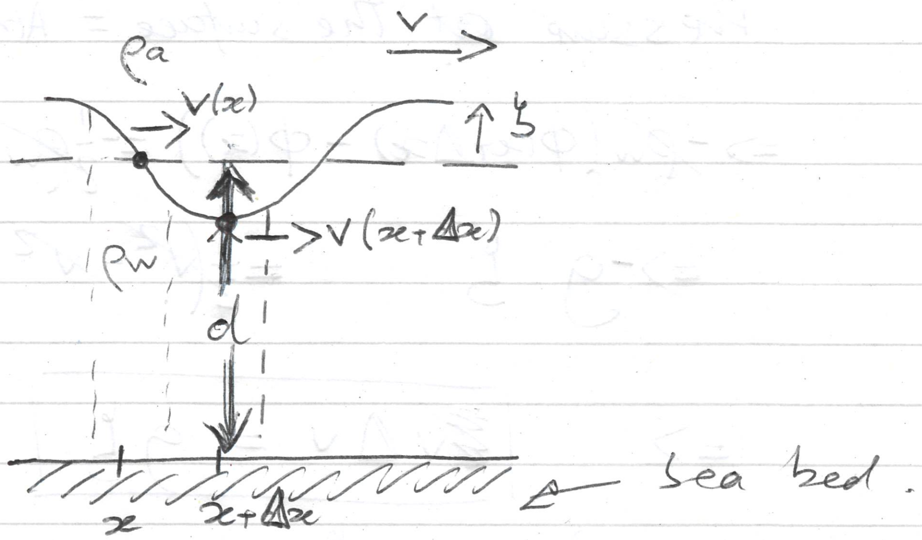
7/5/19

AH PS3 Q.1 : What is the velocity of a water wave?



Consider the wave from the fixed frame of reference of the shore / sea bed.

[a] Mass is conserved

max^m displacement from eq^b = ½ wave height

$$\Rightarrow \rho_w \times (d - \xi) \times \Delta t \, v(x+\Delta x)$$

$$= \rho_w \times (d) \times \Delta t \, v(x)$$

$$v(x+\Delta x) = v + \Delta v$$

$$\Rightarrow (d - \xi)(v + \Delta v) = d \times v$$

$$\Rightarrow \cancel{dv} - \underbrace{\xi \Delta v}_{\text{small}} - \xi v + d \times \Delta v = \cancel{d \times v}$$

$$\Rightarrow \boxed{d \times \Delta v = \xi v} \quad \boxed{1}$$

Bernouilli

$$P_x - \rho_w \phi_x - \frac{1}{2}\rho_w v^2 = P_{x+\Delta x} - \rho_w \phi_{x+\Delta x} - \frac{1}{2}\rho_w (v+\Delta v$$

pto.

Pressure at the surface = Atmospheric pressure

$$\Rightarrow -\cancel{\rho_w}\left(\phi(x+\Delta x) - \phi(x)\right) = -\frac{1}{2}\cancel{\rho_w}\left(v^2 - (v+\Delta v)^2\right)$$

$$\Rightarrow -g\,\zeta \qquad = \frac{1}{2}\left(\cancel{v^2} - \cancel{v^2} - 2v\Delta v - \cancel{\Delta v^2}\right)$$

Negligible

$$\Rightarrow \qquad \boxed{\cancel{2}\,v\Delta v = g\zeta} \qquad \boxed{2}$$

(1) $\Delta v = \frac{\zeta}{d} v$

$$\Rightarrow \quad \cancel{2}\,v^2 \frac{\cancel{\zeta}}{d} = g\cancel{\zeta}$$

$$\Rightarrow \qquad \boxed{v = \sqrt{gd}}$$