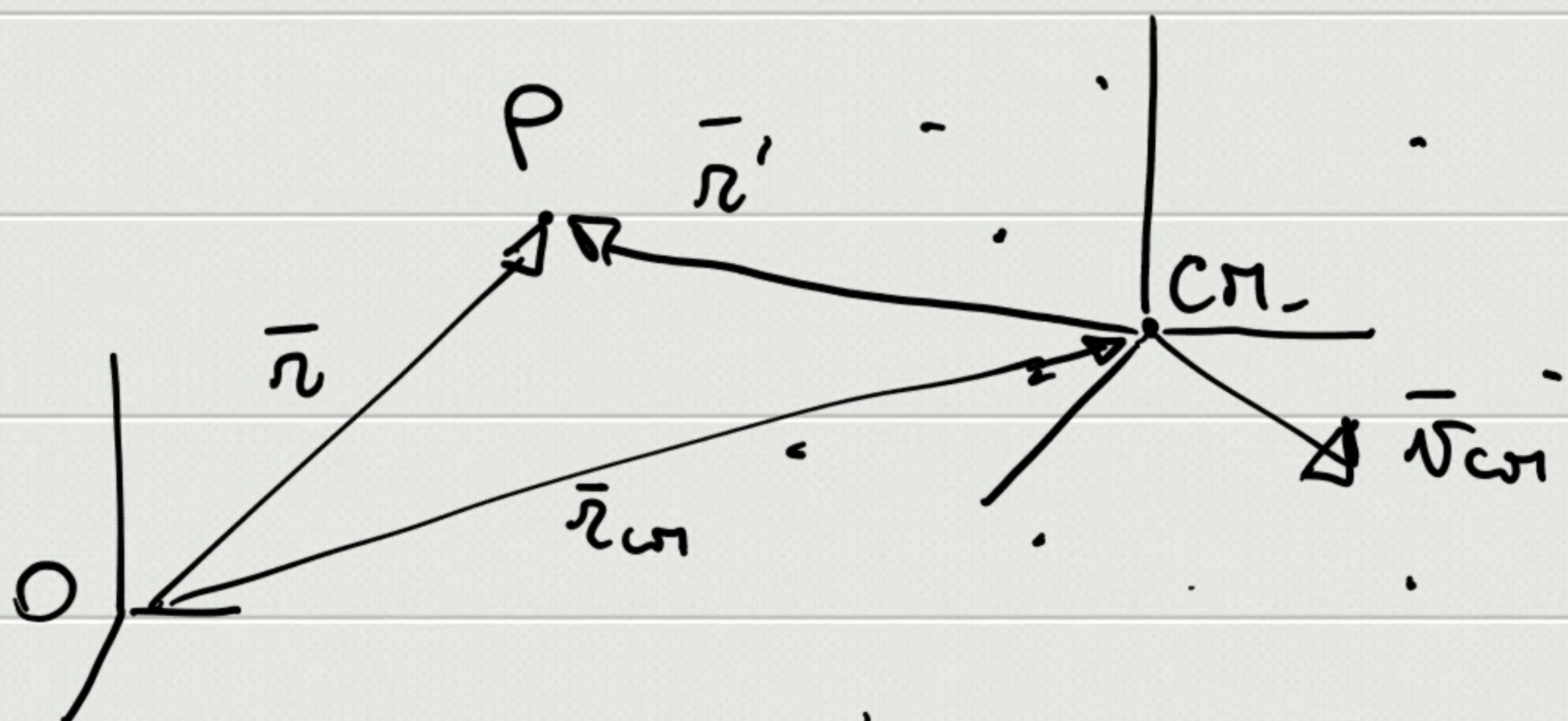


$$\bar{r} = \bar{r}' + \bar{r}_0'$$

$$\bar{v} = \bar{v}' + \bar{v}_0' + \bar{\omega} \times \bar{r}'$$

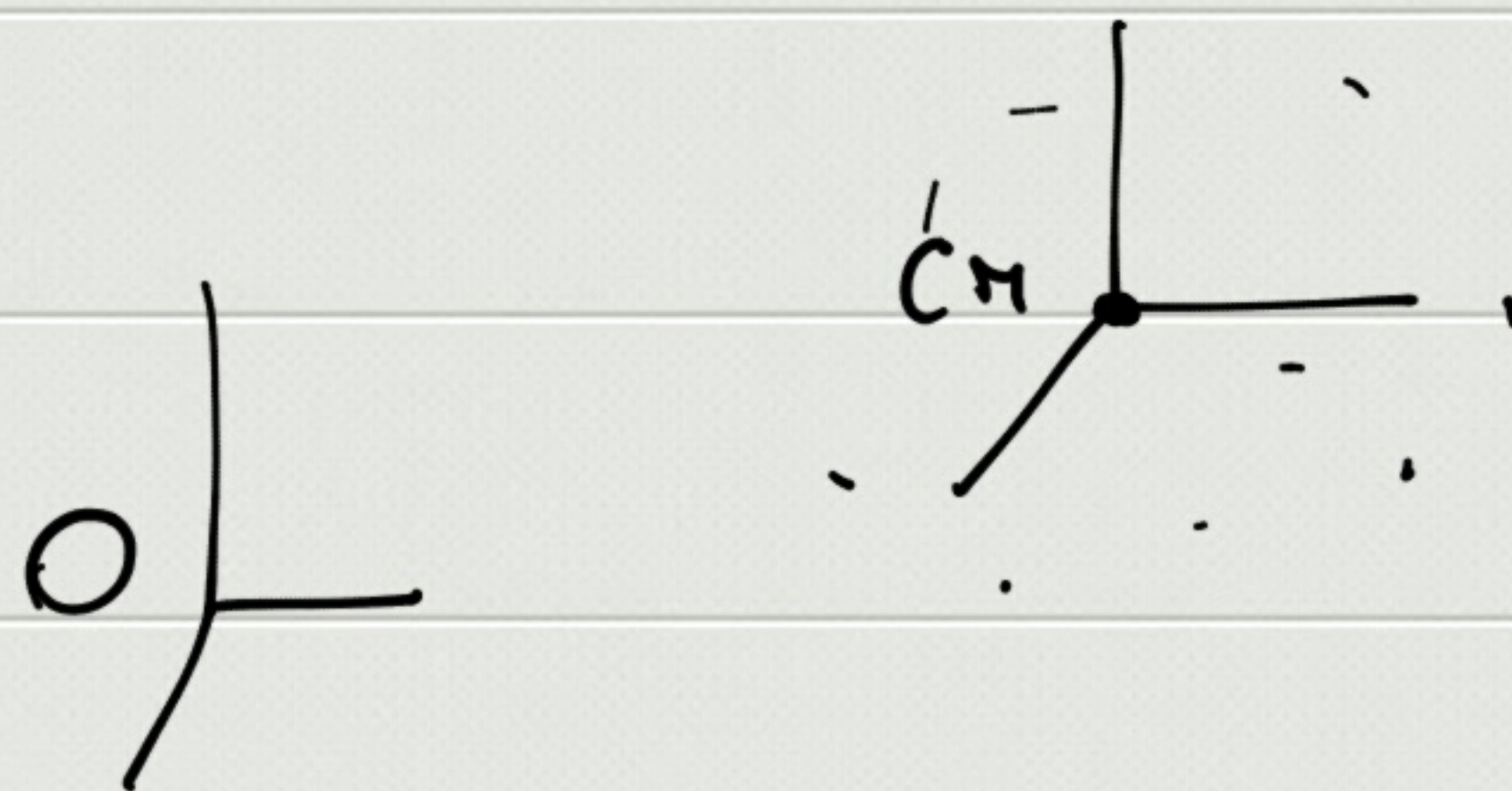
$$\bar{a} = \bar{a}' + \bar{a}_0' + \bar{\omega} \times (\bar{\omega} \times \bar{r}') + \frac{d\bar{\omega}}{dt} \times \bar{r}' + 2\bar{\omega} \times \bar{v}'$$



$$O' \equiv CM \quad \bar{\omega} = 0$$

Sistemi di riferimento del centro di massa

$$\begin{cases} \bar{r} = \bar{r}' + \bar{r}_{cm} \\ \bar{v} = \bar{v}' + \bar{v}_{cm} \\ \bar{a} = \bar{a}' + \bar{a}_{cm} \end{cases} \Rightarrow \begin{cases} \bar{r}' = \bar{r} - \bar{r}_{cm} \\ \bar{v}' = \bar{v} - \bar{v}_{cm} \\ \bar{a}' = \bar{a} - \bar{a}_{cm} \end{cases}$$



$$\begin{aligned} \bar{r}'_{cm} &= 0 \\ &\equiv \frac{\sum_i m_i \bar{r}'_i}{\sum_i m_i} \end{aligned} \left\} \Rightarrow \boxed{\sum_i m_i \bar{r}'_i = 0}$$

$$\begin{aligned} \bar{v}'_{cm} &= 0 \\ &\equiv \frac{\sum_i m_i \bar{v}'_i}{\sum_i m_i} \end{aligned} \left\} \boxed{\sum_i m_i \bar{v}'_i = 0}$$

$$\bar{p}' = \sum_i \bar{p}'_i = \sum_i m_i \bar{v}'_i = 0 \Rightarrow \boxed{\bar{p}' = 0}$$

$$\bar{p}' = \sum_i m_i \bar{v}'_i = \sum_i m_i (\bar{v}_i - \bar{v}_{cm}) =$$

$$= \sum_i m_i \bar{v}_i - \sum_i m_i \bar{v}_{cm} = \bar{p} - m_{tot} \bar{v}_{cm} = 0$$

$$\Rightarrow \bar{p} = m_{tot} \bar{v}_{cm}$$

$$\left. \begin{aligned} \bar{a}'_{cm} &= 0 \\ &= \frac{\sum_i m_i \bar{a}'_i}{\sum_i m_i} \end{aligned} \right\} \Rightarrow \boxed{\sum_i m_i \bar{a}'_i = 0}$$

$$\bar{R}' = \sum_i \bar{F}'_i = \sum_i m_i \bar{a}'_i = 0 \Rightarrow \boxed{\bar{R}' = 0}$$

$$\begin{aligned} \bar{R}' &= \sum_i m_i \bar{a}'_i = \sum_i m_i (\bar{a}_i - \bar{a}_{cm}) = \sum_i m_i \bar{a}_i - \sum_i m_i \bar{a}_{cm} \\ &= \sum_i \bar{F}_i - m_T \bar{a}_{cm} = 0 \Rightarrow \boxed{\bar{R}^E = m_T \bar{a}_{cm}} \end{aligned}$$

$$\bar{L}_{cm} = \sum_i \bar{L}'_{i,cm} =$$

$$= \sum_i \bar{r}'_i \times m_i \bar{v}_i =$$

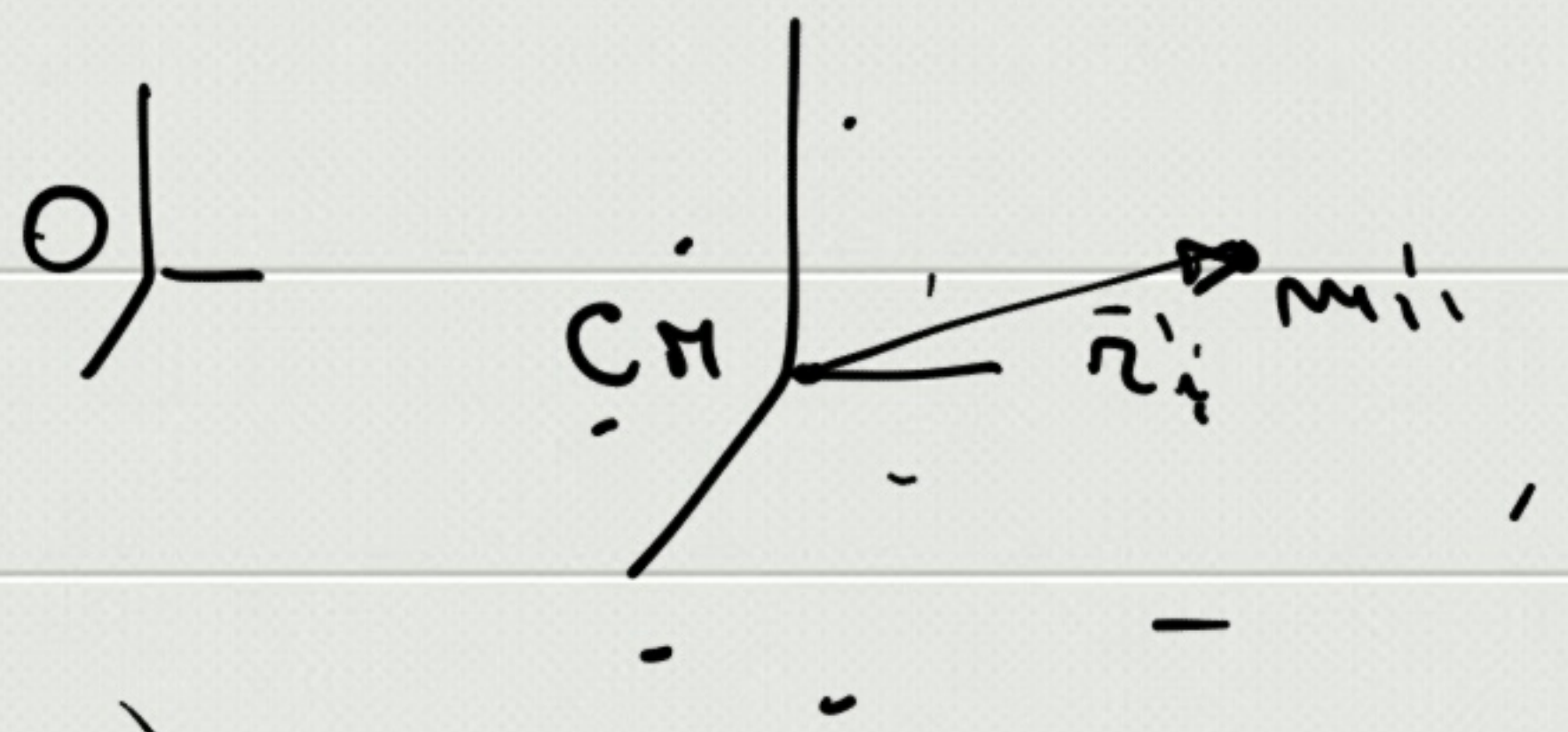
$$= \sum_i \bar{r}'_i \times m_i (\bar{v}'_i + \bar{v}_{cm}) =$$

$$= \sum_i \bar{r}'_i \times m_i \bar{v}'_i + \sum_i \bar{r}'_i \times m_i \bar{v}_{cm} =$$

$$= \sum_i \bar{L}'_{cm,i} + \underbrace{\sum_i m_i \bar{r}'_i}_{=0} \times \bar{v}_{cm} =$$

$$= \bar{L}'_{cm}$$

$$\Rightarrow \boxed{\bar{L}_{cm} = \bar{L}'_{cm}}$$



$$\bar{M}^E_{cm} = \bar{M}'^E_{cm}$$

$$\Rightarrow \text{pob cm} : \frac{d\bar{L}_0}{dt} = -\bar{N}_0 \times \cancel{M_{tot}} \bar{N}_{cm} + \bar{M}_0^E$$

$$\Rightarrow \frac{d\bar{L}_{cm}}{dt} = \bar{M}^E_{cm}$$

$$\Rightarrow \boxed{\frac{d\bar{L}'_{cm}}{dt} = \bar{M}'^E_{cm}}$$