Lezione 24 - 16/05/2024

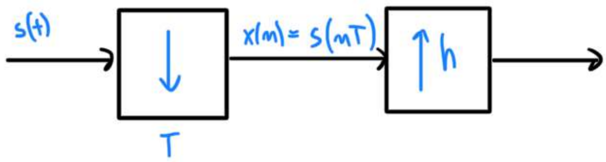
Facciamo qualche esercizio sul teorema del campionamento (slide 120)

Es 1

Proporre uno schema di ricostruzione del segnale $s(t) = \text{sinc}^3(t)$ dai suoi campioni

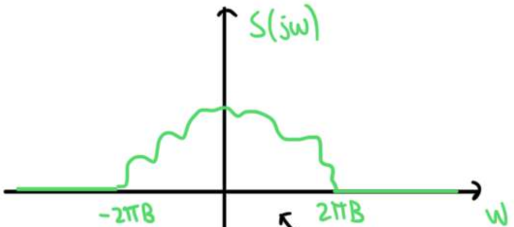
$s(t) = \text{sinc}^3(t)$

Sol.



$$y(t) = \sum_{n=-\infty}^{+\infty} x(n)\, h(t - nT)$$

↳ devo scegliere T tale che $\frac{2\pi}{T} > 2\pi \cdot 2B$



$\Rightarrow T < \frac{1}{2B}$

estensione trasformata di Fourier

una ricostruzione corretta assicura $y(t) = s(t)$

$$\boxed{s(t) = \sum_{n=-\infty}^{+\infty} s(nT)\, \text{sinc}\left(\frac{t - nT}{T}\right)}$$

FORMULA DI RICOSTRUZIONE

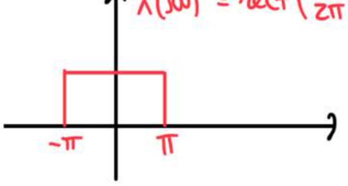
$$\Rightarrow \text{sinc}^3(t) = \sum_{n=-\infty}^{+\infty} \text{sinc}^3(nT)\, \text{sinc}\left(\frac{t - nT}{T}\right)$$

$$s(t) = \text{sinc}^3(t) = \underbrace{\text{sinc}(t)}_{x(t)}\, \underbrace{\text{sinc}^2(t)}_{y(t)}$$

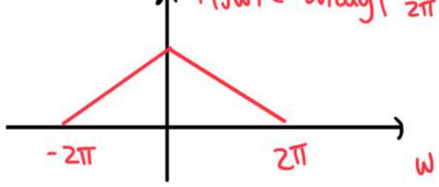
nel tempo é un prodotto, in pulsazione é una convoluzione

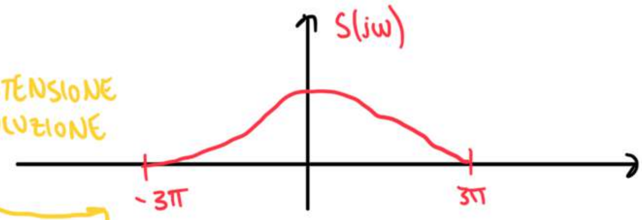
$$S(jw) = \frac{1}{2\pi} X * Y(jw)$$

$X(jw) = \text{rect}\left(\frac{w}{2\pi}\right)$



$Y(jw) = \text{triang}\left(\frac{w}{2\pi}\right)$





REGOLA ESTENSIONE CONVOLUZIONE

$3\pi \leq \frac{\pi}{T} \longrightarrow T \leq \frac{1}{3}$

$3\pi = 2\pi \cdot B \longrightarrow B = \frac{3\pi}{2\pi} = \frac{3}{2} \longrightarrow T \le \frac{1}{2B} = \frac{1}{2 \cdot \frac{3}{2}} = \frac{1}{3}$

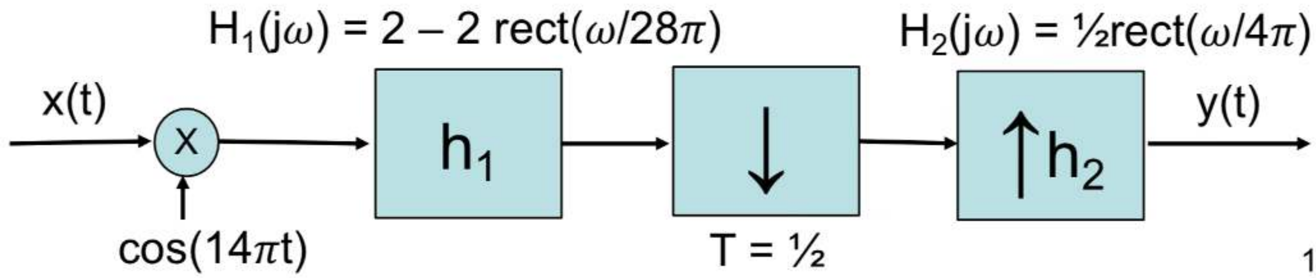
QUALUNQUE $T \le \frac{1}{3}$ FUNZIONA

NELLA PRATICA SI SCEGLIE T PIÙ GRANDE POSSIBILE, CIOÈ $T = \frac{1}{3}$

X CASA: provo con sinc^4 (a maso: $T \le \frac{1}{4}$)
sinc^5 ($T \le \frac{1}{5}$)

Es 3

Calcolare l'uscita con $X(j\omega) = [1 - \text{triang}(\omega/2\pi)]\ \text{rect}(\omega/4\pi)$



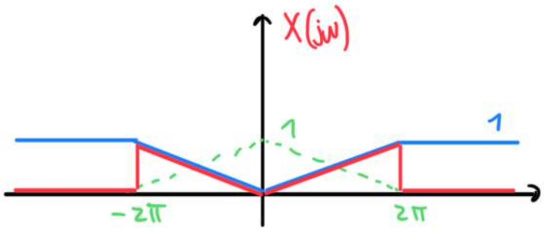
$$X(j\omega) = \text{rect}\left(\frac{\omega}{4\pi}\right)\left(1 - \text{triang}\left(\frac{\omega}{2\pi}\right)\right)$$

$$H_1(j\omega) = 2 - 2\,\text{rect}\left(\frac{\omega}{28\pi}\right)$$

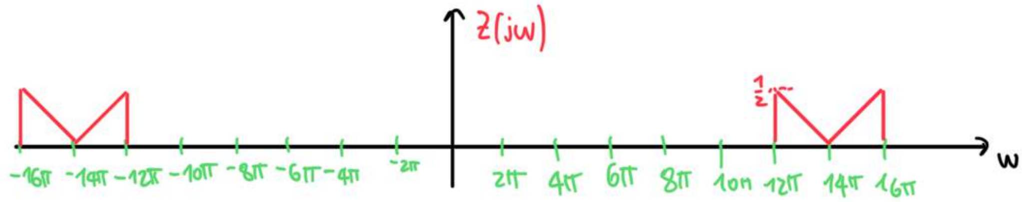
$$H_2(j\omega) = \frac{1}{2}\text{rect}\left(\frac{\omega}{4\pi}\right)$$

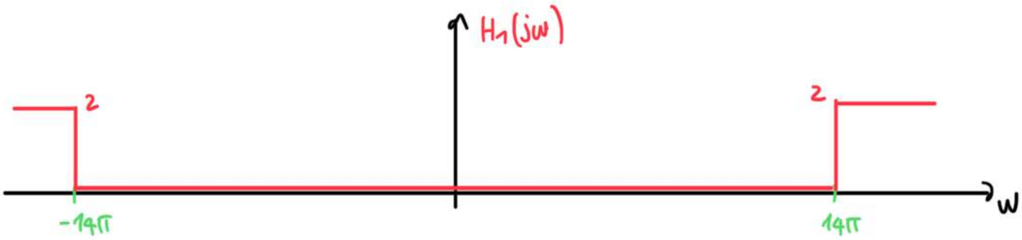
$$y(t) = ?$$

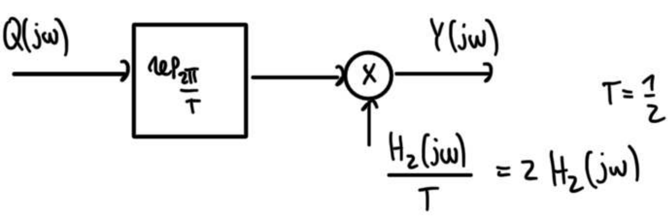
SOL. LAVORIAMO NEL DOMINIO DI FOURIER



$$Z(j\omega) = \frac{1}{2}X(j(\omega-\omega_0)) + \frac{1}{2}X(j(\omega-\omega_0)) \qquad \text{con } \omega_0 = 14\pi$$







(Finiamo domani)