

$$\vec{g} = -g \vec{u}_y \quad (g > 0)$$

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \int_{\vec{v}_0}^{\vec{v}(t)} d\vec{v} = \int_0^t \vec{a} dt \Rightarrow \boxed{\vec{v}(t) = \vec{v}_0 + \vec{g} t}$$

$$x: v_x(t) = v_{0x} = v_0 \cos \theta_0 \quad (= \text{const})$$

$$y: v_y(t) = v_{0y} - g t = v_0 \sin \theta_0 - g t$$

$$\vec{v}(t) = v_x \vec{u}_x + v_y \vec{u}_y = \underline{(v_0 \cos \theta_0) \vec{u}_x + (v_0 \sin \theta_0 - g t) \vec{u}_y}$$

$$\begin{aligned} \vec{r}(t) &= \vec{r}_0 + \int_0^t \vec{v}(t) dt = \\ &= \vec{r}_0 + \int_0^t (\vec{v}_0 + \vec{g} t) dt = \\ &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{g} t^2 \end{aligned}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} \Rightarrow \dots$$

$$(\vec{r}_0 = 0)$$

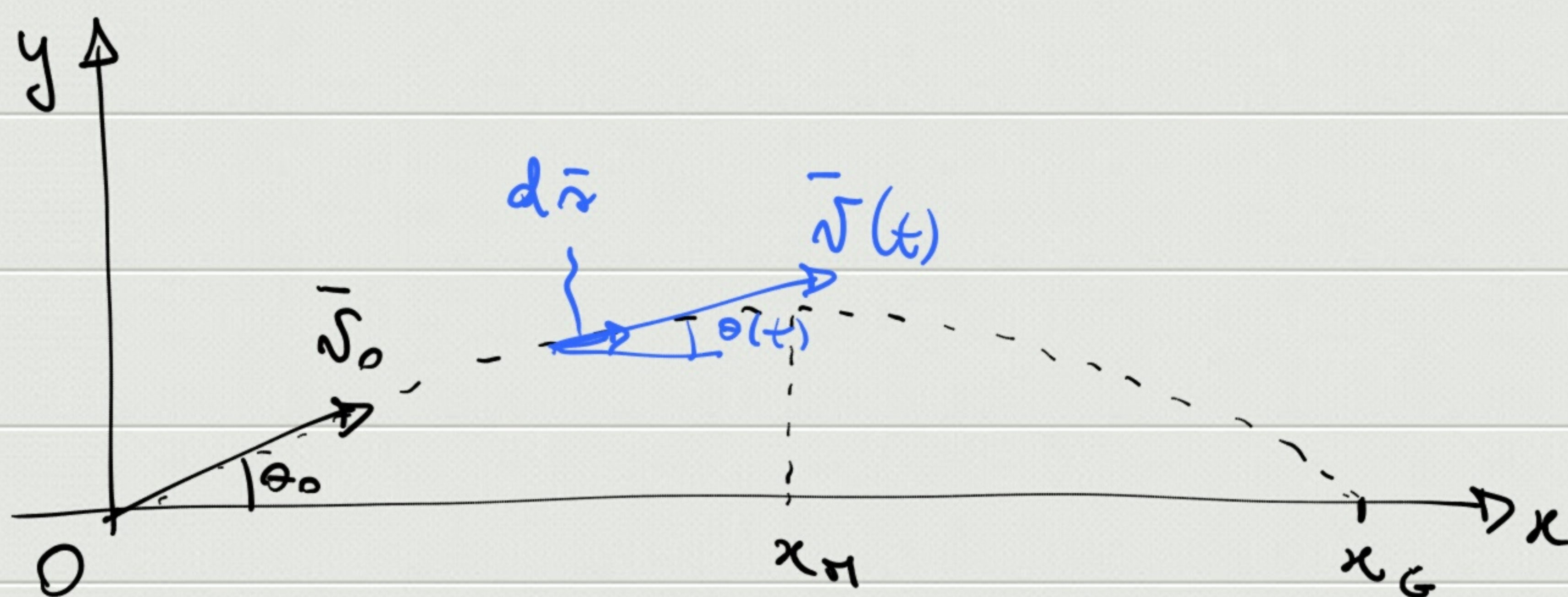
$$\begin{cases} x(t) = v_0 \cos \theta_0 \cdot t \\ y(t) = v_0 \sin \theta_0 \cdot t - \frac{1}{2} g t^2 \end{cases}$$

$$\vec{r}(t) = (v_0 \cos \theta_0 \cdot t) \vec{u}_x + \left(v_0 \sin \theta_0 \cdot t - \frac{1}{2} g t^2 \right) \vec{u}_y$$

$$x(t) \Rightarrow t = \frac{x}{v_0 \cos \theta_0}$$

$$\Rightarrow y[x(t)] = y(x) = \cancel{v_0 \sin \theta_0} \frac{x}{\cancel{v_0 \cos \theta_0}} - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2 \theta_0} =$$

$$y = \tan \theta_0 \cdot x - \frac{g}{2 v_0^2 \cos^2 \theta_0} x^2$$



$$\tan(\theta) = \frac{dy}{dx} = \tan \theta_0 - \frac{g}{v_0^2 \cos^2 \theta_0} x$$

$$x_n: \frac{dy}{dx} = 0 \Rightarrow x_n = \tan \theta_0 \frac{v_0^2 \cos^2 \theta_0}{g} = \frac{v_0^2}{g} \sin \theta_0 \cos \theta_0$$

$$t_n: x_n = x(t_n) \Rightarrow \cancel{v_0 \cos \theta_0} \cdot t_n = \frac{v_0^2}{g} \sin \theta_0 \cancel{\cos \theta_0}$$

$$t_H = \frac{v_0}{g} \sin \theta_0$$

$$y_H = y(t_H) = v_0 \sin \theta_0 \cdot \frac{v_0}{g} \sin \theta_0 - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2 \sin^2 \theta_0 =$$

$$= \frac{1}{2} \frac{v_0^2}{g} \sin^2 \theta_0$$

$$x_G = \frac{2 v_0^2}{g} \sin \theta_0 \cos \theta_0 = \frac{v_0^2}{g} \sin 2\theta_0$$

$$\theta_0 : x_{G, \max} ? \quad \frac{dx_G}{d\theta_0} = 0 \quad 2 \cos 2\theta_0 = 0$$

$$2\theta_0 = \frac{\pi}{2} \Rightarrow \boxed{\theta_0 = \frac{\pi}{4}}$$

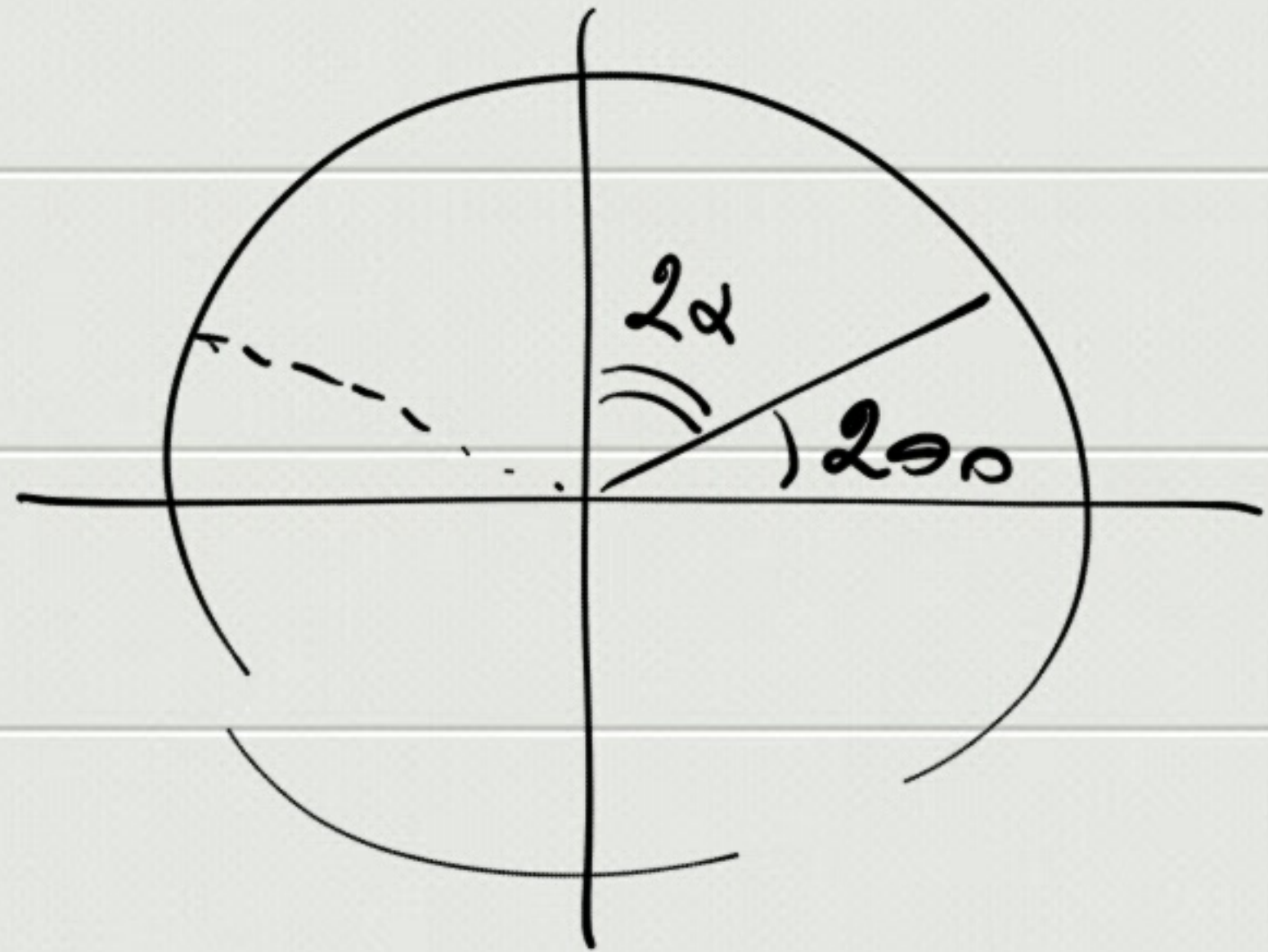
$$t_G = \dots$$



$$|\vec{v}_G| = |\vec{v}_0| \quad \theta_G = -\theta_0$$

$$x_G = \frac{v_0^2}{g} \sin 2\theta_0$$

$$0 < \theta_0 < \frac{\pi}{2}$$



$$2\theta_0 = \frac{\pi}{2} \pm 2\alpha$$

$$\Rightarrow \boxed{\theta_0 = \frac{\pi}{4} \pm \alpha}$$

