

Moto periodico :  $\left[ \begin{array}{l} x(t) = x(t+T) \\ v(t) = v(t+T) \end{array} \right]$

$T \rightarrow$  periodo

Moto armonico semplice :

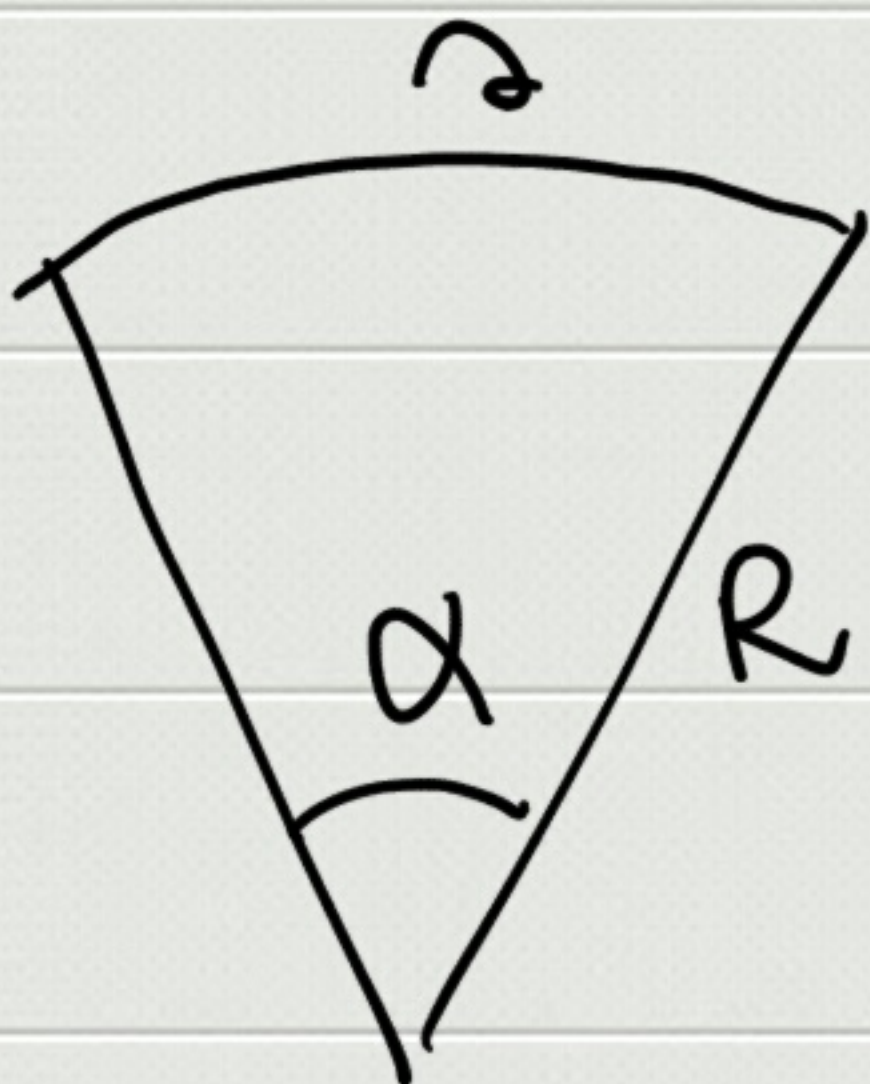
$$x(t) = A \sin(\omega t + \phi)$$

$A$  : ampiezza [m]

$\phi$  : fase iniziale [rad]

$\omega t + \phi$  : fase [rad]

$\omega$  : pulsazione [rad/s]

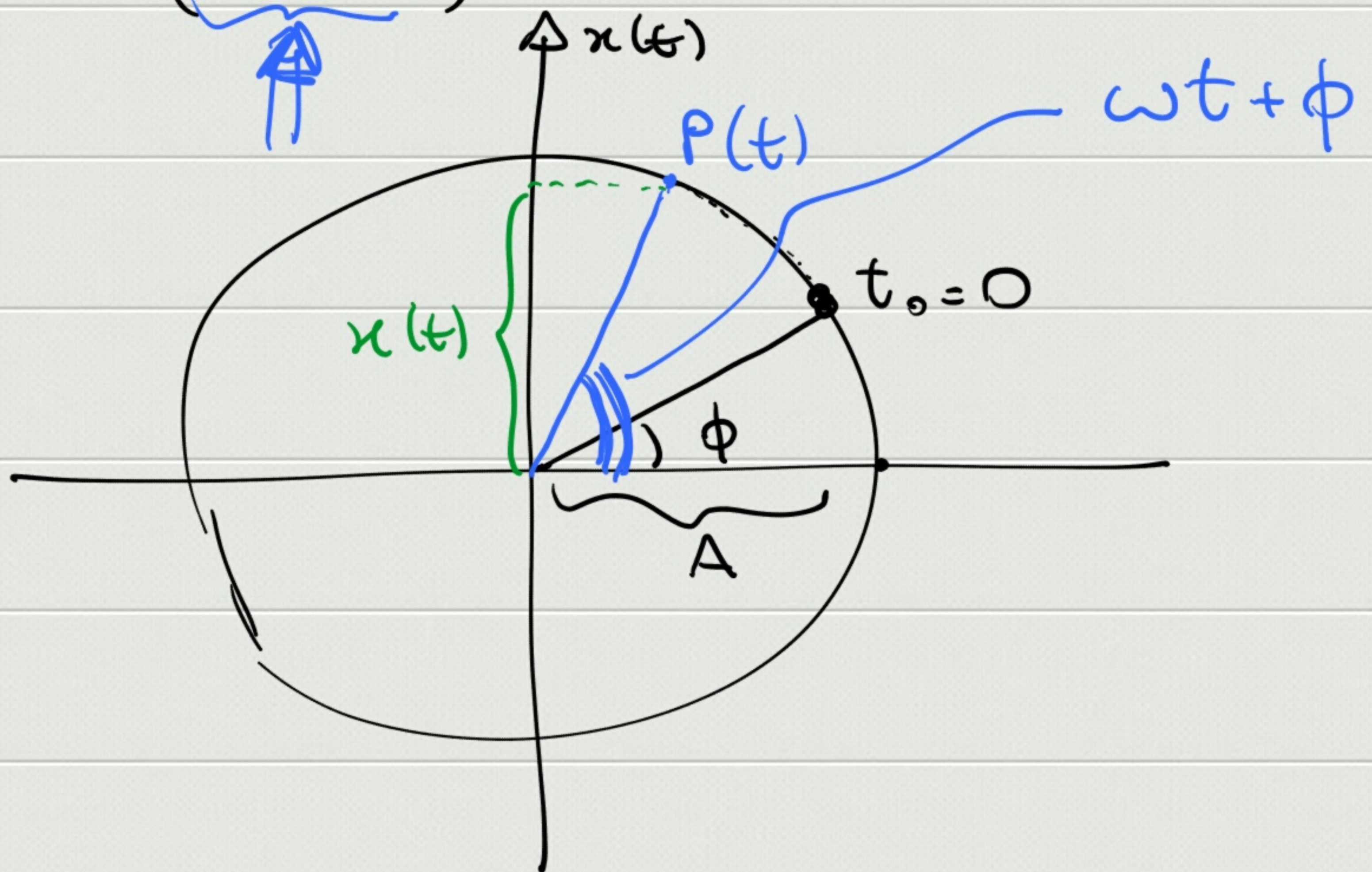


$$\alpha [\text{rad}] = \frac{s}{R}$$

$$s = \alpha R$$



$$x(t) = A \sin(\omega t + \phi)$$



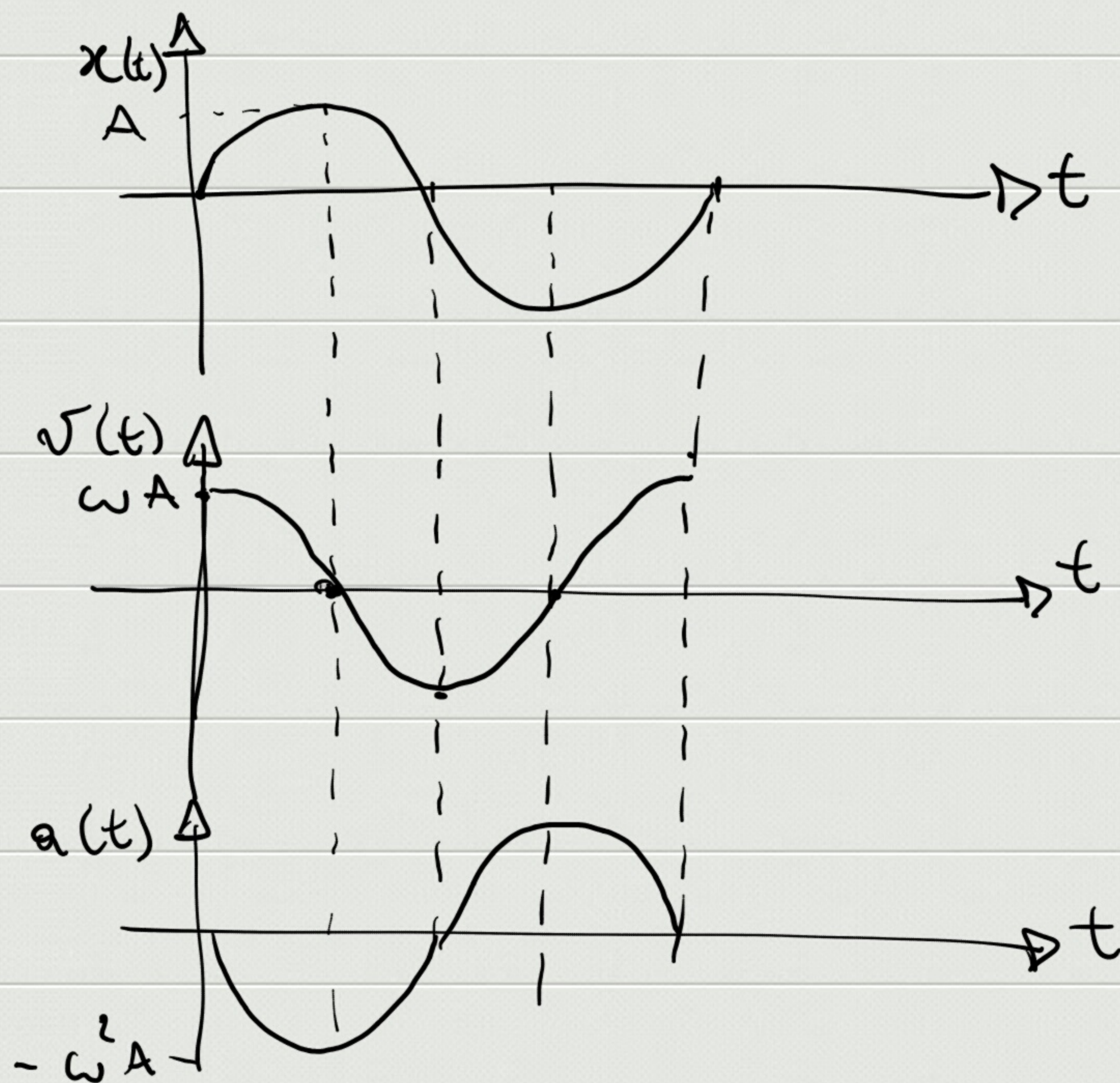
$$v(t) = \frac{dx}{dt} = A \omega \cos(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x(t)$$

$\boxed{a \neq \text{cost}} \Rightarrow \text{moto } \underline{\underline{\text{verio}}}$



$$\phi = 0$$



$$\boxed{x(t) = A \sin(\omega t + \phi)} \quad (\omega \text{ nota})$$

$$\begin{cases} x_0 = x(t=0) = A \sin \phi \\ v_0 = v(t=0) = A \omega \cos \phi \end{cases}$$

$$\frac{x_0}{v_0} = \frac{1}{\omega} \tan \phi \Rightarrow \phi = \arctan\left(\frac{x_0 \omega}{v_0}\right)$$



$$\begin{cases} x_0^2 = A^2 \sin^2 \phi \\ v_0^2 = A^2 \omega^2 \cos^2 \phi \Rightarrow \frac{v_0^2}{\omega^2} = A^2 \cos^2 \phi \end{cases}$$

$$\Rightarrow x_0^2 + \frac{v_0^2}{\omega^2} = A^2 (\sin^2 \phi + \cos^2 \phi) = A^2$$

$$\Rightarrow A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

$$\begin{aligned} v^2(x) &= v_0^2 + 2 \int_{x_0}^x a(x) dx \quad \overset{a = -\omega^2 x}{=} v_0^2 - 2\omega^2 \int_{x_0}^x x dx = \\ &= v_0^2 - 2\omega^2 \frac{1}{2} (x^2 - x_0^2) \end{aligned}$$

$$\Rightarrow \boxed{v^2(x) = v_0^2 + \omega^2 (x_0^2 - x^2)}$$



$$\left. \begin{aligned} a &= -\omega^2 x \\ &= \frac{d^2 x}{dt^2} \end{aligned} \right\} \Rightarrow \boxed{\frac{d^2 x}{dt^2} + \omega^2 x = 0}$$

equazione differenziale del moto armonico

$$\Rightarrow \boxed{x(t) = A \sin(\omega t + \phi)}$$

$$\boxed{x(t) = a \sin(\omega t) + b \cos(\omega t)} \quad (*)$$

$$a = A \cos \phi ; b = A \sin \phi$$

$$\begin{aligned} (*) &= A \sin \omega t \cos \phi + A \cos \omega t \sin \phi = \\ &= A \sin(\omega t + \phi) \end{aligned}$$