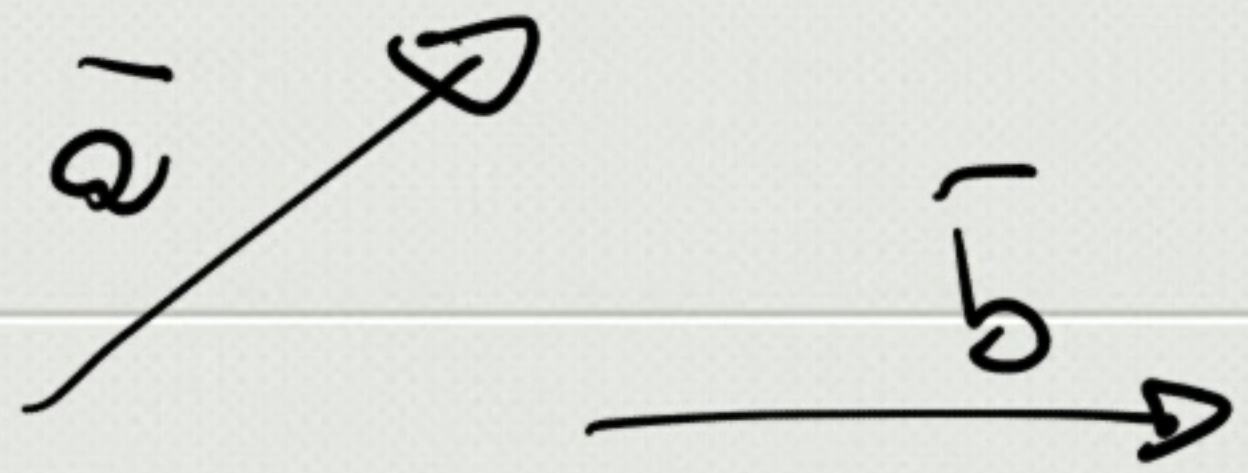
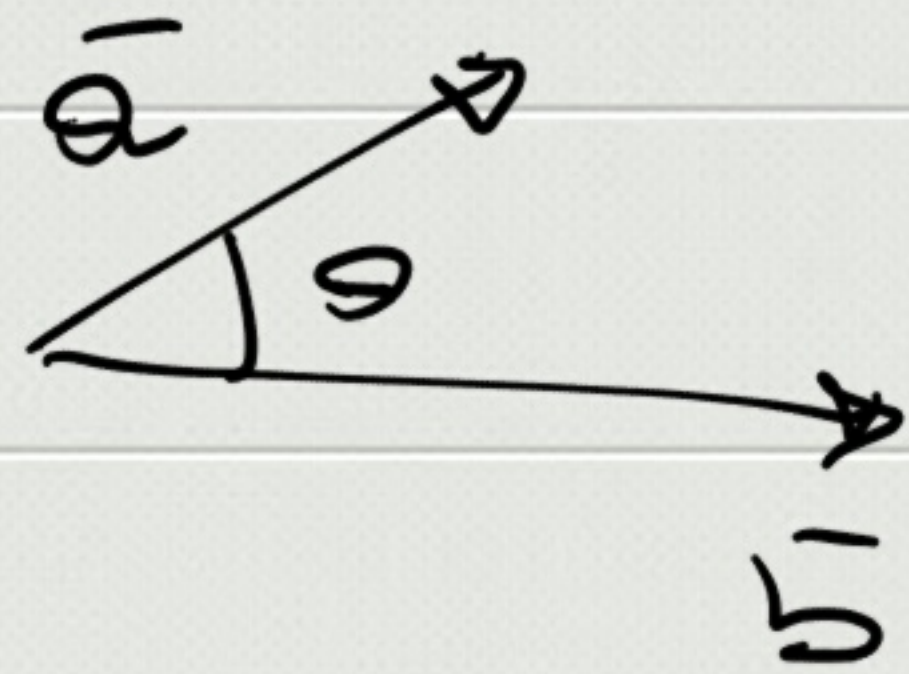


Prodotto scalare

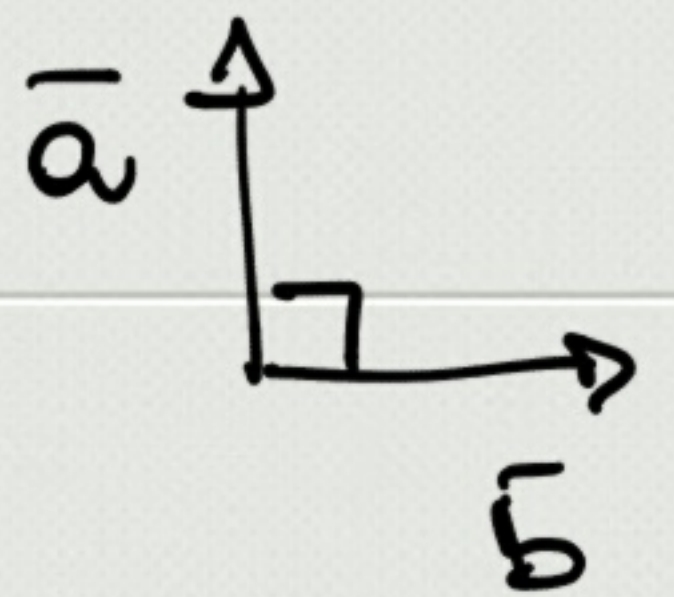


$$\boxed{\bar{a} \cdot \bar{b} = |\bar{a}| |\bar{b}| \cos \theta}$$

$$= ab \cos \theta$$



$$- \bar{a} \cdot \bar{b} = 0 \quad \left\{ \begin{array}{l} a, b \neq 0 \\ \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} \end{array} \right.$$



$$- \bar{a} \bar{b} = ab \cos \theta = ba \cos \theta = \bar{b} \bar{a}$$

$$- \bar{b} = \bar{a} \Rightarrow \bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{a} = aa \cos 0^\circ = a^2$$

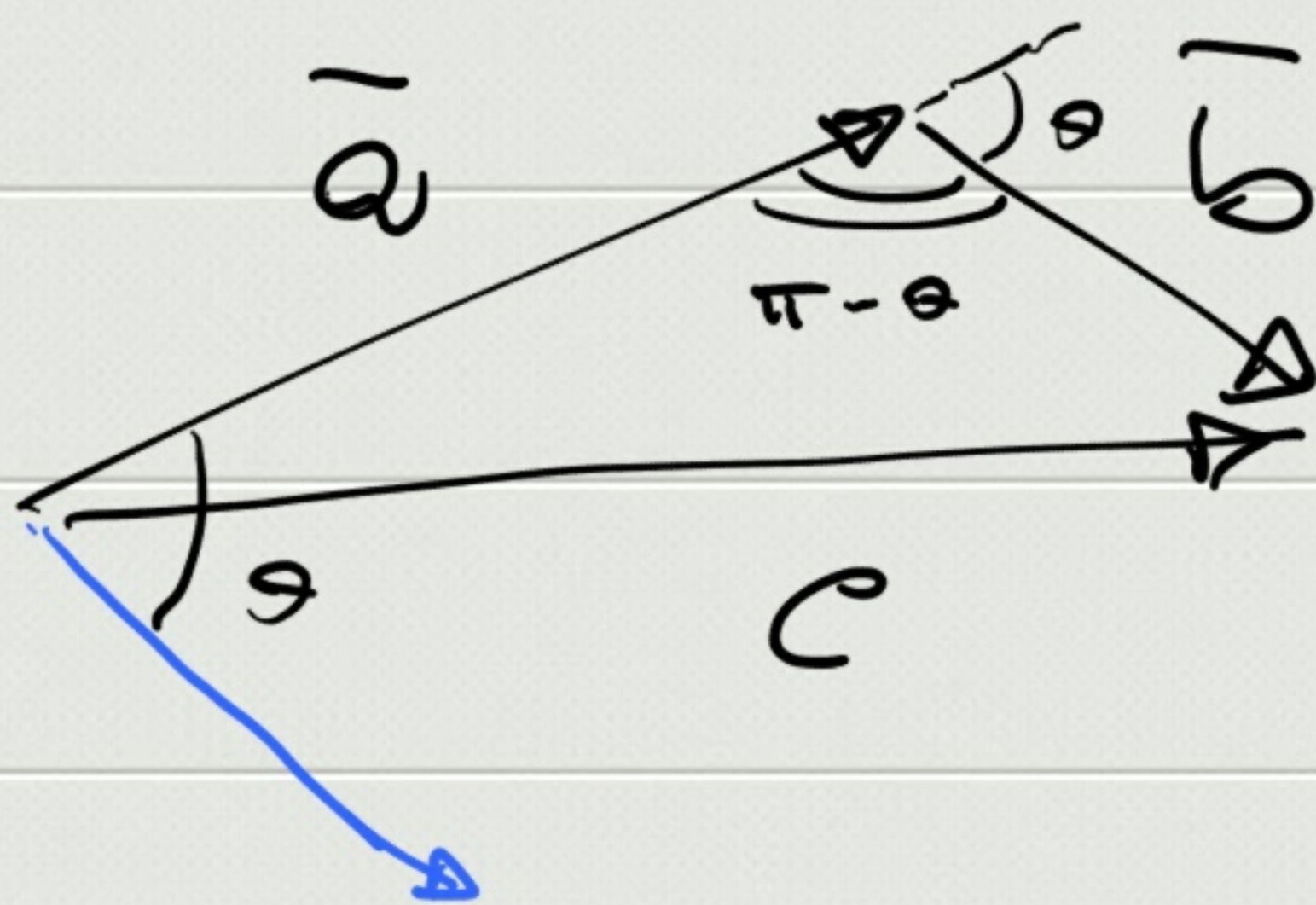
$$\Rightarrow \boxed{\bar{a} \cdot \bar{a} = a^2}$$

~~$$- \bar{a} \cdot \bar{b} \cdot \bar{c}$$~~

$$- \bar{a} \cdot (\bar{b} + \bar{c}) = \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}$$

$$\vec{c} = \vec{a} + \vec{b}$$

$$\begin{aligned} c^2 &= \vec{c} \cdot \vec{c} = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = \\ &= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \\ &= a^2 + b^2 + 2\vec{a} \cdot \vec{b} = \\ &= a^2 + b^2 + 2ab \cos \theta \end{aligned}$$



$$= a^2 + b^2 - 2ab \cos(\pi - \theta)$$

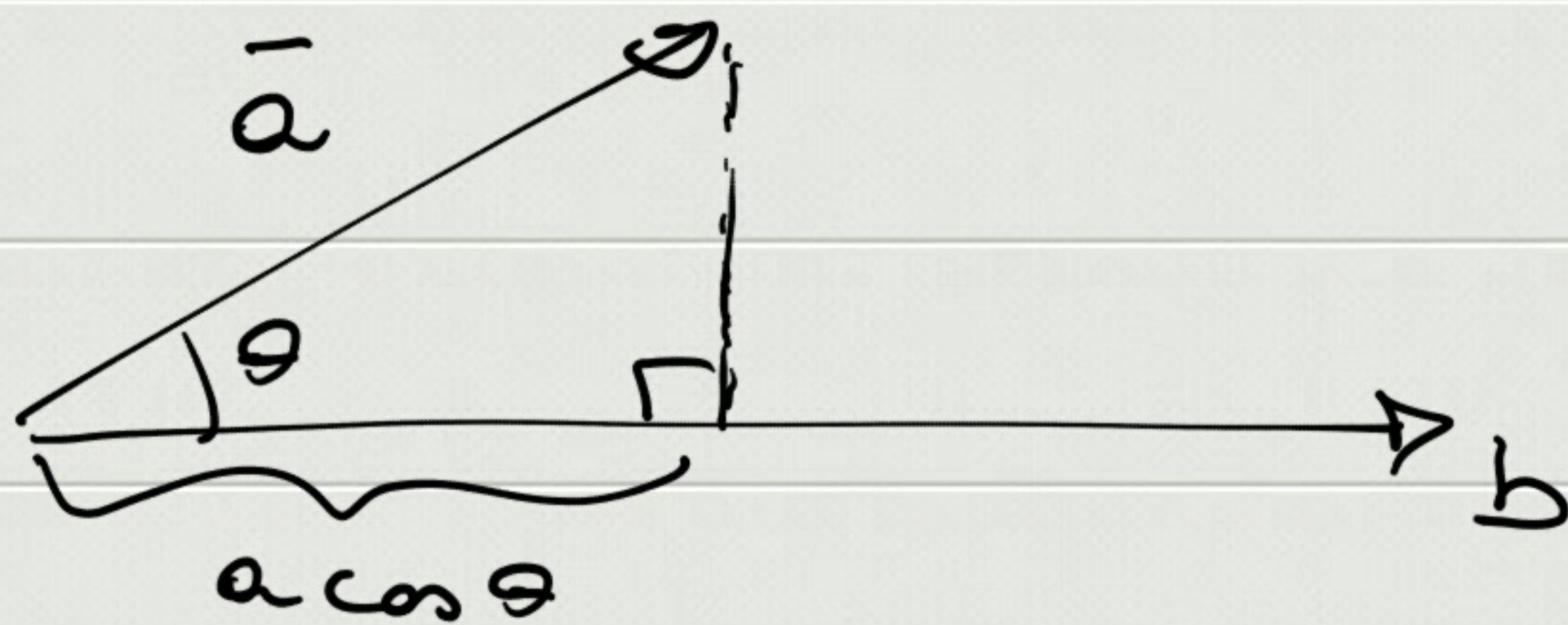
$$\vec{a} = a_x \vec{u}_x + a_y \vec{u}_y + a_z \vec{u}_z$$

$$\vec{b} = b_x \vec{u}_x + b_y \vec{u}_y + b_z \vec{u}_z$$

$$\vec{u}_i \cdot \vec{u}_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\begin{aligned}\bar{a} \cdot \bar{b} &= (a_x \bar{u}_x + a_y \bar{u}_y + \dots) \cdot (b_x \bar{u}_x + b_y \bar{u}_y + \dots) = \\ &= a_x b_x + a_y b_y + a_z b_z\end{aligned}$$

$$\begin{aligned}\bar{a} \cdot \bar{a} &= a_x^2 + a_y^2 + a_z^2 \\ &\stackrel{!}{=} a^2\end{aligned}$$



$$\bar{a} \cdot \bar{b} = a b \cos \theta = (a \cos \theta) b$$

