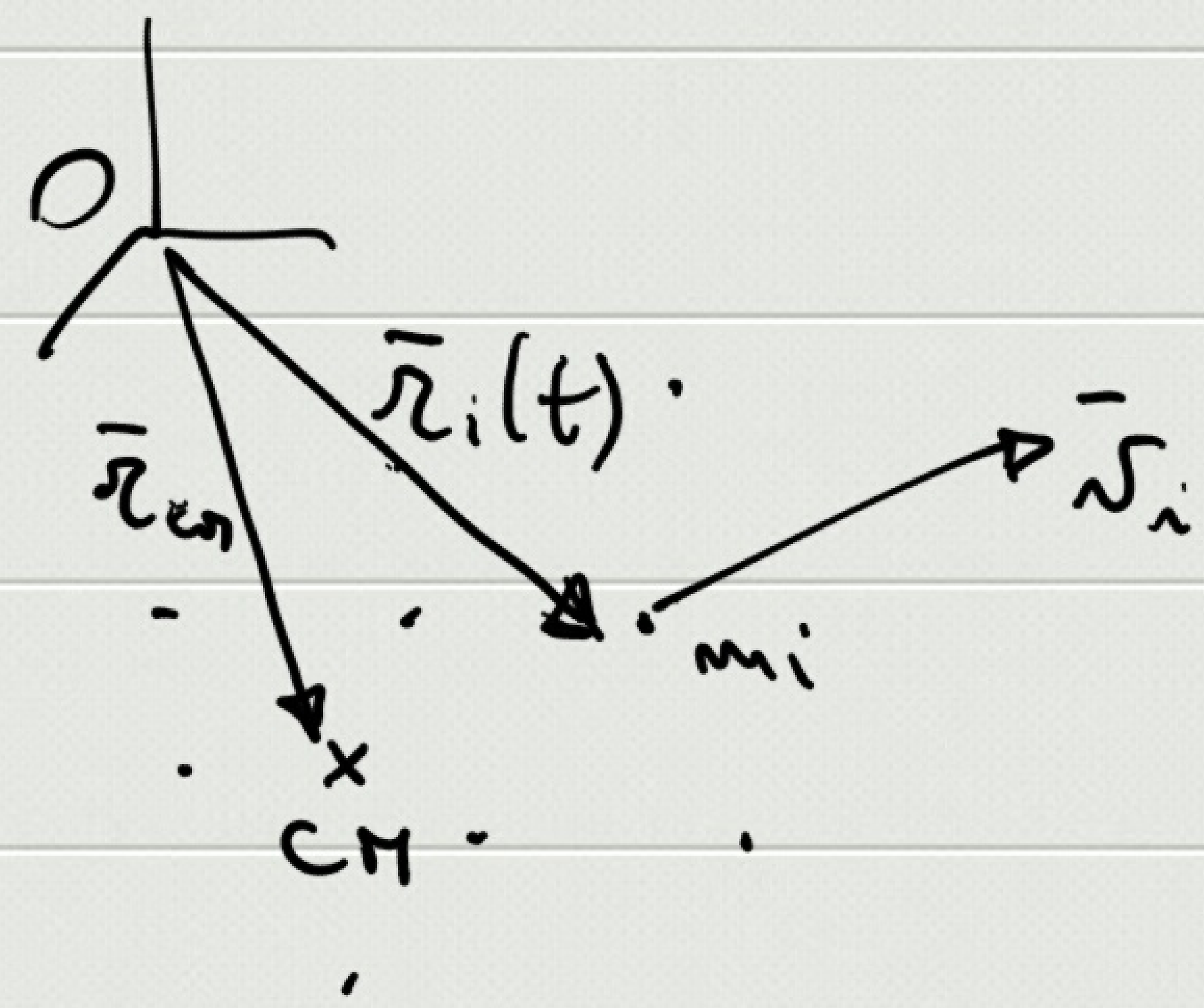
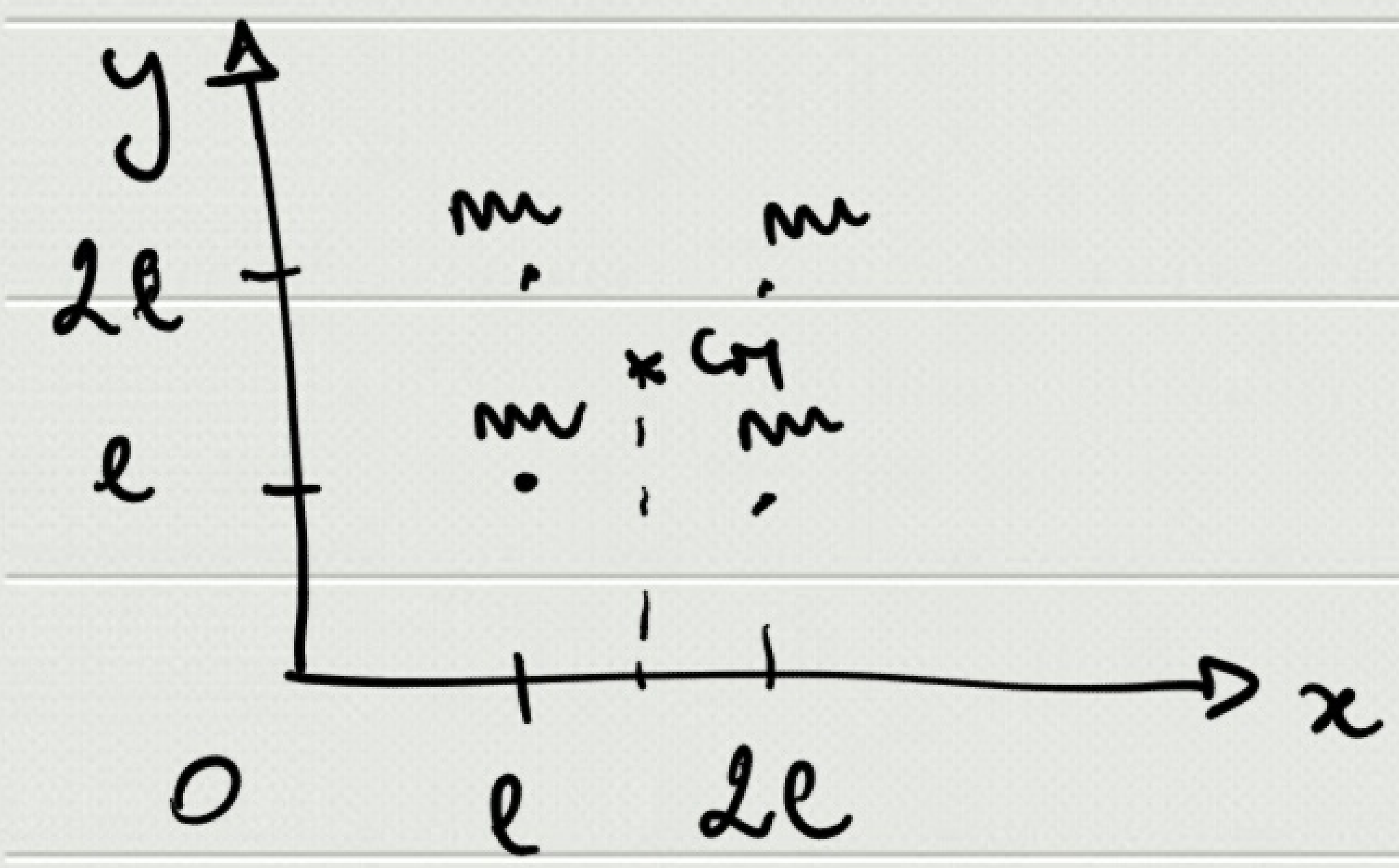
Centro di massa



$$\bar r_{CM} = \frac{m_1\bar r_1 + m_2\bar r_2 + \dots m_i \bar r_i + \dots + m_N \bar r_N}{m_1+m_2+\dots+m_i+\dots+m_N} =$$

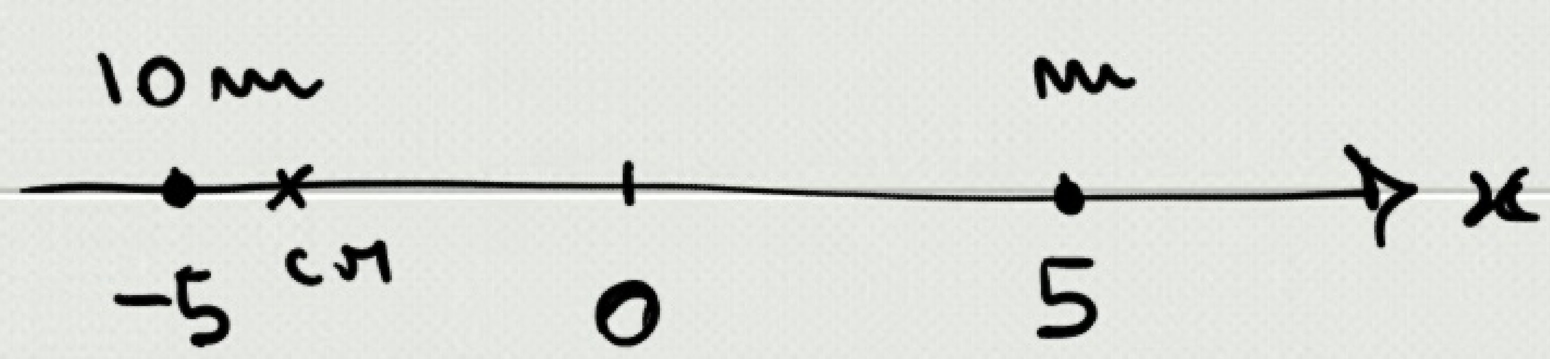
$$= \frac{\sum_i m_i\bar r_i}{\sum_i m_i} = \frac{\sum_i m_i \bar r_i}{m_{TOT}} \Rightarrow \begin{cases} x_{CM} = \dots \\ y_{CM} = \dots \\ z_{CM} = \dots \end{cases}$$



$$x_{CM} = \frac{ml + m2l + m2l + ml}{4m} =$$

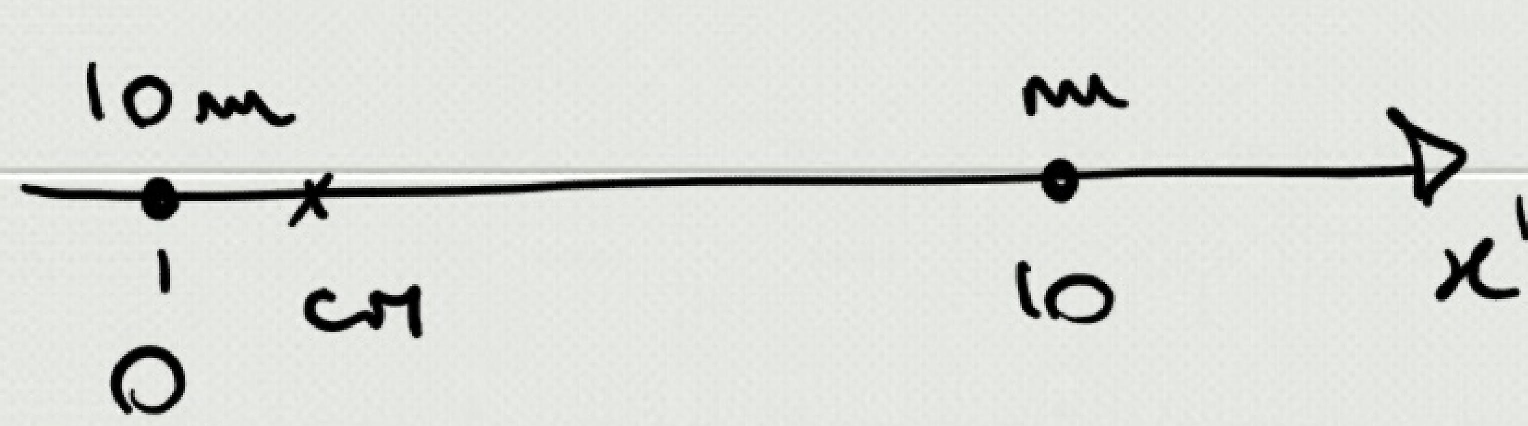
$$= \frac{6\cancel{m}l}{4\cancel{m}} = \frac{3}{2}l$$

$$y_{CM} = \frac{3}{2}l$$



$$x_{CM} = \frac{10m\cdot(-5) + m\cdot 5}{11m} =$$

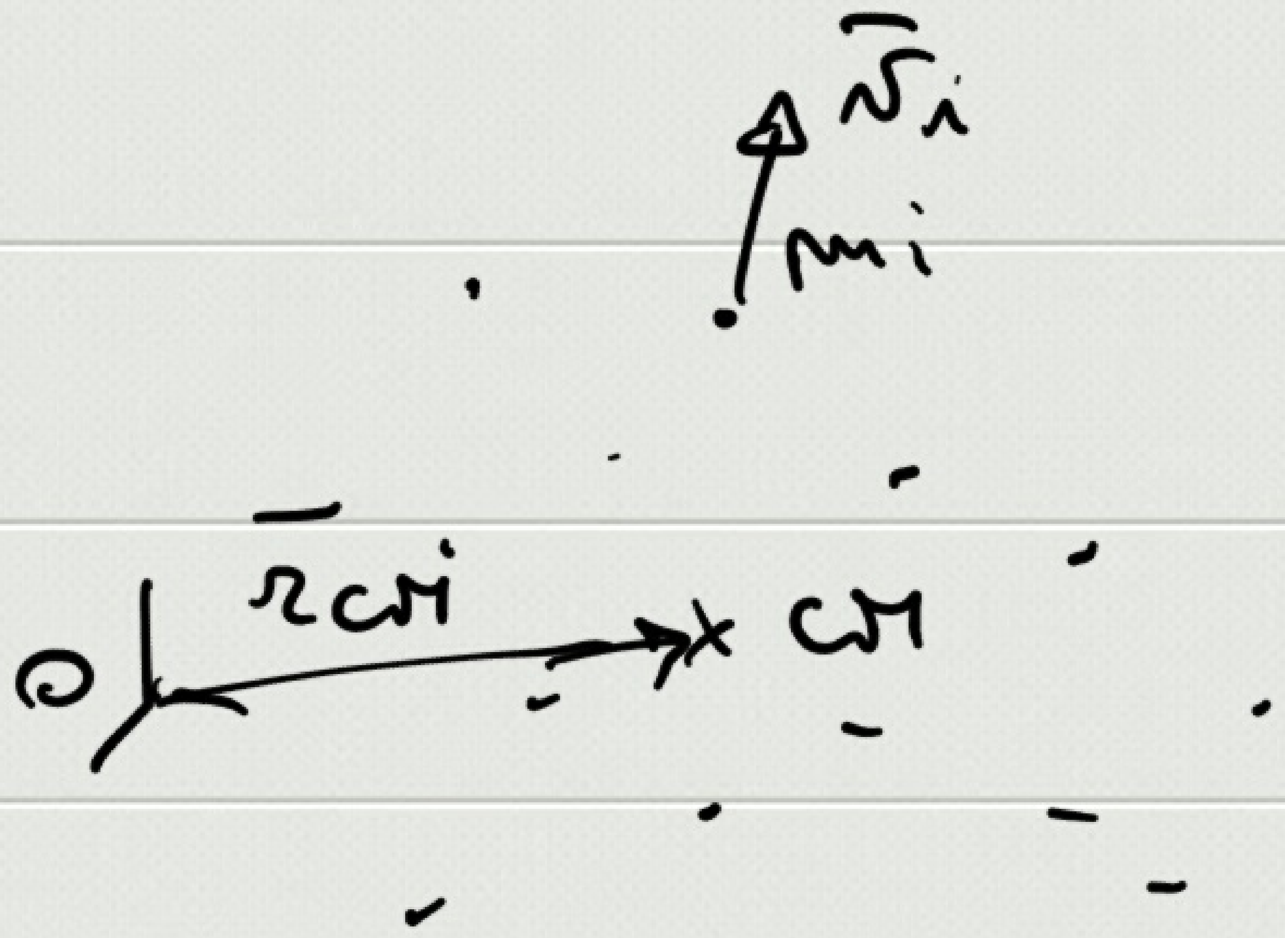
$$= -\frac{45}{11}$$



$$x'_{CM} = \frac{10m\cdot 0 + m\cdot 10}{11m} =$$

$$= \frac{10}{11}$$

$$\boxed{x = x' - 5}$$



$$\bar r_{cm} = \frac{\sum_i m_i \bar r_i}{\sum_i m_i} = \bar r_{cm}(t)$$

$$\bar v_{cm} = \frac{d\bar r_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum_i m_i \bar r_i}{\sum_i m_i}\right) = \frac{\sum_i m_i \frac{d\bar r_i}{dt}}{\sum_i m_i} = \frac{\sum_i m_i \bar v_i}{\sum_i m_i}$$

$$\Rightarrow \bar v_{cm} = \frac{\sum_i \bar p_i}{\sum_i m_i} = \frac{\bar P}{m_{TOT}} \Rightarrow \boxed{\bar P = m_{TOT}\,\bar v_{cm}}$$

$$\bar a_{cm} = \frac{d\bar v_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum_i m_i \bar v_i}{\sum_i m_i}\right) = \frac{\sum_i m_i \frac{d\bar v_i}{dt}}{\sum_i m_i} = \frac{\sum_i m_i \bar a_i}{\sum_i m_i}$$

$$\bar a_{cm} = \frac{\sum_i \bar F_i}{\sum_i m_i} = \frac{\sum_i \left(\bar F_i^{I} + \bar F_i^{E}\right)}{\sum_i m_i} = \frac{\overset{0}{\cancel{\bar R^{I}}} + \bar R^{E}}{\sum_i m_i} = \frac{\bar R^{E}}{m_{TOT}}$$

$$\Rightarrow \boxed{\bar R^{E} = m_{TOT}\,\bar a_{cm}}$$

Teorema del moto del centro di massa

$$\bar v_{cm} = \frac{\bar P}{m_{TOT}}$$

$$\Rightarrow \bar a_{cm} = \frac{d\bar v_{cm}}{dt} = \frac{d}{dt}\left(\frac{\bar P}{m_{TOT}}\right) = \frac{1}{m_{TOT}}\frac{d\bar P}{dt}$$

$$\Rightarrow \boxed{\bar R^{E} = \frac{d\bar P}{dt}}$$

$$\bar{R}^E = m_{TOT}\,\bar{a}_{CM} \qquad \bar{a}_{CM} = \frac{d\bar{v}_{CM}}{dt}$$

$$\boxed{\bar{R}^E = 0} \Rightarrow \bar{a}_{CM} = 0 \Rightarrow \boxed{\bar{v}_{CM} = cost}$$

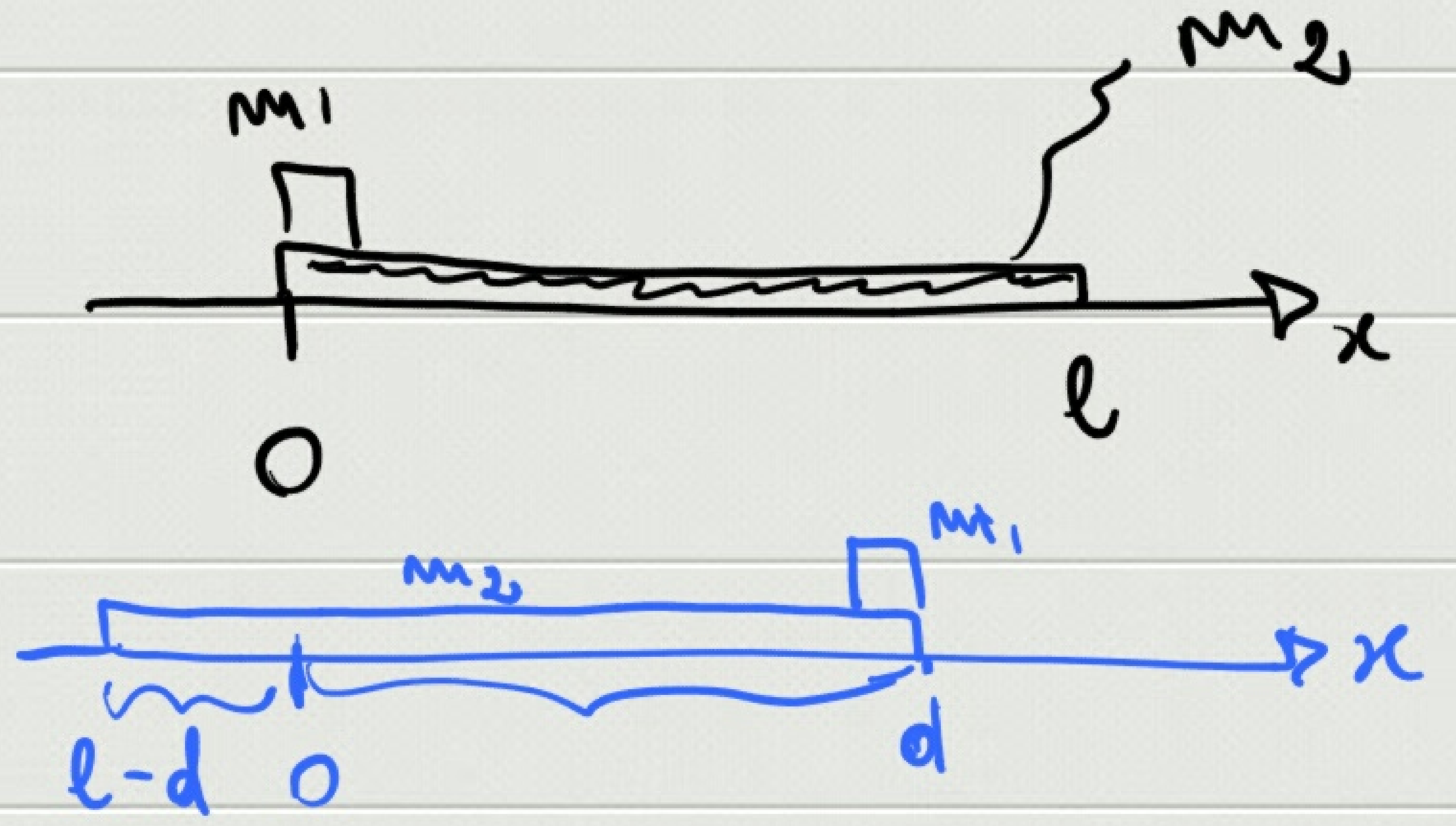
$$\Rightarrow \boxed{\bar{P} = cost}$$

Statica $\Rightarrow \boxed{\bar{v}_{CM} = 0}$

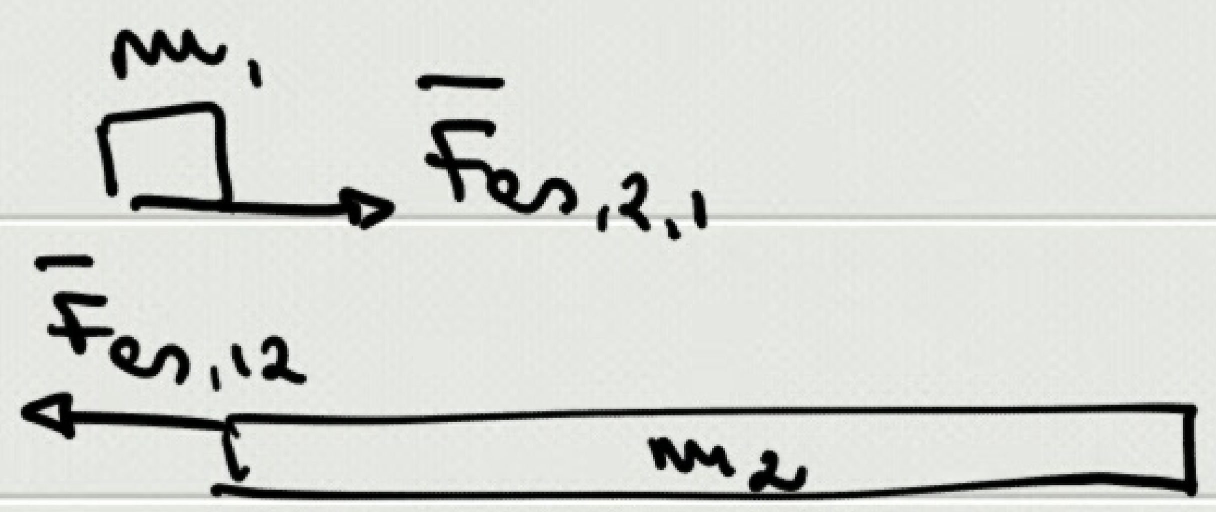
$$\bar{v}_{CM} = \frac{d\bar{r}_{CM}}{dt} = 0 \Rightarrow \boxed{\bar{r}_{CM} = cost}$$

$\bar{R}^E \neq 0$ ma $R^E_{x,y,z} = 0 \Rightarrow P_{x,y,z} = cost$

Sistema isolato



$$\boxed{R^{E}=0} \quad (\bar{P}, \bar{N})$$



$$\Rightarrow \boxed{\bar{P} = cost} = \underbrace{m_1 \bar{v}_1 + m_2 \bar{v}_2}_{=0} \quad (v_{10} = v_{20} = 0) \quad \Rightarrow \boxed{\bar{v}_2 = -\frac{m_1}{m_2}\bar{v}_1}$$

$$\left.\begin{array}{l} \bar{P} = 0 \\ \;\; \parallel m_{TOT}\,\bar{v}_{cm} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \bar{v}_{cm} = 0 \\ \;\; = \dfrac{d\bar{r}_{cm}}{dt} \end{array}\right\} \Rightarrow \boxed{\bar{r}_{cm} = cost}$$

$$\Rightarrow \boxed{x_{cm} = cost}$$

$$x_{cm}^{i} = \frac{m_1 \cdot \emptyset + m_2 \frac{l}{2}}{m_1 + m_2}$$

$$x_{cm}^{f} = \frac{m_1 \cdot d + m_2 \left[\frac{l}{2} - (l-d)\right]}{m_1 + m_2} = \frac{m_1 d + m_2 \left(d - \frac{l}{2}\right)}{m_1 + m_2}$$

$$x_{cm}^{i} = x_{cm}^{f} \Rightarrow m_2 \frac{l}{2} = m_1 d + m_2 d - m_2 \frac{l}{2} \Rightarrow$$

$$\boxed{d = \frac{m_2}{m_1 + m_2}\, l}$$