Murashko Artem B20-02

3.1 Big-O notation

1. $n^2 + 10n\log n + 50n + 100 \Leftrightarrow$

10, 50 - factors and 100 - constant

$\Leftrightarrow n^2 + n\log n + n$

Since $n^2 > n\log n > n$, answer is $\underline{\underline{O(n^2)}}$

$n^2 \vee n\log n$ $n\log n \vee n$

$n \cdot n \vee n\log n$ $\log n > 1$

$n > \log n$

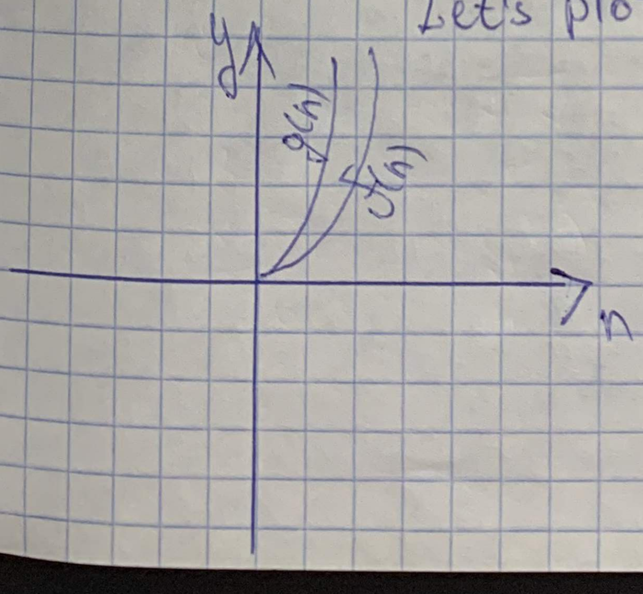
2. $n^{7/2} + 7n^3\log n + n^2 \Leftrightarrow n^{7/2} + n^3\log n + n^2$

7 - factor

- $n^{7/2} > n^2$, since $\frac{7}{2} > 2$
- $n^{7/2} \vee n^3\log n$

Let's plot $y_1 = g(n) = n^{7/2}$

$y_2 = f(n) = n^3\log n$



$g(n) > f(n)$

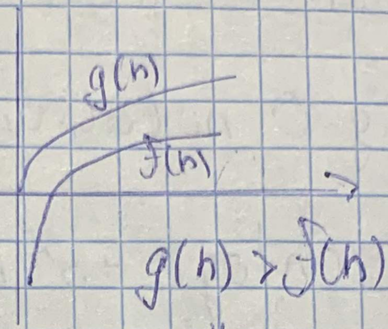
$n^{7/2} > n^3\log(n)$

Another proof:

$n^{7/2} \vee n^3 \log n$

$n^3 \cdot n^{1/2} \vee n^3 \log n$

$g(n) = n^{1/2} \vee \log n = f(n)$



$g(n) > f(n)$

$\Downarrow$

$n^{7/2} > n^3 \log n$

Thus, $n^{7/2} > n^3 \log n > n^2 \Rightarrow O(n^{7/2})$

$3 \cdot 6^{n+1} + 6(n+1)! + 24n^{42} \Leftrightarrow$

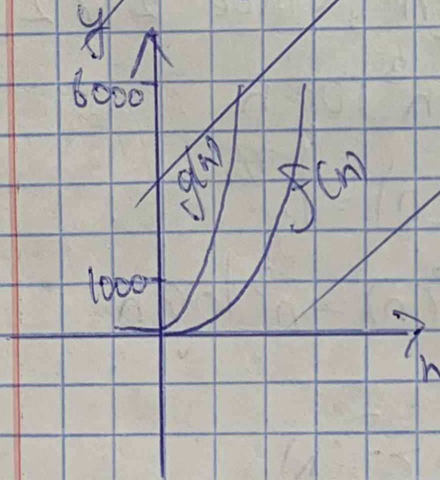
1 - constants; 6,24 - factors

$\Leftrightarrow 6^n + n! + n^{42}$

Let's $g(n) = 6^n$

$f(n) = n!$

$6^n \vee n!$



$3.\ 6^{n+1} + 6(n+1)! + 24n^{42}$

1 - constants; 6, 24 - factors, so:

$6^n + n! + n^{42}$

1) $\lim\limits_{n\to\infty} \frac{6^n}{n!}$

$0 < \frac{6^n}{n!} = \frac{6}{1} \cdot \frac{6}{2} \cdot \frac{6}{3} \cdot \frac{6}{4} \cdot \frac{6}{5} \cdot \underbrace{\frac{6}{6}}_{=1} \cdot \frac{6}{7} \cdots \frac{6}{n} \leq$

$\leq 6\left(\frac{6}{7}\right)^{n-2} = \frac{6 \cdot 49}{36}\left(\frac{6}{7}\right)^n = \frac{49}{6}\left(\frac{6}{7}\right)^n$

$\lim\limits_{n\to\infty} \left(\frac{6}{7}\right)^n = 0$, since $\frac{6}{7} < 1$

$\Downarrow$

$\lim\limits_{n\to\infty} \frac{6^n}{n!} = 0 \Rightarrow \underline{\underline{n! > 6^n}}$

2) $\lim\limits_{x\to\infty} \frac{a^x}{x^b} \Leftrightarrow \lim\limits_{x\to\infty} \frac{x}{\log_a^b x} = +\infty$ (theorem)

So, $a^x > x^b \Rightarrow 6^n > n^{42} \Rightarrow n! > n^{42} \Rightarrow$

$\Rightarrow O(n!)$

Answer: 1) $O(n^2)$ 2) $O(n^{7/2})$ 3) $O(n!)$

3.2.

	Cost	Times
`void calculateComplexity(int n){`		
`for(int i=n; i>=0; i--){`	C_1	$n+1$
`for(int j=0; j<i; j++){`	C_2	$\sum_{i=1}^{n}(i+1)$
`checkpoint(n);`	$(\log_3 n+1)C_3 + \log_3 n\, C_4$	$\sum_{i=1}^{n} i$
`}`		
`}`		
`}`		
`void checkpoint(int number){`		
`for(int i=1; i<number; i*=3){`	C_3	$\log_3 n + 1$
`cout<<("DSA HW "+i);`	C_4	$\log_3 n$
`}`		
`}`		

$$T(n) = C_1(n+1) + C_2\left(\sum_{i=1}^{n}(i+1)\right) + \left((\log_3 n+1)C_3 + \log_3 n\, C_4\right)\sum_{i=1}^{n} i =$$

$$= C_1 n + C_1 + C_2 \frac{n^2}{2} + \frac{3n}{2}C_2 + \frac{n^2+n}{2}\log_3 n \cdot C_3 +$$

$$+ \frac{n^2+n}{2}\log_3 n\, C_4 + C_3\frac{n^2+n}{2} + C_4\frac{n^2+n}{2} =$$

$$= C_1 n + C_1 + n^2\cdot\frac{1}{2}C_2 + n\cdot\frac{3}{2}C_2 + n^2\cdot\log_3 n\cdot\frac{1}{2}\cdot C_3 +$$

$$+ n\log_3 n\cdot\frac{1}{2}\cdot C_3 + n^2\log_3 n\cdot\frac{1}{2}C_4 + n\log_3 n\,\frac{1}{2}\cdot C_4 +$$

$$+ n^2\cdot\frac{1}{2}C_3 + nC_3\cdot\frac{1}{2} + n^2 C_4\frac{1}{2} + n\frac{1}{2}C_4 =$$

$$= n^2\log_3 n\left(\underbrace{\frac{c_3}{2}+\frac{c_4}{2}}_{a}\right) + n\log_3 n\left(\underbrace{\frac{c_3+c_4}{2}}_{b}\right) +$$

$$+ n\left(\underbrace{c_1+\frac{3c_2}{2}+\frac{c_3}{2}+\frac{c_4}{2}}_{c}\right) + c_1 =$$

$$= a n^2\log_3 n + b n\log_3 n + cn + c_1 \Leftrightarrow$$

$$\Leftrightarrow n^2\log n + n\log n + n \Rightarrow O(n^2\log n)$$

$$n^2\log n > n\log n > n$$

Answer: $O(n^2\log n)$

3.3 $T(n) = \sqrt{k}\,T\left(\frac{n}{k^2}\right) + c\cdot\sqrt[4]{n}$, $T(1)=0$

$$T(n) = \sqrt{k}\,T\left(\frac{n}{k^2}\right) + c\sqrt[4]{n} = \sqrt{k}\left(\sqrt{k}\,T\left(\frac{n}{k^4}\right)+c\left(\frac{n}{k^2}\right)^{1/4}\right)+cn^{1/4} =$$

$$= k^{\frac{1}{2}+\frac{1}{2}}\,T\left(\frac{n}{k^4}\right) + 2cn^{1/4} = k^{2/2}\left(\sqrt{k}\,T\left(\frac{n}{k^6}\right) + c\left(\frac{n}{k^4}\right)^{1/4}\right) + 2cn^{1/4} =$$

$$= k^{3\cdot 1/2}\,T(n/k^6) + 3cn^{1/4} = \ldots = k^{m/2}\,T(1) + mcn^{1/4}$$

$$k^{2m} = n \Leftrightarrow m = \frac{\log_k n}{2} = \frac{1}{2}\log_k n$$

We can apply second case of the Master Theorem

$$\underbrace{T(n) = O(n^{1/4}\log n)}_{\text{answer}}$$