AM PS3 Q. 7 Kolmogorov scaling & A cup of Tea

When you stir a cup of tea, how does the energy of the flow become spread over a spectrum of wave numbers?

Kolmogorov Scaling Hypothesis

The kinetic energy density at a given ~~wavelength~~ wavenumber, k, $E(k)$ is proportional to $k^{-5/3}$

$$E(k) \propto k^{-5/3}$$

When you stir a cup of tea you introduce kinetic energy in the form of circular motion of the water. The motion of the water has a scale ~~about~~ equal to the size of the cup.

Fig. 1 - Stirring a cup of t

When you stop stirring, the shear of the velocity of the water introduces a Kelvin-Helmholtz instability which creates smaller vortices. These ~~lesser~~ vortices drive lesser vortices ~~(and so on)~~. which drive lesser vortices still (and so on) at smaller & smaller wavenumbers k. Thus the energy becomes distributed over a range of length scales.

The maximum length scale is the size of the cup. The smallest is determined by the viscosity of the tea. Since viscosity introduces damping which opposes the circular motion.

We describe the amount of spreading across length scales in terms of the ratio of the inertia term & the viscosity term in the ~~Rayleigh-Taylor~~ Navier-Stokes equation

$$Re = \left| \frac{\nabla (u \nabla u)}{\nu \nabla \cdot (\rho \nabla u)} \right|$$

$$Re = \frac{|\rho(\underline{u}\cdot\nabla)\underline{u}|}{|\nabla\cdot(\eta\nabla\underline{u})|}$$

Large Re – viscosity interferes less & smaller vortices formed.