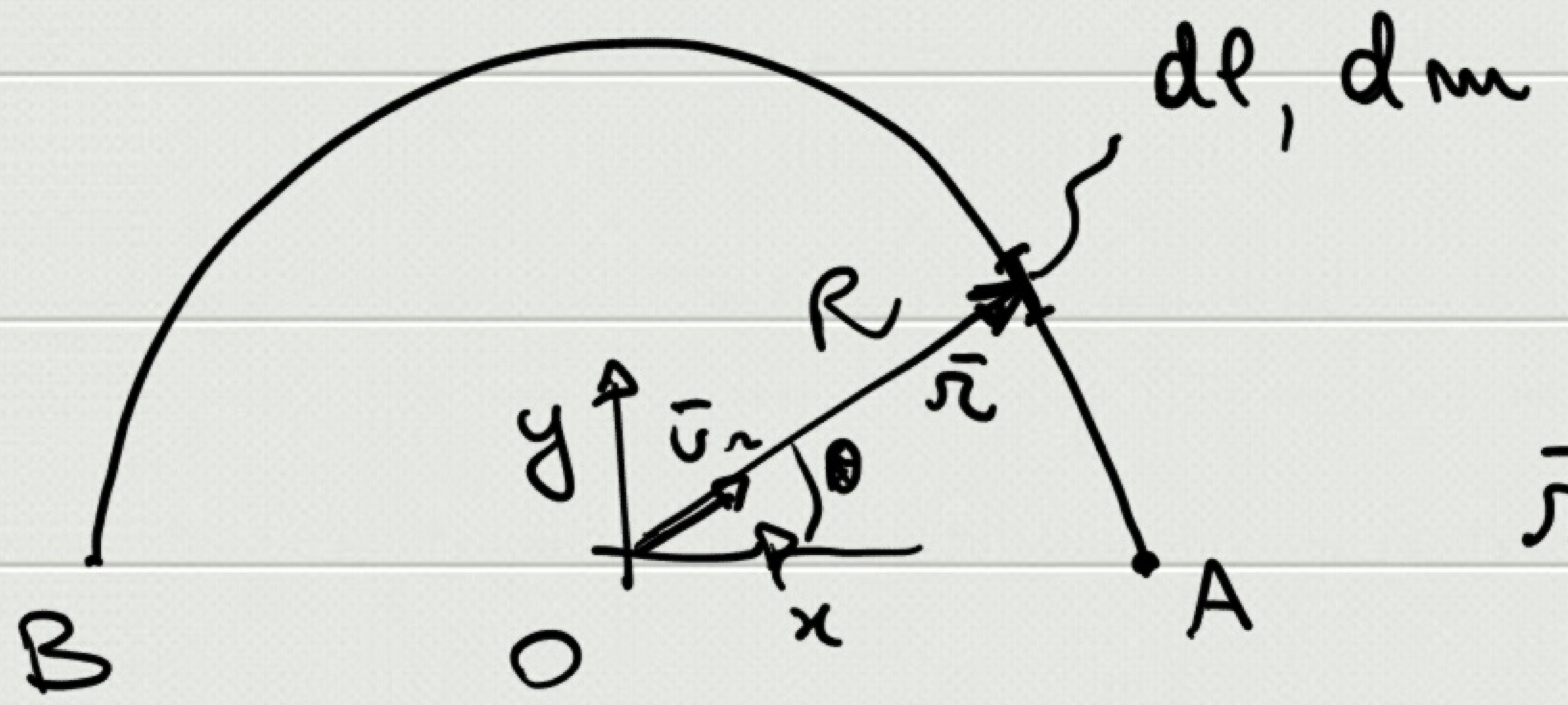


m, R, omogeneo

$\bar{r}_{cm} = ?$

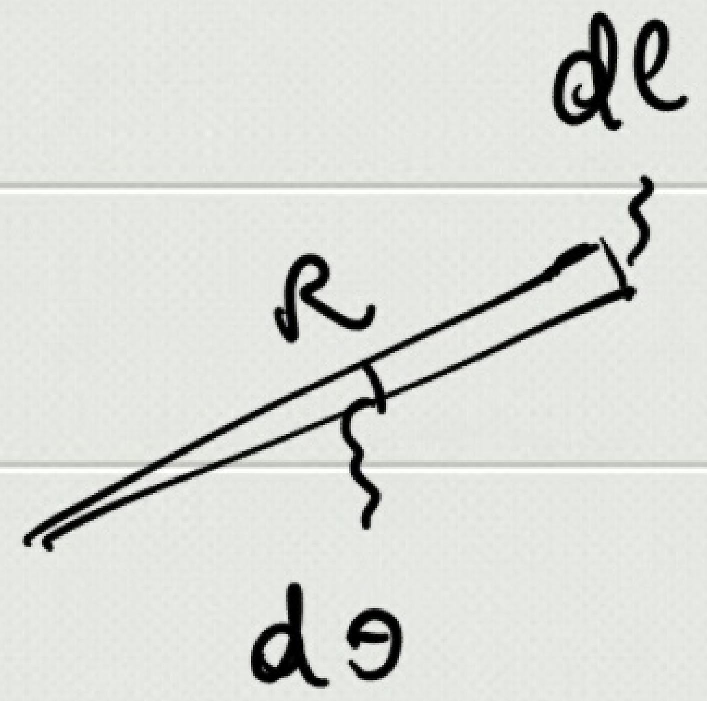
$$\rho_l = \frac{dm}{dl} \qquad \bar{r}_{cm} = \frac{1}{m}\int_{corpo} \bar{r}\, dm$$

$$dm = \rho_l\, dl \qquad \rho_l = cost = \frac{m}{\pi R}$$

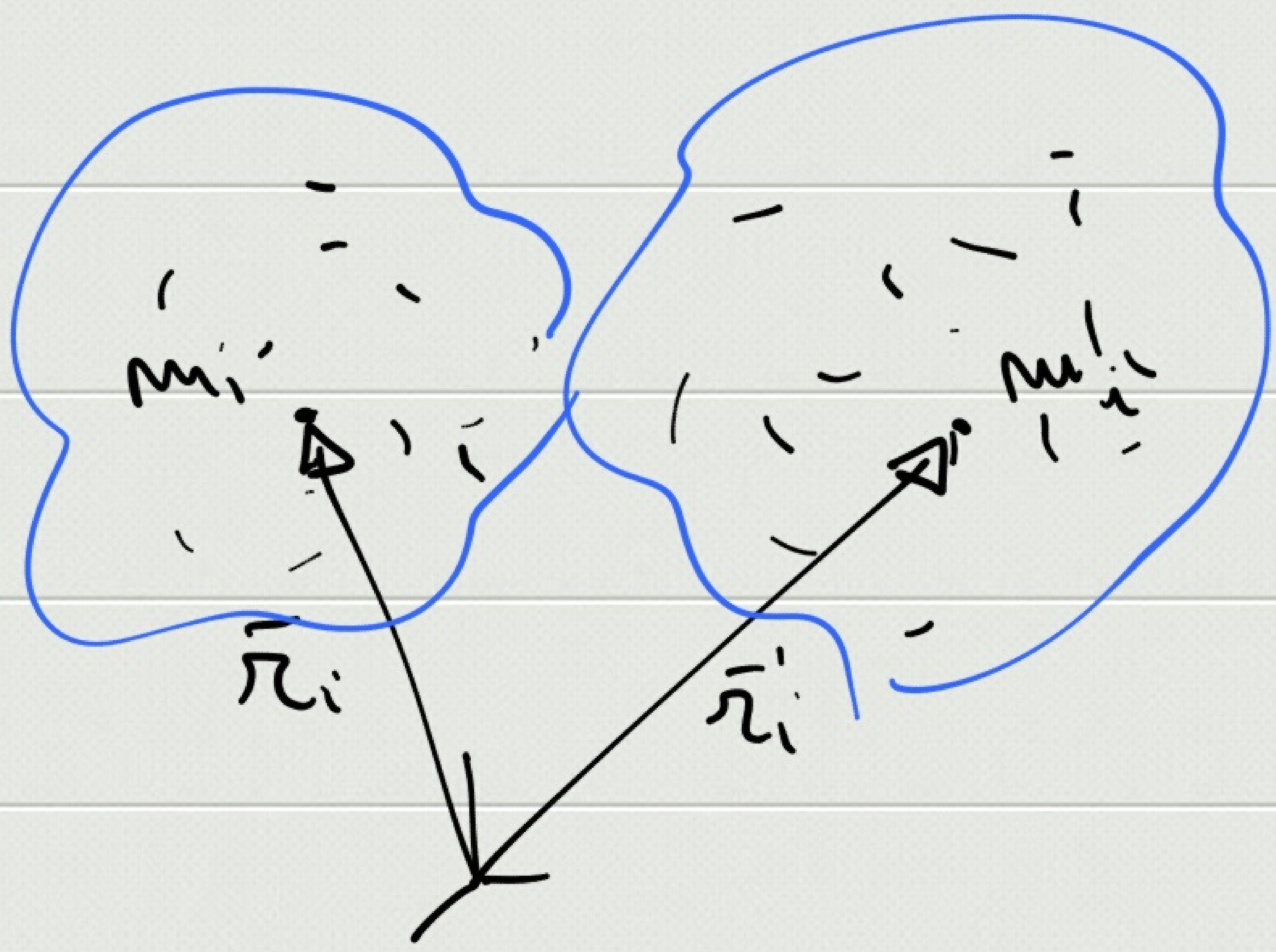
$$\Rightarrow \bar{r}_{cm} = \frac{1}{\cancel{m}}\int_{length} \bar{r}\,\frac{\cancel{m}}{\pi R}\,dl = \frac{1}{\pi \cancel{R}}\int_{length} \cancel{R}\,\bar{u}_r\, dl =$$

$$= \frac{1}{\pi}\int_0^{\pi}\left(\cos\theta\,\bar{u}_x + \sin\theta\,\bar{u}_y\right) R\, d\theta =$$

$$= \frac{R}{\pi}\left[\sin\theta\,\bar{u}_x - \cos\theta\,\bar{u}_y\right]_0^{\pi} =$$



$$= \frac{2R}{\pi}\,\bar{u}_y$$



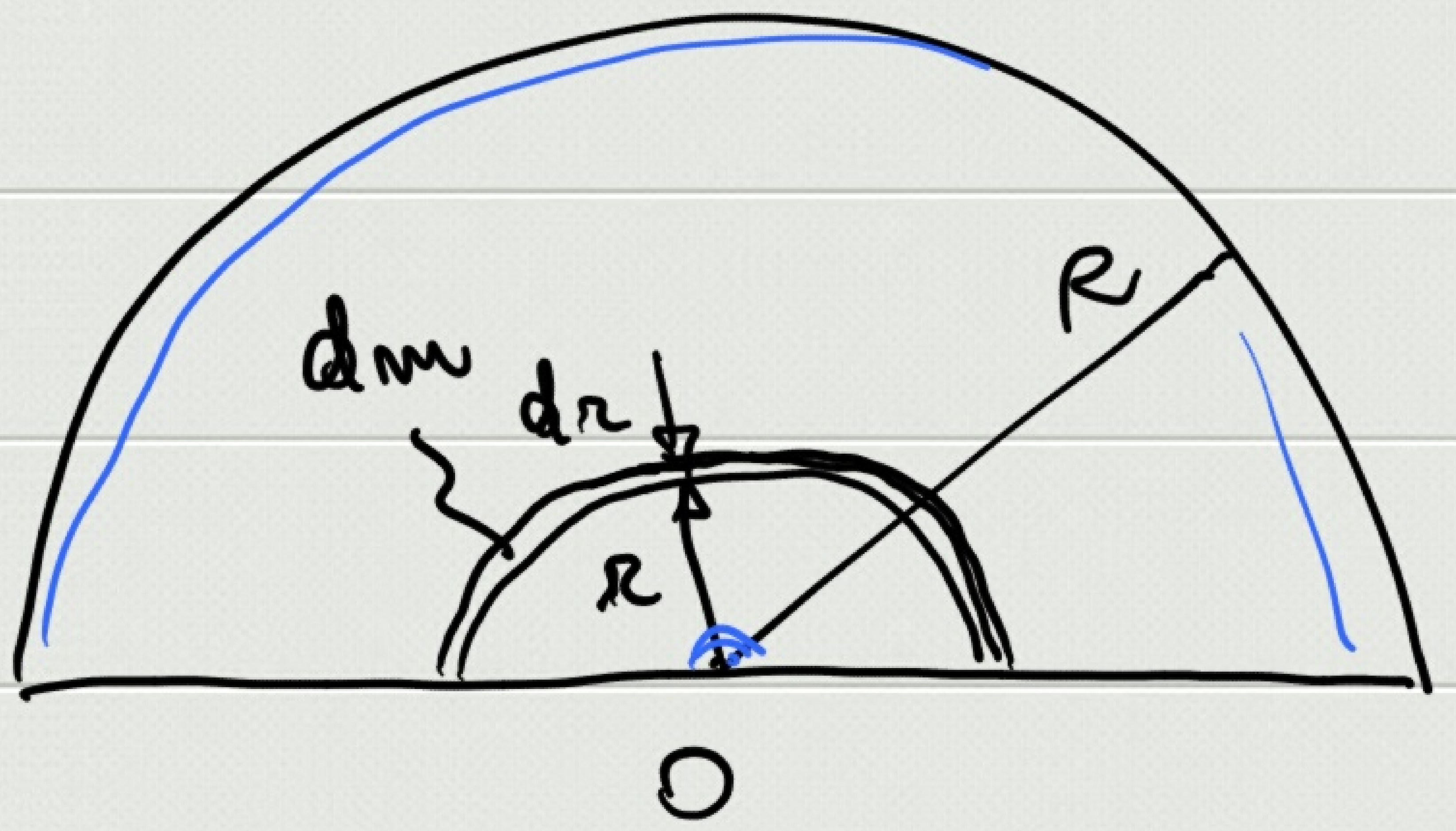
$$\bar{r}_{cm} = \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i} \qquad \bar{r}'_{cm} = \frac{\sum_i m'_i \bar{r}'_i}{\sum_i m'_i}$$

$$\bar{R}_{cm} = \frac{\sum_i \left(m_i \bar{r}_i + m'_i r'_i\right)}{\sum_i \left(m_i + m'_i\right)} =$$

$$= \frac{\sum_i m_i \bar{r}_i + \sum_i m'_i \bar{r}'_i}{\sum_i m_i + \sum_i m'_i \bar{r}'_i} =$$

$$= \frac{m_{TOT} \bar{r}_{cm} + m'_{TOT} \bar{r}'_{cm}}{m_{TOT} + m'_{TOT}} =$$

$$= \frac{\sum_j m_{TOT,j} \bar{r}_{cm,j}}{\sum_j m_{TOT,j}}$$



Semidisco

R, m, omogeneo

$\bar r_{cm} = ?$

$$\bar r_{cm,\cap} = \frac{2r}{\pi}\,\bar u_y$$

$$\bar r_{cm} = \frac{\int \bar r_{cm,\cap}\, dm_\cap}{\int dm_\cap} \qquad (*)$$

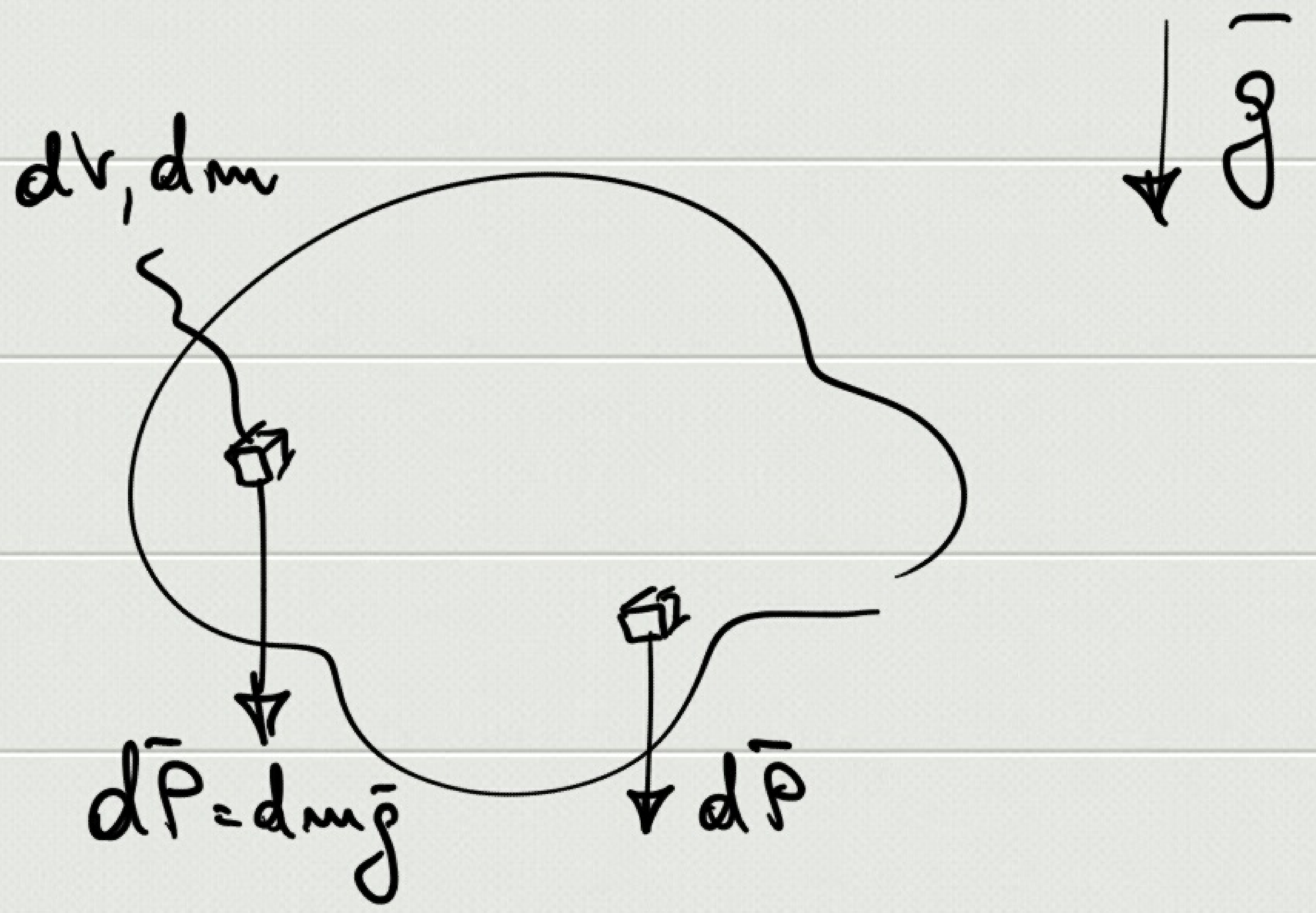
$$\rho_s = \frac{2m}{\pi R^2} \stackrel{!}{=} \frac{dm}{dS} \Rightarrow dm_\cap = \rho_s\, dS = \frac{2m}{\pi R^2}\, dS =$$

$$dS = \pi r\, dr \Rightarrow = \frac{2m}{\cancel{\pi} R^2}\cancel{\pi}\, r\, dr$$

$$(*) \quad \bar r_{cm} = \frac{1}{\cancel{m}} \int_0^R \frac{2r}{\pi}\bar u_y \cdot \frac{2\cancel{m}}{R^2}\, r\, dr =$$

$$= \frac{4}{\pi R^2}\bar u_y \int_0^R r^2\, dr = \frac{4}{\pi \cancel{R^2}}\bar u_y \cdot \frac{1}{3} R^{\cancel{3}} = \underline{\underline{\frac{4R}{3\pi}\,\bar u_y}}$$

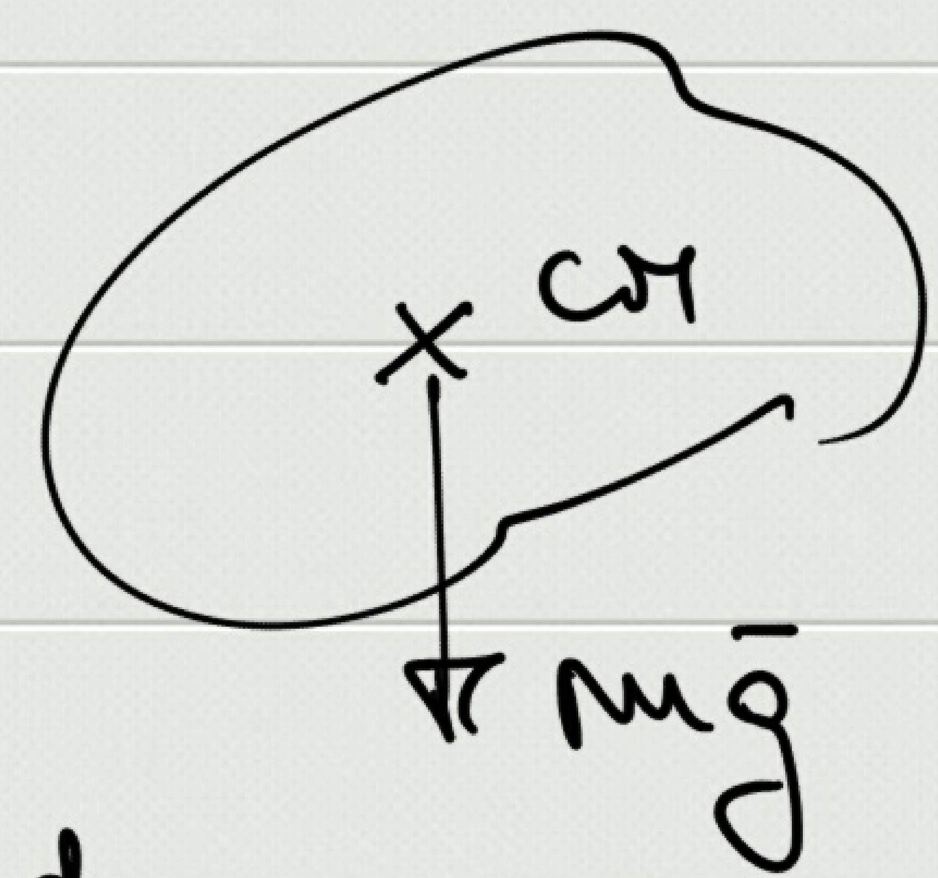
Forza peso



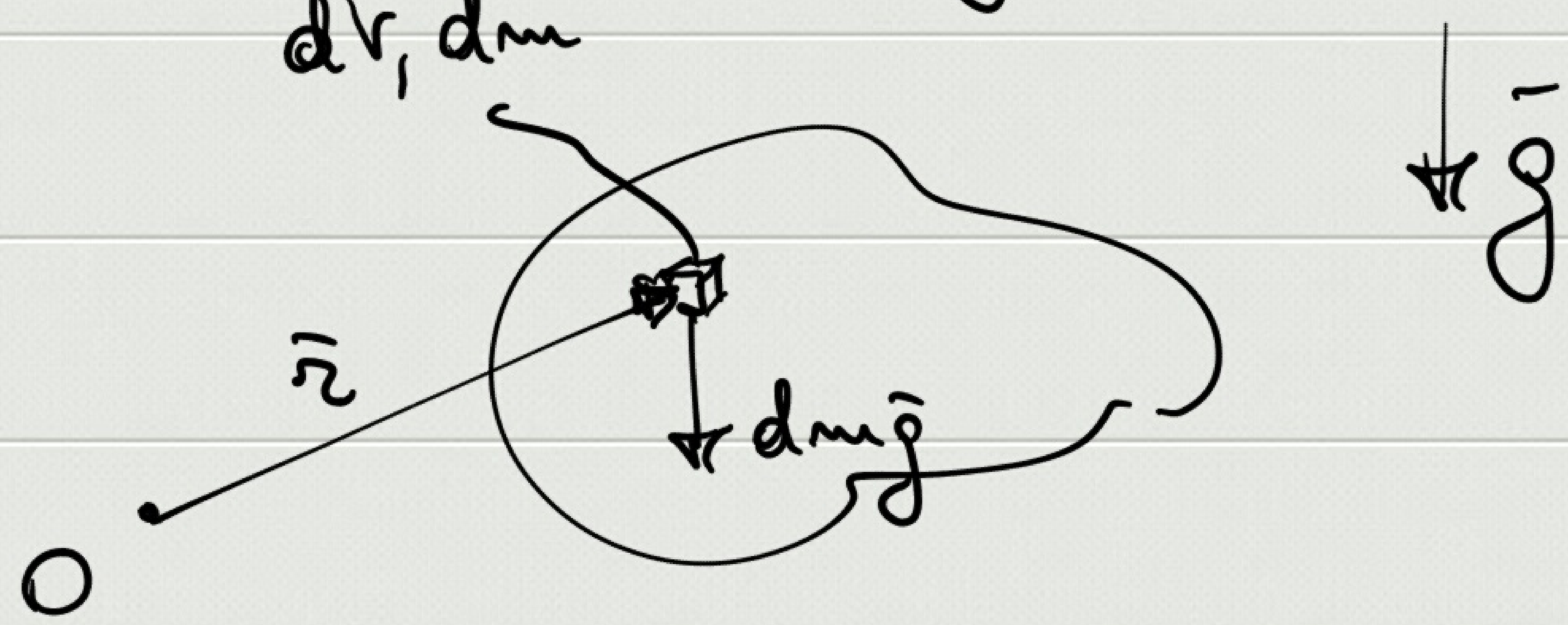
$$\bar{P} = \int_{corpo} d\bar{P} = \int_{corpo} dm\,\bar{g} = \bar{g}\int_{corpo} dm = m\,\bar{g}$$

$$= \bar{R}^{E} = m\,\bar{a}_{CM}$$

$\Rightarrow$



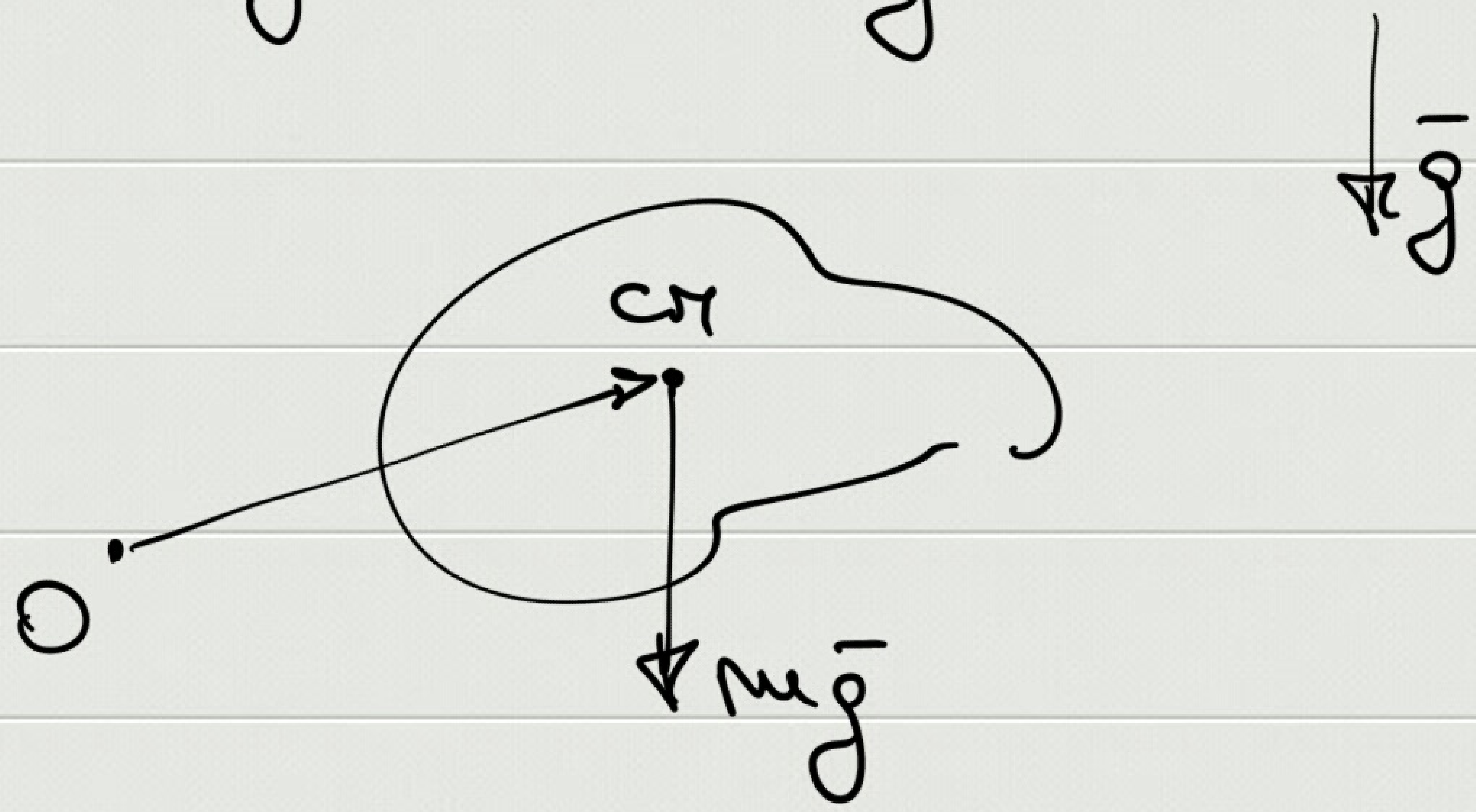
$$d\bar{M}_o = \bar{r} \times dm\,\bar{g}$$

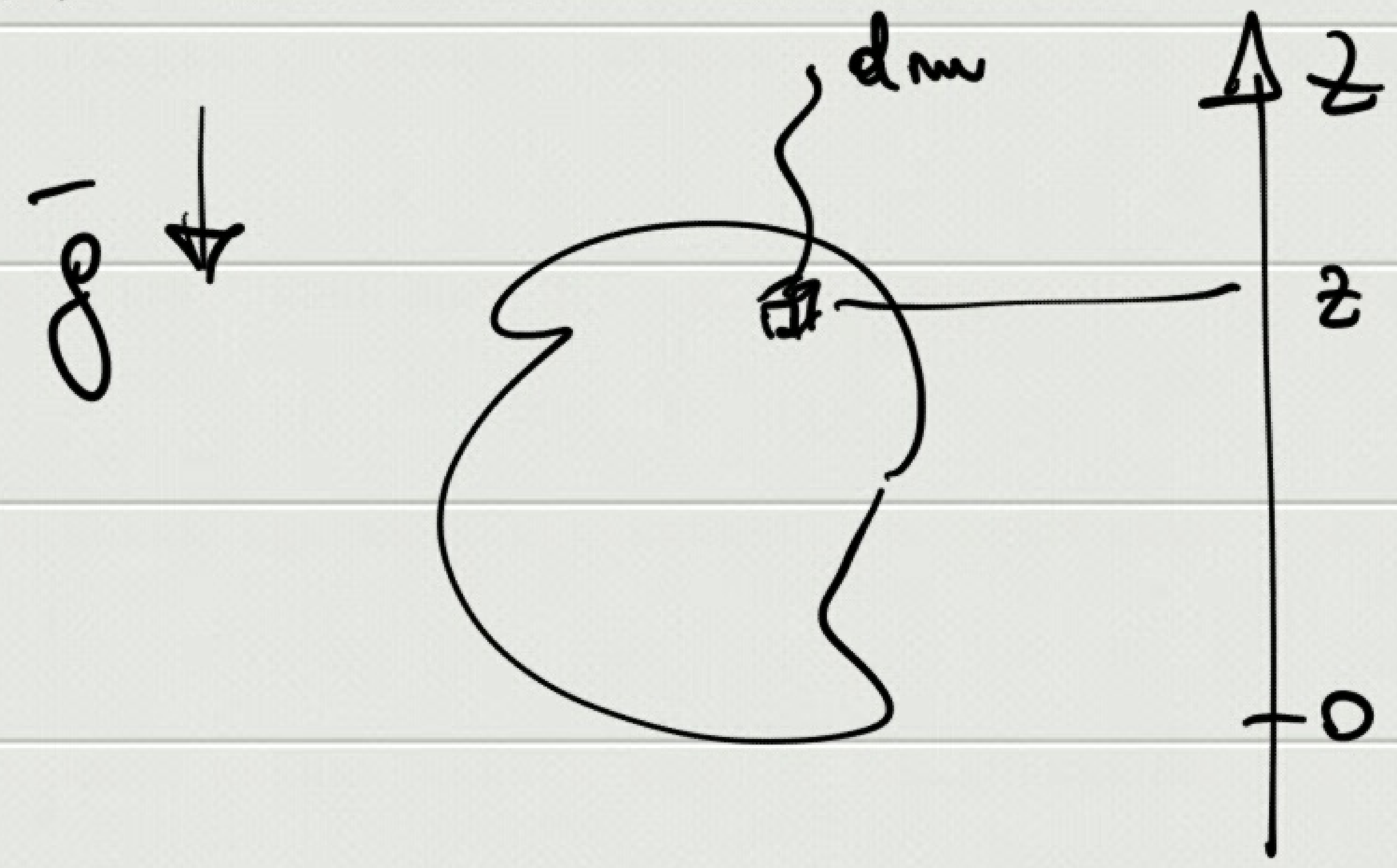


$$\bar{M}_o = \int d\bar{M}_o = \int \bar{r} \times dm\,\bar{g} = \left(\int dm\,\bar{r}\right) \times \bar{g} =$$

$$\bar{r}_{CM} = \frac{\int \bar{r}\,dm}{\int dm} = \frac{1}{m}\int \bar{r}\,dm \qquad\qquad m\,\bar{r}_{CM}$$

$$= m\,\bar{r}_{CM}\times\bar{g} = \bar{r}_{CM}\times m\bar{g} \leftarrow \bar{M}_O$$





$$E_p = \int dE_p = (*)$$

$$dE_p = dm\, g\, z$$

$$(*) = \int dm\, g\, z = g\int z\, dm = g\, m\, z_{CM}$$

$$z_{CM} = \frac{1}{m}\int z\, dm \quad \Rightarrow$$

$$\Longrightarrow \boxed{E_p = m\, g\, z_{CM}}$$