
Vector identities

List of symbols:

- f, g = scalar functions of space coordinates;
- $\mathbf{A}, \mathbf{B}, \mathbf{C}$ = vector functions of space coordinates;
- \cdot = scalar (or internal) product;
- \times = vector (or external) product;
- $(\mathbf{A} \cdot \nabla)$ = direction derivative, along the direction of the vector \mathbf{A} , multiplied by $|\mathbf{A}|$;
- \hat{v} = real unit-vector.

$$\nabla(fg) = f \nabla g + g \nabla f \quad , \quad (\text{C.1})$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \quad , \quad (\text{C.2})$$

$$\nabla \cdot (f\mathbf{A}) = f \nabla \cdot \mathbf{A} + \nabla f \cdot \mathbf{A} \quad , \quad (\text{C.3})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B} \quad , \quad (\text{C.4})$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0 \quad , \quad (\text{C.5})$$

$$\nabla \times (f\mathbf{A}) = f \nabla \times \mathbf{A} + \nabla f \times \mathbf{A} \quad , \quad (\text{C.6})$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B} \quad , \quad (\text{C.7})$$

$$\nabla \times (\nabla f) = 0 \quad , \quad (\text{C.8})$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A} \quad , \quad (\text{C.9})$$

$$\mathbf{A} = (\mathbf{A} \cdot \hat{v}) \hat{v} + \hat{v} \times \mathbf{A} \times \hat{v} \quad , \quad (\text{C.10})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{A} \cdot \mathbf{B}) \mathbf{C} \quad . \quad (\text{C.11})$$