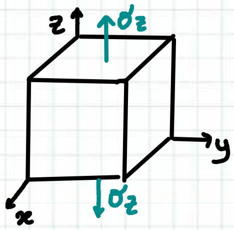
ESEMPI

• Stato di tensione MONOASSIALE



$$\begin{bmatrix} 0 & & \\ & 0 & \\ & & 1 \end{bmatrix} = [\sigma]$$

$\sigma_z = 1\ \text{MPa}$

$E = 10\ \text{GPa}$

$\nu = 0.3$

mat. el. lin. iso. omog.

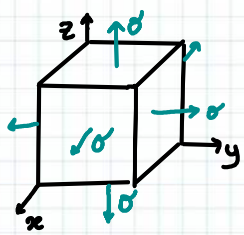
Det. lo stato deformativo

$$\varepsilon_x = \varepsilon_y = -\frac{\nu}{E}\sigma_z = \frac{-0.3}{10^4}\cdot 1 = -0.3\cdot 10^{-4}$$

$$\varepsilon_z = \frac{\sigma_z}{E} = \frac{1}{10^4} = 1\cdot 10^{-4}$$

$$[\varepsilon] = 10^{-4}\cdot \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & -0.3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Stato di tensione triassiale



$$[\sigma] = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

$\sigma = 1\ \text{MPa}$

$E = 10\ \text{GPa}$

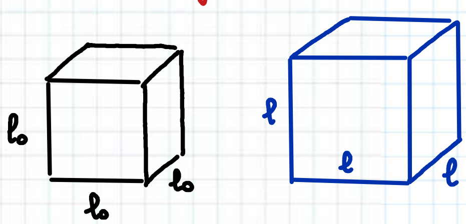
$\nu = 0.3$

$$[\varepsilon] = 10^{-4}\cdot \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}$$

$$\rightarrow \varepsilon_x = \frac{\sigma}{E} - \frac{\nu}{E}\sigma - \frac{\nu}{E}\sigma = \frac{\sigma}{E}(1-2\nu) = \frac{1}{10^4}(0.4) = 0.4\cdot 10^{-4} = \varepsilon_y = \varepsilon_z$$

→ espansione di volume a def costante in x, y, z. $\varepsilon_v = I_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z = 1.2\cdot 10^{-4}$

• stato di deformazione triassiale



$l_0 = 10$ mm

$l = 10.001$ mm

$\nu = 0.3$

$E = 10\ \text{GPa}$

$$\Rightarrow \varepsilon_x = \varepsilon_y = \varepsilon_z = \frac{\Delta l}{l_0} = \frac{0.001}{10} = 1\cdot 10^{-4}$$

$$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$$

$$[\varepsilon] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot 10^{-4}$$

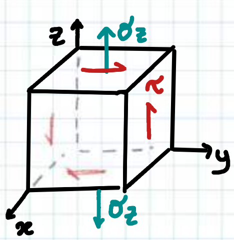
Determinare lo stato di tensione:

$$\sigma_x = \sigma_y = \sigma_z = \sigma = \frac{E}{1+\nu}\varepsilon + \frac{\nu E}{(1+\nu)(1-2\nu)}(3\varepsilon) = 2.5\ \text{MPa}$$

le τ sono nulle

$$[\sigma] = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}$$

• Stato di tensione generico



$\sigma_z = 1\ \text{MPa}$

$\tau_{zy} = 2\ \text{MPa}$

$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$$G = \frac{E}{2(1+\nu)} = \frac{10}{2(1.3)} = 3.85\ \text{GPa}$$

Poiché $\sigma \rightarrow \varepsilon$ e $\tau \rightarrow \gamma$ SOVRAPPOSIZIONE degli EFFETTI

$$\varepsilon_x = -\frac{\nu}{E}\sigma_z = -\frac{0.3}{10^4}\cdot 1\ \text{MPa} = -0.3\cdot 10^{4}\ \text{MPa} = \varepsilon_y$$

$$\varepsilon_z = \frac{\sigma_z}{E} = 10^{-4}\ \text{MPa}$$

$$\gamma_{zy} = \frac{\tau}{G} = \frac{2}{3.85\cdot 10^3} = 5.19\cdot 10^{-4}$$

$$[\varepsilon] = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & -0.3 & \frac{5.19}{2} \\ 0 & \frac{5.19}{2} & 1 \end{bmatrix}\cdot 10^{-4}$$