Atmospheric Physics 2014 Q.1

i Mass of the atmosphere?

Assuming hydrostatic balance

$$\int_{P_s}^{P_{TOA}} \frac{dP}{dz} dz = \int_0^{z_{TOA}} -\rho g \, dz$$

At the top of the atmosphere $P_{TOA} = 0$ & $\rho_{TOA} = 0$.

Integrating, <u>assuming</u> $g \sim$ constant

$$P_{TOA} - P_s = -(0 - \sigma) g$$

where σ = density of a layer of unit thickness $/m^2$.

$$\Rightarrow \quad -P_s = \sigma g$$

Mass of atmosphere, $m = \sigma A$

$A = 4\pi R^2$.

$$\Rightarrow A\sigma = \boxed{m = \frac{P_s}{g} \quad \frac{4\pi R^2 P_s}{g}}$$

$R = 6371$ km, $P_s = 1013$ hPa, $g = 9.81\ ms^{-2}$

$$m = 5.27 \times 10^{18} \text{ kg}$$

pto.

ii

Specific humidity, $q_v = \dfrac{m_v}{m_v + m_d}$ $\boxed{0}$ No ice/water

$e\alpha_v = R_v T$ $\boxed{1}$

$P_d \alpha_d = R_d T$ $\boxed{2}$

Dalton $\Rightarrow$ $P = P_d + e$

$$= \left(\frac{R_d}{\alpha_d} + \frac{R_v}{\alpha_v}\right) T$$

$m_v = \dfrac{V}{\alpha_v}$ $\quad m_d = \dfrac{V}{\alpha_d}$.

$$\Rightarrow q_v = \frac{1/\alpha_v}{1/\alpha_v + 1/\alpha_d} = \frac{1}{1 + \alpha_v/\alpha_d}$$

(1)/(2) $\Rightarrow$ $\dfrac{\alpha_v}{\alpha_d}\dfrac{e}{P_d} = \dfrac{R_v}{R_d} = \dfrac{1}{0.62}$ (given)

$$\Rightarrow q_v = \frac{1}{1 + \frac{1}{0.62} \times \frac{1000}{10}}$$

~~$q_v = 0.98$~~

$= 0.0062$ kg/kg

$= 6.2$ g/kg

iii

The atmosphere rotates at a faster rate than the Earth ~23 hour/revolution rather than 24 hour/revolution

Moment of inertia of ~~atmosphere~~ a solidsphere

$$I = \frac{2}{5} M R^2$$

Angular momentum

$$L = I \omega$$

Angular kinetic energy

$$E = \frac{1}{2} I \omega^2.$$

Angular momentum of an air parcel

East-West velocity

$$L = R \cos \phi \,(u + \Omega R \cos \phi)$$

Latitude

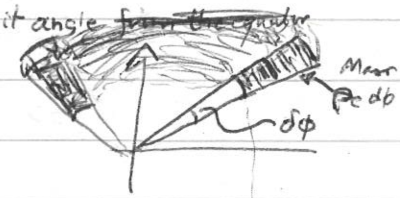
$\Rightarrow$ If ~~Earth~~ atmosphere stopped rotating relative to surface.

$$\Delta L_{parcel} = R \cos(\phi)\, u\, \rho\, dV$$

Integrating, assuming mass of atmosphere is concentrated at the surface $\Rightarrow R \sim R_{\oplus}$

Mass per unit angle ~~from the equator~~

$$\Delta L_{atm.} = \int_{\phi=-\pi/2}^{\pi/2} R_{\oplus} \cos(\phi)\, u \rho_c\, d\phi$$



$$= \left[R_{\oplus} \sin(\phi) \rho_c u \right]_{\phi=-\pi/2}^{\pi/2} = 2 R u M$$

pto see later for iii answer.

at temperature T_a

iv a) ε = Emissivity = $\dfrac{\text{Irradiance of a layer of the atmosphere}}{\text{Irradiance from a blackbody @ temperature } T_a}$

? b) Kirchoff's Law:

$$\varepsilon_\lambda(T_a) = \alpha_\lambda(T_a) \quad \text{Absorptivity}$$

$$\varepsilon_\lambda(T_a) = \frac{I_\lambda^{\cancel{\text{emitted}}\ \text{absorbed}}}{I_\lambda^{\text{incident}}(T_a)}$$ Intensity of radiation ~~emitted~~ absorbed ~~by Earth's surface~~

$$I_\lambda^{\text{incident}} = \text{Intensity of radiation emitted } \cancel{\text{Earth's Irradiance}} \text{ from Earth's surface}$$

$$= \sigma T_a^4$$

$$\Rightarrow I_{\lambda=IR}^{TOA,\uparrow} = \varepsilon \sigma T_a^4$$

$$I_{\lambda=UV}^{TOA\downarrow} = \sigma T_s^4$$

$$= I_{\lambda,uv}^{\text{surface},\downarrow}$$

~~Where~~ we are Assuming that there is no net heating of The Earth's surface

& Assuming there is no net heating of the atmosphere

$$\underbrace{\varepsilon \sigma T_a^4}_{\text{Emitted upward}} = \underbrace{\alpha \sigma T_s^4}_{\text{Absorbed}}$$

(Kirchof) $\Rightarrow \quad = \varepsilon \sigma T_s^4$

$$\Rightarrow T_a^4 = T_s^4$$

$$\Rightarrow I_{\lambda=IR}^{TOA,\uparrow} = \sigma T_s^4 - \varepsilon \sigma T_s^4 = \boxed{(1-\varepsilon)\sigma T_s^4}$$

c) TOA:

$$\boxed{\sigma T_e^4 = (1-\varepsilon)\sigma T_s^4} \quad \boxed{*}$$

Surface

$$\boxed{\varepsilon\sigma T_a^4 + \sigma T_s^4 = \cancel{\varepsilon\sigma T_a^4} + \sigma T_e^4} \quad \boxed{**}$$ ← (Assuming no absorption in shortwave)

d) If $\varepsilon = 0.8$, $T_e = 255K$

~~(**) → (*)~~

~~σT_s^4~~

$$(*) \Rightarrow T_s = \frac{T_e}{\sqrt[4]{1-\varepsilon}}$$

$$= 381\,K$$

e) Cooling of surface via evaporation (latent heat flux) and convection ~~(se~~ + conduction (sensible heat flux).

V) a)

$$fv = \alpha \frac{\partial P}{\partial x}$$

Specific volume of air parcel

Rate of change of pressure in East-West direction

North-South velocity

Coriolis parameter

$f = 2\Omega\sin(\lambda)$

Latitude

Angular velocity of planet

$$0 = -g - \alpha \frac{\partial P}{\partial z}$$

Rate of change of pressure with altitude

Acceleration due to gravity

b) Taking the partial derivative of (1) with height, z,

$$\frac{\partial}{\partial z}(fv) = \frac{\partial}{\partial z}\left(\alpha(P)\frac{\partial P}{\partial x}\right)$$

~~If $\alpha(P)$ is only a function of Pressure~~

$$\cancel{= \alpha(P)\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial z}\right)}$$

$$\cancel{f\frac{\partial v}{\partial z} = \alpha(P)\frac{\partial}{\partial x}\left(-\frac{g}{\alpha(P)}\right)}$$

$$\cancel{= 0}$$

If α is only a fct. of pressure (which is approx. constant at a given latitude.

$$\frac{\partial}{\partial z}(fv) = \frac{\partial}{\partial z}\left(\frac{\partial(\alpha(P)\,P)}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\frac{\partial}{\partial z}(\alpha(P)\,P)$$

$$= \frac{\partial}{\partial x}\left(-\frac{g}{\alpha(P)}\right)$$

$$= 0$$

f is constant

$$\cancel{\Rightarrow \boxed{\frac{\partial f}{\partial v} = 0}} \qquad \Rightarrow \boxed{\frac{\partial v}{\partial z} = 0}$$

Q.1...

iii For a typical East-West wind speed $u = 10 m/s$

$$\Delta L = 2R \underset{10\ m/s}{u} M \leftarrow 5.27\times10^{18} \text{ kg from (i)}$$

$$= 6.75\times10^{26}\ Nm.$$

Because angular momentum is conserved, the change in angular momentum of the Earth will be equal

$$\Rightarrow \Delta\omega = \frac{\Delta L}{I}$$

Mass of Earth, $M = 5.97\times10^{24}$ kg.

$$I = \frac{2}{5}MR^2$$

$$= 9.78\times10^{37}$$

$$\Rightarrow \Delta\omega = \cancel{4.34\times10^{-17}\ s^{-1}}\ \boxed{6.9\times10^{-12}\ rad\,s^{-1}}$$

$$\omega + \Delta\omega = \frac{2\pi}{T+\Delta T}$$

$$\Rightarrow T + \Delta T = \frac{2\pi}{\omega+\Delta\omega}$$

$$\approx \frac{2\pi}{\omega}\left(1 + \frac{\Delta\omega}{\omega}\right)$$

$$\Rightarrow \Delta T \sim \frac{2\pi}{\omega}\frac{\Delta\omega}{\omega}$$

$$= \cancel{4.43\times10^{-12}\ s}\ \boxed{\sim 8\ ms}$$

$\Rightarrow$ Negligible change in length of day.

Atmospheric 2014 Q.2

i ~~a)~~ $$\underbrace{I_\lambda(s) = I_\lambda(s_0)\, e^{-\tau_\lambda(s_0,s)}}_{①} + \underbrace{\int_0^{\tau_\lambda(s_0,s)} B_\lambda(\tau')\, e^{-(\tau_\lambda(s_0,s)-\tau')}\, d\tau'}_{②}$$

① = Intensity of radiation entering the atmosphere at a point s_0 which reaches a point s without being absorbed.

② = Intensity of radiation at point s contributed by the atmosphere via emission and scattering.

ii τ_λ, called the "optical depth" is a measure of the proportion of radiation which is absorbed by the atmosphere as it travels from point s_0 to point s.

ii ~~b)~~ a) • Letting $s_0 = 0$ be the position @ Earth's surface, and assuming the atmosphere is isothermal so $B_\lambda(\tau) = \sigma T^4$ is constant.

• $\tau_\lambda(s_0, s_0) = 0$

$$\Rightarrow I_\lambda(s) = I_\lambda(0)\, e^{-\tau_\lambda(0,s)} + \int_0^{\tau_\lambda(0,s)} B_\lambda(\tau')\, e^{-(\tau_\lambda(0,s)-\tau')}\, d\tau'$$

$$= I_\lambda(0)\, e^{-\tau_\lambda(0,s)} + B_\lambda \left[e^{-(\tau_\lambda(0,s)-\tau')} \right]_0^{\tau_\lambda(0,s)}$$

$$= I_\lambda(0)\, e^{-\tau_\lambda(0,s)} + B_\lambda e^0 - B_\lambda e^{-\tau_\lambda(0,s)}$$

$$\Rightarrow \boxed{I_\lambda(s) = B_\lambda + e^{-\tau_\lambda(0,s)}(I_{\lambda,s} - B_\lambda)}$$

where $I_{\lambda,s} = I_\lambda(0)$

$$\tau_\lambda(s_0,s) = \int_{s_0}^{s} \rho(s')\, k_\lambda(s')\, q_a\, ds'$$

$$\frac{dP}{ds} = -\frac{dP}{dz} = +\rho g$$

$$P = \rho R T$$

Const. T $\Rightarrow \dfrac{dP}{ds} = \dfrac{d\rho}{ds} RT$

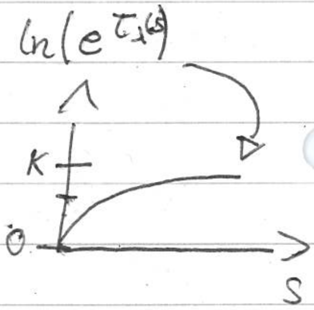
$$\Rightarrow \quad \frac{d\rho}{ds} = \frac{\rho g}{RT}$$

$$\rho \approx \rho_0 e^{-gs/RT}$$

Assume k_λ const & q_a const.

$$\tau_\lambda = \int_0^s F e^{-gs/RT}\, ds'$$

$$\sim K - K e^{-gs/RT}$$



? Effect of T_s :

iii $$I_\lambda^{\downarrow}(s) = I_\lambda(s_{TOA})\, e^{-\tau_\lambda(s_{TOA},s)} + \int_0^{\tau_\lambda(s_{TOA},s)} B_\lambda(\tau')\, e^{-(\tau_\lambda(s_{TOA},s)-\tau')}\, d\tau'$$

$I_\lambda^{\downarrow}(s_{TOA}) = 0$ since density is zero at the TOA
⇒ Nothing to emit radiation.

$$\Rightarrow I_\lambda^{\downarrow}(s) = B_\lambda \left[e^{-(\tau_\lambda(s_{TOA},s)-\tau')} \right]_0^{\tau_\lambda(s_{TOA},s)}$$

$$\Rightarrow \boxed{I_\lambda^{\downarrow}(s) = B_\lambda\left(+1 - e^{-\tau_\lambda(s_{TOA},s)}\right)}$$

iv a)

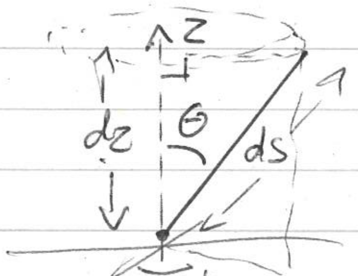


Fig. 1

From fig. 1, $ds = \frac{dz}{\cos\theta} \Rightarrow \tau_\lambda(s_{TOA}, s) = \int \rho k_\lambda \frac{dz}{\cos\theta}$
$= \boxed{\tau_\lambda(TOA, z)/\cos\theta}$

b) $$F_\lambda^{\downarrow}(z) = \iint I_\lambda^{\downarrow}(z)\, \cancel{\sin\cos\theta}\, d\phi\, d\theta$$

Independent of ϕ ⇒ $$= 2\pi B_\lambda \int 1 - e^{-\tau_\lambda(TOA,z)/\cos\theta}\, d\theta$$

Atmospheric Physics 2014 Q. 3

i Tropopause - the altitude at which the temperature is at a minimum.

Assuming lapse rate is constant $= -\frac{dT}{dz}$

$$Z_t = \frac{T_s - T_t}{\Gamma_t}$$

$$= \frac{288 - 210}{7\,K/km}$$

$$\boxed{Z_t = 11.1\ km}$$

ii If Hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g$$

Ideal gas law for dry air

$$P_d = \rho_d R_d T$$

$$\cancel{\frac{dP}{dz} = \frac{d\rho}{dz} R_d T + \rho R_d \frac{dT}{dz}}$$

$$\rho_d = P_d / R_d T$$

$$\frac{dP}{dz} = -\frac{P_d}{R_d T} g$$

$$\int_{P_s}^{P_t} \frac{dP_d'}{P_d'} = \frac{g}{R_d} \int_{z=0}^{z_t} \frac{1}{T_s - \Gamma z} dz$$

$$\ln(P_t/P_s) = +\frac{g}{R_d}\frac{1}{\Gamma}\left[\ln(T_s - \Gamma z)\right]_0^{z_t}$$

pto.

$a = F/m$ $[E] = Fd$

$= [E] \times \frac{M}{L}$

$\frac{[E] L^{-1} M^{-1}}{K L^{-1} [E] M^{-1} K^{-1}} = 1$

J/kg

$$\frac{P_t}{P_s} = \left(\frac{T_s - \Gamma z_t}{T_s} \right)^{+\frac{1}{\Gamma}\frac{g}{R_d}}$$

$g = 9.8\ m/s^2$
$\Gamma = 7 \times 10^{-3}\ K/m$
$R_d = 287\ J kg^{-1} K^{-1}$
$T_s - \Gamma z_t = 78$, $T_s = 288$

$$P_t = 1.7\ hPa$$ (very small)

iii

$$\Gamma_d = \frac{g}{c_{p,d}} \qquad [1]$$

$$s_d = s_{ref} + c_{p,d} \ln(T/T_{ref}) - R_d \ln(P/P_{ref}) \qquad [2]$$

Brunt - Vaisala

$$N^2 = \frac{g}{c_{p,d}} \frac{ds_d}{dz} \qquad [3]$$

Differentiating (2) =>

$$= \frac{g}{c_{p,d}} \left(\frac{c_{p,d}}{T} \frac{dT}{dz} - \frac{R_d}{P} \frac{dP}{dz} \right)$$

(Sub. Ideal gas) =>

$$= -\frac{g}{T} \Gamma_t + \frac{R_d}{P} \frac{P}{R_d T} \frac{g^2}{c_{p,d}}$$

$$= \frac{g}{T} \left(-\Gamma_t - \frac{g}{c_{p,d}} \right)$$

(1) => $$\boxed{N^2 = \frac{g}{T} \left(\Gamma_d - \Gamma_t \right)}$$

For $\Gamma_t = 7\,K/km$, $c_{p,d} =$ for the atmosphere, $g = 9.8$.

$$\Gamma_d = \frac{g}{c_{p,d}} = 7\ K/km.$$

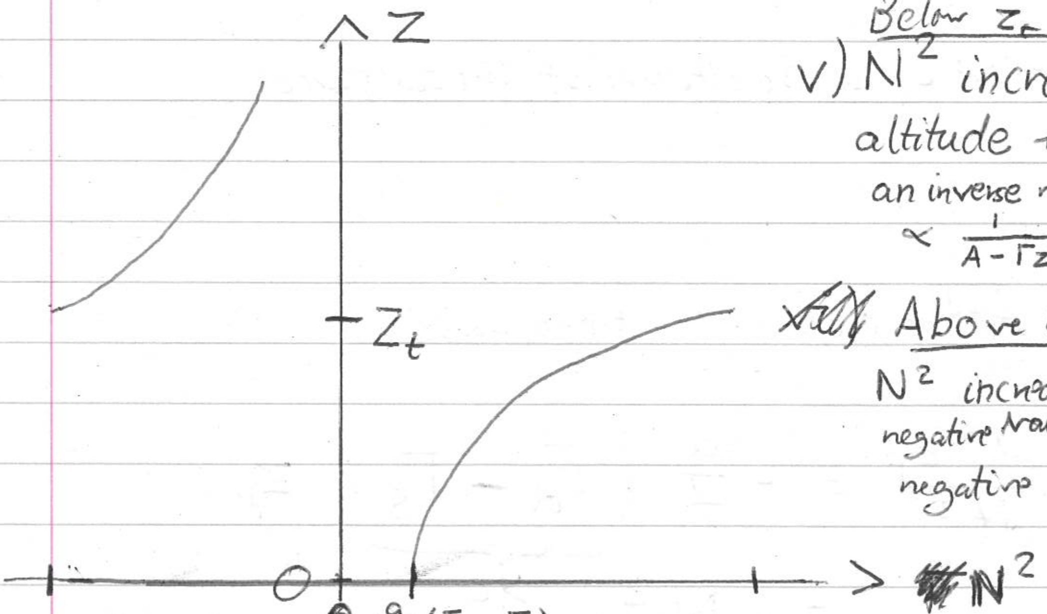
$$\Rightarrow \boxed{N^2 = 0}$$ in the troposphere.

iv
$$\boxed{N^2 = \frac{g}{T}(\Gamma_d - \Gamma_s)}$$ in the stratosphere

since
$$N^2 = -\frac{g}{T}\left(\frac{c_{p,a}}{T}\frac{dT}{dz} - \frac{R_d}{P}\frac{dP}{dz}\right)$$

Same Different Same as ~~bef~~ for troposphere

For $\Gamma_s < 0$ $N^2 > 0$ => Stable to convection



Below z_t:
v) N^2 increases with altitude ~~up the~~ in a an inverse relationship to z.
$\propto \frac{1}{A - \Gamma z}$

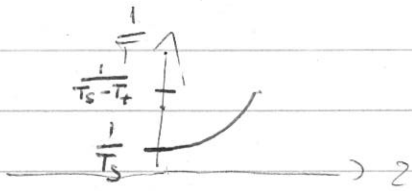
Above z_t:
N^2 increases from a large negative value but remains negative

0 $\frac{g}{T_t}(\Gamma_d - \Gamma_s)$

At zt:
v) If the tropopause & stratopause both have constant lapse rates, N^2 must be discontinuous @ the tropopause since $T > 0$, but Γ flips sign.

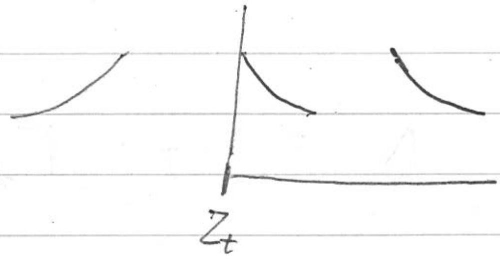
$(T)_{tropo} = T_s - \Gamma z$

$\left(\frac{1}{T}\right)_{tropo} = (T_s - \Gamma_t z)^{-1}$



@ $z = z_t$ $\quad T = T_t > 0$

$\left(\frac{1}{T}\right) = -\frac{1}{T_t + |\Gamma_s| z}$



vi) a) $\frac{d^2 z_p}{dt^2} = -N_t^2 z_p$

where z_p is the altitude of the air parcel.

b) After crossing the tropopause

$$\frac{d^2 z_p}{dt^2} = -\frac{g}{T}\underbrace{(\Gamma_d - \Gamma_s)}_{>0} z_p$$

$$= -N_s^2 z_p$$

$$\boxed{z_p = z_t + A\cos(N_s t)}$$

$\frac{dw_p}{dt} = g\,\frac{\theta_p - \theta_e}{\theta_e} \qquad N_s^2 = \frac{g}{\theta_e}\frac{d\theta_e}{dz}$

In the stratosphere θ_e decreases with height
=> while it rises $\frac{dw_p}{dt}$ decreases until eventually
=> $N^2 < 0$ at which point ~~so~~ the convection will be stable. The parcel will lose kinetic energy as it does work against the pressure gradient.