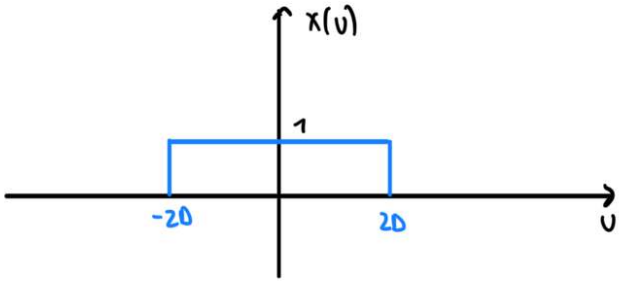
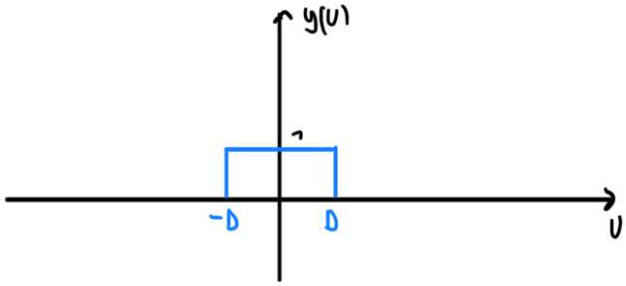
Lezione 9 - 15/03/2024

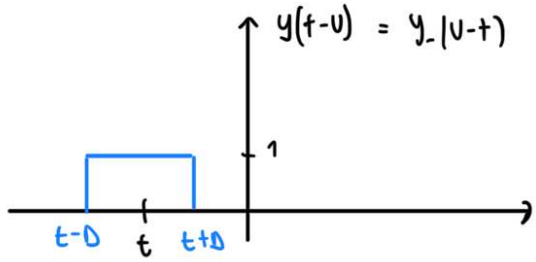
Es 2
Calcolare la convoluzione tra un rettangolo di base 4D ed uno di base 2D

ESERCIZIO 2 SLIDE 24

CALCOLARE LA CONVOLUZIONE $z(t) = x * y(t)$ CON

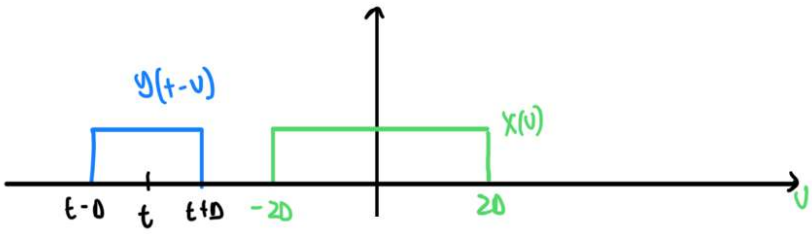






SOL. OCCORRE DISTINGUERE 5 CASI, A SECONDA DELLA POSIZIONE RECIPROCA DEI 2 SEGNALI

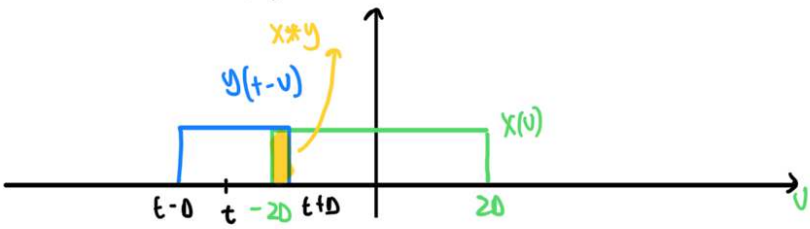
A) $y(t-v)$ A SINISTRA



$t+D < 2D \longrightarrow t < -3D \qquad z(t) = x * y(t) = 0$

(SI RICORDA CHE L'INTERPRETAZIONE DELLA CONVOLUZIONE È L'AREA IN COMUNE TRA I 2 SEGNALI)

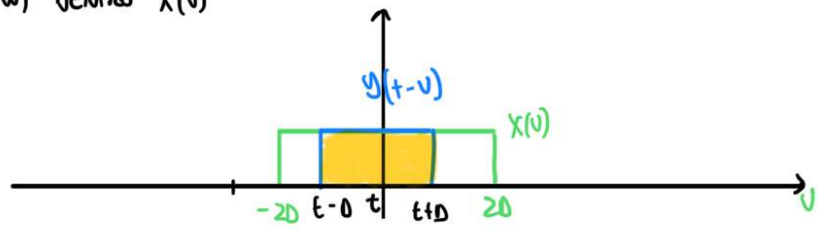
B) $y(t-w)$ ENTRA DA SINISTRA



$t-D < -2D < t+D \rightarrow -3D < t < -D$

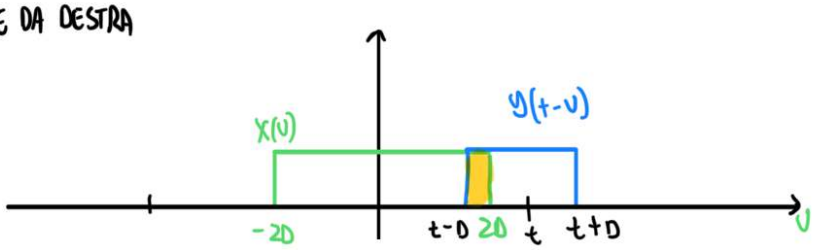
$\rightarrow z(t) = t+D-(-2D) = t+3D$

c) $y(t-v)$ DENTRO $x(v)$



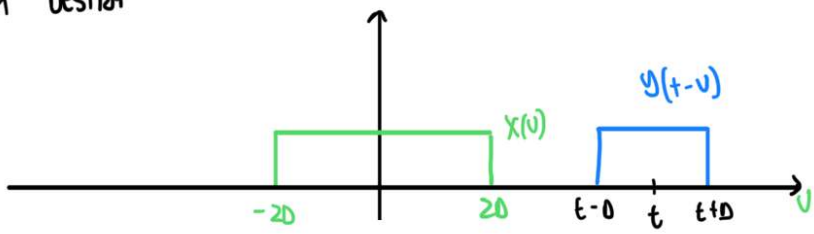
$-D < t < D \longrightarrow z(t) = 2D$

D) $y(t-v)$ ESCE DA DESTRA



$D < t < 3D \rightarrow z(t) = 2D - (t-D)$
$= 3D - t$

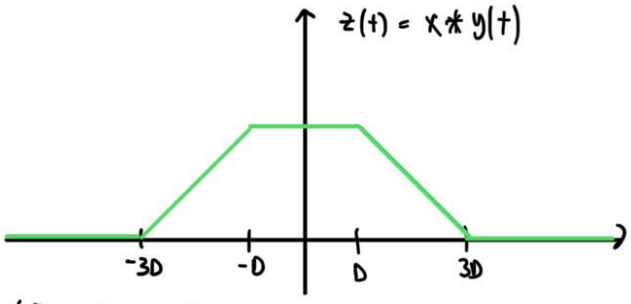
E) $y(t-v)$ A DESTRA



$t > 3D \longrightarrow z(t) = x * y(t) = 0$

IN CONCLUSIONE

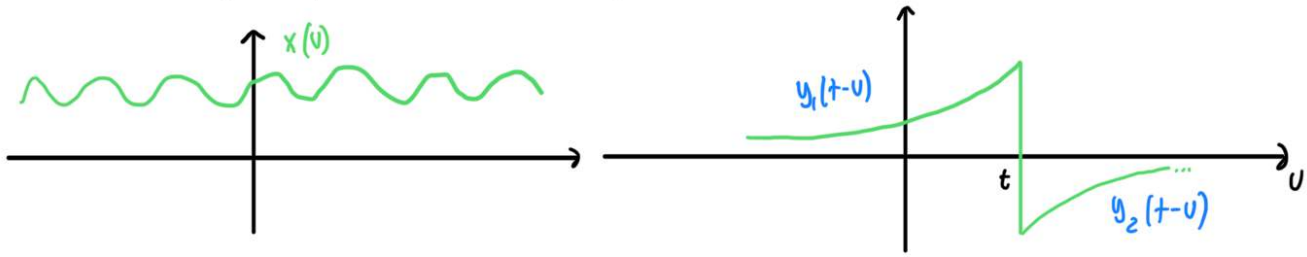
$$z(t) = \begin{cases} 0 & t < 3D \\ t + 3D & -3D < t < -D \\ 2D & -D < t < D \\ 3D - t & D < t < 3D \\ 0 & t > 3D \end{cases}$$



(è un trapezio)

NOTA: SULLE REGIONI DELL'INTEGRALE DI CONVOLUZIONE (CONVOLUZIONE A TRATTI)

Facciamo un esempio. Supponiamo di avere 2 segnali

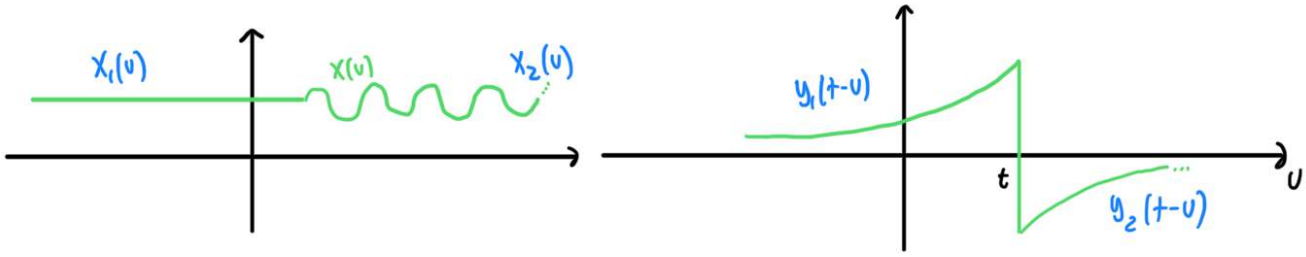


in questo caso per y(t) l'integrale va diviso in 2 parti, perché l'espressione analitica di y(t) cambia per i valori prima di t e dopo t

$$z(t) = \int_{-\infty}^{t} x(u)\,\underbrace{y_1(t-u)}_{\text{espressione 1}}\,du + \int_{t}^{+\infty} x(u)\,\underbrace{y_2(t-u)}_{\text{espressione 2}}\,du$$

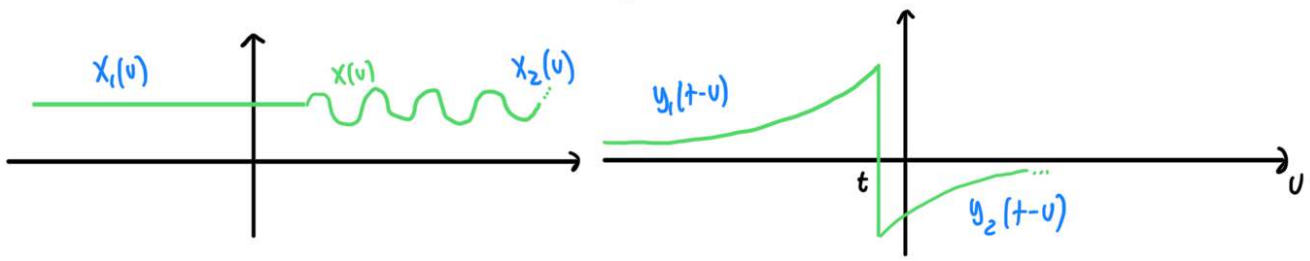
2 INTEGRALI DI CONVOLUZIONE
1 REGIONE

SUPPONIAMO CHE ENTRAMBI I SEGNALI ABBIANO 2 DIFFERENTI ESPRESSIONI ANALITICHE



$$z(t) = \int_{-\infty}^{0} x_1(u)\,y_1(t-u)\,du + \int_{0}^{t} x_2(u)\,y_1(t-u)\,du + \int_{t}^{+\infty} x_2(u)\,y_2(t-u)\,du$$

QUESTO È QUELLO CHE SUCCEDE SE $t>0$. SE $t<0$?



PER $t<0$: $$z(t) = \int_{-\infty}^{t} x_1(u)\,y_1(t-u)\,du + \int_{t}^{0} x_1(u)\,y_2(t-u)\,du + \int_{0}^{+\infty} x_2(u)\,y_2(t-u)\,du$$

2 REGIONI
3 INTEGRALI

Su Wikipedia possiamo veder l'azione della convoluzione tramite un disegno animato:
https://en.wikipedia.org/wiki/Convolution

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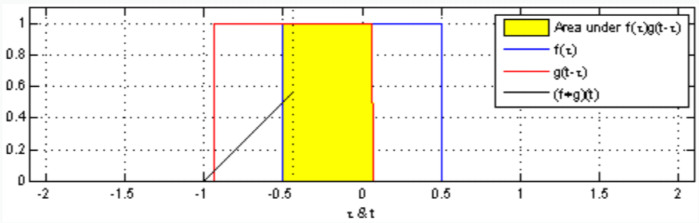
$$\int_{-\infty}^{\infty} \delta(\tau) g(t-\tau)\, d\tau = g(t)$$

In this example, the red-colored "pulse", $g(\tau)$, is an even function ($g(-\tau) = g(\tau)$), so convolution is equivalent to correlation. A snapshot of this "movie" shows functions $g(t-\tau)$ and $f(\tau)$ (in blue) for some value of parameter t, which is arbitrarily defined as the distance along the τ axis from the point $\tau = 0$ to the center of the red pulse. The amount of yellow is the area of the product $f(\tau) \cdot g(t-\tau)$, computed by the convolution/correlation integral. The movie is created by continuously changing t and recomputing the integral. The result (shown in black) is a function of t, but is plotted on the same axis as τ, for convenience and comparison.



In this depiction, $f(\tau)$ could represent the response of a resistor-capacitor circuit to a narrow pulse that occurs at $\tau = 0$. In other words, if