ESERCIZI SCHEDA 8

ESERCIZIO 1

ⓐ

$$f(x) = \cot(x) = \frac{1}{\tan x} = (\tan x)^{-1}$$

$$f'(x) = -(\tan x)^{-2} \cdot \frac{1}{\cos^2 x} = -\frac{\cancel{\cos^2 x}}{\sin^2 x} \cdot \frac{1}{\cancel{\cos^2 x}} = \boxed{-\frac{1}{\sin^2 x}}$$

E se non ricordo la derivata della tangente?

$$f(x) = \cot(x) = \frac{\cos x}{\sin x}$$

$$f'(x) = [\text{derivazione del quoziente}] = \frac{-\sin x \cdot \sin x - \cos x \cos x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

ⓑ

$$f(x) = \tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{\cancel{2}} \cdot \frac{\cancel{2}}{e^x + e^{-x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$f'(x) = [\text{derivazione del quoziente}]$$

$$= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} + 2 - \cancel{e^{2x}} - \cancel{e^{-2x}} + 2}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} = \boxed{\frac{1}{\cosh^2 x}}$$

ⓒ

$$f(x) = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$f'(x) = -(\cos x)^{-2} \cdot (-\sin x) = \frac{\sin x}{\cos^2 x} = \boxed{\frac{\tan x}{\cos x}}$$

(d) $f(x) = \frac{1}{\sin x} = (\sin x)^{-1}$

$f'(x) = -(\sin x)^{-2} \cdot \cos x = -\frac{\cos x}{\sin^2 x} = -\frac{\cot x}{\sin x}$

ESERCIZIO 2

$$f(x) = \begin{cases} x^n |x| & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases} \Rightarrow f(x) = \begin{cases} x^n x = x^{n+1} & \text{se } x > 0 \\ 0 & \text{se } x = 0 \\ x^n(-x) = -x^{n+1} & \text{se } x < 0 \end{cases}$$

La funzione è continua, ma in $x=0$ ci può essere un punto di discontinuità:

- $f(0) = 0$
- $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^{n+1} = \begin{cases} 0 & \text{se } n > -1 \\ 1 & \text{se } n = -1 \\ +\infty & \text{se } n < -1 \end{cases}$
- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x^{n+1}) = \begin{cases} 0 & \text{se } n > -1 \\ -1 & \text{se } n = -1 \\ +\infty & \text{se } n < -1 \end{cases}$

La funzione è continua in $\mathbb{R}$ $\left(f(x) \in C^{(0)}(\mathbb{R}) \right)$ se $n > -1$.

Non studio la derivabilità per $n \leq -1$, in quanto la funzione è discontinua $\Rightarrow$ Non derivabile.

$$f'(x) = \begin{cases} (n+1)x^n & \text{se } x > 0 \\ -(n+1)x^n & \text{se } x < 0 \end{cases}$$

- $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (n+1)x^n = \begin{cases} 0 & \text{se } n>0 \lor n=-1 \\ 1 & \text{se } n=0 \\ -\infty & \text{se } n<-1 \end{cases}$

- $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -(n+1)x^n = \begin{cases} 0 & \text{se } n>0 \lor n=-1 \\ -1 & \text{se } n=0 \\ -\infty & \text{se } n<-1 \end{cases}$

$f(x) \in C^{(1)}(\mathbb{R}) \iff n>0$

Esercizio 3

$$f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$$

$$\left.\begin{aligned} \lim_{x\to 0^+} \frac{2}{3\sqrt[3]{x}} &= +\infty \\ \lim_{x\to 0^-} \frac{2}{3\sqrt[3]{x}} &= -\infty \end{aligned}\right\} \text{Si ha una cuspide.}$$

Esercizio 4

$$f(x) = \begin{cases} \arctan(x^2+x) & \text{se } x\geq 0 \\ x & \text{se } x<0 \end{cases}$$

Studio la continuità in $x=0$:

- $f(0) = \arctan(0) = 0$
- $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \arctan(x^2 + x) = 0$
- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$

$f(x)$ è continua in $x=0$

In $x \neq 0$ la funzione è composizione di funzioni continue.

$$\Rightarrow \boxed{f(x) \in C^{(0)}(\mathbb{R})}$$

$$f'(x) = \begin{cases} \dfrac{2x+1}{1+(x^2+x)^2} & \text{se } x>0 \\ 1 & \text{se } x<0 \end{cases}$$

Studio la derivabilità in $x=0$:

- $\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \dfrac{2x+1}{1+(x^2+x)^2} = 1$
- $\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} 1 = 1$

$f(x)$ è derivabile in $x=0$

Per $x \neq 0$, la funzione è composizione di funzioni derivabili.

$$\Rightarrow \boxed{f(x) \in C^{(1)}(\mathbb{R})}$$

ESERCIZIO 5

$$f_n(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right) & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

(a)

$$f_0(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases} \qquad f_0'(x) = \begin{cases} -\dfrac{\cos\left(\frac{1}{x}\right)}{x^2} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

Studio la continuità in $x=0$:

$$\left.\begin{array}{l} \bullet\ f_0(0) = 0 \\ \bullet\ \lim\limits_{x \to 0^+} f_0(x) = \lim\limits_{x \to 0^+} \sin\left(\frac{1}{x}\right) \quad \nexists \\ \bullet\ \lim\limits_{x \to 0^-} f_0(x) = \lim\limits_{x \to 0^-} \sin\left(\frac{1}{x}\right) \quad \nexists \end{array}\right\} \text{discontinuità in } x=0$$

$\Rightarrow f_0 \notin C^{(0)}(\mathbb{R})$

(b)

$$f_1(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

$$f_1'(x) = \begin{cases} \sin\left(\frac{1}{x}\right) - \dfrac{\cos\left(\frac{1}{x}\right)}{x} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

Studio la continuità in $x=0$:

$$\left.\begin{aligned} & \bullet\ f_1(0) = 0 \\ & \bullet\ \lim_{x\to 0^+} f_1(x) = \lim_{x\to 0^+} x\sin\left(\frac{1}{x}\right) = 0 \\ & \bullet\ \lim_{x\to 0^-} f_1(x) = \lim_{x\to 0^-} x\sin\left(\frac{1}{x}\right) = 0 \end{aligned}\right\} \text{continuità in } x=0$$

Per $x\neq 0$, la funzione è composizione di funzioni continue.

$$\Rightarrow f_1 \in C^{(0)}(\mathbb{R})$$

Studio la derivabilità in $x=0$

$$\left.\begin{aligned} & \bullet\ \lim_{x\to 0^+} f_1'(x) = \lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) - \frac{\cos\left(\frac{1}{x}\right)}{x} \ \nexists \\ & \bullet\ \lim_{x\to 0^-} f_1'(x) = \lim_{x\to 0^-} \sin\left(\frac{1}{x}\right) - \frac{\cos\left(\frac{1}{x}\right)}{x} \ \nexists \end{aligned}\right\} \begin{gathered}\text{Non derivabilità} \\ \text{in } x=0\end{gathered}$$

$$\Rightarrow f_1(x) \notin C^{(1)}(\mathbb{R})$$

Analogamente si dimostrano c e d.

ESERCIZIO 6

$f(x) = \sin x \rightarrow f'(x) = \cos x$

formula della retta tangente:

$y = f'(x_0)(x-x_0) + f(x_0)$ con $x_0 = 1$

$= \cos(1)(x-1) + \sin 1$

$$\Rightarrow \boxed{y = \cos 1\, x + \sin 1 - \cos 1}$$

ESERCIZIO 7

$f(x) = x^2 \rightarrow f'(x) = 2x$

rette passanti per $(1,-3)$

$y = f'(x_0)(x-x_0) + f(x_0)$

$y = 2x_0(x-x_0) + x_0^2$

$y = 2xx_0 - 2x_0^2 + x_0^2 \quad \Rightarrow \quad y = 2xx_0 - x_0^2$

Sostituisco $y=-3,\ x=1$:

$-3 = 2x_0 - x_0^2 \Leftrightarrow x_0^2 - 2x_0 - 3 = 0$

$\Leftrightarrow (x_0+1)(x_0-3) = 0 \quad \Leftrightarrow \quad x_0 = -1 \ \vee \ x_0 = 3$

$y_1 = -2x-1 \qquad y_2 = 6x-9$

ESERCIZIO 8

$f(x) = ax^2 + bx + c \rightarrow f'(x) = 2ax + b$

$(1,2) \in f(x) \Rightarrow$ [sostituendo]: $2 = a+b+c$

$(0,0) \in f(x) \Rightarrow$ [sostituendo]: $0 = c$

retta tangente:

$y = f'(0)(x-0) + f(0) = x$

$\Leftrightarrow bx + c = x$

$$\begin{cases} a+b+\cancel{c}=2 \\ c=0 \\ bx+\cancel{c}=x \end{cases} \qquad \begin{cases} a+b=2 \\ c=0 \\ bx=x \end{cases} \qquad \begin{cases} a=1 \\ c=0 \\ b=1 \end{cases}$$

$$\Rightarrow \boxed{f(x)=x^2+x}$$

Esercizio 9

1

$$f(x)=\frac{x-\sqrt{x}}{x+\sqrt{x}}=\frac{x-x^{\frac{1}{2}}}{x+x^{-\frac{1}{2}}}$$

$$f'(x)=\frac{\left(1-\frac{1}{2\sqrt{x}}\right)\left(x+\sqrt{x}\right)-\left(1+\frac{1}{2\sqrt{x}}\right)\left(x-\sqrt{x}\right)}{\left(x+\sqrt{x}\right)^2}$$

$$=\frac{\cancel{x}+\sqrt{x}-\frac{\sqrt{x}}{2}-\cancel{\frac{1}{2}}-\cancel{x}+\sqrt{x}-\frac{\sqrt{x}}{2}+\cancel{\frac{1}{2}}}{\left(x+\sqrt{x}\right)^2}$$

$$=\boxed{\frac{\sqrt{x}}{\left(x+\sqrt{x}\right)^2}}$$

2

$$f(x)=\left(\frac{1+x^2}{1+x}\right)^5$$

$$f'(x)=5\left(\frac{1+x^2}{1+x}\right)^4\cdot\frac{2x(1+x)-(1+x^2)}{(1+x)^2}$$

$$=5\left(\frac{1+x^2}{1+x}\right)^4\cdot\frac{x^2+2x-1}{(1+x)^2}=\boxed{\frac{5(1+x^2)^4(x^2+2x-1)}{(1+x)^6}}$$

③ $$f(x)=x\sqrt{\frac{1-x}{1+x^2}}$$

$$f'(x)=\sqrt{\frac{1-x}{1+x^2}}+x\cdot\frac{1}{2\sqrt{\frac{1-x}{1+x^2}}}\cdot\frac{-(1+x^2)-2x(1-x)}{(1+x^2)^2}$$

$$=\boxed{\sqrt{\frac{1-x}{1+x^2}}+\frac{x(x^2-2x-1)}{(1+x^2)^2}\sqrt{\frac{1+x^2}{4(1-x)}}}$$

④ $$f(x)=\sqrt{x+\sqrt{x+\sqrt{x}}}$$

$$f'(x)=\boxed{\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\left(1+\frac{1}{2\sqrt{x}}\right)\right)}$$

⑤ $$f(x)=(x\cot x)^2$$

$$f'(x)=2x\cot x\cdot\left(\cot x-\frac{x}{\sin^2 x}\right)$$

$$=2x\frac{\cos x}{\sin x}\left(\frac{\sin x\cos x-x}{\sin^2 x}\right)=\boxed{\frac{2x\cos x(\sin x\cos x-x)}{\sin^3 x}}$$

⑥ $$f(x)=\sqrt{\log x}$$

$$f'(x)=\frac{1}{2\sqrt{\log x}}\cdot\frac{1}{x}=\boxed{\frac{1}{2x\sqrt{\log x}}}$$

(7) $f(x)=\sqrt[3]{\dfrac{1}{1+x^2}}$

$$f'(x)=\frac{1}{3}\left(\frac{1}{1+x^2}\right)^{-\frac{2}{3}}\cdot\left(-\frac{1}{(1+x^2)^2}\right)\cdot 2x$$

$$=-\frac{2x\sqrt[3]{(1+x^2)^2}}{3\,(1+x^2)^2}=\boxed{-\frac{2x}{3(1+x^2)\sqrt[3]{1+x^2}}}$$

(8) $f(x)=\dfrac{1}{2\sin x\cos x}$

Per chi si ricorda le formule goniometriche: $2\sin x\cos x=\sin(2x)$

$$f'(x)=\frac{1}{2}\cdot\left(-\frac{1}{\sin^2x\cos^2x}\right)\cdot\left(\cos^2x-\sin^2x\right)$$

$$=\boxed{\frac{\sin^2x-\cos^2x}{2\sin^2x\cos^2x}}$$

(9) $f(x)=\cos x\cdot\sqrt{1+\sin^2x}$

$$f'(x)=-\sin x\cdot\sqrt{1+\sin^2x}+\cos x\cdot\frac{1}{\cancel{2}\sqrt{1+\sin^2x}}\cdot\cancel{2}\sin x\cdot\cos x$$

$$=-\sin x\sqrt{1+\sin^2x}+\frac{\sin x\cos^2x}{\sqrt{1+\sin^2x}}$$

$$=\frac{-\sin x(1+\sin^2x)+\sin x\cos^2x}{\sqrt{1+\sin^2x}}$$

$$=\boxed{\frac{\sin x\left(\cos^2x-\sin^2x-1\right)}{\sqrt{1+\sin^2x}}}$$

(10)

$$f(x) = \log_3 x^2 - 1$$

$$f'(x) = \frac{1}{\cancel{x^2} \ln 3} \cdot 2\cancel{x} = \boxed{\frac{2}{x \ln 3}}$$

(11)

$$f(x) = \log\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right) = \frac{1}{2} \log\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$f'(x) = \frac{1}{2} \cdot \frac{1-\sin x}{1+\sin x} \cdot \frac{\cos x\,(1-\sin x) - (-\cos x)(1+\sin x)}{(1-\sin x)^2}$$

$$= \frac{1}{2} \cdot \frac{\cancel{1-\sin x}}{1+\sin x} \cdot \frac{\cos x - \cancel{\sin x \cos x} + \cos x + \cancel{\sin x \cos x}}{(1-\sin x)^{\cancel{2}}}$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cos x}{1-\sin^2 x} = \frac{\cos x}{\cos^2 x} = \boxed{\frac{1}{\cos x}}$$

(12)

$$f(x) = e^{kx} \sin(\omega x + \varphi)$$

$$f'(x) = k e^{kx} \sin(\omega x + \varphi) + e^{kx} \cos(\omega x + \varphi) \cdot \omega$$

$$= \boxed{e^{kx}\left[k \sin(\omega x + \varphi) + \omega \cos(\omega x + \varphi)\right]}$$

(13)

$$f(x) = \arcsin\left(\sqrt{\frac{1-\cos x}{2}}\right)$$

$$f'(x) = \frac{1}{\sqrt{1 - \frac{1-\cos x}{2}}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1-\cos x}{2}}} \cdot \frac{\sin x}{2}$$

$$= \sqrt{\frac{2}{1+\cos x}} \cdot \frac{1}{2} \cdot \sqrt{\frac{2}{1-\cos x}} \cdot \frac{\sin x}{2}$$

$$= \sqrt{\frac{4}{1-\cos^2 x}} \cdot \frac{1}{2} \cdot \frac{\sin x}{2}$$

$$= \sqrt{\frac{4}{\sin^2 x}} \cdot \frac{1}{2} \cdot \frac{\sin x}{2}$$

$$= \frac{\cancel{2}}{|\sin x|} \cdot \frac{1}{\cancel{2}} \cdot \frac{\sin x}{2} = \frac{\operatorname{sgn}(\sin x)}{2}$$

→ La funzione $\operatorname{sgn}(x)$ è detta "funzione SEGNO". Restituisce 1 quando l'argomento è positivo e restituisce -1 quando l'argomento è negativo.

Dall'esercizio:

$$\frac{\sin x}{|\sin x|} = \begin{cases} \frac{\sin x}{\sin x} = 1 & \text{se } \sin x \geq 0 \\ \frac{\sin x}{-\sin x} = -1 & \text{se } \sin x < 0 \end{cases}$$

14

$$f(x) = x \arctan\left(\frac{x-1}{x+1}\right) - \log\left(\sqrt{x^2+1}\right)$$

$$= x \arctan\left(\frac{x-1}{x+1}\right) - \frac{1}{2}\log\left(x^2+1\right)$$

$$f'(x) = \arctan\left(\frac{x-1}{x+1}\right) + x \cdot \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2} \cdot \frac{\cancel{x}+1-(\cancel{x}-1)}{(x+1)^2} - \frac{\cancel{2}x}{\cancel{2}(x^2+1)}$$

$$=$$

$$= \arctan\left(\frac{x-1}{x+1}\right) + x \cdot \frac{\cancel{(x+1)^2}}{x^2+\cancel{2x}+1+x^2-\cancel{2x}+1} \cdot \frac{2}{\cancel{(x+1)^2}} - \frac{x}{x^2+1}$$

$$= \arctan\left(\frac{x-1}{x+1}\right) + \frac{\cancel{2}^{1}x}{\cancel{2x^2+2}_{x^2+1}} - \frac{x}{x^2+1}$$

$$= \arctan\left(\frac{x-1}{x+1}\right) + \cancel{\frac{x}{x^2+1}} - \cancel{\frac{x}{x^2+1}}$$

$$= \arctan\left(\frac{x-1}{x+1}\right)$$

15

$$f(x) = \log\left(\sin\left(\sqrt[3]{\arctan(e^{2x})}\right)\right) = \log\left(\sin\left(\left(\arctan(e^{2x})\right)^{\frac{1}{3}}\right)\right)$$

$$f'(x) = \frac{\cos\left(\sqrt[3]{\arctan(e^{2x})}\right)}{\sin\left(\sqrt[3]{\arctan(e^{2x})}\right)} \cdot \frac{1}{3} \cdot \frac{1}{\left(\arctan(e^{2x})\right)^{\frac{2}{3}}} \cdot \frac{1}{1+e^{4x}} \cdot 2e^{2x}$$

$$= \frac{2e^{2x}}{3\tan\left(\sqrt[3]{\arctan(e^{2x})}\right) \cdot \sqrt[3]{\arctan^2(e^{2x})} \cdot (e^{4x}+1)}$$

16

$$f(x) = x \sinh x = x \frac{e^x - e^{-x}}{2} = \frac{1}{2} x (e^x - e^{-x})$$

$$f'(x) = \frac{1}{2}\left[e^x - e^{-x} + x(e^x + e^{-x})\right]$$

$$= \frac{e^x - e^{-x}}{2} + x \frac{e^x + e^{-x}}{2} = \sinh x + x \cosh x$$

17

$$f(x) = \cosh(\sinh x)$$

$$f'(x) = \sinh(\sinh x) \cdot \cosh x$$

18

$$f(x) = \arccos(\tanh x)$$

$$f'(x) = -\frac{1}{\sqrt{1-\tanh^2 x}} \cdot \frac{1}{\cosh^2 x}$$

$$= -\frac{1}{\sqrt{1-\frac{\sinh^2 x}{\cosh^2 x}}} \cdot \frac{1}{\cosh^2 x}$$

$$= -\sqrt{\frac{\cosh^2 x}{\underbrace{\cancel{\cosh^2 x - \sinh^2 x}}_{1}}} \cdot \frac{1}{\cosh^2 x}$$

$$= -\cancel{\cosh x} \cdot \frac{1}{\cosh^{\cancel{2}} x} = \boxed{-\frac{1}{\cosh x}}$$

19

$$f(x) = x^x = \exp\{\log(x^x)\} = e^{x\log x}$$

$$f'(x) = e^{x\log x}(\log x + 1) = \boxed{x^x(\log x + 1)}$$

20

$$f(x) = (\cos x)^{\sin x} = \exp\{\log[(\cos x)^{\sin x}]\} = e^{\sin x \cdot \log(\cos x)}$$

$$f'(x) = e^{\sin x \cdot \log(\cos x)}\left(\cos x \cdot \log(\cos x) + \frac{\sin x}{\cos x}(-\sin x)\right)$$

$$= \boxed{(\cos x)^{\sin x}\left(\log[(\cos x)^{\cos x}] - \tan x \cdot \sin x\right)}$$

21

$$f(x) = \left(1+\frac{1}{x}\right)^x = \exp\left\{\log\left[\left(1+\frac{1}{x}\right)^x\right]\right\} = e^{x\log\left(1+\frac{1}{x}\right)}$$

$$f'(x) = e^{x\log\left(1+\frac{1}{x}\right)}\left[\log\left(1+\frac{1}{x}\right) + \frac{x}{1+\frac{1}{x}}\left(-\frac{1}{x^2}\right)\right]$$

$$= \left(1+\frac{1}{x}\right)^x \left[\log\left(1+\frac{1}{x}\right)+\frac{\cancel{x^2}}{x+1}\left(-\frac{1}{\cancel{x^2}}\right)\right]$$

$$= \boxed{\left(1+\frac{1}{x}\right)^x \left[\log\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right]}$$

22

$$f(x)=\left(\sqrt{\tan x}\right)^{x+1} = (\tan x)^{\frac{x+1}{2}} = e^{\frac{x+1}{2}\log(\tan x)}$$

$$f'(x)= e^{\frac{x+1}{2}\log(\tan x)}\left(\frac{1}{2}\log(\tan x)+\frac{x+1}{2}\cdot\frac{1}{\tan x}\cdot\frac{1}{\cos^2 x}\right)$$

$$= \boxed{\frac{1}{2}\left(\sqrt{\tan x}\right)^{x+1}\left[\log(\tan x)+\frac{x+1}{\sin x\cos x}\right]}$$

23

$$f(x)=\sqrt{\frac{1-\arcsin x}{1+\arcsin x}}$$

$$f'(x)=\frac{1}{2}\sqrt{\frac{1+\arcsin x}{1-\arcsin x}}\cdot\frac{-\frac{1}{\sqrt{1-x^2}}(1+\arcsin x)-(1-\arcsin x)\left(\frac{1}{\sqrt{1-x^2}}\right)}{(1+\arcsin x)^2}$$

$$=\frac{1}{2}\sqrt{\frac{\cancel{1+\arcsin x}}{1-\arcsin x}}\cdot\frac{\frac{1}{\sqrt{1-x^2}}\left(-1+\cancel{\arcsin x}-1-\cancel{\arcsin x}\right)}{(1+\arcsin x)^{\cancel{2}\ \frac{3}{2}}}$$

$$=\frac{1}{\cancel{2}}\cdot\frac{-\cancel{2}}{\sqrt{(1-\arcsin)(1+\arcsin x)^3(1-x^2)}}$$

$$=\boxed{-\frac{1}{\sqrt{(1-\arcsin)(1+\arcsin x)^3(1-x^2)}}}$$

24 $f(x) = 1 + \dfrac{2^{x+1}}{2^x - 1} = \dfrac{2^x - 1 + 2 \cdot 2^x}{2^x - 1} = \dfrac{3 \cdot 2^x - 1}{2^x - 1}$

$$f'(x) = \frac{3 \cdot 2^x \ln 2 \, (2^x - 1) - (3 \cdot 2^x - 1) \cdot 2^x \ln 2}{(2^x - 1)^2}$$

$$= \frac{\cancel{3 \cdot 4^x \ln 2} - 3 \cdot 2^x \ln 2 - \cancel{3 \cdot 4^x \ln 2} + 2^x \ln 2}{(2^x - 1)^2}$$

$$= \boxed{-2 \, \frac{2^x \ln 2}{(2^x - 1)^2}}$$

ESERCIZIO 10 (Non studio la continuità, scrivo solo l'esito)

1 $f(x) = \log|x-3| = \begin{cases} \log(x-3) & \text{se } x \geq 3 \\ \log(3-x) & \text{se } x < 3 \end{cases}$

$|x-3| > 0 \iff x \neq 3$

$\Rightarrow \text{dom} f = \mathbb{R} \setminus \{3\}$

$\{3\} \notin \text{dom} f \rightarrow$ Non ha senso studiare la derivabilità in $x = 3$.

2

$$f(x) = x^2 e^{\frac{|x|-1}{x}} = \begin{cases} x^2 e^{\frac{x-1}{x}} & \text{se } x>0 \\ x^2 e^{-\frac{x+1}{x}} & \text{se } x<0 \end{cases}$$

DISCONTINUITÀ DI II SPECIE

$$f'(x) = \begin{cases} 2x e^{\frac{x-1}{x}} + \cancel{x^2} e^{\frac{x-1}{x}} \cdot \frac{1}{\cancel{x^2}} = e^{\frac{x-1}{x}}(2x+1) & \text{se } x>0 \\ 2x e^{-\frac{x+1}{x}} + \cancel{x^2} e^{-\frac{x+1}{x}} \cdot \frac{1}{\cancel{x^2}} = e^{-\frac{x+1}{x}}(2x+1) & \text{se } x<0 \end{cases}$$

$$= e^{\frac{|x|-1}{x}}(2x+1)$$

$$f'_+(0) = \lim_{x\to 0^+} e^{\frac{|x|-1}{x}}(2x+1) = 0$$

$$f'_-(0) = \lim_{x\to 0^-} e^{\frac{|x|-1}{x}}(2x+1) = +\infty$$

La funzione NON presenta un punto angoloso in $x=0$. Semplicemente, è discontinua.

3

$$f(x) = \frac{x|\log x|}{(\log x - 1)^2} = \begin{cases} \frac{x\log x}{(\log x-1)^2} & \text{se } x \geq 1 \\ -\frac{x\log x}{(\log x-1)^2} & \text{se } x<1 \end{cases}$$

CONTINUA

$$\frac{d}{dx}\left(\frac{x\log x}{(\log x-1)^2}\right) = \frac{(\log x+1)(\log x-1)^{\cancel{2}} - 2\cancel{(\log x-1)}\left(\frac{1}{x}\right)(x\log x)}{(\log x-1)^{\cancel{4}\,3}}$$

$$= \frac{\log^2 x - 1 - 2\log x}{(\log x-1)^3}$$

$$f'(x) = \begin{cases} \frac{\log^2 x - 1 - 2\log x}{(\log x-1)^3} & \text{se } x>1 \\ -\frac{\log^2 x - 1 - 2\log x}{(\log x-1)^3} & \text{se } x<1 \end{cases}$$

$$f'_+(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{\log^2 x - 1 - 2\log x}{(\log x - 1)^3} = \frac{-1}{(-1)^3} = 1$$

$$f'_-(x) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(-\frac{\log^2 x - 1 - 2\log x}{(\log x - 1)^3}\right) = \frac{1}{(-1)^3} = -1$$

La funzione presenta un punto angoloso in $x=1$.

4

$$f(x) = \frac{1}{\sqrt{x + |x^2 - x|}} = \begin{cases} \dfrac{1}{\sqrt{x^2}} = \dfrac{1}{|x|} & \text{se } x \le 0 \vee x \ge 1 \\[2ex] \dfrac{1}{\sqrt{2x - x^2}} & \text{se } 0 < x < 1 \end{cases}$$

$$= \begin{cases} \dfrac{1}{x} & \text{se } x \ge 1 \\[2ex] \dfrac{1}{\sqrt{2x - x^2}} & \text{se } 0 < x < 1 \\[2ex] -\dfrac{1}{x} & \text{se } x \le 0 \end{cases}$$

CONTINUA

$\{0\} \notin \mathrm{dom} f$

limiti a $\pm\infty$

$$f'(x) = \begin{cases} -\dfrac{1}{x^2} & \text{se } x > 1 \\[2ex] -\dfrac{1}{\cancel{2}} \cdot \dfrac{\cancel{2} - \cancel{2}x}{\sqrt{(2x - x^2)^3}} = \dfrac{x - 1}{\sqrt{(2x - x^2)^3}} & \text{se } 0 < x < 1 \\[2ex] \dfrac{1}{x^2} & \text{se } x < 0 \end{cases}$$

$$f'_-(1) = \lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} \frac{x-1}{\sqrt{(2x-x^2)^3}} = 0$$

$$f'_+(1) = \lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} \left(-\frac{1}{x^2}\right) = -1$$

La funzione presenta un punto angoloso in $x = 1$.

5

$$f(x) = e^{\sin x}|\sin x| = \begin{cases} \sin x \cdot e^{\sin x} & \text{se } 2k\pi < x < \pi + 2k\pi \\ -\sin x \cdot e^{\sin x} & \text{se } \pi + 2k\pi < x < 2\pi + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

CONTINUA

$$f'(x) = \begin{cases} \cos x\, e^{\sin x} + \sin x\, e^{\sin x} \cdot \cos x & \text{se } 2k\pi < x < \pi + 2k\pi \\ -(\cos x\, e^{\sin x} + \sin x\, e^{\sin x} \cdot \cos x) & \text{se } \pi + 2k\pi < x < 2\pi + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

$$= \begin{cases} \cos x\, e^{\sin x}(\sin x + 1) & \text{se } 2k\pi < x < \pi + 2k\pi \\ -\cos x\, e^{\sin x}(\sin x + 1) & \text{se } \pi + 2k\pi < x < 2\pi + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

$$f'_+(\pi + 2k\pi) = \lim_{x\to(\pi+2k\pi)^+} \left(-\cos x\, e^{\sin x}(\sin x + 1)\right) = 1$$

$$f'_-(\pi + 2k\pi) = \lim_{x\to(\pi+2k\pi)^-} \cos x\, e^{\sin x}(\sin x + 1) = -1$$

$$f'_+(2k\pi) = \lim_{x\to(2k\pi)^+} \cos x\, e^{\sin x}(\sin x + 1) = 1$$

$$f'_-(2k\pi) = \lim_{x\to(2k\pi)^-} \left(-\cos x\, e^{\sin x}(\sin x + 1)\right) = -1$$

La funzione presenta un punto angoloso per $x = k\pi \ \forall k \in \mathbb{Z}$.