Optics- and Radar-based Observations

F7003R

General Radar Theory

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In this Assignment, ***Problem 2 - general radar theory***, ...

1 Operational frequency for scientific radar

The operational frequency for a scientific radar is highly depending on the application and the type of the target. If for example the ionosphere, a so-called soft-target is to be observed, the range resolution of the radar plays a more important role than for example when a hard target is wanted.

1.a Frequency ranges

According to ISR uses frequencies between 50MHz and 2 GHz

add reference ti that oslo presentation

3-300mhz csr

Radar stands for ***Radio Detection and Ranging***, so Radar generally operates in the RF band, going from about 3MHz to 300GHz [2]. Therefore, any of the Radar techniques explained above can be operated in any of those ranges. However, some specific frequency ranges may have advantages and disadvantages for the particular radar technique applied, which is why sometimes ISR, CSR and Doppler Radar Applications are categorized in different bands. Unfortunately, different literature and papers state different ranges, so for example

1.b Information gathered by radar measurement

2 The radar equation

To derive the radar equation one starts with the assumption of an isotropic radiation source. Therefore the power density at a distance R is denoted as

$$Q_i = \frac{P_t}{4\pi R^2} \qquad \left[\frac{W}{m^2}\right] \tag{1}$$

Since usually radar is focused in one specific direction, the equation is multiplied with the Gain G of the antenna

$$Q_i = \frac{P_t G}{4\pi R^2} \qquad \left[\frac{W}{m^2}\right] \tag{2}$$

When the radiated power now hits a target, a fraction of the power is reflected, or *re-radiated*. This reradiated power is depending on the cross-section σ of the hit target. So the received Power density at the antenna is

$$Q_{re} = \frac{Q_i \sigma}{4\pi R^2} = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} \qquad \left[\frac{W}{m^2}\right] \tag{3}$$

But this received Power is now again dependent on the antenna gain, which is in this case also depending on the antenna effective area A_{eff}

$$P_r = Q_{re} A_{eff} = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} A_{eff} \qquad [W] \tag{4}$$

The antenna effective area can now be expressed with the help of the antenna gain. The derivation of the gain is not shown here, but it can be derived using the effective area of a Hertzian dipole and the assumption that the antenna perfectly absorbs all received power.

This leads to the equation for the antenna effective area, which is depending on the antenna gain and the wavelength λ of the used frequency.

$$A_{eff} = G \frac{\lambda^2}{4\pi} \qquad [m^2] \tag{5}$$

Inserting this relation in equation 4 leads to the so-called Radar Equation, the total received power of the antenna.

$$P_r = P_t \frac{\rho_a^2 A^2}{4\pi \lambda^2 R^2} \sigma \qquad [W] \tag{6}$$

where ρ_a is the antenna efficiency, defined as

$$\rho_a = \frac{\lambda^2 G}{4\pi A} = \frac{A_{eff}}{A} \tag{7}$$

with A being the total antenna area.

This derivation was done keeping in mind that only one target, a so-called hard target is being observed. In case of a distributed, or soft target the equation 6 is further enhanced.

To generalize the equation 6 for more than one target over a volume, one can use the integral relation of cross sectino areas in a Volume (10).

$$\int \sigma(\vec{x}) dV_s \equiv \sigma \tag{8}$$

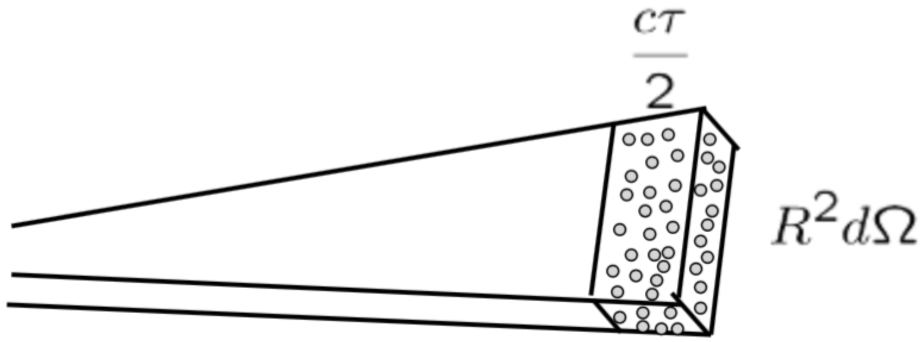

Figure 1: Soft Target (image source: lecture by Philip Erickson [1])

Inserting this into Eq. 6 leads to a generalized version Equation.

$$P_r = \int P_t \frac{\rho_a^2 A^2}{4\pi\lambda^2 R^2} \sigma(\vec{x}) dV_s \qquad [W] \tag{9}$$

Assuming an isotropic scatter distribution for the volume (c.f. **??**), one can solve the integral to:

$$\int \sigma(\vec{x}) dV_s = \frac{c\tau}{2} R^2 \sigma \tag{10}$$

Combining (10) and the generalized radar equation (9) leads to the radar equation for a soft target:

$$P_r = P_t \frac{c\rho_a^2 A^2 \tau}{8\pi\lambda^2 R^2} \sigma \qquad [W] \tag{11}$$

3 Cross section calculation

To calculate the minimum cross section that is detectable for the given values

Object Location	$R = 100$km
Wavelength	$\lambda = 6$m
Transmit Power	$P_t = 10$ kW
Antenna Gain	$G = 20$ dB $= 100$ W
System Noise Temperature	$T_s = 10^3$ K
Bandwith	$B_w = 1$ MHz

the Signal-to-Noise ratio (SNR) is used. The SNR is the ratio between the received power versus the total system noise Power, where the System Noise Power is, according to , denoted as

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$$P_n = N_0 = k_B T_s B_w \qquad [W] \tag{12}$$

Using equations 4 and 12, the SNR can be written as

$$SNR = \frac{P_r}{P_n} = \frac{P_t G A_{eff} \sigma}{4\pi R^2 k_B T_s B_w} \tag{13}$$

Substituting A_{eff} with eq. 5 and re-ordering 14 to the cross section area σ leads to

$$\sigma = SNR \frac{(4\pi)^3 R^4 k_B T_s B_w}{P_t G^2 \lambda^2} \qquad [m^2] \tag{14}$$

To calculate the smallest cross section area that is detectable at a range of 100km, one has to assume a minimum SNR, at which the minimum cross section is still detectable.

The minimum SNR, not taking into account LNA's or other instruments to increase the SNR, is obviously 1, since if the signal is stronger than the noise the signal is theoretically detectable.

Inserting $SNR_{min} = 1$ for SNR in eq. 14 this leads to a minimum cross section

$$\sigma_{min} = 0.76105 \qquad [m^2] \tag{15}$$

This is the same result as is given in the assignment sheet.

4 MST radars

4.a SNR as function of universal time and altitude

To plot the Signal to Noise ratio as a function of time and altitude, data from 28th of February 2006 is used. The plot can be seen in fig. **??**, the MATLAB Code used to plot these heatmaps can be found in Appendix A.1.

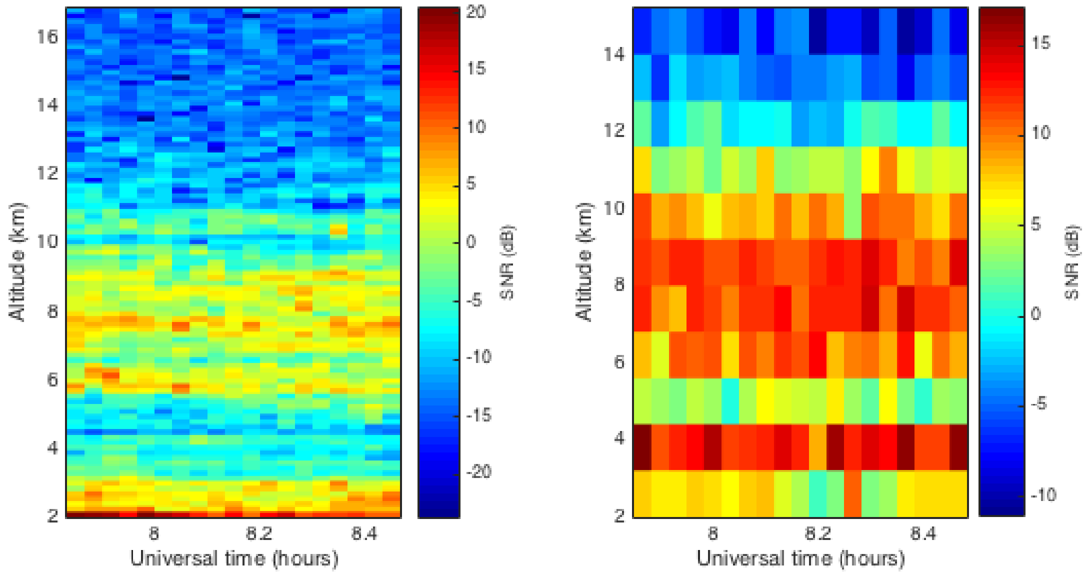

Figure 2: SNR as function of altitude and time, data from 28Feb06

4.b Pulse calculations

To calculate the pulse length, the inter-pulse period (IPP), pulse repetition frequency (PRF)and maximum unambiguous range R_{max} for these height resolutions, the following formulas are used:

$$\Delta R = \frac{c\tau_p}{2} \qquad [km] \qquad \text{, with } \Delta R \text{ as Range resolution and } \tau_p \text{ as pulse length} \tag{16}$$

$$IPP = \frac{\tau_p}{D} \qquad [s] \qquad \text{, with D as duty cycle} \tag{17}$$

$$PRF = \frac{1}{IPP} \qquad [Hz] \tag{18}$$

$$R_{max} = \frac{cIPP}{2} \qquad [km] \tag{19}$$

Using these equations the appropriate values for each height resolution is calculated. For the Duty Cycle a value of 5% is assumed.

4.c Transmitted pulse length and received signal strength

Using equations 17 and 18 as well as the relation of Peak to average power (20) one can combine those and get the transmitted Power in relation of to the transmitted pulse length τ_p (c.f. eq. 21).

Table 1: Calculated values using eq. (16) to (19)

Range resolution	**150m**	**1200m**
Pulse Length	1 μs	8 μs
IPP	20 μs	160 μs
PRF	50 kHz	6250 Hz
$\mathbf{R}_{max}$	3 km	24 km

$$P_{avg} = P_t D = (\tau_p \text{PRF}) \tag{20}$$

$$P_t = \frac{P_{avg} T_d}{\tau_p n_p} \tag{21}$$

With n_p being the number of pulses, in this case one, and the dwell-time

$$T_d = \frac{n_p}{\text{PRF}} \tag{22}$$

the transmitted power P_t can be substituted in eq. 6, which leads to the total received power related to the transmitted pulse length ([c.f. 2, chap. 2.10]):

$$P_r = \frac{P_{avg} T_d \rho^2 A^2 \sigma}{\tau_p 4\pi \lambda^2 R^2} \qquad [W] \tag{23}$$

4.d Atmospheric parameters

The radar observes the atmosphere up to a height of about 17 km, thus the radar is observing the troposphere (0 to 12km) and lower parts of the stratosphere (12 to 50km).

One can observe that up to height of about 3km the SNR is quite high, presumably this is the case of the dense atmosphere near ground, for example clouds or fog. Between 3 and 6 the SNR gets lower, with then having a region with higher SNR up to about 11km.

It seems that the wind is causing this increase in SNR, since plotting the wind data provided in addition to the SNR and altitude data (when available) reveals that the wind in these attitude is quite strong, with absolute wind speeds up to 10 m/s

A Appendix

A.1 MATLAB Code

Listing 1: Matlabcode for plotting SNR vs. altitude

```
close all;clear all;clc;
%% Load Data
data{1} = load('TXT_20060228_test1.fca');
data{2} = load('TXT_20060228_test2.fca');

% data files with format
% UT - Altitude - Signal amplitude (linear) - SNR, dB

% Dataset 1 - 150m resolution, Dataset 2 - 1200m resolution
intervall = [100,12];

%% Parse Test Data & Plot
parsedData = cell(2,1);
n = 1;
while n < 3
    intervallQuantity = cellfun('length',data(1,n))/intervall(n);
    parsedData{1,n} = zeros(1,intervallQuantity);
    k =1;
    while k < intervall(n)+1
        parsedData{1,n}(k,:) = (data{1,n}(k:intervall(n):cellfun('length',data(1,n)),4))';
        k = k+1;
    end

    subplot(1,2,n);
    pcolor(data{n}(1:(intervall(n)):(cellfun('length',data(1,n))),1),data{n}(1:intervall(n),2),parsedData{1,n})
    colormap jet;
    shading flat;
    grid on;
    xlabel('Universal time (hours)');
    xlabel(colorbar, 'SNR (dB)');
    ylabel('Altitude (km)');

    n = n+1;
end

```

```
%% plot wind
figure()
plot(data{1}(1:end,6))
```

References

[1] Philip J. Erickson. Lecture: General radar theory. University Lecture, 2016.

[2] M. A. Richards. *Principles of modern radar.* SciTech Publishing, Raleigh, NC, 2010.