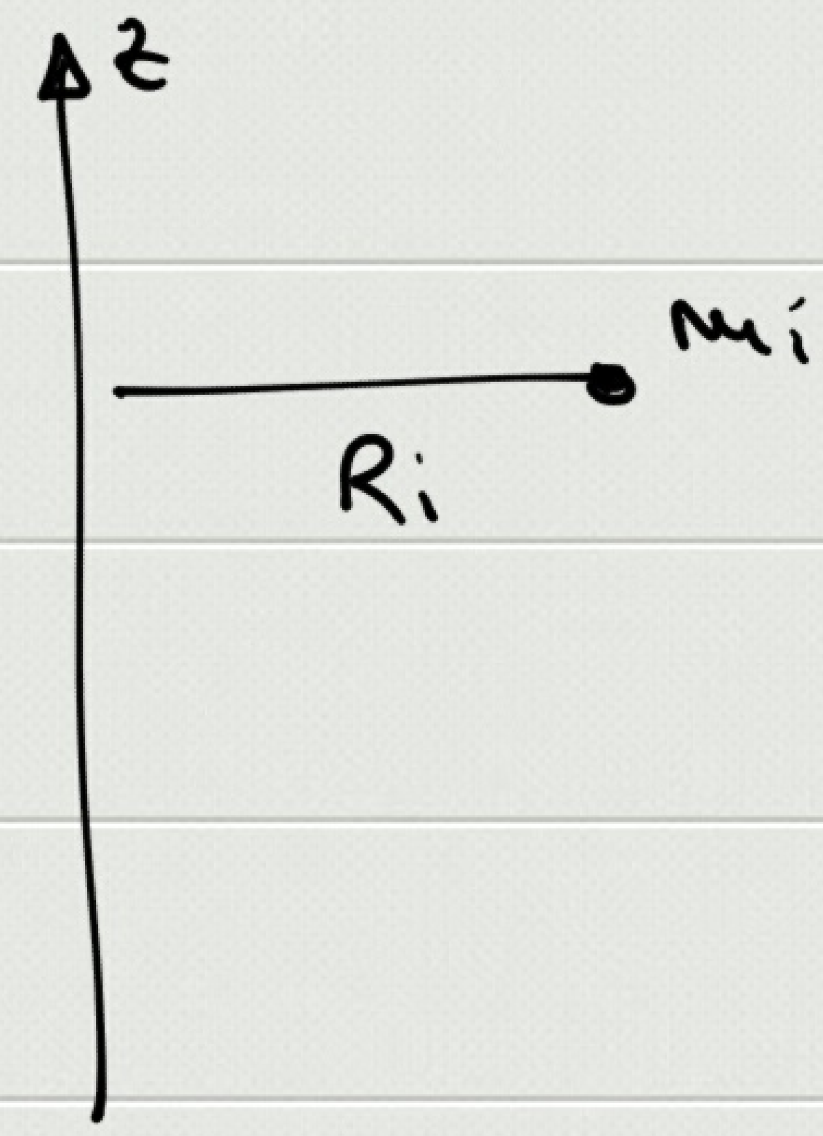
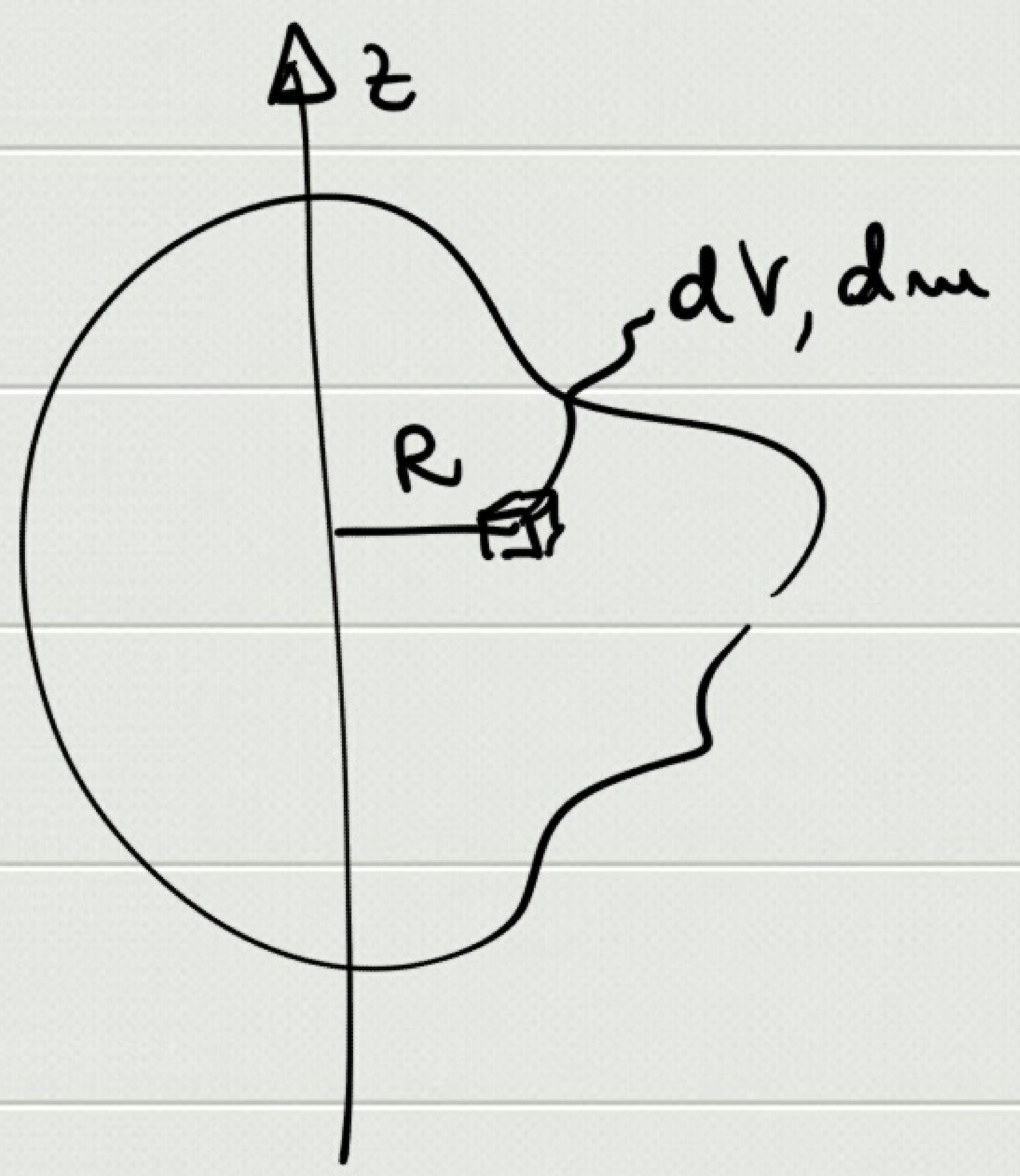
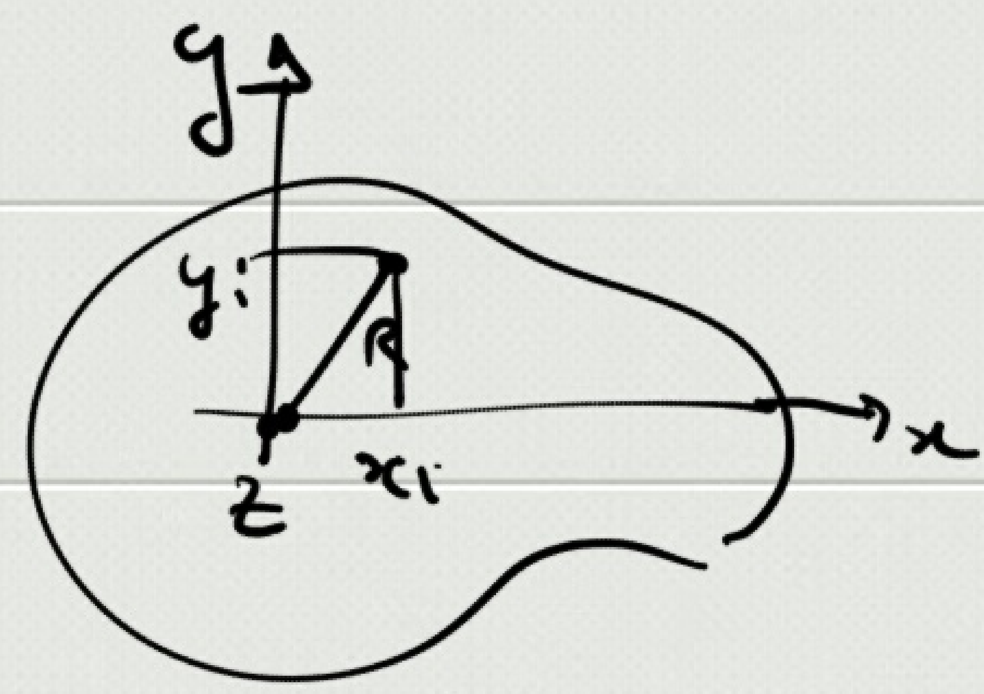
$$I_z = \sum_i m_i R_i^2$$





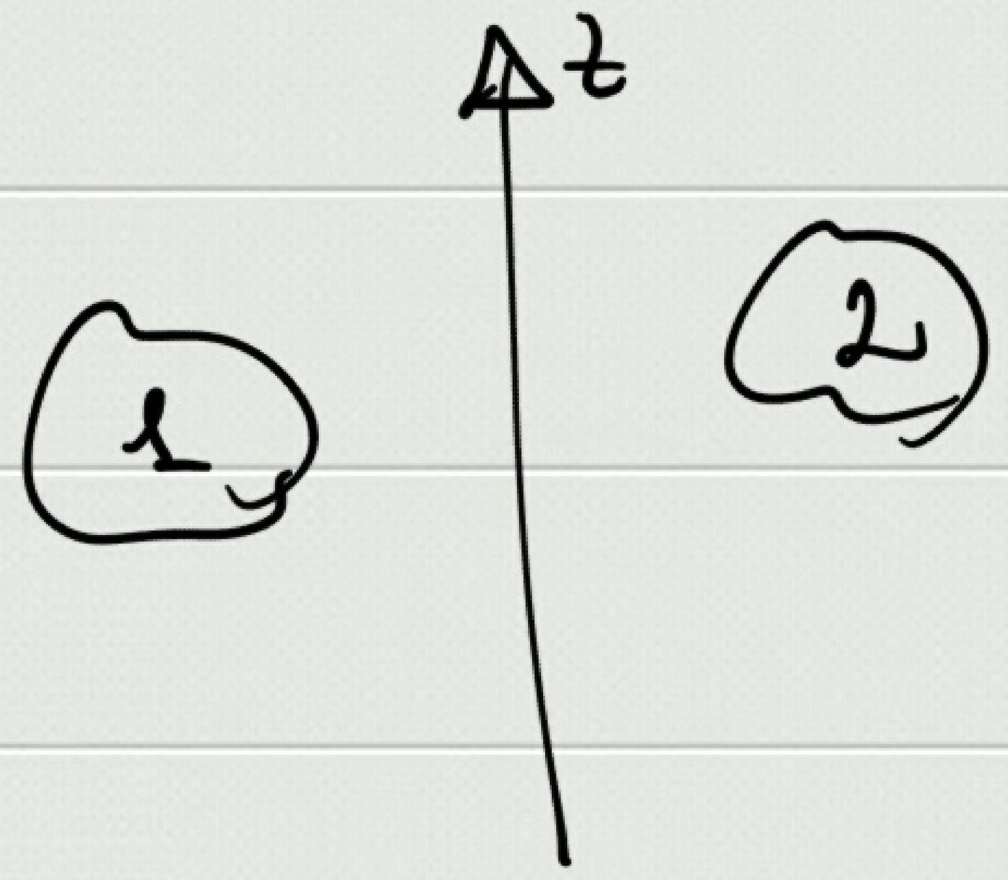
$$\boxed{I_z = \int_{corpo} R^2 \, dm}$$

$$\rho = \frac{dm}{dV}$$

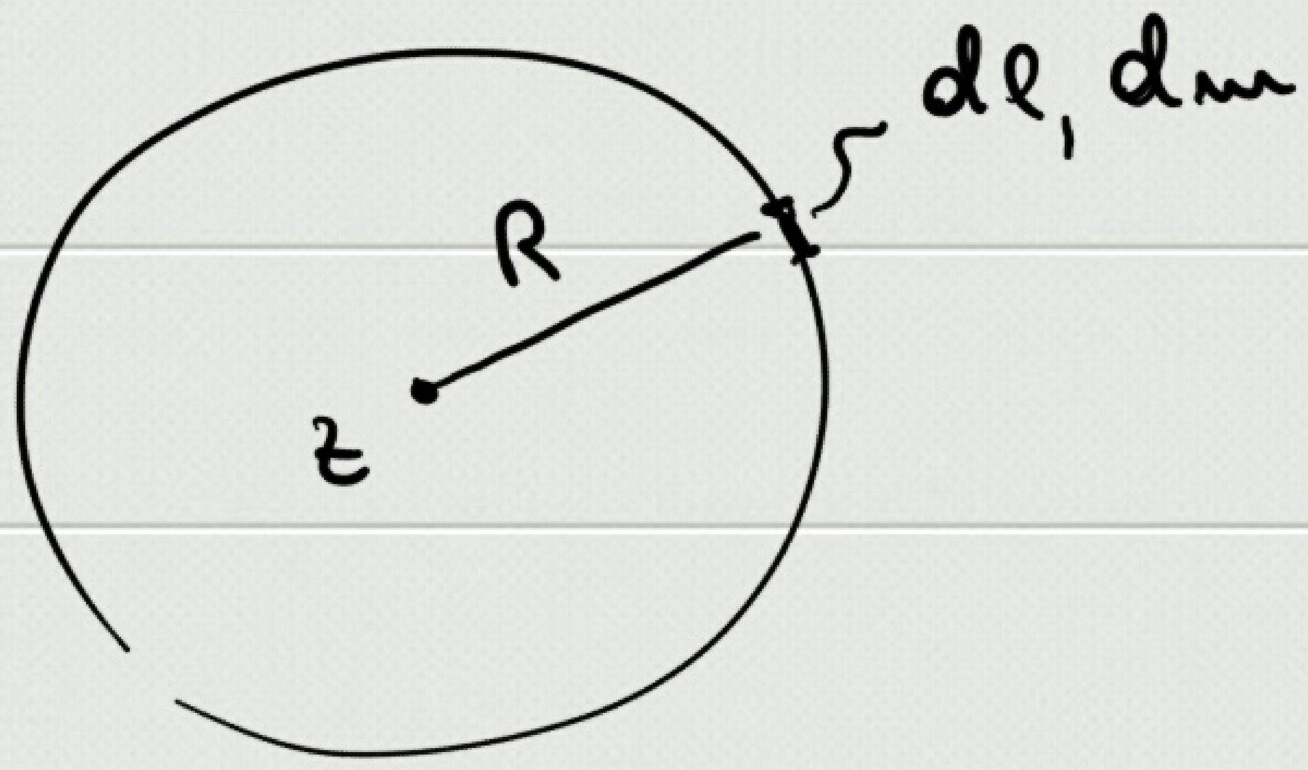


$$I_z = \int R^2 \rho \, dV = \int (x^2 + y^2) \rho \, dV$$

Il momento di inerzia è additivo



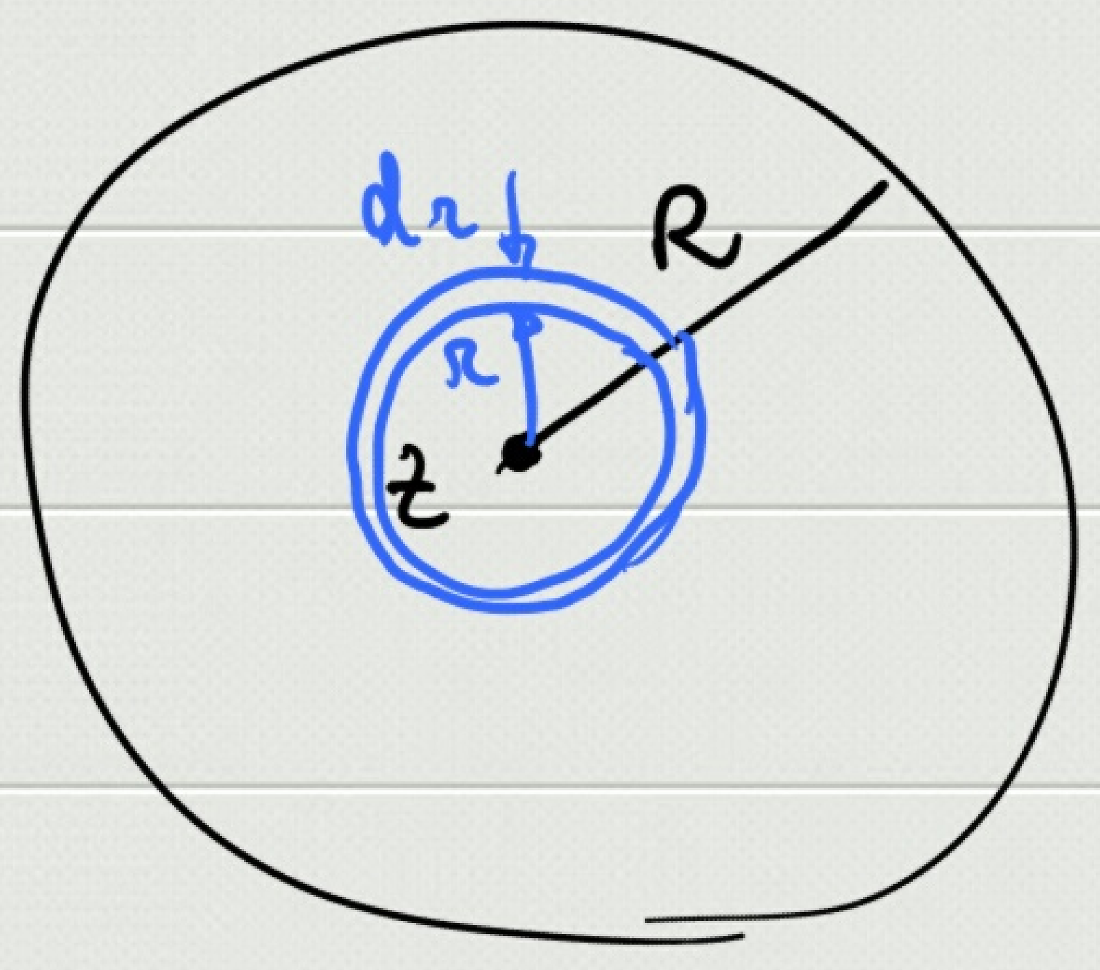
$$I_{z,1+2} = I_{z,1} + I_{z2}$$



Anello sottile omogeneo

R, m

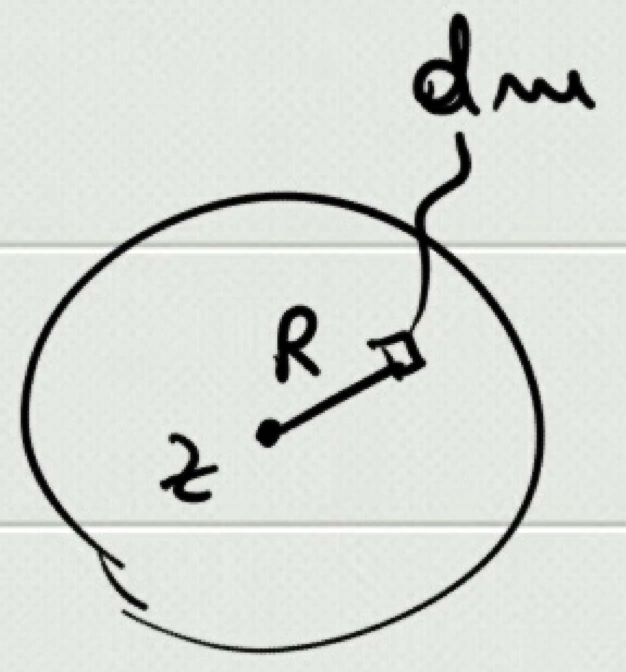
$$I_z = \int_{corpo} R^2 \, dm = R^2 \int_{corpo} dm = m R^2$$



Disco sottile omogeneo

m, R

$$I_z = \int R^2 \, dm$$



$$dI_{z,anello} = r^2 \, dm$$

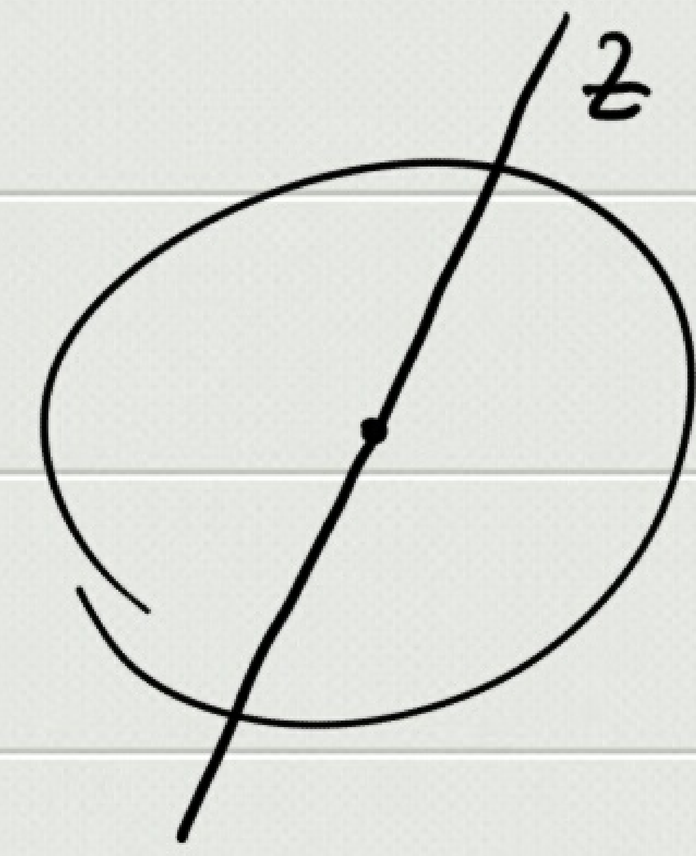
$$dm = \rho_S \, dS = \frac{m}{\cancel{\pi} R^2} 2 \cancel{\pi} r \, dr$$

$$\Rightarrow dI_{z,anello} = \frac{2m}{R^2} r^3 \, dr$$

$$I_{z,disco} = \int dI_{z,anello} = \int_0^R \frac{2m}{R^2} r^3 \, dr = \frac{2m}{\cancel{R^2}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} = \frac{1}{2} m R^2$$

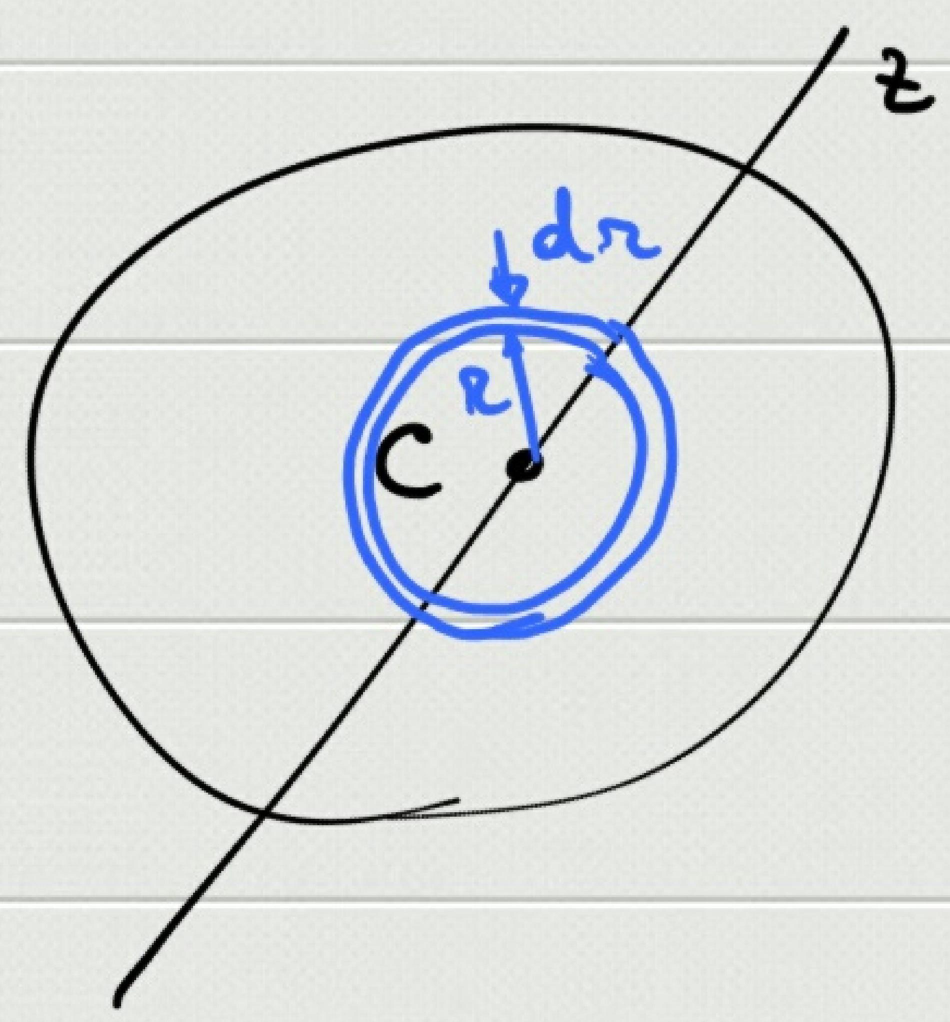
guscio sferico sottile m, R

$$I_z = \frac{2}{3} m R^2$$



— o —

I_z sfera piena, m, R omogenea



$$dI_{z,guscio} = \frac{2}{3} dm\, r^2$$

$$I_{z,sfera} = \int dI_{z,guscio}$$

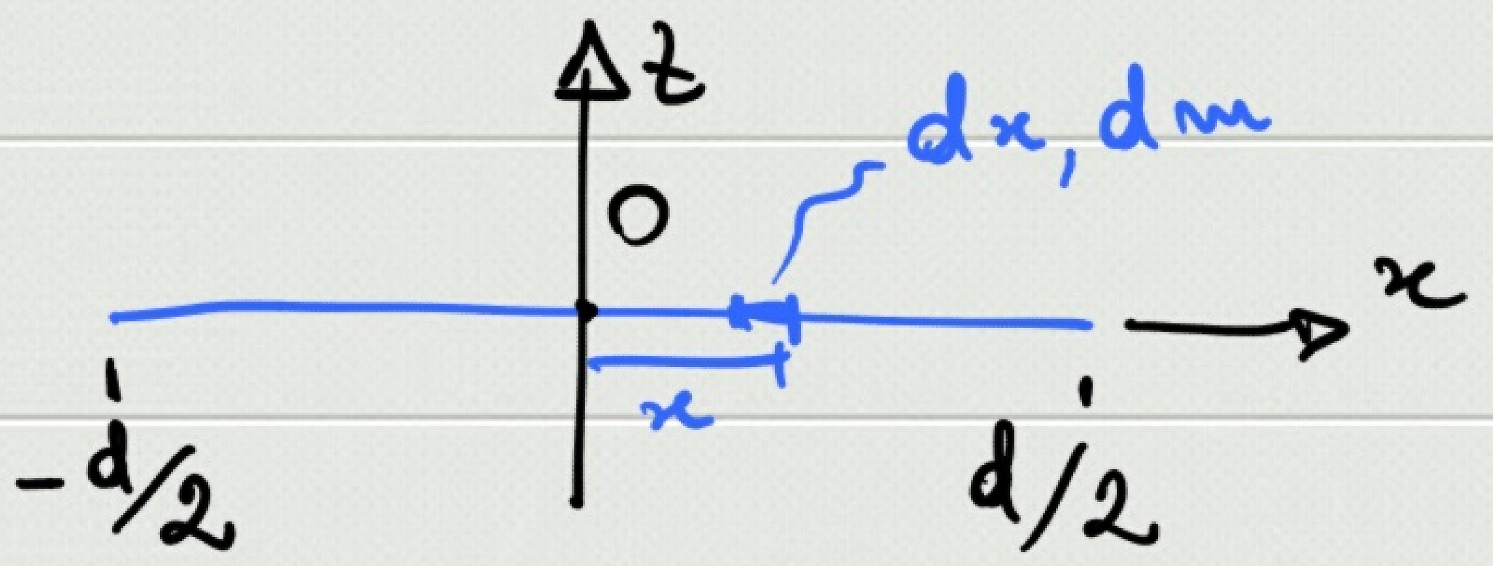
$$\left.\begin{array}{l} \rho = \dfrac{m}{V} = \dfrac{m}{\frac{4}{3}\pi R^3} \\ \quad = \dfrac{dm}{dV} \end{array}\right\} \Rightarrow dm = \frac{3m}{4\pi R^3} dV$$

$$dV = 4\pi r^2 dr$$

$$\Rightarrow dm = \frac{3m}{\cancel{4\pi} R^3} \cancel{4\pi} r^2 dr = \frac{3m}{R^3} r^2 dr$$

$$I_{z,sfera} = \int dI_{z,guscio} = \int_0^R \frac{2}{\cancel{3}} r^2 \cdot \frac{\cancel{3}m}{R^3} r^2 dr = \frac{2m}{\cancel{R^3}} \frac{R^{\cancel{5}\,2}}{5} = \underline{\underline{\frac{2}{5} m R^2}}$$

Sbarra sottile omogenea d, m



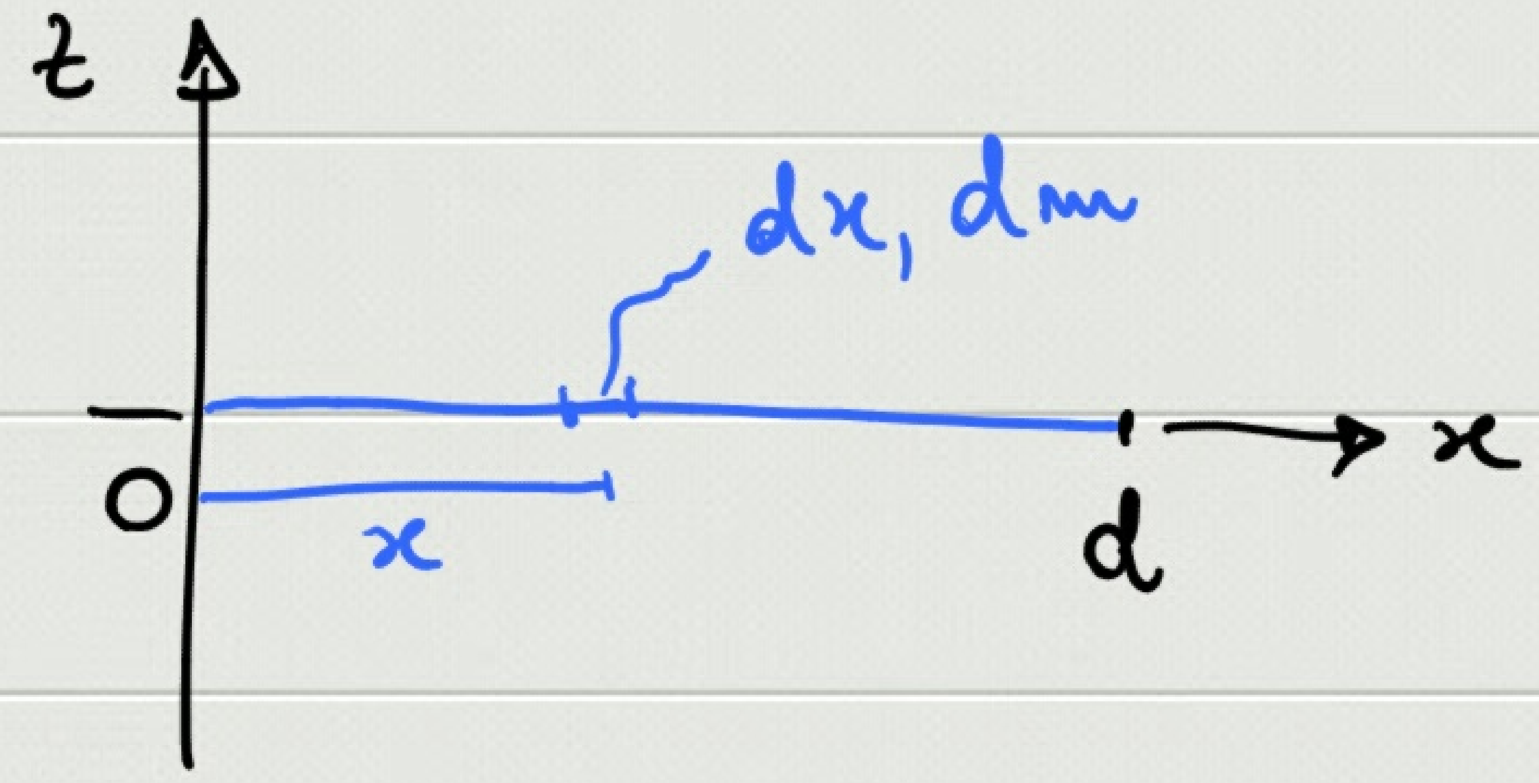
$$I_z = \int R^2 \, dm$$

$$I_z = \int x^2 \, dm =$$

$$\left.\begin{aligned} \rho_\ell &= \frac{m}{d} \\ &\parallel \\ &= \frac{dm}{dx} \end{aligned}\right\} \Rightarrow dm = \frac{m}{d} dx$$

$$= \int_{-d/2}^{d/2} x^2 \frac{m}{d} \, dx = \frac{m}{d} \left[\frac{x^3}{3} \right]_{-d/2}^{d/2} =$$

$$= \frac{m}{3d} \left(\frac{d^3}{8} + \frac{d^3}{8} \right) = \frac{m}{3\cancel{d}} \frac{d^{\cancel{3}\,2}}{4} = \underline{\underline{\frac{1}{12} m d^2}}$$

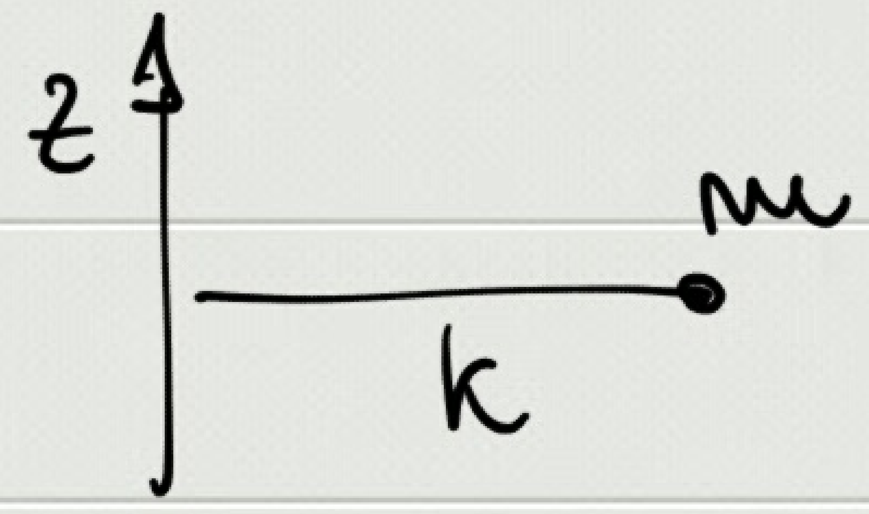


$$I_z = \int R^2 \, dm = \int x^2 \, dm = \qquad dm = \frac{m}{d} dx$$

$$= \int_0^d x^2 \frac{m}{d} dx = \frac{m}{d} \left[\frac{x^3}{3} \right]_0^d = \frac{1}{3} m d^2$$

$$I_z = m k^2$$

$$\overset{!}{=} \sum_i m_i R_i^2$$



$k \rightarrow$ raggio giratore