M2 - TSI

UE31 Laboratory Report

Signal Estimation

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1 Introduction

In this report various methods for the estimation of parameters of a given data set are evaluated and compared. To cut down the problem's complexity, we use a simple model for the flux of an elliptic galaxy, as is described by the Sersic profile, which provides us an initial data set. This data set - a noisy image of a elliptic galaxy as it would have been taken by a ground-based telescope - is then used to evaluate different estimation approaches as the least square estimation, maximum likelihood estimation and Bayesian estimation.

2 Modelling a galaxy

The Sersic profile is very common amongst astrophysicists to model the flux of observed elliptic galaxies in a simple way, and is given by the equation

$$I(l,c) = exp(-R(l,c)^{\frac{1}{n}}) \tag{1}$$

which describes the variation of intensity with respect to the distance of the galaxy's centre. The distance R of a pixel with the coordinates (l,c) from the galaxies centre is given by

$$R(l,c)^2 = \left(\frac{(l-l_0)\sin(\alpha) - (c-c_0)\cos(\alpha)}{\sigma_l}\right)^2 + \left(\frac{(l-l_0)\cos(\alpha) - (c-c_0)\sin(\alpha)}{\sigma_c}\right)^2 \tag{2}$$

with (l_0, c_0) being the galaxy's centre coordinates, (σ_l, σ_c) the two galaxy's axes length and the horizontal angle α.

This leads to the following equation modelling the data

$$d(l,c) = s + aI(l,c) + n(l,c) \tag{3}$$

with a as the amplitude of the galaxy, s the amplitude of the sky's background and $n(l,c)$ as noise.

By assuming a white Gaussian noise with the known variance σ_n^2 and using the given Sersic-function (see Appendix) we can create a "initial image" that will be used as the initial data set as previously mentioned, see figure **??**.

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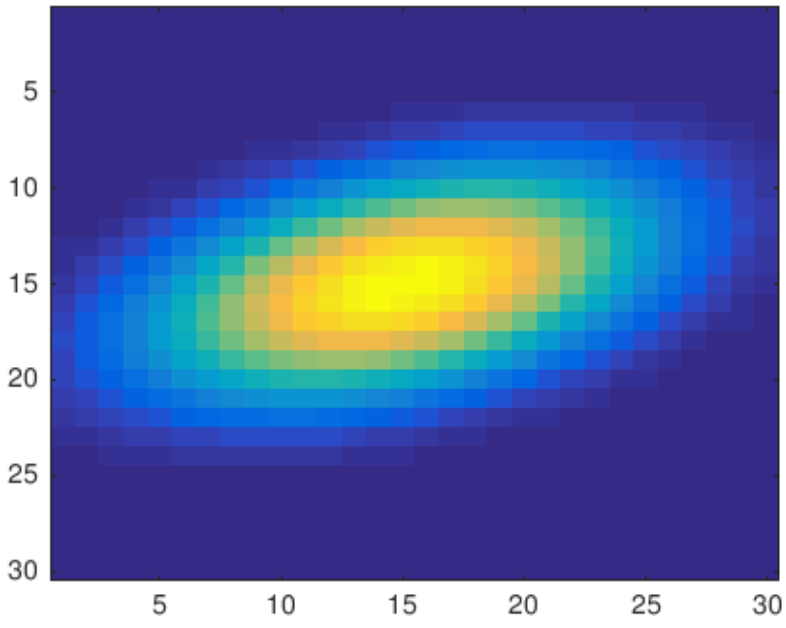
The parameters used to create fig. **??** are given below.



s

Figure 1: Initial Data Set, with a artificial elliptic galaxy

$$\begin{array}{cccc} L, C = 30px & l_0, c_0 = 15 & \sigma_l = 10 & \sigma_c = 5 \\ \alpha = 0.3 & n = 0.4 & a = 10 & s = 3 \end{array} \tag{4}$$

with L, C as height respectively width of image in pixels, l_0, c_0 the galaxy's centre in pixel coordinates and σ_l, σ_c as length parameters.

probalby don't need this explanation here, see above text

3 Estimation with known Galaxy's location and shape parameters

Having successfully modelled a simple elliptic galaxy

4 Maximum likelihood estimation of all parameters

5 Estimation in the Bayesian framework

5.1 Maximum a posteriori estimator

5.2 Posterior mean estimator

A Sersic function

B Least Square estimation