Lezione 22 - 2/05/2024

Si ringrazia Giulia Selvestrel per il materiale dato che purtroppo a quella lezione non ero presente

Es 1d

$$s(n) = A\cos(n\theta_0 + \theta_1) = \frac{A}{2} e^{j\theta_1} \cdot e^{jn\theta_0} + \frac{A}{2} e^{-j\theta_1} \cdot e^{-jn\theta_0}$$

$s(e^{j\theta})$?

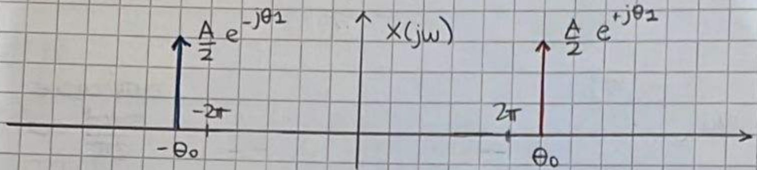
$s(n) = x(t)\big|_{t=n}$ → campionamento di segnale continuo

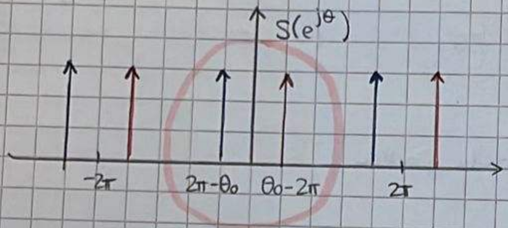
$$x(t) = A\cos(\theta_0 t + \theta_1) \longrightarrow \boxed{\downarrow} \longrightarrow s(n) = A\cos(\theta_0 n + \theta_1)$$

$$\downarrow \mathcal{F} \qquad\qquad\qquad \downarrow \mathcal{F}$$

$$X(j\omega) \longrightarrow \boxed{rep_{2\pi}} \longrightarrow S(e^{j\theta})$$

$$X(j\omega) = \frac{A}{2} e^{j\theta_1} \delta(\omega - \theta_0) + \frac{A}{2} e^{-j\theta_1} \delta(\omega + \theta_0)$$





Dimostrazione

Dimostrare che i segnali $x(n) = e^{jn\theta_0}$ sono autofunzioni dei filtri discreti

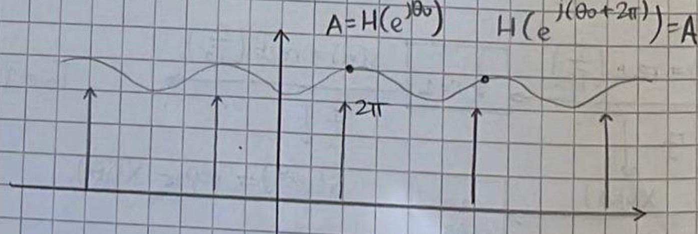
$$x(n) \longrightarrow \boxed{h(\cdot)} \longrightarrow y(n) = \lambda x(n)$$

$$\downarrow \mathcal{F} \qquad\qquad \downarrow \mathcal{F}$$

$$X(e^{j\theta}) \longrightarrow \otimes \longrightarrow Y(e^{j\theta})$$

$X(e^{j\theta})$ periodico 2π; $H(e^{j\theta})$ periodico 2π

$$X(e^{j\theta}) = 2\pi \, rep_{2\pi} \, \delta(\theta - \theta_0)$$



$$Y(e^{j\theta_0}) = \underbrace{H(e^{j\theta_0})}_{\text{costante } A} X(e^{j\theta_0})$$

$$y(n) = \underset{A}{H(e^{j\theta_0})} \; x(n)$$

$$e^{jn\theta_0} \longrightarrow \boxed{h(\cdot)} \longrightarrow H(e^{j\theta_0}) e^{jn\theta_0}$$

I filtri rea...

x(n) = co...

x(n) = ...

y(n) = ...

=

=

cos(nθ...

I filtri reali non distorcono le sinusoidi

$$x(n) = \cos(n\theta_0 + \theta_1) \longrightarrow \boxed{h(\cdot)} \xrightarrow{y(n)}$$

REALE

Simmetria Reale nel tempo

$\Downarrow$

simmetria Hermitiana nella trasformata

$$x(n) = \frac{e^{j\theta_1}}{2} \cdot e^{jn\theta_0} + \frac{e^{-j\theta_1}}{2} \cdot e^{-jn\theta_0}$$

$$y(n) = \underbrace{\frac{e^{j\theta_1}}{2} H(e^{j\theta_0}) e^{jn\theta_0}} + \underbrace{\frac{e^{-j\theta_1}}{2} \cdot \underset{H^*(e^{j\theta_0})}{\underset{\parallel}{H(e^{-j\theta_0})}} e^{-jn\theta_0}}_{\text{complesso coniugato}}$$

$$= \mathrm{Re}\left[H(e^{j\theta_0}) e^{j(n\theta_0 + \theta_1)} \right] = \mathrm{Re}\left[|H(e^{j\theta_0})| e^{j\angle H(e^{j\theta_0})} e^{j(n\theta_0+\theta_1)} \right]$$

$$= |H(e^{j\theta_0})| \cos(n\theta_0 + \theta_1 + \angle H(e^{j\theta_0}))$$

$$\cos(n\theta_0 + \theta_1) \longrightarrow \boxed{h(\cdot)} \qquad |H(e^{j\theta_0})| \cos(n\theta_0 + \theta_1 + \angle H(e^{j\theta_0}))$$

REALE