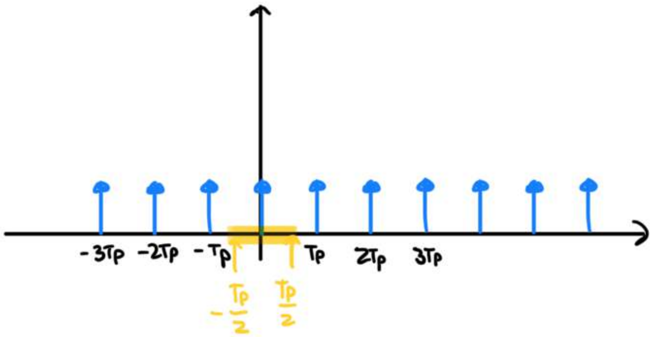
# Lezione 13 - 4/04/2024

## ESERCIZIO 1a

$s(t) = comb_{T_p}(t) = rep_{T_p}\,\delta(t)$ (treno di impulsi)




- $S_K = ?$
- SERIE DI FOURIER?

SOL. Uso la DEFINIZIONE DI $S_K$

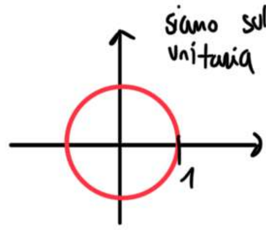
$$\boxed{S_K = \frac{1}{T_p}\int_{t_0}^{t_0+T_p} s(t)\, e^{-jKW_0 t}\, dt}$$

$$W_0 = \frac{2\pi}{T_p}$$

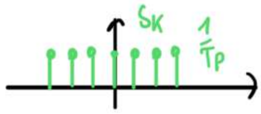
CI METTIAMO IN UN INTERVALLO CENTRATO IN 0 DI AMPIEZZA $T_p$

$$S_K = \frac{1}{T_p}\int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} \delta(t)\, e^{-jKW_0 t}\, dt = \frac{1}{T_p}\, e^{-jKW_0 \cdot 0} = \frac{1}{T_p}\cdot 1 = \frac{1}{T_p}$$

siamo sulla circonferenza unitaria




$$\Rightarrow \boxed{\underbrace{s(t) = rep_{T_p}\,\delta(t)}_{\text{delta periodico}} \xrightarrow{\mathcal{F}} \underbrace{S_K = \frac{1}{T_p}}_{\text{segnale costante}}}$$

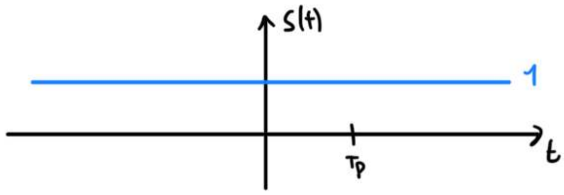



$$s(t) = \sum_{K=-\infty}^{+\infty} S_K\, e^{jKW_0 t} = \underbrace{\frac{1}{T_p}\sum_{K=-\infty}^{+\infty} e^{j2\pi K\frac{t}{T_p}}}_{\text{SERIE DI FOURIER}}$$

, converge alla $rep_{T_p}\,\delta(t)$

ESERCIZIO 1b (sarebbe l'esercizio 1a inverso)

$s(t) = 1$

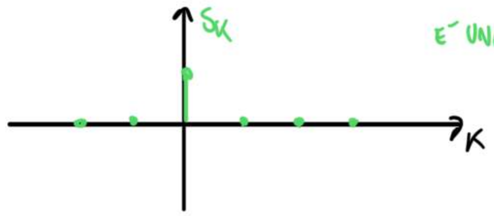



$$S_K = \frac{1}{T_p}\int_0^{T_p} 1 e^{-jk\omega_0 t}\,dt \qquad \omega_0 = \frac{2\pi}{T_p}$$

ricorda: questo è un esponenziale complesso a fase lineare, cioè una sinusoide

$$= \begin{cases} \frac{1}{T_p}\cdot T_p = 1 & K = 0 \\ 0 & K \neq 0 \end{cases}$$

$$\frac{1}{T_p}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_0^{T_p} = \frac{e^{-jk\omega_0 T_p} - 1}{-jk\omega_0 T_p} = \frac{1 \cdot 1}{-jk\omega_0 T_p} = 0$$




E' UNA $\delta(K)$!

$$s(t) = 1 \xrightarrow{\mathcal{F}} S_K = \delta(K)$$

SEGNALE COSTANTE

DELTA DISCRETA

ABBIAMO SCOPERTO CHE C'E' UNA DUALITÁ TRA DELTA E SEGNALI COSTANTI

$$\boxed{\text{delta} \overset{\mathcal{F}}{\longleftrightarrow} \text{SEGNALE COSTANTE}}$$

DUALITÁ DELTA-COSTANTE

$$s(t) = \sum_{K=-\infty}^{+\infty} S_K e^{jk\omega_0 t} = 1$$

## VALORE MEDIO E POTENZA (alla luce dei risultati delle slide 25 e 26)

a) $s(t) = \text{rep}_{T_P}\, \delta(t) \xrightarrow{\mathcal{F}} S_K = \frac{1}{T_P}$

$m_S = S_0 = \frac{1}{T_P}$

$P_S = \sum_K |S_K|^2 = \sum_{K=-\infty}^{+\infty} \frac{1}{T_P^2} = \infty$

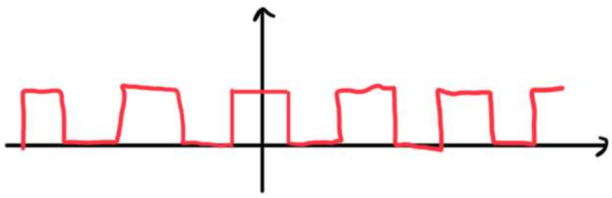
b) $s(t) = 1 \longrightarrow S_K = \delta(K)$

$m_S = S_0 = 1$

$P_S = \sum_K |S_K|^2 = 1$

c) $s(t) = \text{rep}_{T_p}\, \text{rect}\left(\frac{t}{2a}\right) \longrightarrow S_K = d\, \text{sinc}(Kd) \qquad d = \frac{2a}{T_P}$

$m_S = S_0 = d$



$P_S = \sum_{K=-\infty}^{+\infty} |S_K|^2 = \sum_{K=-\infty}^{+\infty} d^2 \text{sinc}^2(Kd) = ?$

(questa serie non è risolvibile)

$= d$ (ma calcolandolo dal dominio del tempo)

- la **sinusoide** $s(t) = A\cos(\omega_0 t + \varphi_0)$ con $\omega_0 = 2\pi/T_p$

## ESERCIZIO 1c

$$s(t) = \cos(\omega_0 t + \varphi_0) \qquad \omega_0 = \frac{2\pi}{T_p}$$

a) $S_K = ?$
b) $m_s = ?$
c) $P_s = ?$

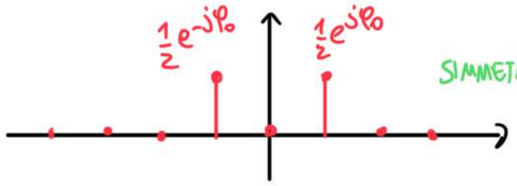
SOL. a. SCRIVO IL COSENO SOTTO FORMA DI ESPONENZIALI COMPLESSI CON EULERO

$$s(t) = \underbrace{\frac{e^{j\varphi_0}}{2}}_{S_1} e^{j\omega_0 t} + \underbrace{\frac{e^{-j\varphi_0}}{2}}_{S_{-1}} e^{-j\omega_0 t}$$

$K=1$ $\qquad$ $K=-1$

$$= \sum_K S_K e^{jK\omega_0 t}$$

$$S_K = \begin{cases} \dfrac{e^{j\varphi_0}}{2} & K = 1 \\ \dfrac{e^{-j\varphi_0}}{2} & K = -1 \\ 0 & \text{ALTROVE} \end{cases}$$




SIMMETRIA HERMITIANA: $S_{-K} = S_K$

ABBIAMO AGITO PER ISPEZIONE: abbiamo guardato la struttura del segnale e abbiamo visto che era già espresso sotto forma di SDF con 2 coefficienti attivi

b) $m_s = S_0 = 0$

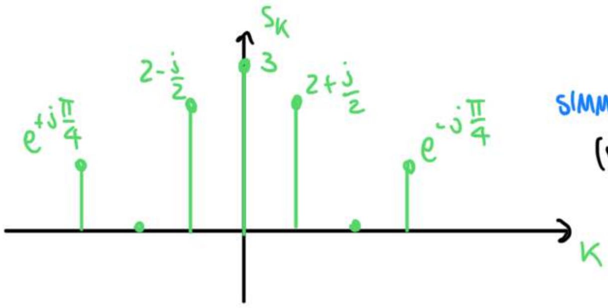
c) $P_s = \sum_K |S_K|^2 = \left|\frac{e^{j\varphi_0}}{2}\right|^2 + \left|\frac{e^{-j\varphi_0}}{2}\right|^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

## ESERCIZIO

$$s(t) = \overset{0\cdot\omega_0 = 0}{3} - \sin(\overset{\omega_0 = 2}{2t}) + 4\cos(2t) + 2\cos(\overset{3\omega_0}{6t} - \frac{\pi}{4})$$

$$s(t) = 3 - \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + 4\,\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + 2\,\frac{e^{j3\omega_0 t}e^{-j\frac{\pi}{4}} + e^{-j3\omega_0 t}e^{j\frac{\pi}{4}}}{2}$$

$$= 3 + \left(\frac{j}{2} + 2\right)e^{j\omega_0 t} + \left(-\frac{j}{2} + 2\right)e^{-j\omega_0 t} + e^{-j\frac{\pi}{4}}e^{j3\omega_0 t} + e^{j\frac{\pi}{4}}e^{-j3\omega_0 t}$$




SIMMETRIA HERMITIANA: $S_{-K} = S_K^*$

(regola di controllo)

$m_s = 3$

$$P_s = 3^2 + 2\left|2 + \frac{j}{2}\right|^2 + 2\left|e^{-j\frac{\pi}{4}}\right|$$

$$= 9 + 2\left(4 + \frac{1}{4}\right) + 2\cdot 1 = 9 + 8 + \frac{1}{2} + 2 = 19 + \frac{1}{2} = 19,5$$