length = 0
    return rss, values


def encode_header(qts, hts, cop_infos, height, width):
    writer = BytesWriter()
    add_bytes = writer.add_bytes
    add_bytes(
        MARKER.SOI,
        MARKER.APP0,
        b"\x00\x10",  # length = 16
        b"JFIF\x00",  # identifier = JFIF0
        b"\x01\x01",  # version
        b"\x00",  # unit
        b"\x00\x01",  # x density
        b"\x00\x01",  # y density
```

```
        b"\x00\x00",  # thumbnail data
    )
    for id_, qt in enumerate(qts):
        add_bytes(
            MARKER.DQT,
            b"\x00C",  # length = 67
            # precision (8 bits), table id, = 0, id_
            int2bytes(id_, 1),
            qt.astype(np.uint8).tobytes(),
        )
    cop_num = len(cop_infos)
    add_bytes(
        MARKER.SOF0,
        int2bytes(8 + 3 * cop_num, 2),  # length
        int2bytes(8, 1),  # 8 bit precision
        int2bytes(height, 2),
        int2bytes(width, 2),
        int2bytes(cop_num, 1),
    )
    add_bytes(*[info.encode_SOF0_info() for info in cop_infos])

    # type << 4 + id, (type 0: DC, 1 : AC)
    type_ids = [b"\x00", b"\x10", b"\x01", b"\x11"]
    for type_id, ht in zip(type_ids, hts):
        count, weigh = convert_huffman_table(ht)
        ht_bytes = count.tobytes() + weigh.tobytes()
        add_bytes(
            MARKER.DHT,
            int2bytes(len(ht_bytes) + 3, 2),  # length
            type_id,
            ht_bytes,
        )

    add_bytes(
        MARKER.SOS,
        int2bytes(6 + cop_num * 2, 2),  # length
        int2bytes(cop_num, 1),
    )
```

```
    add_bytes(*[info.encode_SOS_info() for info in cop_infos])
    add_bytes(b"\x00\x3f\x00")
    return writer


def encode_mcu(mcu, hts):
    bit_stream = BitStreamWriter()
    for cur in mcu:
        for dct, (dc_ht, ac_ht) in zip(cur, hts):
            dc_code = encode_2s_complement(dct[0])
            container = [dc_ht[len(dc_code)], dc_code]
            rss, values = run_length_encode(dct[1:])
            for rs, v in zip(rss, values):
                container.append(ac_ht[rs])
                container.append(encode_2s_complement(v))
            bitstring = "".join(container)
            bit_stream.write_bitstring(bitstring)
    return bit_stream.to_bytes()


def encode_jpeg(im, quality=95, subsample="4:2:0", use_rm_ht=True):
    im = np.expand_dims(im, axis=-1) if im.ndim == 2 else im
    height, width, depth = im.shape

    mh, mw = mcu_sizes[subsample] if depth == 3 else (BH, BW)
    im = padding(im, mh, mw)
    im = RGB2YCbCr(im) if depth == 3 else im

    # DC level shift for luminance,
    # the shift of chroma was completed by color conversion
    Y_im = im[:, :, 0] - 128
    # divide image into MCUs
    mcu = divide_blocks(Y_im, mh, mw)
    # MCU to blocks, for luminance there are more than one blocks in each MCU
    Y = scan_blocks(mcu, mh, mw)
    Y_dct = DCT(Y)
    # the quantization table was already processed by zigzag scan,
    # so we apply zigzag encoding to DCT block first
```

```python
    Y_z = zigzag_encode(Y_dct)
    qt_y = load_quantization_table(quality, "lum")
    Y_q = quantization(Y_z, qt_y)
    Y_p = DPCM(Y_q)
    # whether to use recommended huffman table
    if use_rm_ht:
        Y_dc_ht, Y_ac_ht = reverse(RM_Y_DC), reverse(RM_Y_AC)
    else:
        Y_dc_ht = jpegCreateHuffmanTable(np.vectorize(bits_required)(Y_p[:, 0]))
        Y_ac_ht = jpegCreateHuffmanTable(
            flatten(run_length_encode(Y_p[i, 1:])[0] for i in range(Y_p.shape[0]))
        )
    qts, hts = [qt_y], [Y_dc_ht, Y_ac_ht]
    cop_infos = [ComponentInfo(1, mw // BW, mh // BH, 0, 0, 0)]
    # the number of Y DCT blocks in an MCU
    num = (mw // BW) * (mh // BH)
    mcu_hts = [(Y_dc_ht, Y_ac_ht) for _ in range(num)]
    # assign DCT blocks to MCUs
    mcu_ = Y_p.reshape(-1, num, BH * BW)

    if depth == 3:
        # chroma subsample
        ch = im[:: mh // BH, :: mw // BW, 1:]
        Cb = divide_blocks(ch[:, :, 0], BH, BW)
        Cr = divide_blocks(ch[:, :, 1], BH, BW)
        Cb_dct, Cr_dct = DCT(Cb), DCT(Cr)
        Cb_z, Cr_z = zigzag_encode(Cb_dct), zigzag_encode(Cr_dct)
        qt_c = load_quantization_table(quality, "chr")
        Cb_q, Cr_q = quantization(Cb_z, qt_c), quantization(Cr_z, qt_c)
        Cb_p, Cr_p = DPCM(Cb_q), DPCM(Cr_q)
        if use_rm_ht:
            C_dc_ht, C_ac_ht = reverse(RM_C_DC), reverse(RM_C_AC)
        else:
            ch_ = np.concatenate([Cb_p, Cr_p], axis=0)
            C_dc_ht = jpegCreateHuffmanTable(np.vectorize(bits_required)(ch_[:, 0]))
            C_ac_ht = jpegCreateHuffmanTable(
                flatten(run_length_encode(ch_[i, 1:])[0] for i in range(ch_.shape[0]))
            )
```

```python
        qts.append(qt_c), hts.extend([C_dc_ht, C_ac_ht])
        cop_infos.extend(
            [ComponentInfo(2, 1, 1, 1, 1, 1), ComponentInfo(3, 1, 1, 1, 1, 1)]
        )
        mcu_hts.extend((C_dc_ht, C_ac_ht) for _ in range(2))
        mcu_ = np.concatenate([mcu_, Cb_p[:, None], Cr_p[:, None]], axis=1)

    writer = encode_header(qts, hts, cop_infos, height, width)
    bytes_ = encode_mcu(mcu_, mcu_hts)
    writer.write(bytes_.replace(b"\xff", b"\xff\x00"))
    writer.write(MARKER.EOI)
    return writer.getvalue()


def write_jpeg(filename, im, quality=95, subsample="4:2:0", use_rm_ht=True):
    bytes_ = encode_jpeg(im, quality, subsample, use_rm_ht)
    Path(filename).write_bytes(bytes_)


def main():
    im = Image.open("./data/villeLyon.jpg")
    write_jpeg("data/villeLyonLow.jpg", np.array(im), 5, "4:1:1", False)


if __name__ == "__main__":
    main()

```

encoder/comparator.py

```python
import numpy as np
import encoder
import sys, os
from pathlib import Path
```

```python
from PIL import Image
import cv2
import pandas as pd
import time as t
import shutil
import utils
import matplotlib.pyplot as plt
from encoder import DCT, padding
from scipy.fftpack import dct
import random as rd

LIM = 2  # number of files to test to


def compare(
    qualities=None,
    dataDirectory=None,
    outputDirectory=None,
    subsamples=None,
    useStdHuffmanTable=None,
    DeleteFilesAfterward=True,
):
    if qualities is None:
        qualities = [np.random.randint(0, 101)]
    if subsamples is None:
        subsamples = ["4:2:2"]
    if dataDirectory is None:
        dataDirectory = "./data/datasetBmp"
    if useStdHuffmanTable is None:
        useStdHuffmanTable = [False]
    stat = np.zeros(
        (LIM * len(qualities) * len(subsamples) * len(useStdHuffmanTable), 6),
        dtype=object,
    )  # dim 0 : quality factor, dim 1 : subsample method, dim 2 : usage of std Hf Tables, dim 3 : size before compression, dim 4 : size after compression, dim 5 : time to compress
    i = 0
    i_max = LIM * len(qualities) * len(subsamples) * len(useStdHuffmanTable)
    filesTreated = rd.choices(os.listdir(dataDirectory), k=LIM)
```

```
    for quality in qualities:
        for subsample in subsamples:
            for hfTables in useStdHuffmanTable:
                outputDirectory = f"./data/treated/quality{quality}-subsample{subsample}-stdHf{hfTables}"
                for filename in filesTreated:
                    f = os.path.join(dataDirectory, filename)
                    if not os.path.exists(outputDirectory):
                        os.makedirs(outputDirectory)
                    f_out = os.path.join(outputDirectory, filename + ".jpg")
                    if os.path.isfile(f):
                        previousSize = os.stat(f).st_size
                        image = Image.open(f)
                        time = t.time()
                        encoder.write_jpeg(
                            f_out, np.array(image), quality, subsample, hfTables
                        )
                        time = t.time() - time
                        newSize = os.stat(f_out).st_size
                        stat[i][0] = quality
                        stat[i][1] = subsample
                        stat[i][2] = hfTables
                        stat[i][3] = previousSize
                        stat[i][4] = newSize
                        stat[i][5] = time
                    i += 1
                    print(f"{i}/{i_max}", end="\r")
                if DeleteFilesAfterward:
                    shutil.rmtree(outputDirectory)
    return stat


def write_stat(statFile, stat, quality, subsample, standHuffTables):
    with open(statFile, "a+") as f:
        f.write("\n" * 2)
        f.write("New sample \n")
        f.write(f"Size of sample : {LIM} images \n")
        f.write(
```

```python
            f"Parameters of compression : (quality) {quality}, (subsample) {subsample}, (usage of standard HuffTables) {'Yes' if standHuffTables else 'No'} \n"
        )
        avgPreviousSize = np.average(stat[:, 0])
        avgNewSize = np.average(stat[:, 1])
        f.write(
            f"Average size of image before compression : {avgPreviousSize} bytes \n"
        )
        f.write(f"Average size of images after compression : {avgNewSize} bytes \n")
        f.write(f"Ratio is {avgPreviousSize / avgNewSize:.2f}")


def write_stat_csv(output, stat):
    if os.path.isfile(output):
        pd.DataFrame(stat).to_csv(output, mode="a", index=False, header=False)
    else:
        pd.DataFrame(stat).to_csv(
            output,
            index=False,
            header=[
                "quality",
                "subsample",
                "stdHuffmanTables",
                "oldSize",
                "newSize",
                "time",
            ],
        )


def csv_to_stat(csvFile):
    stat = pd.read_csv(csvFile)
    return stat


def dataInterpreation(dataFrame):
    df = dataFrame
    qualities = df["quality"].unique()
```

```
    qualitySize = {}
    qualityTime = {}
    for quality in qualities:
        qualitySize[quality] = int(df[df["quality"] == quality]["newSize"].mean())
        qualityTime[quality] = round(df[df["quality"] == quality]["time"].mean(), 3)
    stdSize = int(df[df["stdHuffmanTables"] == True]["newSize"].mean())
    stdTime = round(df[df["stdHuffmanTables"] == True]["time"].mean(), 3)
    nonStdSize = int(df[df["stdHuffmanTables"] == False]["newSize"].mean())
    nonStdTime = round(df[df["stdHuffmanTables"] == False]["time"].mean(), 3)

    plt.rcParams["figure.figsize"] = [10, 5]

    fig, (ax1, ax3) = plt.subplots(1, 2)
    ax2 = ax1.twinx()

    fig.suptitle("Comparaison des compressions en fonction du facteur de qualité")

    width = 0.25

    initialSize = 786486
    xaxis = list(qualitySize.keys())
    yaxisSize = np.array(list(qualitySize.values()))
    yaxisTime = np.array(list(qualityTime.values()))
    yaxisRatio = (initialSize - yaxisSize) / yaxisTime

    color1 = "tab:red"
    color2 = "tab:blue"
    color3 = "tab:green"

    ax1.bar(
        np.arange(len(qualitySize)) - width,
        yaxisSize,
        width,
        tick_label=xaxis,
        color=color1,
        label="Taille après compression",
    )
    ax2.bar(
```

```python
        np.arange(len(qualityTime)),
        yaxisTime,
        width,
        tick_label=xaxis,
        color=color2,
        label="Temps de compression",
    )
    ax3.bar(
        np.arange(len(qualitySize)),
        yaxisRatio,
        width,
        tick_label=xaxis,
        color=color3,
        label="Octets gagnés par seconde",
    )

    ax1.legend(loc="upper left")
    ax2.legend(loc="upper left", bbox_to_anchor=(0, 0.9))
    ax3.legend(loc="upper right")

    ax3.yaxis.tick_right()

    ax1.set_xlabel("Facteur de qualité")
    ax1.set_ylabel("Taille (en octets)", color=color1)
    ax2.set_ylabel("Temps (en secondes)", color=color2)
    ax3.set_xlabel("Facteur de qualité")
    ax3.set_ylabel("Taille gagné par unité de temps (octets.Hz)", color=color3)
    ax3.yaxis.set_label_position("right")

    plt.savefig("./data/treated/compressionComparaison", transparent=True)

    plt.rcParams["figure.figsize"] = [7, 5]
    plt.clf()

    fig = plt.figure()
    ax1 = fig.add_subplot(111)
    ax2 = ax1.twinx()
```

```python
    fig.suptitle(
        "Comparaison des compressions en fonction des tables de Huffman utilisées"
    )

    yaxisSize = [stdSize, nonStdSize]
    yaxisTime = [stdTime, nonStdTime]

    labels = ["Tables Standards", "Tables Optimales"]
    ax1.bar(
        np.arange(2) - width / 2,
        yaxisSize,
        width,
        tick_label=labels,
        color=color1,
        label="Taille après compression",
    )
    ax2.bar(
        np.arange(2) + width / 2,
        yaxisTime,
        width,
        tick_label=labels,
        color=color2,
        label="Temps de compression",
    )

    ax1.legend(loc="upper center", bbox_to_anchor=(0.45, 1))
    ax2.legend(loc="upper center", bbox_to_anchor=(0.45, 0.9))

    ax1.set_ylabel("Taille (en octets)", color=color1)
    ax2.set_ylabel("Temps (en secondes)", color=color2)

    plt.savefig("./data/treated/compressionComparaison2", transparent=True)


def energyCompaction(imgPath):
    img = cv2.imread(imgPath)

    imgYCrCB = cv2.cvtColor(
```

```python
        img, cv2.COLOR_RGB2YCrCb
    )  # Convert RGB to YCrCb (Cb applies V, and Cr applies U).

    Y, Cr, Cb = cv2.split(padding(imgYCrCB, 8, 8))
    Y = Y.astype("int") - 128
    blocks_Y = utils.divide_blocks(Y, 8, 8)
    dctBlocks_Y = np.zeros_like(blocks_Y)
    for i in range(len(blocks_Y)):
        dctBlocks_Y[i] = dct(
            dct(blocks_Y[i], axis=0, norm="ortho"), axis=1, norm="ortho"
        )
    avg_Y = utils.averageMatrix(blocks_Y)
    avgDct_Y = utils.averageMatrix(dctBlocks_Y)

    x = np.random.randint(blocks_Y.shape[0])
    arr1 = blocks_Y[x]
    arr2 = dctBlocks_Y[x]

    fig, (ax1, ax2) = plt.subplots(1, 2)

    valueMax, valueMin = max(np.max(arr1), np.max(arr2)), min(
        np.min(arr1), np.min(arr2)
    )
    # fig.suptitle('Matrice de la luminance de "villeLyon.jpg"')

    ax1.matshow(arr1, cmap="cool", vmin=valueMin, vmax=valueMax)
    ax1.set_title("avant DCT")

    ax2.matshow(arr2, cmap="cool", vmin=valueMin, vmax=valueMax)
    ax2.set_title("après DCT")

    for i in range(arr1.shape[0]):
        for j in range(arr1.shape[1]):
            cNormal = int(arr1[i, j])
            cDct = int(arr2[i, j])
            ax1.text(i, j, str(cNormal), va="center", ha="center")
            ax2.text(i, j, str(cDct), va="center", ha="center")
    plt.savefig("./data/energyCompaction.png", transparent=True)
```

```python
def rgbToYCbCr_channel_bis():
    img = cv2.imread("./data/villeLyon.jpg")  # Read input image in BGR format

    imgYCrCB = cv2.cvtColor(
        img, cv2.COLOR_BGR2YCrCb
    )  # Convert RGB to YCrCb (Cb applies V, and Cr applies U).

    Y, Cr, Cb = cv2.split(imgYCrCB)

    # Fill Y and Cb with 128 (Y level is middle gray, and Cb is "neutralized").
    onlyCr = imgYCrCB.copy()
    onlyCr[:, :, 0] = 128
    onlyCr[:, :, 2] = 128
    onlyCr_as_bgr = cv2.cvtColor(
        onlyCr, cv2.COLOR_YCrCb2BGR
    )  # Convert to BGR - used for display as false color

    # Fill Y and Cr with 128 (Y level is middle gray, and Cr is "neutralized").
    onlyCb = imgYCrCB.copy()
    onlyCb[:, :, 0] = 128
    onlyCb[:, :, 1] = 128
    onlyCb_as_bgr = cv2.cvtColor(
        onlyCb, cv2.COLOR_YCrCb2BGR
    )  # Convert to BGR - used for display as false color

    cv2.imshow("img", img)
    cv2.imshow("Y", Y)
    cv2.imshow("onlyCb_as_bgr", onlyCb_as_bgr)
    cv2.imshow("onlyCr_as_bgr", onlyCr_as_bgr)
    cv2.waitKey()
    cv2.destroyAllWindows()

    cv2.imwrite("./data/treated/villeLyon_Y.jpg", Y)
    cv2.imwrite("./data/treated/villeLyon_Cb.jpg", onlyCb_as_bgr)
    cv2.imwrite("./data/treated/villeLyon_Cr.jpg", onlyCr_as_bgr)
```

```python

if __name__ == "__main__":
    # compare()
    # rgbToYCbCr_channel_bis()
    # energyCompaction("./data/villeLyon.jpg")
    # test = np.array([[93, 90, 83, 68, 61, 61, 46, 21],
    #                  [102, 92, 95, 77, 65, 60, 49, 32],
    #                  [69, 55, 47, 57, 65, 60, 72, 65],
    #                  [55, 55, 40, 42, 23, 1, 11, 38],
    #                  [55, 57, 47, 53, 35, 59, -2, 26],
    #                  [64, 41, 42, 55, 60, 57, 25, -8],
    #                  [77, 87, 58, -2, -5, 14, -10, -35],
    #                  [38, 14, 33, 33, -21, -23, -43, -34]])
    # print(dct(dct(test, axis=0, norm="ortho"), axis=1, norm='ortho'))

    # stat = compare(qualities = list(range(1, 101, 10)), subsamples=['4:4:4', '4:2:0', '4:1:1', '4:2:2'], useStdHuffmanTable=[True, False], DeleteFilesAfterward=True)
    # write_stat_csv("./data/treated/stat.csv", stat)
    stat = csv_to_stat("./data/treated/stat.csv")
    dataInterpreation(stat)

```