Imperial College London

BSc/MSci EXAMINATION June 2014

This paper is also taken for the relevant Examination for the Associateship

ATMOSPHERIC PHYSICS

For 3rd and 4th-Year Physics Students

28 May 2014: 14:00 to 16:00

The paper consists of two sections: A & B. Section A contains one question. Section B contains three questions.
Answer ALL parts of Section A and TWO questions from Section B.
Marks shown on this paper are indicative of those the Examiners anticipate assigning.

General Instructions

Complete the front cover of each of the 4 answer books provided.

If an electronic calculator is used, write its serial number at the top of the front cover of each answer book.

USE ONE ANSWER BOOK FOR EACH QUESTION.

Enter the number of each question attempted in the box on the front cover of its corresponding answer book.

Hand in 4 answer books even if they have not all been used.

You are reminded that Examiners attach great importance to legibility, accuracy and clarity of expression.

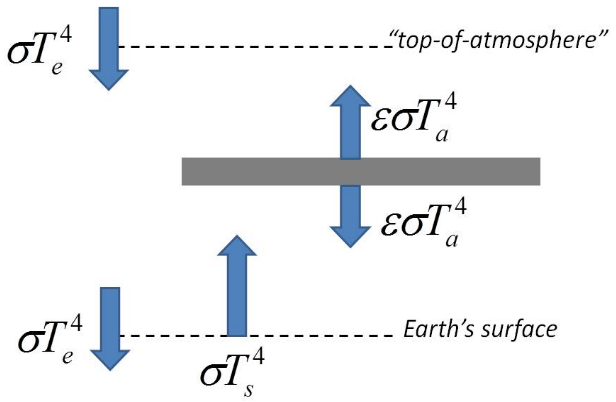

Figure 1: Schematic of a model of the greenhouse effect. **Note that not all energy fluxes are shown in this diagram.**

SECTION A

1. (i) Show that the mass m of the atmosphere can be approximately computed from $m = 4\pi R^2 P_s/g$ where $R = 6371km$ is the Earth's radius, $P_s = 1013hPa$ the surface pressure and $g = 9.81ms^{-2}$ the gravity at the Earth's surface. State any assumptions made and compute m from the values given above. [4 marks]

(ii) The vapour pressure in a sample of air is found to be $e = 10hPa$ while the total pressure is $P = 1000hPa$. No liquid or ice water was found in the sample. Compute the associated specific humidity q_v.The ratio of the ideal gas constant for dry air and water vapour is $R_d/R_v = 0.62$. [4 marks]

(iii) Compute the change in the length of the day if the relative winds were to stop and the atmosphere simply rotated en masse with the same angular speed as the Earth. [Info needed: mass of the Earth $M = 6 \times 10^{24} kg$; moment of inertia of a solid sphere of radius R and mass M, $I = 2MR^2/5$; see also numerical values in (i)]. [6 marks]

(iv) A simple model of the greenhouse effect is depicted in Fig. 1. The atmosphere, at temperature T_a, is assumed to radiate in the infrared according to $\epsilon\sigma T_a^4$ in which σ is Stefan-Boltzmann's constant (grey slab in Fig. 1). The Earth's surface is assumed to be a blackbody at temperature T_s. For simplicity, absorption of solar energy by the atmosphere is neglected. The solar flux is written in the form σT_e^4 in which $T_e = 255K$ is the emission temperature of the Earth.

(a) Interpret physically the constant ϵ. [2 marks]

(b) Using Kirchoff's law, explain why the amount of longwave radiation emitted by the Earth's surface which reaches the "top-of-the-atmosphere" (TOA) is $(1-\epsilon)\sigma T_s^4$. [2 marks]

[This question continues on the next page . . .]

(c) Write the conservation of energy at the "top-of-the-atmosphere" and at the surface. [6 marks]

(d) Compute the surface temperature T_s and the atmospheric temperature T_a when $\epsilon = 0.8$. [4 marks]

(e) What are the surface cooling mechanisms omitted in this model? [2 marks]

(v) The geostrophic and hydrostatic approximations can be written in the form,

$$fv = \alpha \frac{\partial P}{\partial x} \tag{1}$$

$$0 = -g - \alpha \frac{\partial P}{\partial z} \tag{2}$$

(a) Give the meaning of all the symbols appearing in these equations. [4 marks]

(b) Use these two equations to show that if $\alpha = \alpha(P)$ (i.e., α is just a function of P) the wind component v cannot vary with height. [6 marks]

[Total 40 marks]

SECTION B

2. In this question we analyze infrared radiation in an isothermal atmosphere at $T = T_o$. In absence of scattering, we accept that the intensity of radiation I_λ at wavelength λ (either upward or downward) through a path measured by the distance s from a reference start s_o satisfies,

$$I_\lambda(s) = I_\lambda(s_o)e^{-\tau_\lambda(s_o,s)} + \int_0^{\tau_\lambda(s_o,s)} B_\lambda(\tau')e^{-(\tau_\lambda(s_o,s)-\tau')}d\tau' \tag{1}$$

(i) Interpret physically the two terms on the r.h.s of (1) and explain in simple terms what $\tau_\lambda(s_o, s)$ is. [6 marks]

(ii) We focus here on the upward infrared radiation.

(a) Show that it can be calculated as,

$$I_\lambda^{\uparrow}(s) = B_\lambda + e^{-\tau_\lambda(0,s)}(I_{\lambda,s} - B_\lambda) \tag{2}$$

where $I_{\lambda,s}$ is the intensity of upward radiation emitted at the Earth's surface ($s_o = 0$). [4 marks]

(b) Sketch as a function of s the contribution to $I^{\uparrow}(s)$ coming from the Earth's surface and that coming from the atmosphere. Discuss qualitatively the shape of $I^{\uparrow}(s)$ as a function of the Earth's surface temperature T_s. [6 marks]

(iii) Show that the downward infrared radiation reaching s, namely $I_\lambda^{\downarrow}(s)$, obeys,

$$I_\lambda^{\downarrow}(s) = B_\lambda(1 - e^{-\tau_\lambda(TOA,s)}) \tag{3}$$

in which $s_o = TOA$ denotes the"top-of-the-atmosphere". [2 marks]

(iv) We now wish to integrate (3) over solid angle to obtain the downward irradiance $F_\lambda^{\downarrow}$.

(a) Show that the optical thickness measured along a path at an angle θ with the vertical, namely $\tau_\lambda(TOA, s)$, is related to the optical thickness measured along the vertical $\tau_\lambda(TOA, z)$ by,

$$\tau_\lambda(TOA, s) = \tau_\lambda(TOA, z)/\cos\theta \tag{4}$$

[2 marks]

(b) Hence show that

$$F_\lambda^{\downarrow} = 2\pi B_\lambda \int_0^1 (1 - e^{-\tau_\lambda(TOA,z)/\mu})\mu d\mu \tag{5}$$

in which $\mu = \cos\theta$. State any assumptions made. [4 marks]

[This question continues on the next page . . .]

(v) We assume that the Earth's surface temperature is $T_s = T_o$. By using the approximation,

$$2\int_0^1 e^{-\tau_\lambda(TOA,z)/\mu}\mu d\mu \approx e^{-\tau_\lambda(TOA,z)/\overline{\mu}} \tag{6}$$

in which $\overline{\mu} = 1/1.66$, and after considering both upward and downward irradiances, show that the net heating rate $Q_\lambda(z)$ (in Wm^{-3} per wavelength) satisfies,

$$|Q_\lambda(z)| \propto e^{-\tau_\lambda(TOA,z)/\overline{\mu}} \tag{7}$$

Discuss whether Q_λ is a heating or a cooling. [6 marks]

[Total 30 marks]

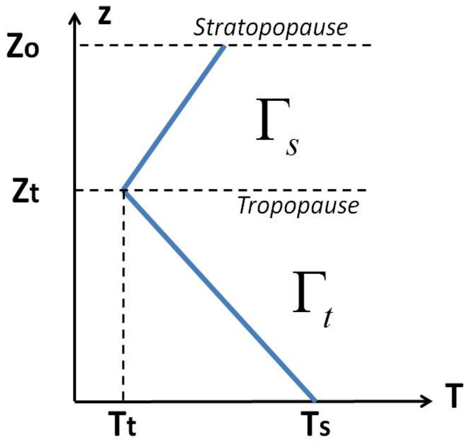

Figure 1: Schematic of temperature variations with height for Q3.

3. In this question we study the buoyancy force experienced by a parcel of air of troposheric origin as it crosses the tropopause. The effects of moisture are entirely neglected and the troposphere and stratosphere are assumed to have constant lapse rates, $\Gamma_t > 0$ and $\Gamma_s < 0$, respectively (Fig. 1). The surface temperature T_s, the surface pressure P_s, the tropopause temperature T_t and the lapse rates are assumed to be known.

(i) Express the height z_t of the tropopause as a function of T_s, T_t and Γ_t. Compute its numerical value for $T_s = 288K$, $T_t = 210K$, $\Gamma_t = 7K/km$. [2 marks]

(ii) Compute the pressure P_t of the tropopause if $P_s = 1000hPa$. You may assume the hydrostatic equation holds. [Ideal gas constant for dry air: $R_d = 287Jkg^{-1}K^{-1}$.] [6 marks]

(iii) Compute the Brunt-Vaisala frequency $N^2(z) = (g/c_{p,d})ds_d/dz$ throughout the troposphere ($0 \leq z \leq z_t$) and show that it can be expressed as,

$$N^2 = \frac{g}{T}(\Gamma_d - \Gamma_t) \quad (1)$$

in which $\Gamma_d = g/c_{p,d}$ is the dry adiabatic lapse rate. [Specific entropy of dry air: $s_d = s_{ref} + c_{p,d} \ln T/T_{ref} - R_d \ln P/P_{ref}$ in which the subscript *ref* indicates a reference state, $c_{p,d}$ is the specific heat capacity at constant pressure and R_d was introduced in (ii).] [6 marks]

(iv) Find likewise an expression for N^2 as a function of height for the stratosphere. Sketch its variations from the Earth's surface ($z = 0$) to the stratopause ($z = z_o$) (Fig. 1). [6 marks]

(v) Qualitatively describe how potential temperature θ varies with height ($0 \leq z \leq z_o$). Is it discontinuous at the tropopause? [2 marks]

(vi) We now study the motion of an air parcel originating from the troposphere as it crosses the tropopause. We simplify by taking a constant Brunt-Vaisala frequency in each layer, i.e., $N^2 = N_t^2$ for $z < z_t$ and $N^2 = N_s^2$ for $z > z_t$.

[This question continues on the next page . . .]

(a) Write the equation describing the one dimensional motion of an air parcel in the troposphere when the Brunt-Vaisala frequency is constant.

[2 marks]

(b) We assume that the initial velocity of the parcel is large enough that it reaches the tropopause. Write the equation of motion describing its fate after it crosses the tropopause and discuss qualitatively its solution assuming that the parcel conserves its potential temperature. [You might use without proof that $dw_p/dt = g(\theta_p - \theta_e)/\theta_e$ in which w_p is the parcel's upward velocity, θ_p its potential temperature and θ_e that of the environment.] [6 marks]

[Total 30 marks]

4. We study in this question the modulation of the Jet Stream by a stationary Rossby wave.

(i) A form of the linearized vorticity equation for the vertical component of vorticity (ζ) in Cartesian coordinates is,

$$\frac{\partial \zeta'}{\partial t} + U_o \frac{\partial \zeta'}{\partial x} + \beta v' = 0 \tag{1}$$

in which ζ' is the perturbation relative vorticity of the flow, v' is the north-south perturbation velocity, $U_o > 0$ a background zonal velocity and β is the gradient of the Coriolis parameter with latitude ($\beta = df/dy$). Interpret physically this equation. [4 marks]

(ii) Explain briefly why Rossby wave motion can be considered in geostrophic balance and justify briefly why, as a result, the horizontal velocity components can be expressed as a function of a streamfunction ψ' from,

$$(u', v') = (-\frac{\partial \psi'}{\partial y}, +\frac{\partial \psi'}{\partial x}) \tag{2}$$

[3 marks]

(iii) Express ζ' as a function of a perturbation streamfunction ψ'. [2 marks]

(iv) Find the dispersion relation satisfied by waves for a perturbation streamfunction of the form $\psi' \propto e^{i(kx+ly-\omega t)}$, where k, l are the zonal and meridional wavenumbers, respectively, and ω is the angular frequency. [4 marks]

(v) By considering a solution $\psi' \propto \sin(kx + ly - \omega t)$ compute the average value of $F = u'v'$ over a latitude circle and interpret its physical meaning. Illustrate your answer with a plot of the perturbation streamfunction when $|k| = |l|$, discussing both the case where $kl > 0$ and $kl < 0$. [6 marks]

(vi) The presence of the Himalayas and the Rocky mountains tend to generate a zonal (east-west) wavenumber 2 in the Northern Hemisphere at $40^\circ N$.

(a) Compute the associated values of f and β. [Earth's radius: $R = 6371 km$]. [2 marks]

(b) Compute the meridional wavenumber l required for a stationary wave when $U_o = 15 m/s$. Why can't stationary waves exist when $U_o < 0$? [4 marks]

(c) We take the stationary wave to be confined to a North-South channel of width $\Delta y = \pi/l$, i.e., $\psi'(x, y) = A\cos(\pi y/\Delta y)\cos(kx)$, with $-\Delta y/2 \leq y \leq \Delta y/2$. Sketch the total flow (perturbation + mean) for $A \ll U_o \Delta y$, $A \simeq U_o \Delta y$, and $A \gg U_o \Delta y$, paying attention to the strength of the westerly wind. [5 marks]

[Total 30 marks]