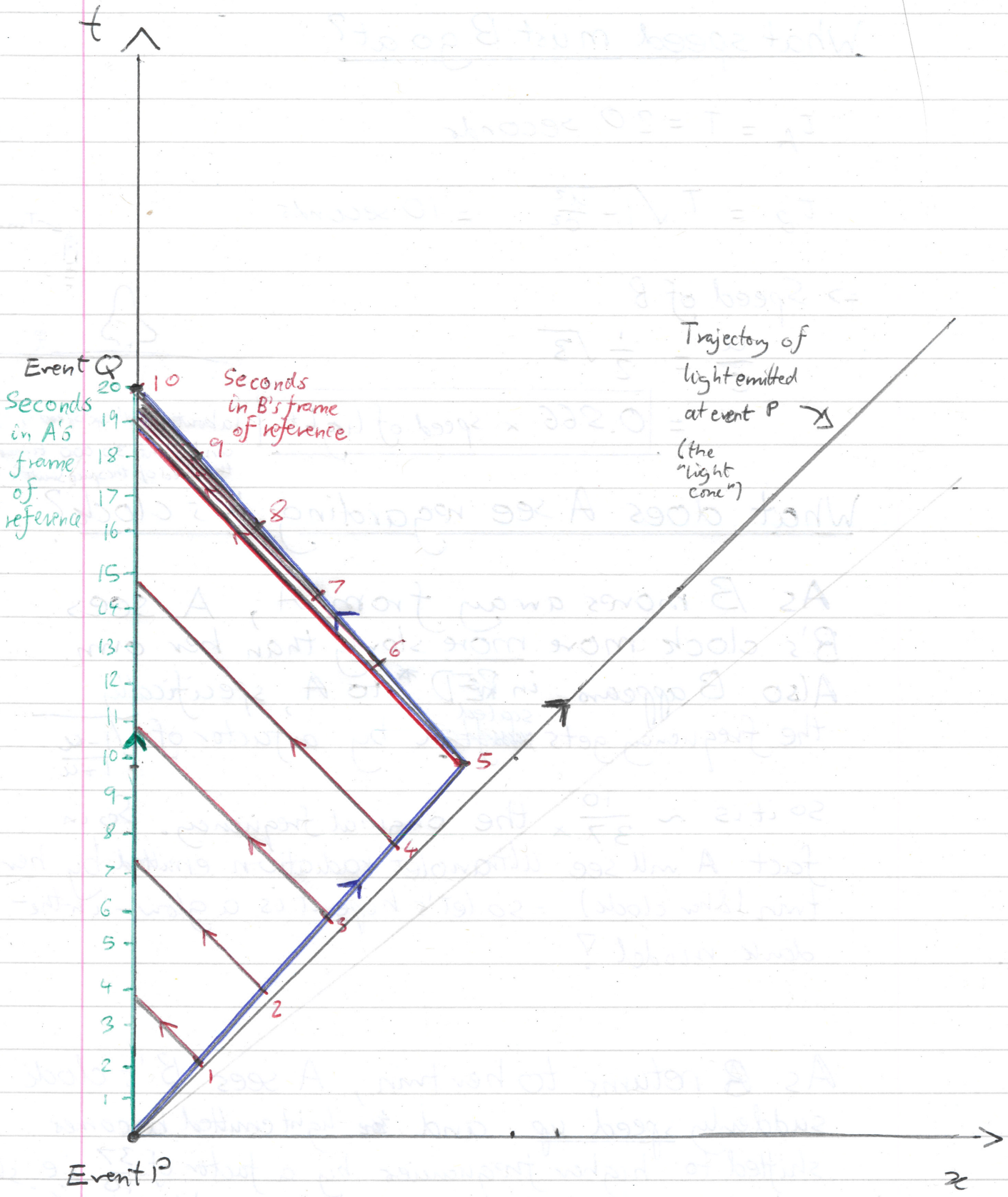
GR PS4 Q. 1 The Twin Paradox: What the twins really see

Here is another way to think about the twin paradox. Often when the puzzle is presented, it is put in a confusing way in which people may say "observer A sees observer B's clocks going slow". What is meant by this statement is nothing to do with twin A "seeing" anything. Remember to see something ~~an~~ you need to receive a ~~signal~~ photon - and a photon travels at the speed of light (a 45° angle on a space-time diagram). ~~What is~~

When someone says "observer A sees observer B's clocks going slow" what they mean is that in the rest frame of A, ~~the~~ tick's of B's clock ~~are~~ at every second of proper time along her worldline are separated by coordinate time intervals greater than one second.

Consider the usual situation: one twin A, stays on Earth at rest in an inertial frame. The other twin goes off in the x direction at speed v and then turns around and comes back at speed v. Suppose ~~the time~~ between B's departure (event P) and return (event Q) ~~is~~ 20 seconds of proper time have passed for A, and ten for B. That means 5 seconds on the outward leg and 5 seconds on the return leg.

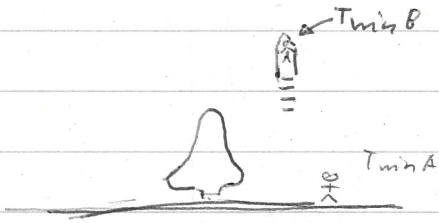
pto.


t
Event Q
Seconds in A's frame of reference
Seconds in B's frame of reference
Trajectory of light emitted at event P
(the "light cone")
Event P
x
A's worldline
B's worldline
Worldline of light emitted at each tick of B's clock

What speed must B go at?

$t_A = T = 20$ seconds

$t_B = T\sqrt{1 - \frac{u^2}{c^2}} = 10$ seconds



=> Speed of B

$$\frac{u}{c} = \frac{1}{2}\sqrt{3}$$

$$= \boxed{0.866 \times \text{speed of light}}$$

(about 1 million× speed of a jet and about 30,000 times the speed of the space Shuttle)

What does A see regarding B's clock?

As B moves away from A, A sees B's clock move more slowly than her own. Also B appears in ~~RED~~ * to A, specifically the frequency gets ~~shifted~~ scaled by a factor of $\sqrt{\frac{1-u}{1+u}}$

so it is $\sim \frac{10}{37} \times$ the original frequency. So in fact A will see ultraviolet radiation emitted by her twin (& her clock) - so let's hope it is a glow-in-the-dark model ∇

As B returns to her twin, A sees B's clock suddenly speed up and ~~the~~ light emitted becomes shifted to higher frequencies by a factor of $\frac{37}{10}$ i.e. she gets to see her sister's body heat as visible light (without a dumb smartphone app)

* ~~or other~~ cover ~~high~~ frequency radiation.