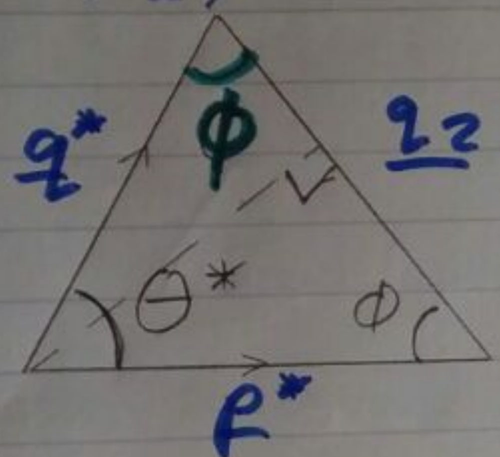
Angles & K.E. in
3.5 Collisions with a Stationary Target

Things we want to know:

1. COM & lab frame scattering angles
2. Max. scattering angle
3. K.E. transfer (what ratio of masses maximises this)

1.



$|p^*| = |q^*|$ If no resultant external force.

$\Rightarrow$ Isosceles triangle!

$\Rightarrow$ $2\phi + \theta^* = \pi$, Σ Angles $= \pi$

$$\boxed{\phi = \frac{\pi - \theta^*}{2}}$$ Eq. 3.5-1, q_2

Consider diagram 3.5-1

$$\tan\theta = \frac{|q^*| \sin(\theta^*)}{\frac{m_1}{m_2}|p^*| + |q^*|\cos\theta^*}$$

$$\Rightarrow \tan\theta = \frac{\sin\theta^*}{\frac{m_1}{m_2} + \cos\theta^*}$$

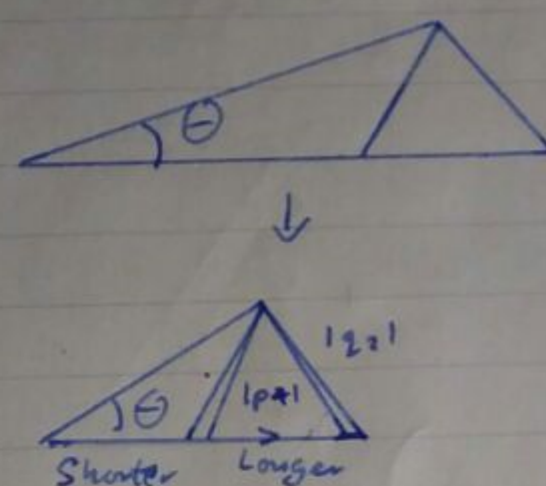
How do you find θ^*?

Maximum scattering angle

If $m_1 < m_2$

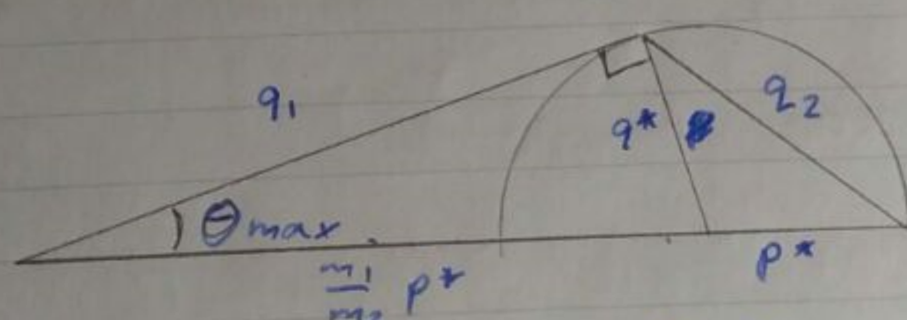
$$\frac{m_1}{m_2} < 1$$

$$|p^*| > |p^*|\frac{m_1}{m_2}$$



Derivation tips:
- **Tangent**
- **Use $|p^*| = |q^*|$.**

At max. scattering angle = θ = Angle incoming particle is deflected by



$\underline{q_1}$ tangential to $\underline{q^*}$ @ max. scattering angle

$$\Rightarrow \sin\theta_{max.} = \frac{q^*}{\frac{m_1}{m_2}p^*} = \frac{m_2}{m_1} \quad \text{If } q^* = p^*$$

when not true?

$$\boxed{\theta_{max.} = \arcsin\left(\frac{m_2}{m_1}\right)}$$

for stationary target

Other conditions?

eg. Fast ~~e⁻ scatt~~ proton scattering

$$p \longrightarrow \underset{\text{stationary}}{e^-}$$

$m_2 = m_{e^-}$
$m_1 = m_p$

$$\frac{m_2}{m_1} \approx \frac{1}{1836}$$

Max. scattering angle, $\theta_{max.} = \arcsin\left(\frac{1}{1836}\right) \approx 0.031^\circ$

(If e^- stationary) $\nearrow$

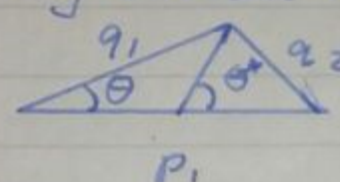
~~K.E.~~,

Kinetic Energy Transfer

$$K.E._1 = \cancel{|p_1|} \; \frac{1}{2} m_1 |\underline{v}_1|^2 = \frac{|p_1|^2}{2m_1}$$

$$\boxed{\Rightarrow KE_1 = \frac{\left(|p^*| \left(\frac{m_1}{m_2} + 1\right)\right)^2}{2m_1}}$$

Derivation tip:
Using ultimate triangle



$$K.E._2{}' = \frac{1}{2} m_2 |\underline{v}_2|^2 = \frac{|q_2|^2}{2m_2}$$

$$\boxed{|q_2| = 2|p^*| \sin\left(\tfrac{1}{2}\theta^*\right)}$$

Derivation tip
· Sub ~~trian~~ ultimate triangle of stationary target collisions.