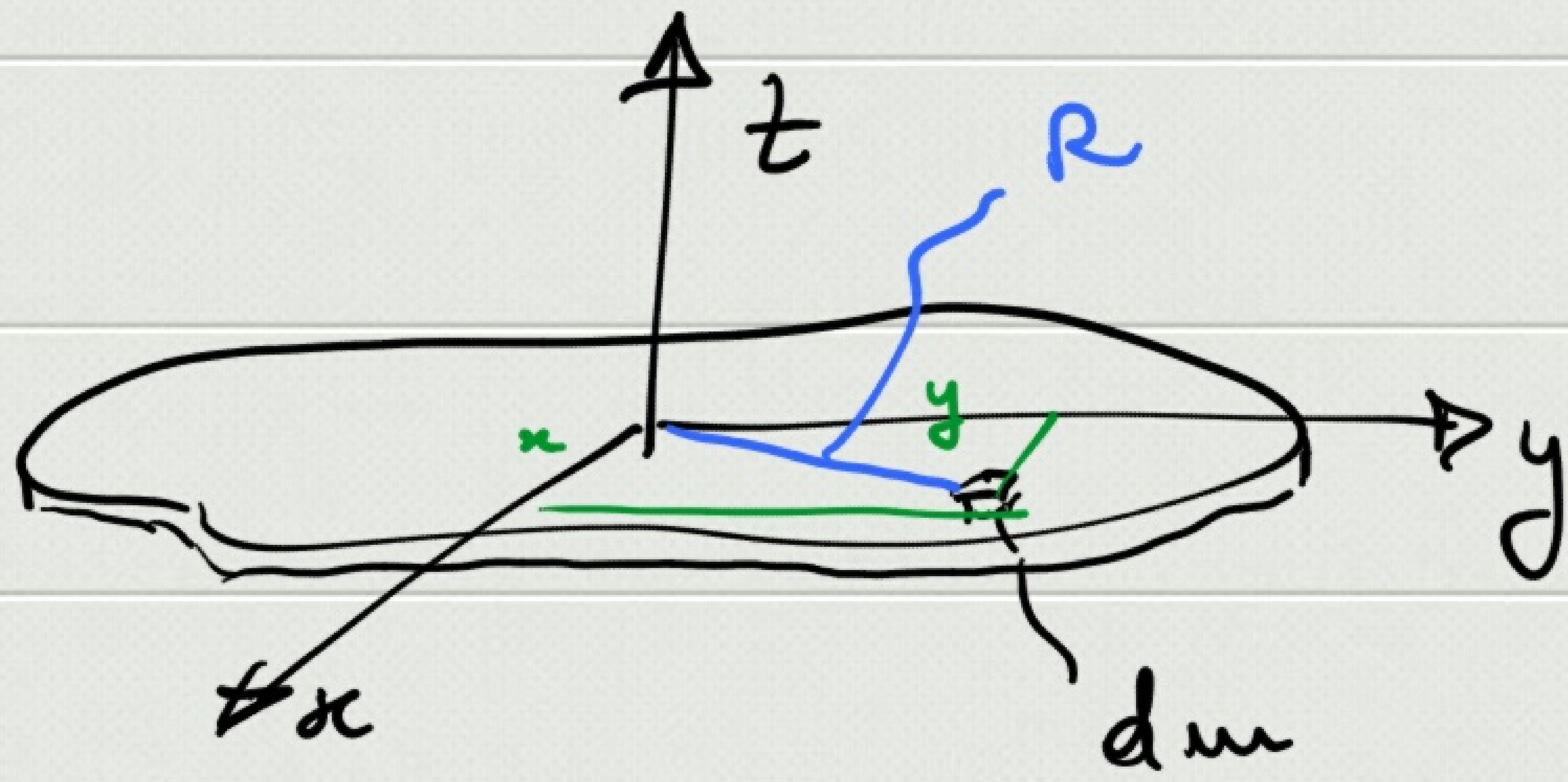



Figure piane

$$\underline{I} = \int R^2 \, dm$$

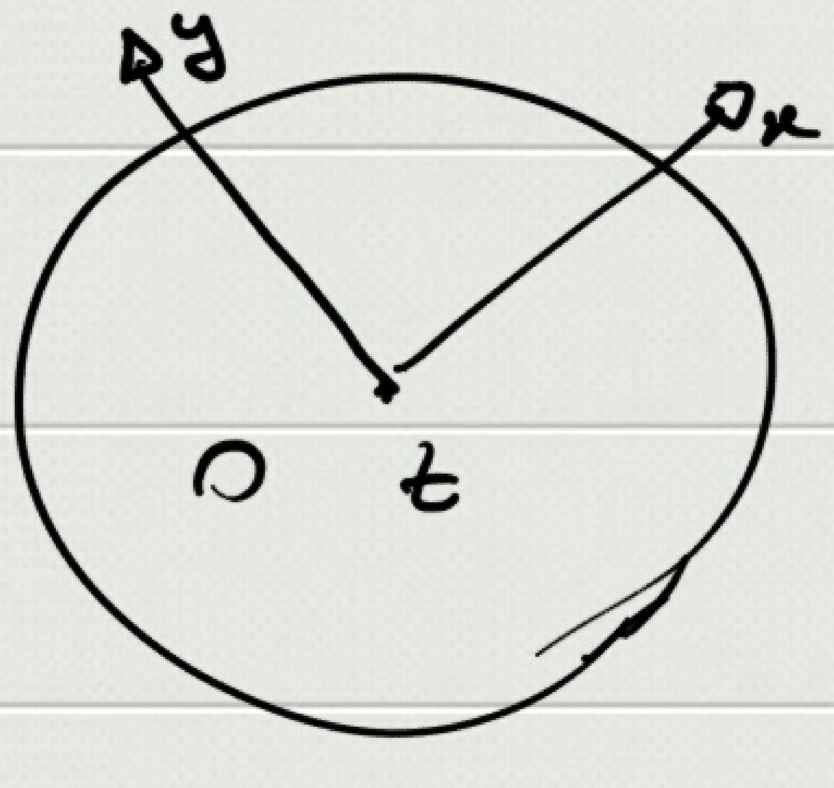
$$I_z = \int (x^2 + y^2) \, dm$$

$$I_x = \int R_x^2 \, dm = \int (y^2 + \underset{\underset{0}{\parallel}}{z^2}) \, dm = \int y^2 \, dm$$

$$I_y = \int R_y^2 \, dm = \int (x^2 + \underset{\underset{0}{\parallel}}{z^2}) \, dm = \int x^2 \, dm$$

$$\boxed{I_z = I_x + I_y}$$

Teorema delle figure piane



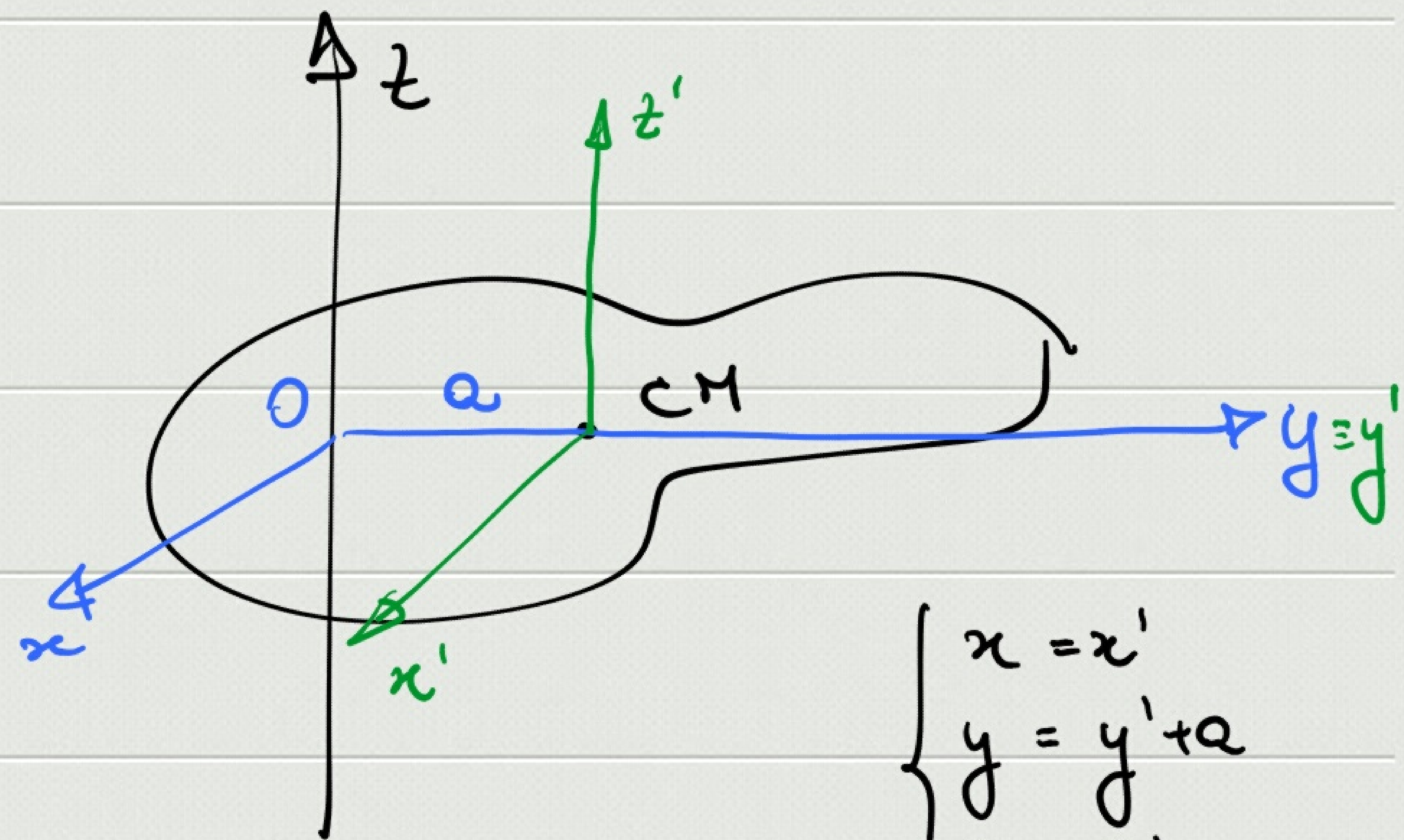

Anello$(m, R)$ $\quad I_z = mR^2$

$$I_x = I_y$$

$$\Rightarrow I_z = I_x + I_y = 2 I_x$$

$$\Rightarrow I_x = \frac{1}{2} I_z = \frac{1}{2} m R^2$$

$I_z = ?$




$$\begin{cases} x = x' \\ y = y' + a \\ z = z' \end{cases}$$

$$I_z = \sum_i m_i R_i^2 = \sum_i m_i (x_i^2 + y_i^2) =$$

$$= \sum_i m_i \left[x_i'^2 + (y_i' + a)^2\right] =$$

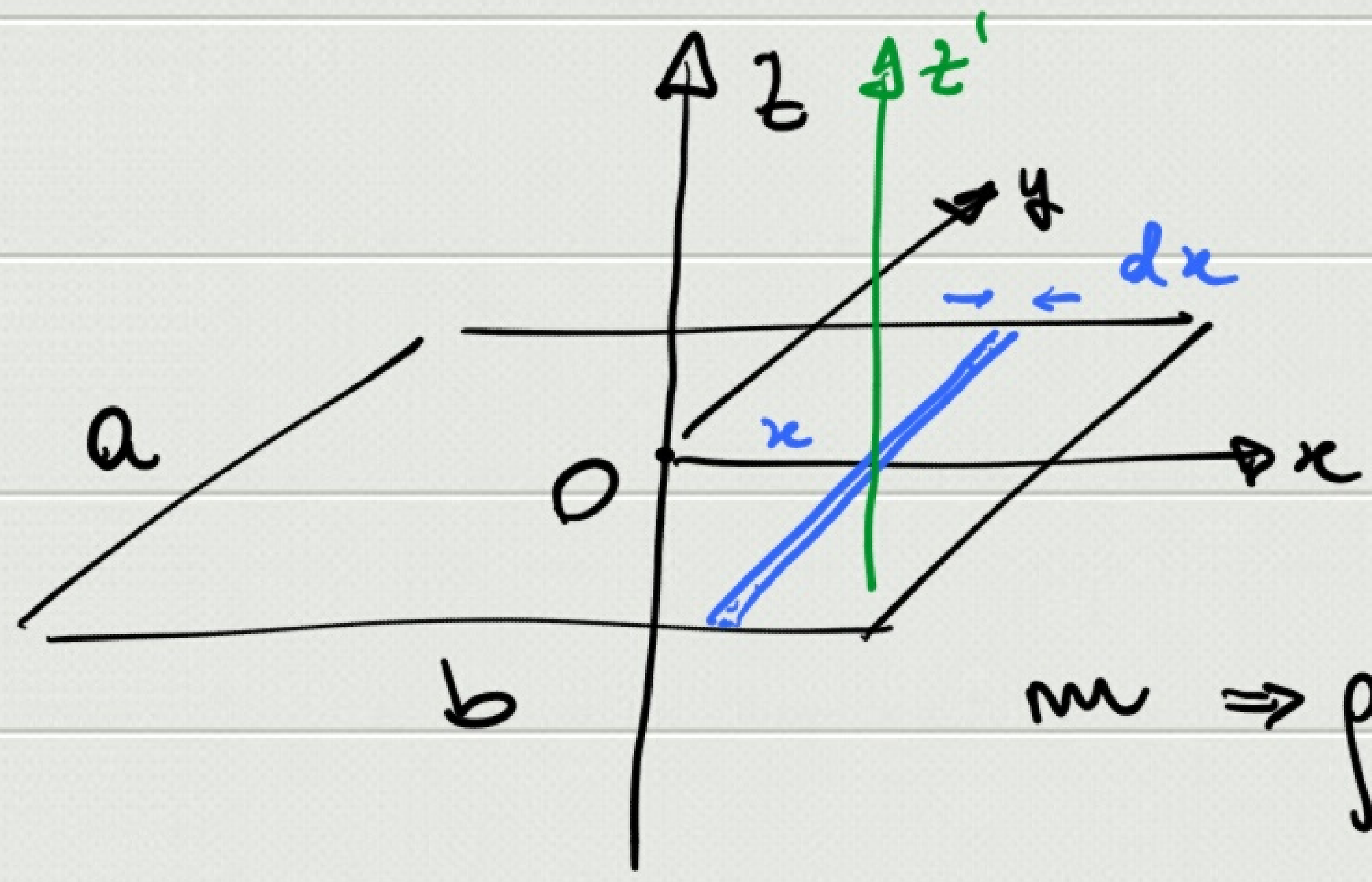
$$= \sum_i m_i (x_i'^2 + y_i'^2 + a^2 + 2a y_i') =$$

$$= \sum_i m_i \underbrace{(x_i'^2 + y_i'^2)}_{R_i'^2} + \sum_i m_i a^2 + \cancel{2a \underbrace{\sum_i m_i y_i'}_{y'_{CM} \cdot m \to = 0}} =$$

$$= I_{z'} + m a^2$$

$$\boxed{I_z = I_{z'} + m a^2}$$

Teorema di Huygens Steiner




$$I_z = \int R^2 \, dm$$

$$m \Rightarrow \rho_S = \frac{m}{ab}$$

$$I_z = \int dI_z \qquad \left[ I_{sbarra} = \frac{1}{12} m d^2 \right]$$

$$I_{z, lastra} = \int dI_{z, sbarra} \overset{H-S}{\stackrel{!}{=}} \int \left( dI_{z', sbarra} + x^2 dm \right) =$$

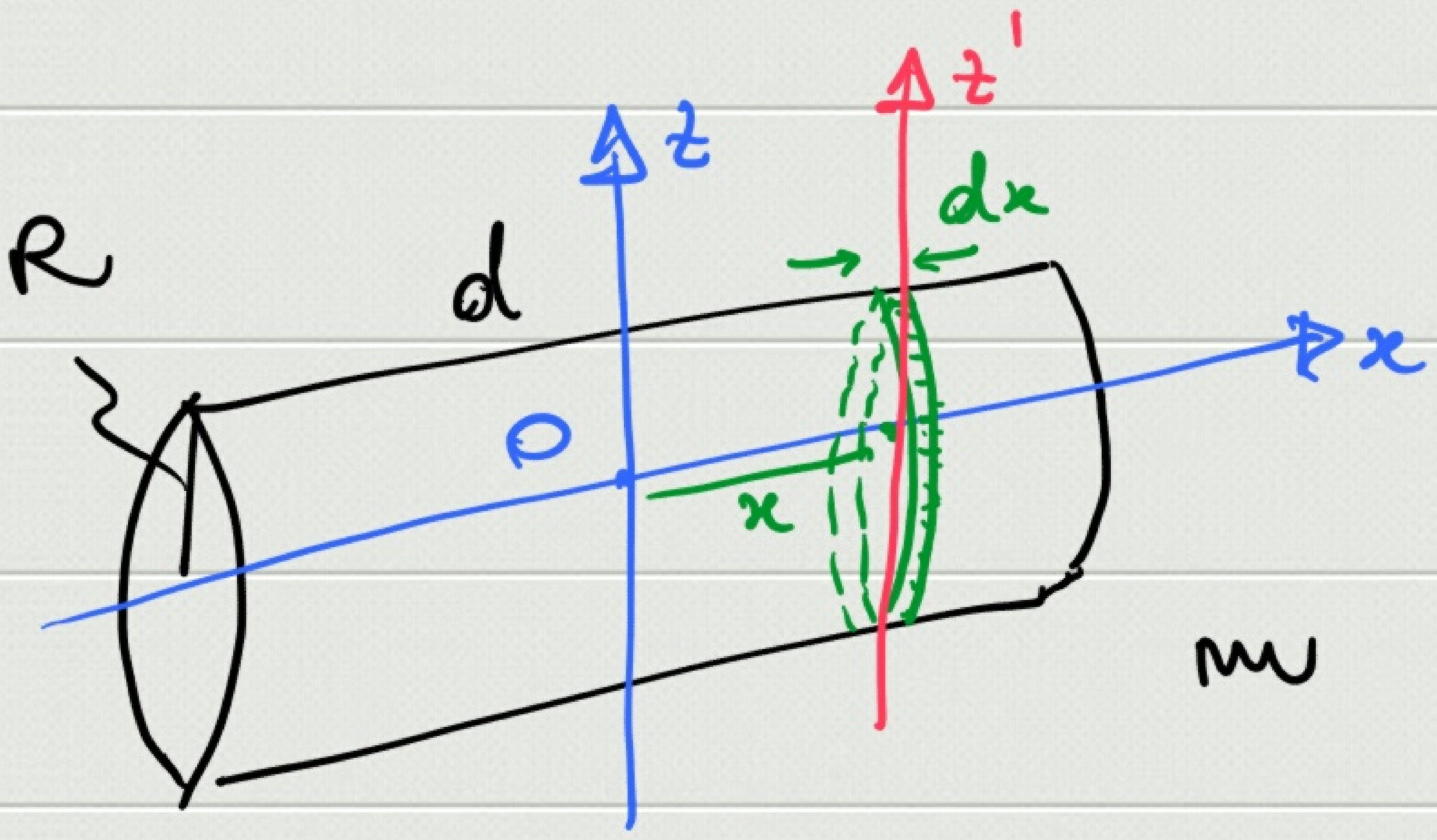
$$= \int \left( \frac{1}{12} dm\, a^2 + x^2 \rho_S \, dS \right) =$$

$$= \int \frac{1}{12} a^2 dm + \int_{-b/2}^{b/2} x^2 \frac{m}{\cancel{a} b} \cancel{a} \, dx =$$

$$= \frac{1}{12} m a^2 + \frac{m}{b} \left[ \frac{x^3}{3} \right]_{-b/2}^{b/2} =$$

$$= \frac{1}{12} m a^2 + \frac{m}{3b} \left( \frac{b^3}{8} + \frac{b^3}{8} \right) =$$

$$= \frac{1}{12} m a^2 + \frac{m}{3\cancel{b}} \frac{b^{\cancel{3}2}}{4} = \frac{1}{12} m \left( a^2 + b^2 \right)$$




guscio sottile

$$I_z = \int dI_{z\,\text{anello}} \overset{H-S}{=} \int \left( dI_{z'} + x^2\, dm \right) =$$

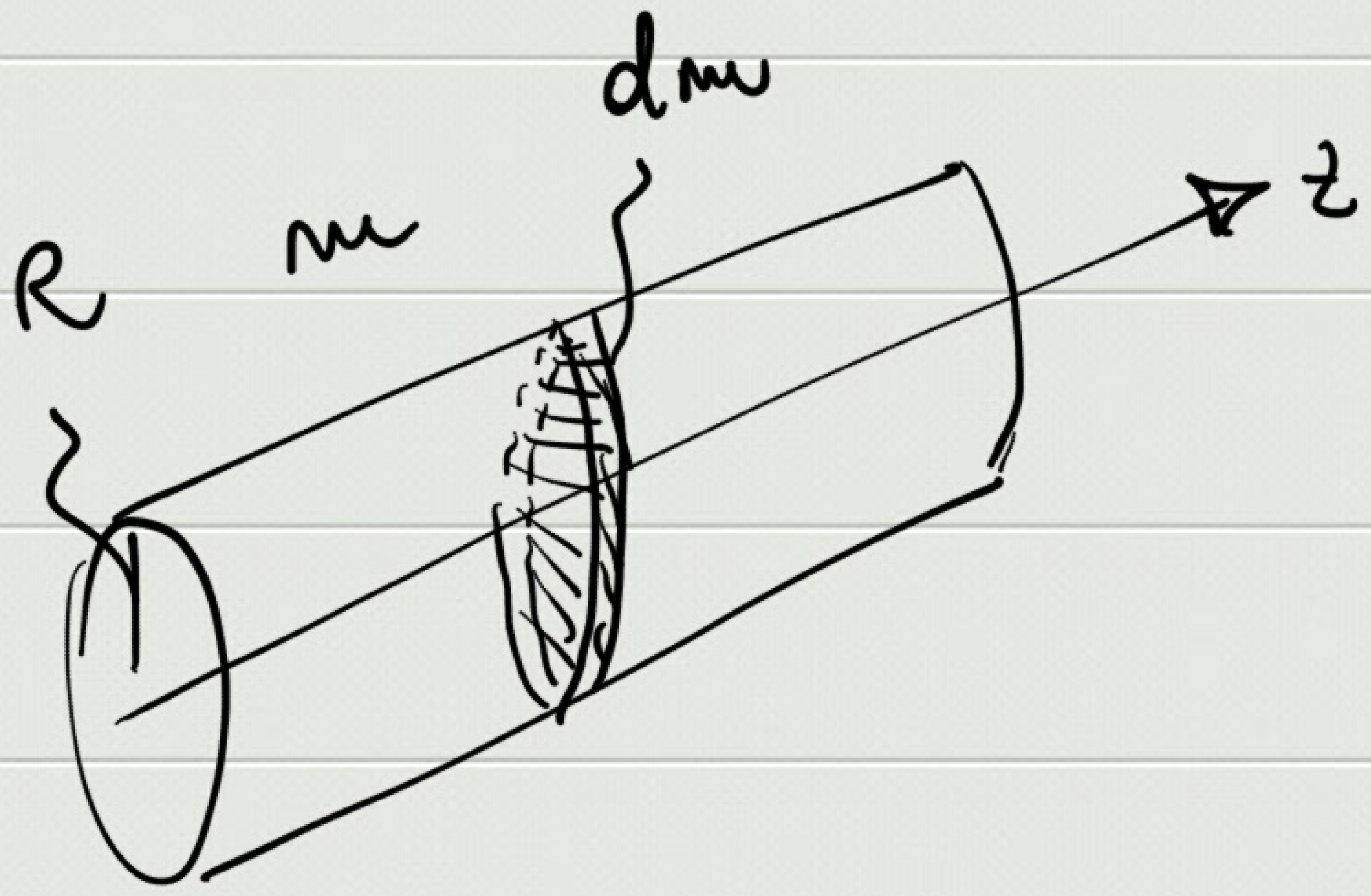
$$= \int \left( \frac{1}{2}\, dm\, R^2 + x^2\, dm \right) = \frac{1}{2} R^2 m + \int x^2\, dm =$$

$$dm = \rho_S\, dS = \frac{m}{\cancel{2\pi R}\, d} \cancel{2\pi R}\, dx = \frac{m}{d}\, dx$$

$$= \frac{1}{2} m R^2 + \int_{-d/2}^{+d/2} x^2 \frac{m}{d}\, dx =$$

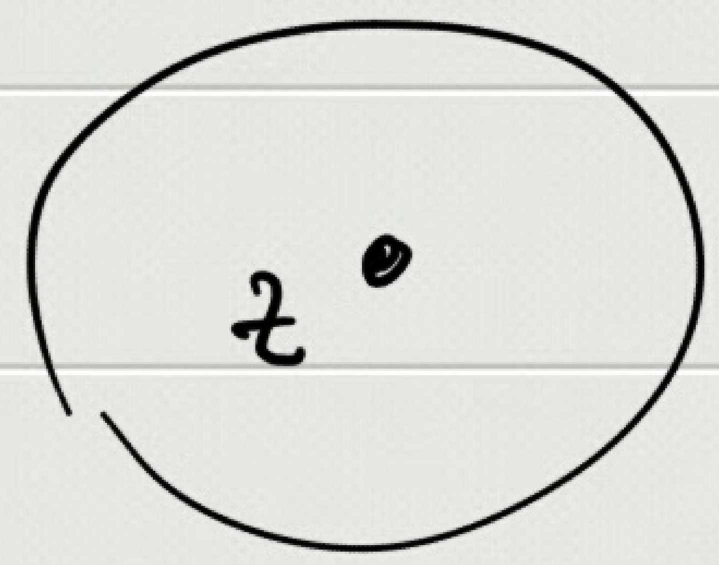
$$= \frac{1}{2} m R^2 + \frac{m}{d} \left[ \frac{x^3}{3} \right]_{d/2}^{d/2} =$$

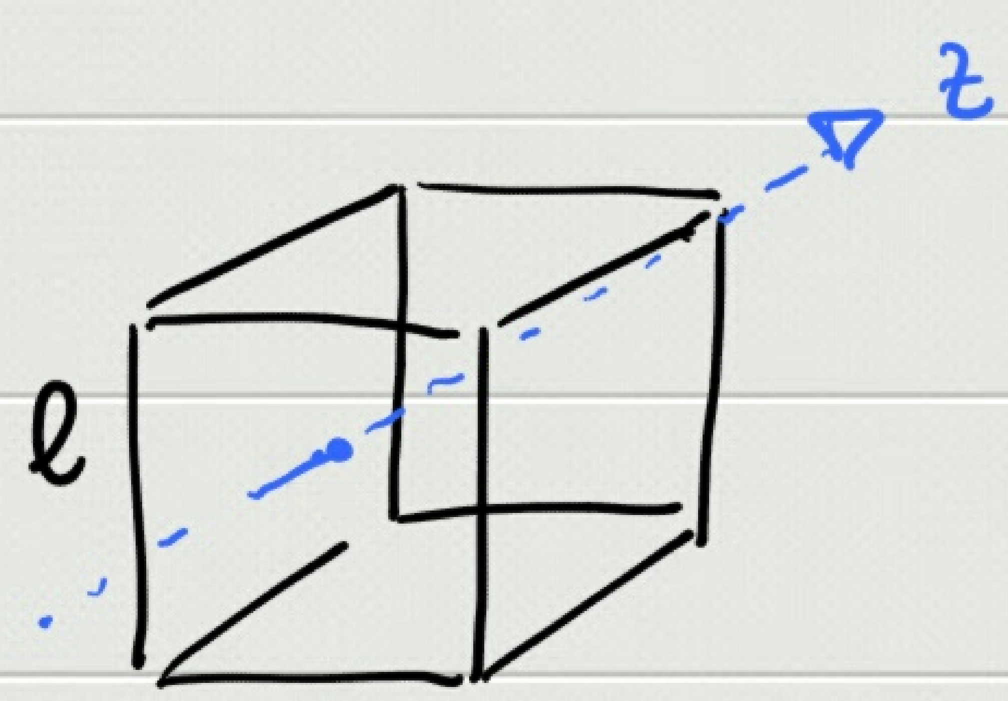
$$= \frac{1}{2} m R^2 + \frac{m}{3\cancel{d}} \frac{d^{\cancel{3}\,2}}{4} = \frac{1}{2} m R^2 + \frac{1}{12} m d^2$$

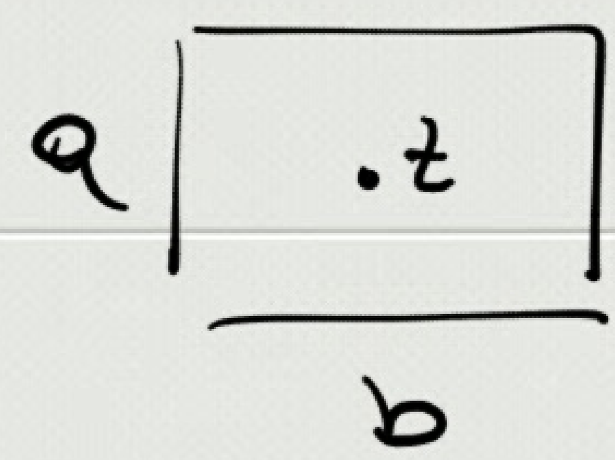



cilindro pieno

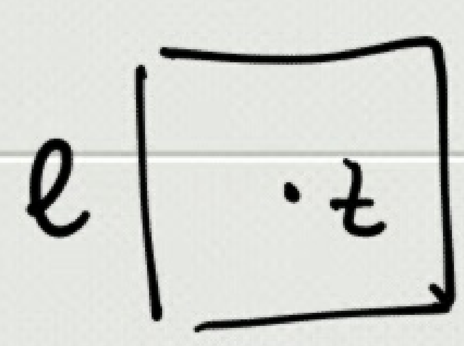
$$\underset{\text{cilindro}}{I_z} = \int \underset{\substack{\downarrow \\ \text{disco}}}{dI_z} = \int \frac{1}{2} dm\, R^2 = \frac{1}{2} R^2 \int dm = \frac{1}{2} m R^2$$




$$I_z = \frac{1}{12} m (a^2 + b^2)$$




$$I_z = \frac{1}{12} m (l^2 + l^2) = \frac{1}{6} m l^2$$

$$\boxed{I_{z,\text{cubo}} = \frac{1}{6} m l^2}$$