TDT4171 - Assignment 4
Making Simple Decisions

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Exercise 1

The Surprise Candy Company makes candy in two flavors: 70% are strawberry flavor and 30% are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves along the production line, a machine randomly selects a certain percentage to be trimmed into a square. 80% of the strawberry candies are round, while 90% of the anchovy candies are square. Then each piece is wrapped in a wrapper whose color is either red or brown. 80% of the strawberry candies are chosen randomly to have a red wrapper, while 90% of the anchovy candies are chosen to have a brown wrapper. All candies are sold individually in sealed, identical, black boxes.

Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box. Consider the three Bayes nets in Figure 1.

a. *Which network(s) can correctly represent $P(Flavor, Wrapper, Shape)$? Consider if each network can represent all dependencies between the variables required to fit with the story?*

Network (i) implies that $Wrapper$ and $Shape$ are independent. Let's write $F = Flavor$, $S = Shape$ and $W = Wrapper$. From the statement, we can infer that S and W are conditionally independent given F. Thus,

$$P(S, W|F) = P(S|F)P(W|F)$$

Let's show that S and W are not independent, i.e

$$P(S, W) \neq P(S)P(W)$$

On the one hand, because of the conditionally independence given F between S and W,

$$P(S, W) = \sum_f P(f)P(S, W|f) = \sum_f P(f)P(S|f)P(W|f)$$
$$P(S = round, W = red) = 0.7 \cdot 0.8 \cdot 0.8 + 0.3 \cdot 0.1 \cdot 0.1 = 0.451$$

On the other hand,

$$P(S = round) = \sum_f P(f)P(S = round|f) = 0.7 \cdot 0.8 + 0.3 \cdot 0.1 = 0.59$$
$$P(W = red)) = \sum_f P(f)P(W = red|f) = 0.7 \cdot 0.8 + 0.3 \cdot 0.1 = 0.59$$
$$P(S = round)P(W = red) = 0.59 \cdot 0.59 = 0.3481$$

We notice that,

$$P(S = round, W = red) \neq P(S = round)P(W = red)$$

Thus we have the desired result,

$$P(S, W) \neq P(S)P(W)$$

This implies that network (i) can not represent $P(Flavor, Wrapper, Shape)$.

Because network (ii) is fully connected, thus it can represent $P(Flavor, Wrapper, Shape)$.

Network (iii) represents well the dependencies between the variables to fit the story, the flavor is set, and then, the wrapping and the shape are set independently knowing the flavor.

Thus, networks (ii) and (iii) can represent the joint distribution of the problem while network (i) doesn't.

b. *Which network is the best representation for this problem? Consider the size of the representation, and how easily you can deduce the numbers required by the conditional probability tables in your chosen model.*

Network (iii) is a better representation for this problem than network (ii). There are less edges and the representation fit the problem description, then the conditional probability tables are easy to infer with that one.

c. *Does network (i) assert that $Wrapper$ is independent of $Shape$?*

Yes it does because they are d-separated.

d. *In the box is a round candy with a red wrapper. What is the probability that its flavor is strawberry?*

We want $P(F = strawberry|S = round, W = red)$. We have, using bayes,

$$P(F|S,W) = \frac{P(F)P(S,W|F)}{\sum_f P(S,W|f)}$$

$$P(F|S,W) = \frac{P(F)P(S|F)P(W|F)}{\sum_f P(S,W|f)}$$

$$P(F = strawberry|S = round, W = red) = \frac{0.7 \cdot 0.8 \cdot 0.8}{0.7 \cdot 0.8 \cdot 0.8 + 0.3 \cdot 0.1 \cdot 0.1} \approx 0.993$$

e. *A unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the expected value of an unopened candy box.*

The expected value of an unopened candy box is the value of the expected alue of the random variable F, ie:

$$E(F) = P(F = s) \cdot s + P(F = a) \cdot a = 0.7s + 0.3a$$

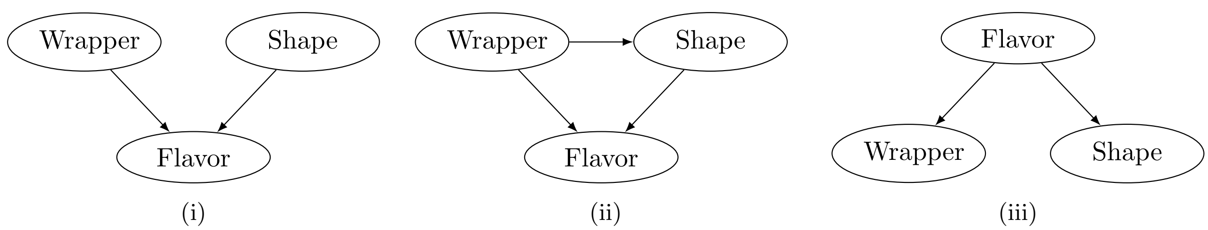

Figure 1: Three proposed Bayes nets for exercise 1

Exercise 2

Economists often make use of an exponential utility function for money: $U(x) = -e^{-x/R}$, where R is a positive constant representing an individual's risk tolerance. Risk tolerance reflects how likely an individual is to accept a lottery with a particular expected monetary value (EMV) versus some certain payoff. As R (which is measured in the same units as x) becomes larger, the individual becomes less risk-averse.

a. *Assume Mary has an exponential utility function with $R = \$500$. Mary is given the choice between receiving \$500 with certainty (probability 1) or participating in a lottery which has a 60% probability of winning \$5000 and a 40% probability of winning nothing. Assuming Mary acts rationally, which option would she choose? Show how you derived your answer.*

In the first case, the value of the utility function is:

$$-e^{-500/500} = e^{-1} = 0.368$$

In the second case, the value of the utility function is:

$$0.6 \cdot (-e^{-5000/500}) + 0.4 \cdot (-e^{0/500}) = -(0.6e^{-10} + 0.4) \approx 0.4$$

Supposing Mary acts rationally, she choose option 2, the one with 60% to earn \$5000 and \$0 otherwise.

b. *Consider the choice between receiving* \$100 *with certainty (probability 1) or participating in a lottery which has a* 50% *probability of winning* \$500 *and a* 50% *probability of winning nothing. Approximate the value of R (to 3 significant digits) in an exponential utility function that would cause an individual to be indifferent to these two alternatives.*

We want R such that utility value from both cases is equal, ie:

$$-e^{-100/R} = -0.5e^{-500/R} - 0.5e^{-0/R}$$
$$-e^{-100/R} = -0.5e^{-500/R} - 0.5$$

Let $x = e^{-100/R}$, then our equation can be written:

$$\begin{aligned}
&x^5 - 2x + 1 = 0 \\
&(x^5 - x) - (x - 1) = 0 \\
&x(x^4 - 1) - (x - 1) = 0 \\
&x(x^2 - 1)(x^2 + 1) - (x - 1) = 0 \\
&x(x - 1)(x + 1)(x^2 + 1) - (x - 1) = 0 \\
&(x - 1)[x(x + 1)(x^2 + 1) - 1] = 0 \\
&(x - 1)(x^4 + x^3 + x^2 + x - 1) = 0
\end{aligned}$$

We then notice that $x \neq 1$, because $\forall R^*$, $-100/R \neq 0$. We thus have to resolve $x^4 + x^3 + x^2 + x - 1 = 0$ but it seems really hard so we will use Python to find one solution (if there is one).

We first notice that $x \in]0, 1[$, because $\forall R$, $e^{-100/R} > 0$ and $-100/R < 0$ so, $x = e^{-100/R} < 1$. So our research will be in that section.

First thing we plot the curve of $f : x \mapsto x^4 + x^3 + x^2 + x - 1$ on Figure 2. We notice that there is an unique solution that satisfies our conditions on x (because f is continuous on $]0, 1[$).

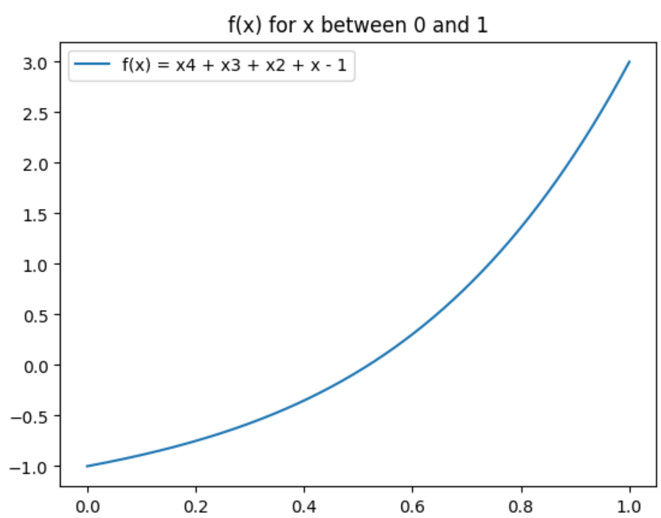

Figure 2: Representative curve of f between 0 and 1 on Python

We thus do a little program to find the value of x with a precision of 10^{-6}. Thus we find $R = -100/log(x)$, which gives us $R = 152$, for a precision on 3 digits.