Atmospheric Physics 2014 Q.1

i Mass of the atmosphere?

Assuming hydrostatic balance

$$\int_{P_s}^{P_{TOA}} \frac{dP}{dz}\, dz = \int_{0}^{z_{TOA}} -\rho g \, dz$$

At the top of the atmosphere $P_{TOA} = 0$ & $\rho_{TOA} = 0$.

Integrating, assuming $g \sim$ constant

$$P_{TOA} - P_s = -(0 - \sigma) g$$

where σ = density of a layer of unit thickness /m²

$$\Rightarrow \quad -P_s = \sigma g$$

Mass of atmosphere, $m = \sigma A$

$$A = 4\pi R^2$$

$$\Rightarrow A\sigma = \boxed{m = \frac{P_s}{g} \qquad \frac{4\pi R^2 P_s}{g}}$$

$R = 6371$ km, $P_s = 1013$ hPa, $g = 9.81\,\text{m s}^{-2}$

$$m = 5.27 \times 10^{18}\ \text{kg}$$

pto

ii

Specific humidity, $q_v = \frac{m_v}{m_v + m_d}$ $\boxed{0}$ No ice/water

$$e\alpha_v = R_v T \quad \boxed{1}$$

$$P_d \alpha_d = R_d T \quad \boxed{2}$$

Dalton $\Rightarrow$ $P = P_d + e$

$$= \left(\frac{R_d}{\alpha_d} + \frac{R_v}{\alpha_v} \right) T$$

$$m_v = \frac{V}{\alpha_v} \qquad m_d = \frac{V}{\alpha_d}$$

$$\Rightarrow q_v = \frac{1/\alpha_v}{1/\alpha_v + 1/\alpha_d} = \frac{1}{1 + \alpha_v/\alpha_d}$$

$(1)/(2) \Rightarrow \frac{\alpha_v}{\alpha_d} \frac{e}{P_d} = \frac{R_v}{R_d} = \frac{1}{0.62}$ (given)

$$\Rightarrow q_v = \frac{1}{1 + \frac{1}{0.62} \times \frac{1000}{10\text{[illegible]}}}$$

~~$q_v = 0.98$~~ ?

$$= 0.0062 \text{ kg/kg}$$

$$= \boxed{6.2 \text{ g/kg}}$$

iii

The atmosphere rotates at a faster rate than the Earth ~23 hours/revolution rather than 24 hour/revolution

Moment of inertia of ~~atmosphere~~ a solidsphere

$$I = \frac{2}{5} M R^2$$

Angular momentum

$$L = I\omega$$

Angular kinetic energy

$$E = \frac{1}{2} I \omega^2 .$$

Angular momentum of an air parcel

East -West velocity

$$L = R \cos \phi (u + \Omega R \cos \phi)$$

Latitude

⇒ If ~~Eart~~ atmosphere stopped rotating relative to surface.

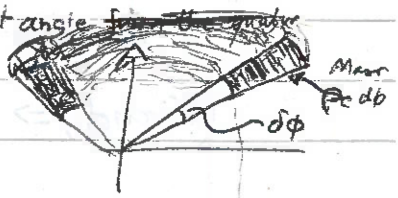
$$\Delta L_{parcel} = R \cos(\phi)\, u\, \rho\, dV$$

of atmosphere

Integrating, assuming mass of atmosphere is concentrated at the surface ⇒ $R \sim R_\oplus$

Mass per unit angle ~~from the equator~~

$$\Delta L_{atm.} = \int_{\phi=-\pi/2}^{\pi/2} R_\oplus \cos(\phi)\, u \rho_c\, d\phi$$



$$= \left[R_\oplus \sin(\phi) \rho_c u \right]_{\phi=-\pi/2}^{\pi/2} = 2 R u M$$

pto see later for iii answer.

at temperature T_a ↓

iv a) ε = Emissivity = $\dfrac{\text{Irradiance of a layer of the atmosphere}}{\text{Irradiance from a blackbody @ temperature } T_a}$

? b) Kirchoff's Law:

$\varepsilon_\lambda(T_a) = \alpha_\lambda(T_a)$ ← Absorptivity

$\varepsilon_\lambda(T_a) = \dfrac{I_\lambda^{\text{absorbed}}}{I_\lambda^{\text{incident}}(T_a)}$ ~~emitted~~ ← Intensity of radiation ~~emitted~~ absorbed ~~by Earth's surface~~

~~$I_\lambda^{\text{incident}}$ = Intensity of radiation emitted ~~Irradiance~~ from Earth's surface~~

~~$= \sigma T_a^4$~~

~~$\Rightarrow I_{\lambda=IR}^{TOA,\uparrow} = \varepsilon \sigma T_a^4$~~

~~$I_{\lambda=UV}^{TOA\downarrow} = \sigma T_s^4$~~

~~$= I_{\lambda,UV}^{\text{surface},\downarrow}$~~

~~Where we are Assuming that there is no net heating of The Earth's surface~~

& Assuming there is no net heating of the atmosphere

$$\underbrace{\varepsilon\sigma T_a^4}_{\text{Emitted upward}} = \underbrace{\alpha\sigma T_s^4}_{\text{Absorbed}}$$

(Kirchof) ⇒ $= \varepsilon\sigma T_s^4$

$\Rightarrow T_a^4 = T_s^4$

$$\Rightarrow I_{\lambda=IR}^{TOA,\uparrow} = \sigma T_s^4 - \varepsilon\sigma T_s^4 = \boxed{(1-\varepsilon)\sigma T_s^4}$$

c) TOA :

$$\boxed{\sigma T_e^4 = (1-\varepsilon)\sigma T_s^4 \quad \boxed{*}}$$

Surface

$$\boxed{\varepsilon\sigma T_a^4 + \sigma T_s^4 = \cancel{\sigma T_a^4} + \sigma T_e^4} \quad \boxed{**}$$

← (Assuming no absorption in shortwave)

d) If $\varepsilon = 0.8$, $T_e = 255K$

~~[illegible] => (*)~~

~~σT_s^4~~

$$(*) \Rightarrow T_s = \frac{T_e}{\sqrt[4]{1-\varepsilon}}$$

$$= \boxed{381K}$$

e) Cooling of surface via evaporation (latent heat flux) and convection ~~(se~~ + conduction (sensible heat flux).

V) a)

$$fv = \alpha \frac{\partial P}{\partial x}$$

Specific volume of air parcel

Rate of change of pressure in East - West direction

North South velocity

Coriolis parameter

$f = 2\Omega \sin(\lambda)$

Latitude

Angular velocity of planet

$$0 = -g - \alpha \frac{\partial P}{\partial z}$$

Rate of change of pressure with altitude

Acceleration due to gravity.

b) Taking the partial derivative of (1) with height, z,

$$\frac{\partial}{\partial z}(f v) = \frac{\partial}{\partial z}\left(\alpha(P)\frac{\partial P}{\partial x}\right)$$

~~If $\alpha(P)$ is only a function of Pressure~~

$$\cancel{= \alpha(P)\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial z}\right)}$$

$$\cancel{f\frac{\partial v}{\partial z} = \alpha(P)\frac{\partial}{\partial x}\left(-\frac{g}{\alpha(P)}\right)}$$

$$\cancel{= 0}$$

If α is only a fct. of pressure (which is approx. constant at a given latitude.

$$\frac{\partial}{\partial z}(f v) = \frac{\partial}{\partial z}\left(\frac{\partial(\alpha(P)\,P)}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\frac{\partial}{\partial z}\left(\alpha(P)\,P\right)$$

$$= \frac{\partial}{\partial x}\left(-\frac{g}{\alpha(P)}\right)$$

$$= 0$$

f is constant

$$\cancel{\Rightarrow \boxed{\frac{\partial f}{\partial v} = 0}} \qquad \Rightarrow \boxed{\frac{\partial v}{\partial z} = 0}$$

Q.1...

iii For a typical East-West wind speed $u = 10 m/s$

$$\Delta L = 2R\overset{}{\underset{\nwarrow\ 10\ m/s}{u}} M \leftarrow 5.27\times10^{18}\ \text{kg from (i)}$$

$$= 6.75\times10^{26}\ Nm.$$

Because angular momentum is conserved, the change in angular momentum of the Earth will be equal

$$\Rightarrow \Delta\omega = \frac{\Delta L}{I}$$

Mass of Earth, $M = 5.97\times10^{24}$ kg.

$$I = \frac{2}{5}MR^2$$

$$= 9.78\times10^{37}$$

$$\Rightarrow \Delta\omega = \cancel{4.34\times10^{-17}\ s^{-1}}\ \boxed{6.9\times10^{-12}\ rads^{-1}}$$

$$\omega + \Delta\omega = \frac{2\pi}{T+\Delta T}$$

$$\Rightarrow T + \Delta T = \frac{2\pi}{\omega+\Delta\omega}$$

$$\approx \frac{2\pi}{\omega}\left(1+\frac{\Delta\omega}{\omega}\right)$$

$$\Rightarrow \Delta T \sim \frac{2\pi}{\omega}\frac{\Delta\omega}{\omega}$$

$=$ [illegible] $\boxed{\sim 8\,ms}$

$\Rightarrow$ Negligible change in length of day.

Atmospheric 2014 Q.2

i ~~a)~~ $$I_\lambda(s) = \underbrace{I_\lambda(s_0)\,e^{-\tau_\lambda(s_0,s)}}_{\textcircled{1}} + \underbrace{\int_0^{\tau_\lambda(s_0,s)} B_\lambda(\tau')\,e^{-(\tau_\lambda(s_0,s)-\tau')}\,d\tau'}_{\textcircled{2}}$$

① = Intensity of radiation entering the atmosphere at a point s_0 which reaches a point s without being absorbed.

② = Intensity of radiation at point s contributed by the atmosphere via emission and scattering.

& τ_λ, called the "optical depth" is a measure of the proportion of radiation which is absorbed by the atmosphere as it travels from point s_0 to point s.

ii ~~b)~~ a) • Letting $s_0 = 0$ be the position @ Earth's surface, and assuming the atmosphere is isothermal so $B_\lambda(\tau) = \sigma T^4$ is constant.

• $\tau_\lambda(s_0, s_0) = 0$

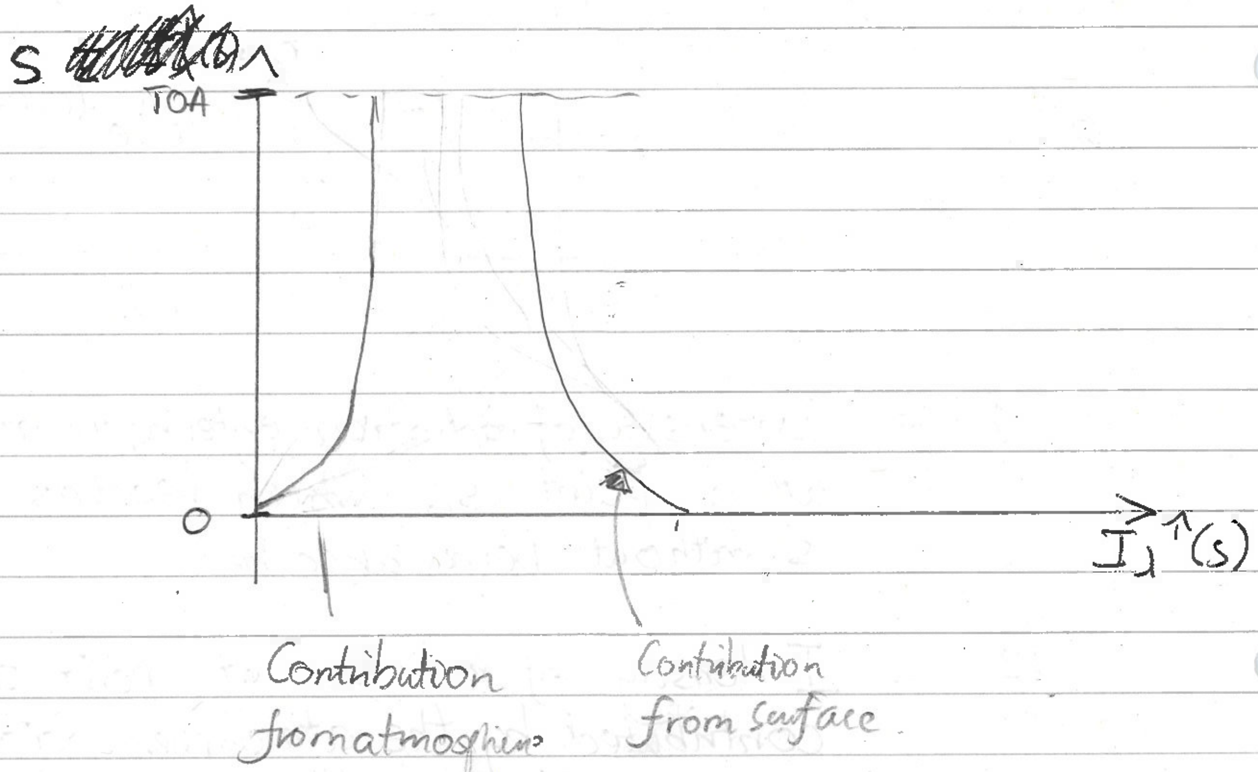
$$\Rightarrow I_\lambda(s) = I_\lambda(0)\,e^{-\tau_\lambda(0,s)} + \int_0^{\tau_\lambda(0,s)} B_\lambda(\tau')\,e^{-(\tau_\lambda(0,s)-\tau')}\,d\tau$$

$$= I_\lambda(0)\,e^{-\tau_\lambda(0,s)} + B_\lambda\left[e^{-(\tau_\lambda(0,s)-\tau')}\right]_0^{\tau_\lambda(0,s}$$

$$= I_\lambda(0)\,e^{-\tau_\lambda(0,s)} + B_\lambda e^0 - B_\lambda e^{-\tau_\lambda(0,s)}$$

$$\Rightarrow \boxed{I_\lambda(s) = B_\lambda + e^{-\tau_\lambda(0,s)}(I_{\lambda,s} - B_\lambda)}$$

where $I_{\lambda,s} = I_\lambda(0)$



$$\tau_\lambda(s_0,s) = \int_{s_0}^{s} \rho(s')\, k_\lambda(s')\, q_a \, ds'$$

$$\frac{dP}{ds} = -\frac{dP}{dz} = +\rho g$$

$$P = \rho RT$$

Const. T $\Rightarrow \frac{dP}{ds} = \frac{d\rho}{ds} RT$

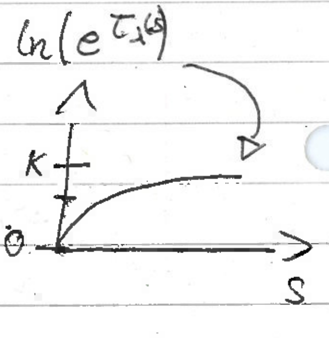
$$\Rightarrow \quad \frac{d\rho}{ds} = \frac{\rho g}{RT}$$

$$\rho \approx \rho_0 e^{-gs/RT}$$

Assume k_λ const & q_a const

$$\tau_\lambda = \int_0^s F e^{-gs/RT} ds'$$

$$\sim K - K e^{-gs/RT}$$



?

Since I is proportional to T_S^4, if the surface temperature T_S increases, intensity of radiation emitted from the surface $I_{lambda,s}$ will also increase, shifting I_{up}(s) to higher values.

Convection will cause the temperature higher up in the tropopause to increase more than the surface temperature, causing B_{lambda} to increase and hence the contribution from the upper troposphere to also increase.

Therefore I_{lambda} will be shifted to higher values..

Note that since the downward flux will also increase and will partially counter this effect. Unless there is a change in the composition of the atmosphere the long term temperature will tend towards the original equilibrium temperature.

iii $$I_\lambda^{\downarrow}(s) = I_\lambda(S_{TOA})\,e^{-\tau_\lambda(S_{TOA},s)} + \int_0^{\tau_\lambda(S_{TOA},s)} B_\lambda(\tau')\,e^{-(\tau_\lambda(S_{TOA},s)-}$$

$I_\lambda^{\downarrow}(S_{TOA}) = 0$ since density is zero at the TOA
⇒ Nothing to emit radiation.

$$\Rightarrow I_\lambda^{\downarrow}(s) = B_\lambda\left[e^{-(\tau_\lambda(S_{TOA},s)-\tau')}\right]_0^{\tau_\lambda(S_{TOA},s)}$$

$$\Rightarrow \boxed{I_\lambda^{\downarrow}(s) = B_\lambda\left(+1 - e^{-\tau_\lambda(S_{TOA},s)}\right)}$$

iv a)

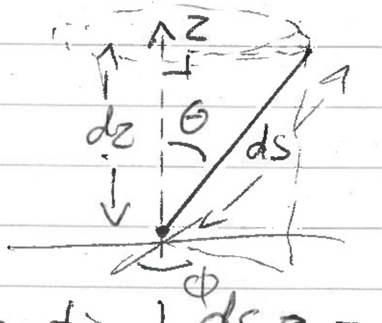


Fig. 1

From fig. 1, $ds_z = \dfrac{dz}{\cos\theta} \Rightarrow \tau_\lambda(S_{TOA},s) = \int \rho k g \dfrac{dz}{\cos\theta}$
$= \boxed{\tau_\lambda(TOA,z)/\cos\theta}$

b) $F_\lambda^{\downarrow}(z) = \iint I_\lambda^{\downarrow}(z)\cancel{\sin\theta\cos\theta}\,d\phi\,d\theta$

Independent of φ ⇒ $= 2\pi \quad B_\lambda \int 1 - e^{-\tau_\lambda(TOA,z)/\cos\theta}\,d\theta$

Atmospheric Physics 2014 Q.3

i Tropopause - the altitude at which the temperature is at a minimum.

Assuming lapse rate is constant $= -\frac{dT}{dz}$

$$Z_t = \frac{T_S - T_t}{\Gamma_t}$$

$$= \frac{288-210}{7\,K/km}$$

$$\boxed{Z_t = 11.1 \text{ km}}$$

ii If Hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g$$

Ideal gas law for dry air

$$P_d = \rho_d R_d T$$

$$\cancel{\frac{dP}{dz} = \frac{d\rho}{dz} R_d T + \rho R_d \frac{dT}{dz}}$$

$$\rho_d = P_d / R_d T$$

$$\frac{dP}{dz} = -\frac{P_d}{R_d T} g$$

$$\int_{P_s}^{P_t} \frac{dP_d'}{P_d'} = \frac{g}{R_d} \int_{z=0}^{z_t} \frac{1}{T_s - \Gamma z} \, dz$$

$$\ln(P_t/P_s) = + \frac{g}{R_d} \frac{1}{\Gamma} \Big[\ln(T_s - \Gamma z) \Big]_0^{z_t}$$

pto.

$E \equiv Fd$

$a = F/m$

$= [E] \times \frac{M}{L}$

J/kg

$$\frac{[E]L^{-1}M^{-1}}{KL^{-1}[E]M^{-1}K^{-1}} = 1$$

$$\frac{P_t}{P_s} = \left(\frac{T_s - \Gamma z_t}{T_s}\right)^{+\frac{1}{\Gamma}\frac{g}{R_d}}$$

$g = 9.8\,m/s^2$

$\Gamma = 7\times10^{-3}\,K/m$

$R_d = 287\,J\,kg^{-1}K^{-1}$

$T_s - \Gamma z_t = 78$, $T_s = 288$

$$P_t = 1.7\,hPa$$ (very small)

iii

$$\Gamma_d = \frac{g}{c_{p,d}} \qquad \boxed{1}$$

$$s_d = s_{ref} + c_{p,d}\ln(T/T_{ref}) - R_d\ln(P/P_{ref}) \qquad \boxed{2}$$

Brunt - Vaisala

$$N^2 \equiv \frac{g}{c_{p,d}}\frac{ds_d}{dz} \qquad \boxed{3}$$

Differentiating (2) =>

$$= \frac{g}{c_{p,d}}\left(\frac{c_{p,d}}{T}\frac{dT}{dz} - \frac{R_d}{P}\frac{dP}{dz}\right)$$

(Sub. Ideal gas) =>

$$= -\frac{g}{T}\Gamma_t + \frac{R_d}{P}\frac{P}{R_d T}\frac{g^2}{c_{p,d}}$$

$$= \frac{g}{T}\left(-\Gamma_t - \frac{g}{c_{p,d}}\right)$$

(1) =>

$$\boxed{N^2 = \frac{g}{T}\left(\Gamma_d - \Gamma_t\right)}$$

For $\Gamma_t = 7 K/km$, $c_{p,d} =$ for the atmosphere, $g = 9.8$

$$\Gamma_d = \frac{g}{c_{p,d}} = 7 \; K/km$$

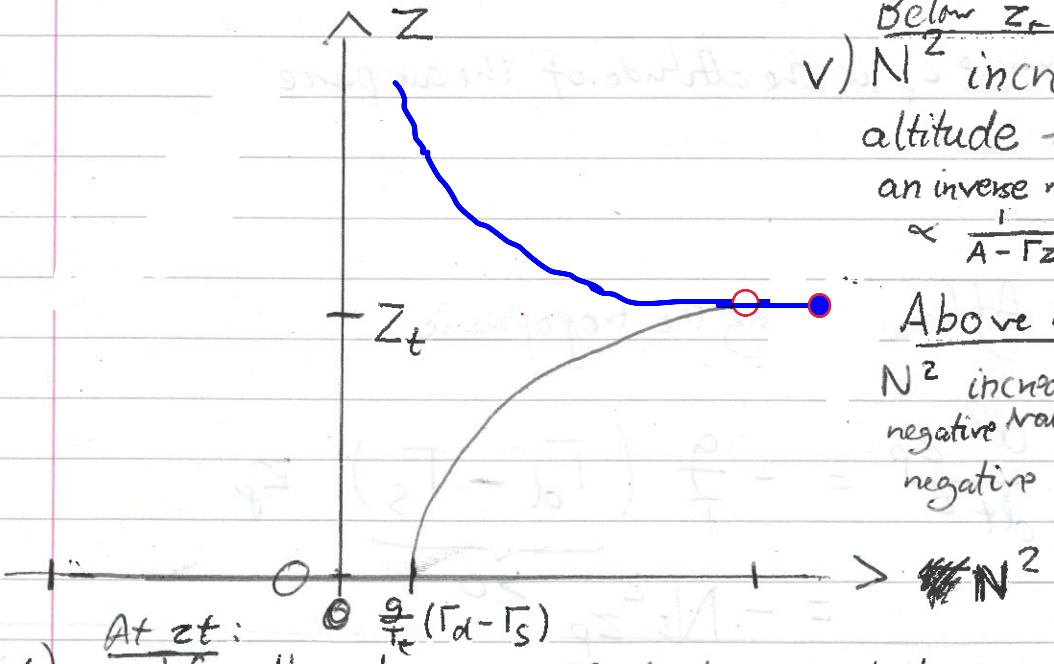
$\Rightarrow \boxed{N^2 = 0}$ in the troposphere.

iv $\boxed{N^2 = \frac{g}{T}(\Gamma_d - \Gamma_s)}$ in the stratosphere

since

Same

For $\Gamma_s < 0$ $\quad N^2 > 0$ => Stable to convection



Below z_t:

v) N^2 increases with altitude ~~up the~~ in a an inverse relationship to z.

$\propto \frac{1}{A - \Gamma z}$

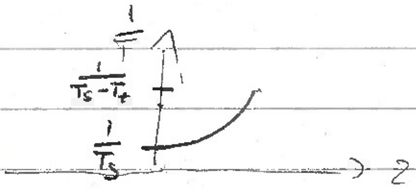
Above z_t:

N^2 increases from a large negative value but remains negative

At zt:

v) If the tropopause & stratopause both have constant lapse rates, N^2 must be discontinuous @ the tropopause since $T > 0$, but Γ flips sign.

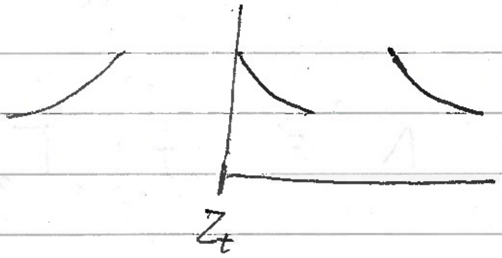
$$(T)_{tropo} = T_s - \Gamma Z$$



$$\left(\frac{1}{T}\right)_{tropo} = (T_s - \Gamma_t z)^{-1}$$

@ $z = Z_t$ $\quad T = T_t > 0$

$$\left(\frac{1}{T}\right) = -\frac{1}{T_t + \Gamma_s \Gamma z}$$



vi) a) $$\frac{d^2 z_p}{dt^2} = -N_t^2 z_p$$

where z_p is the altitude of the air parcel.

b) After crossing the tropopause

$$\frac{d^2 z_p}{dt^2} = -\frac{g}{T}\underbrace{(\Gamma_d - \Gamma_s)}_{>0} z_p$$

$$= -N_s^2 z_p$$

$$\boxed{z_p = z_t + A\cos(N_s t)}$$

$$\frac{dw_p}{dt} = g\,\frac{\theta_p - \theta_e}{\theta_e} \qquad N_s^2 = \frac{g}{\theta_e}\frac{d\theta_e}{dz}$$

Above the tropopause potential temperature continues to increases with height and will dominate over the potential temperature of the parcel. Therefore the parcel will decelerate and begin to fall.

This results in sinusoidal motion with a frequency equal to g/T * (Gamma_d-Gamma_s)