Optics- and Radar-based Observations

F7003R

Signal Processing for Radar Applications

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In this Assignment, ***Problem 1 - Signal Processing for radar applications***, the Fourier transform, the discrete Fourier Transform and Detection in noise is investigated using MATLAB.

1 Preparations

1.a Fourier Transform and Series

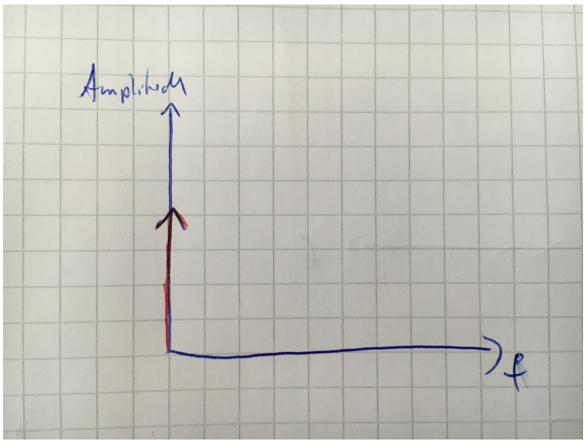

Figure 1: DC Level function

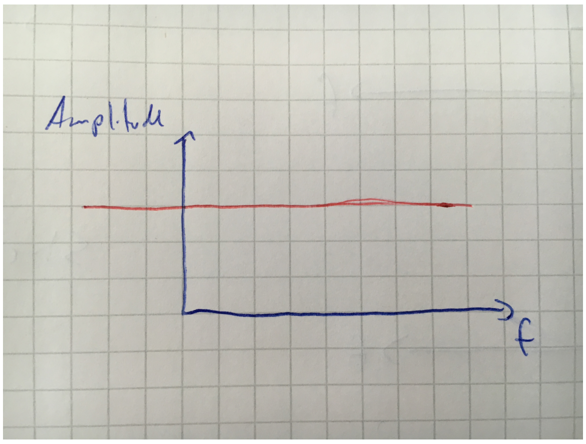

Figure 2: Dirac Function

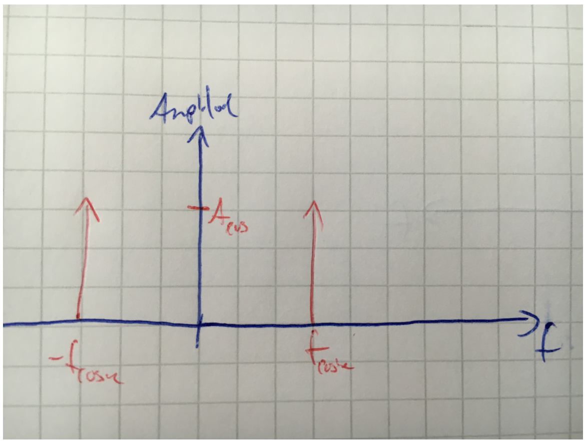

Figure 3: cosine with phase zero

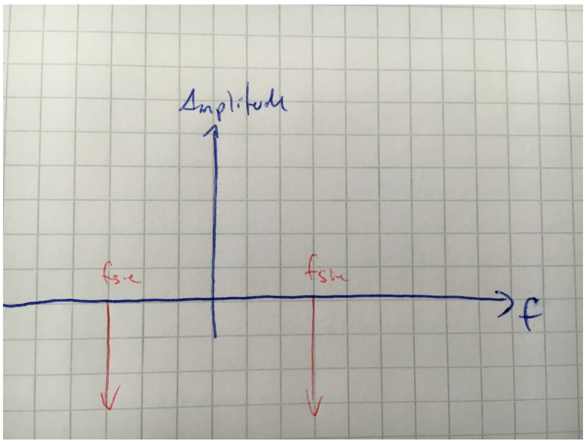

Figure 4: sine with phase $\pi/2$

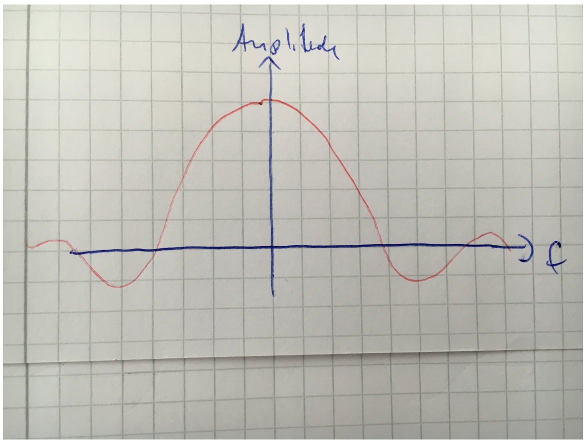

Figure 5: Narrow rectangular function

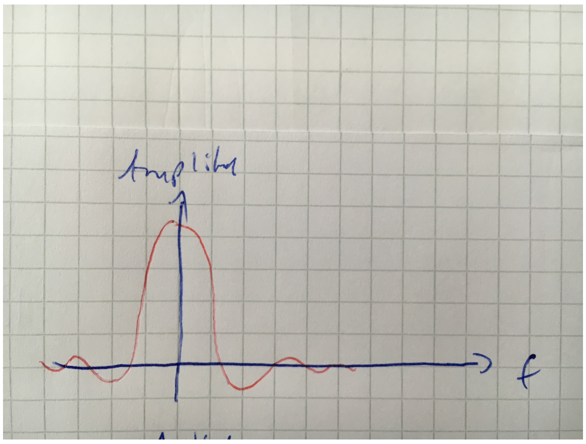

Figure 6: Broad rectangular function

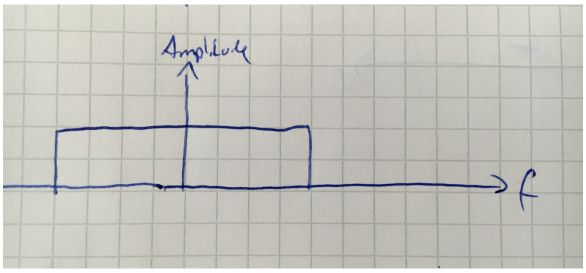

Figure 7: Narrow sinc function

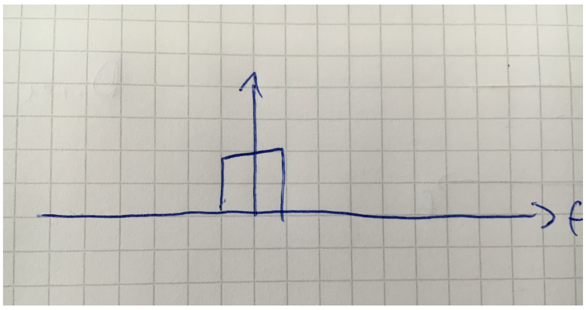

Figure 8: Broad sinc function

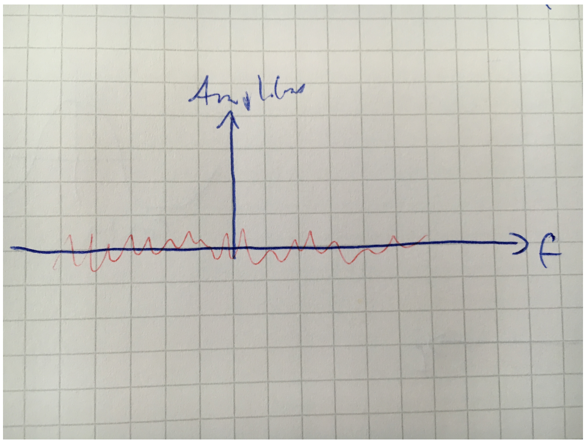

Figure 9: White Noise

1.b Discussion

The spectrum of real valued, even functions has no imaginary part, whereas the spectrum of non-even function might have imaginary parts.

2 Sampling

Three harmonic functions, on the form x(t) = Acos($2\pi ft + \varphi$) are plotted in figure 10.

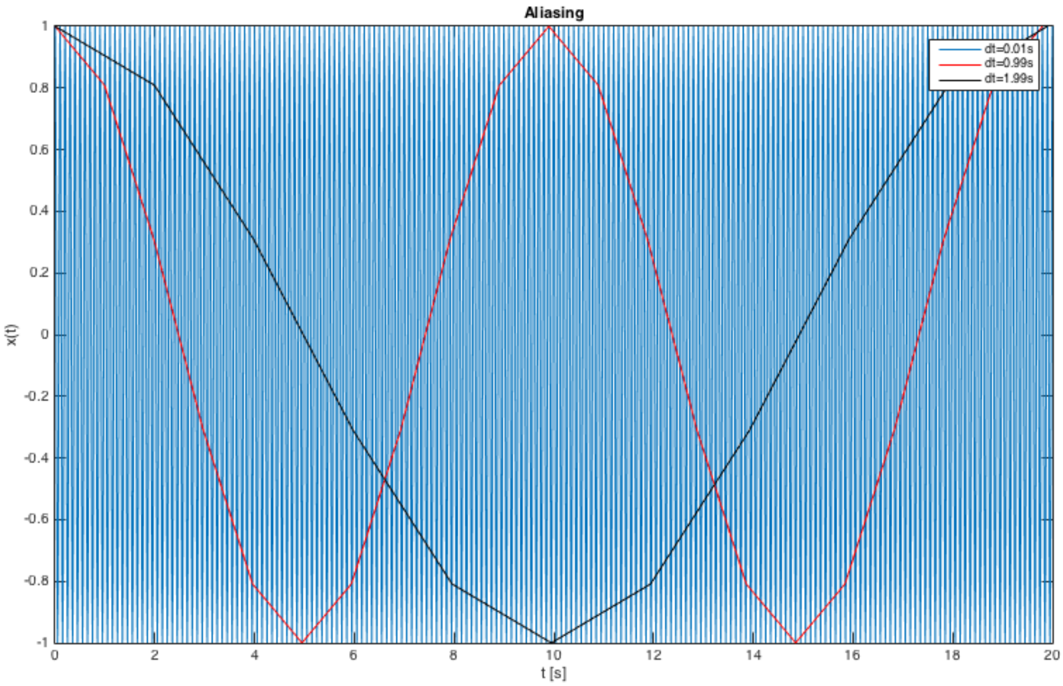

Figure 10: Aliased harmonic functions

The two Aliasing frequencies are determined by reading the time period for one period of the respective harmonic function. The two Aliasing frequencies are:

- 0.1 Hz (red line)
- 0.05 Hz (blue line)

An appropriate expression for the aliasing frequency is given as follows:

$$f_{alias}(N) = f - Nf_S \tag{1}$$

where f is the signal frequency and N represents an appropriate integer value.

4 Amplitude Diagram

Power of Sum is like summing the power of each In Figure 11 the DFT of three harmonic functions with different amplitudes is shown. In MATLAB the *fft*-function[1] was used.

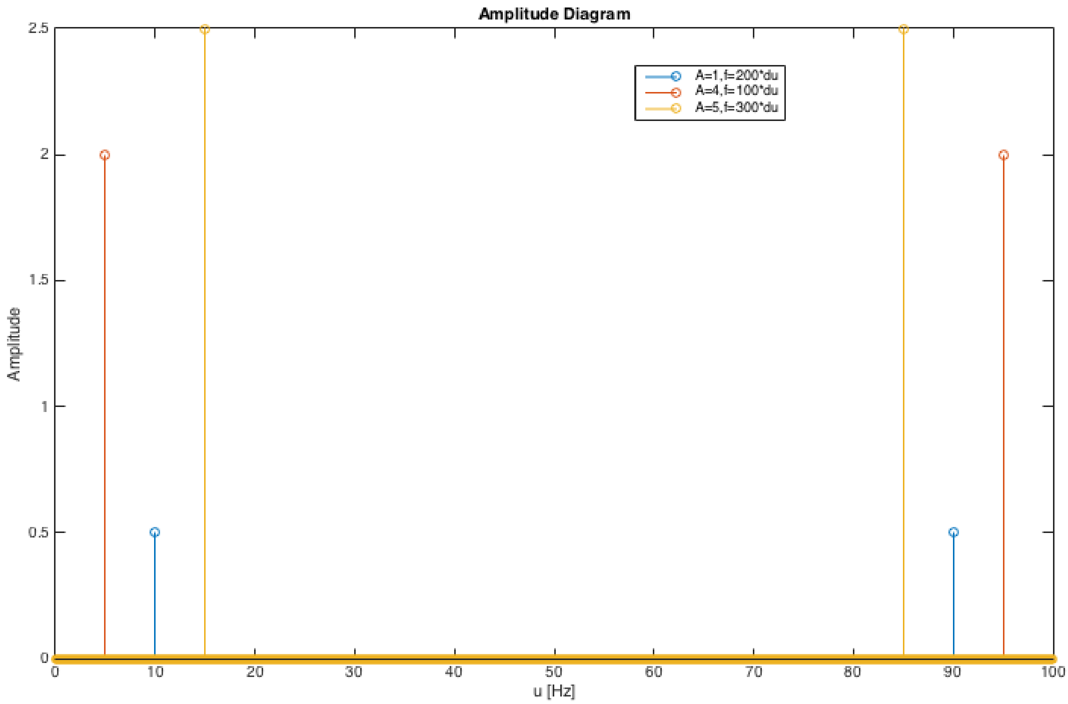

Figure 11: Three functions with different Amplitudes

As one can see, it does not matter in which order the requested operation, the power of the sum of two functions, is processed. The power of the sum of two functions is the same as the sum of the power of two functions, in the DFT. Whereas the power of the signal is the squared magnitude of the frequency (*note:* The x axis in the plot is shifted to the left. An appropriate double side diagram would have the origin point at, in this case, 50u).

[1]Fast Fourier Transform

5 Leakage

5.a Explanation of Leakage

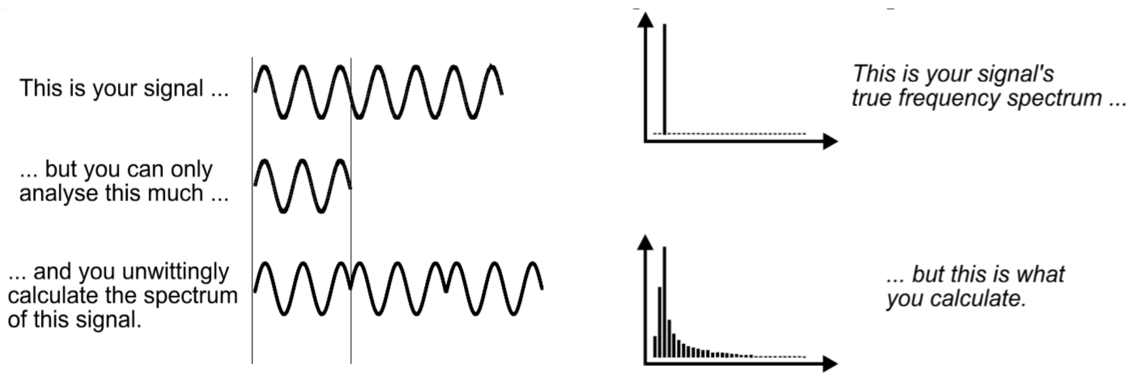

Figure 12: Leakage from truncation, (*Source: Lab Instructions*)

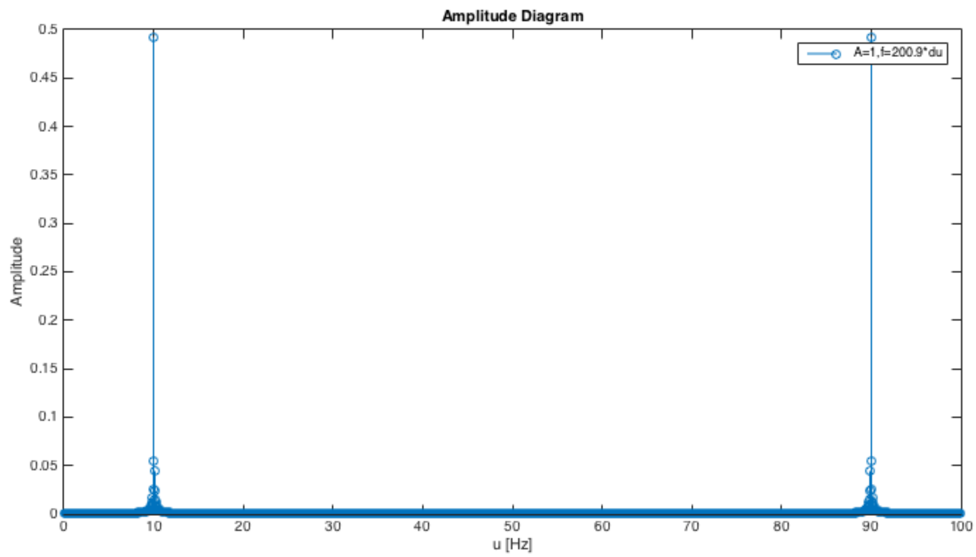

Figure 13: Harmonic function with value for C = 200.9

The problem that happens, when the truncation does not happen exactly at the period is, that the spectrum becomes "less sharp" (c.f. fig. 13), which means that the main peak is accompanied by smaller peaks at the bottom part of the diagram. Also, the main peak is a little bit lower. This phenomenon is called leakage.

5.b Straddle Loss

The problem with leakage for radar signals, is that leakage significantly reduces the Signal-to-Noise-Ratio which makes it "harder" to detect the proper signal, as can be seen in fig. 13 and fig. 12. This is also called "straddle-loss". It is basically a consequence of not appropriate or good enough sampling.

8 Rectangular Function

Plotting the DFT of the rectangular function

$$x(t) = \begin{cases} 1 & \text{if } 0 \leq t < 3s \\ 0 & \text{if } 3s \leq t \end{cases}$$

for t $\leq$ 6 with sampling distance of 1 second shows the sinc-function (in the frequency domain), but due to the very less given samples the sinc-function is not very clear, means the frequency resolution is quite bad. Also, since the given number of samples is even, the left and right halves of the transform are not "mirrored" at the y-axis but show different peaks (right side one less than left side), see red parts in fig. 14. To increase this frequency resolution a technique called *zeropadding* is applied, see next chapter.

9 Zeropadding

In fig. 14 the black peaks represent the zeropadded rectangular function from task 8.

a) Compared to the main peak lobe, the peak side lobe (Amplitude 0.16) is lower by about 0.33.

b) Again, reading from fig. 14, the amplitude is reduced by 50% at a frequency of ± 0.2 Hz.

c) Expressing the previous items as 10*log(amplitude) results in:

- 10 * log(0.33) = -4.81 dB
- 10 * log(0.2) = -3 dB

10 Dirac's delta function δ(t)

The Fourier Transform of a constant function (constant level in the time domain, DC-Level) is the Dirac's delta function in the frequency domain, and vice versa. This is also addressed as

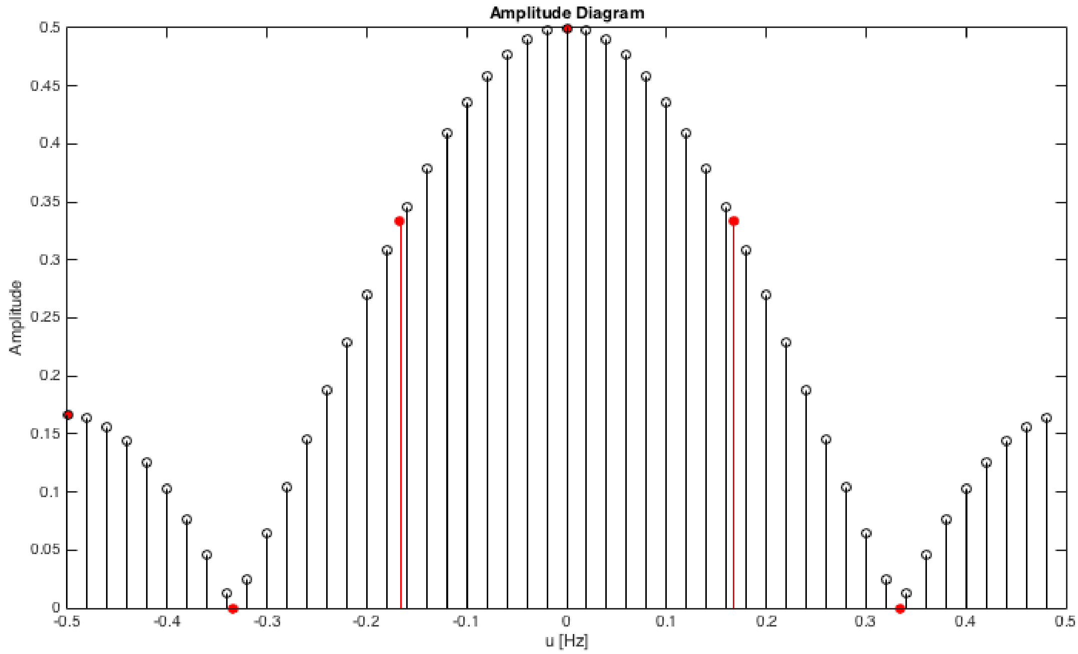

Figure 14: Sinc-function, zeropadded

the duality of transform pairs.

11 Sinc Function

Plotting and varying the values leads to the conclusion, that the broader the function is in time domain, the narrower it is in the frequency domain and vice versa.

12 Noise, auto correlation and cross correlation

12.a Mean of Many Realizations

Taking the mean of many realization leads to more averaging, and thus to more noise cancellation. Also, the auto correlation peak is standing out more, since one takes multiple realizations.

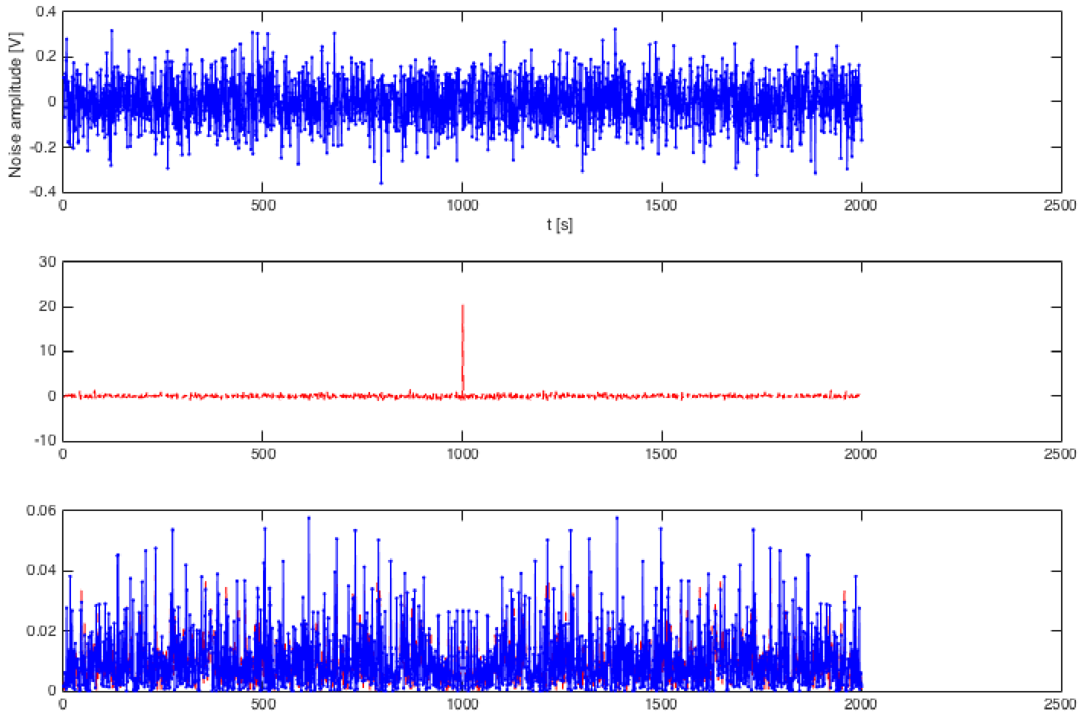

Figure 15: Noise, Auto-correlation and Power Spectrum

12.b Power spectrum and autocorrelation function

In the power spectrum graphs the DC component is always visible, with the ideal plot being an exception, the red line in the plotted functions of the provided MATLAB-file *pulseNoise2016.m*. This DC component can not get rid off by simple averaging. Whereas the noise, also always present can be averaged and thus more or less cancelled.

In the autocorrelation functions the noise and DC is also always visible, as well as the cross-correlation triangle, which has features of auto-correlation.

12.c Integration in the signal Path

The auto-correlation function and the power spectrum differs depending on where in the signal path the integration is performed, since one get rid of noise by averaging, which makes it better to apply FFT in the end (after the integration), than for each single sample, where the noise is present and obviously influences the FFT, since it is not cancelled out.