Lagrange Assignment in LaTeX Format

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1 Introduction

This lagrange assignment looks at a pulley system with a spring and a massless node.

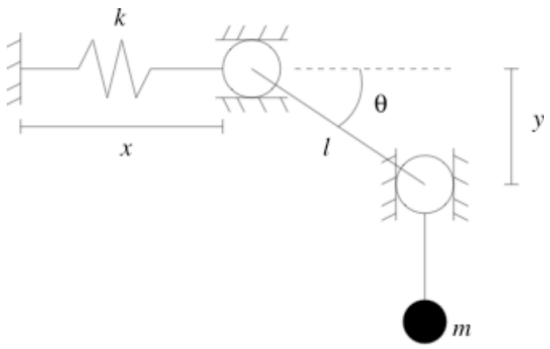

Figure 1. Pulley System

Maths The following maths and the idea for this assignment was taken from the examples on the connex page.

Constraint equations:

$$f = x - l(1 - cos(\theta)) \tag{1}$$
$$g = y - lsin(\theta) \tag{2}$$

Potential Energy:

$$T = \frac{1}{2}m\dot{y}^2 \tag{3}$$

Kinetic Energy:

$$U = \frac{1}{2}kx^2 - mgy \tag{4}$$

Equation of L:

$$L = T - U = \frac{1}{2}m\dot{y}^2 - \frac{1}{2}kx^2 + mgy \tag{5}$$

Derivatives from L for Lagrange Equations:

$$\frac{dL}{dx} = -kx \tag{6}$$
$$\frac{dL}{dy} = mg \tag{7}$$
$$\frac{dL}{d\theta} = \frac{d}{dt}\frac{dL}{d\dot{x}} = \frac{d}{dt}\frac{dL}{d\dot{\theta}} = 0 \tag{8}$$
$$\frac{d}{dt}\frac{dL}{d\dot{y}} = m\ddot{y} \tag{9}$$

Plug into Lagrange Equations:

$$-kx + \lambda_1 = 0 \tag{10}$$
$$mg - m\ddot{y} + \lambda_2 = 0 \tag{11}$$
$$-\lambda_1 lsin(\theta) - \lambda_2 lcos(\theta) = 0 \tag{12}$$

Solving for λ_1, λ_2:

$$\lambda_1 = kx = kl(1 - cos(\theta)) = kl\left(1 - \sqrt{1 - \frac{y}{l}^2}\right) \tag{13}$$
$$\lambda_2 = -\lambda_1 tan(\theta) \tag{14}$$
$$= -kl\left(1 - \sqrt{1 - (\frac{y}{l})^2}\right) \cdot \frac{y}{l\sqrt{1 - (\frac{y}{l})^2}} \tag{15}$$
$$= -ky\left(\left[1 - \frac{y}{l}^2\right]^{\frac{-1}{2}} - 1\right) \tag{16}$$

Final Lagrange Equation:

$$\ddot{y} - g + \frac{k}{m}y\left(\left[1 - \frac{y}{l}^2\right]^{\frac{-1}{2}} - 1\right) = 0 \tag{17}$$
$$\ddot{y} = g - \frac{k}{m}y\left(\left[1 - \frac{y}{l}^2\right]^{\frac{-1}{2}} - 1\right) \tag{18}$$

2 Screenshots

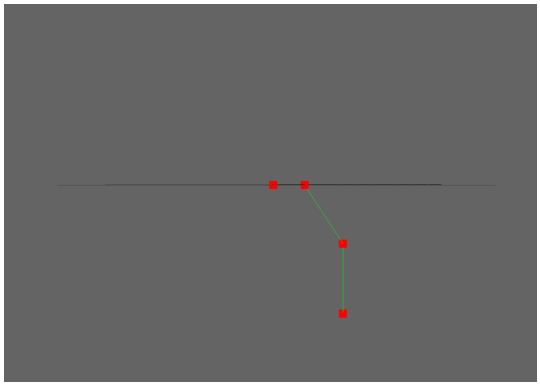

Figure 2. Pulley system at rest

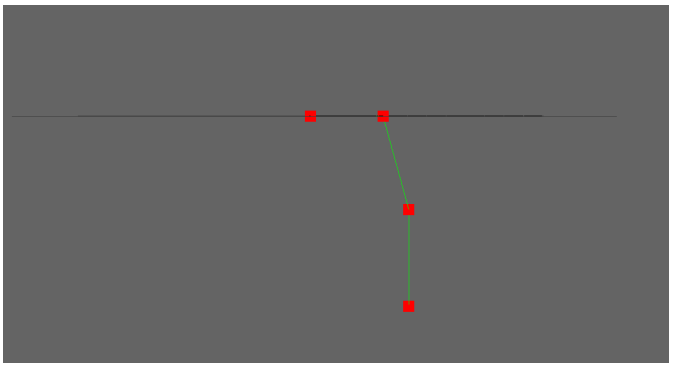

Figure 3. Pulley system at max

References

https://connex.csc.uvic.ca/access/content/group/9fa028b3-83b7-4ff4-b125-6862fcf2de1a/Lagrange-examples/examplesLagrange.pdf