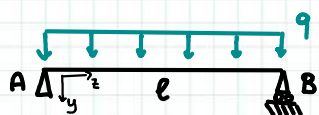


ES 1



M, T, N continui in tutto l

1° step: devo conoscere le REAZ. VINCOLARI

2° step: calcolo gli andamenti M, T, N.

METODO 1

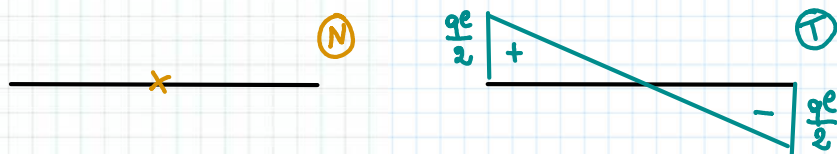
$$\begin{cases} \frac{dN}{dz} = 0 & N = \text{cost} = 0 \\ N(z=0) = 0 \end{cases}$$

$$\begin{cases} \frac{dT}{dz} = -q & T = -qz + C \\ T(z=0) = \frac{ql}{2} \end{cases} \quad T = \frac{ql}{2} - qz$$

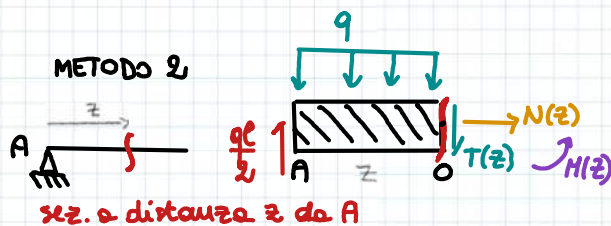
$\frac{ql}{2} \quad z=0$
 $-\frac{ql}{2} \quad z=l$

$$\begin{cases} \frac{dM}{dz} = T(z) = -qz + \frac{ql}{2} & M = -q\frac{z^2}{2} + \frac{ql}{2}z + C \\ M(z=0) = 0 & C = 0 \end{cases} \quad M(z) = -q\frac{z^2}{2} + \frac{ql}{2}z$$

Rappresentazione grafica



METODO 2



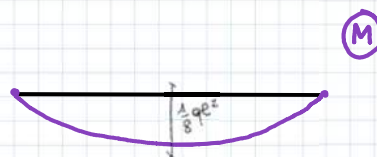
Eq. orizz di AO $N(z) = 0$

Eq. verticale di AO $T(z) - \frac{ql}{2} + qz = 0$

$$T(z) = \frac{ql}{2} - qz$$

Eq. momento in O $M(z) = M(z) + \frac{qz^2}{2} - \frac{ql}{2}z = 0$

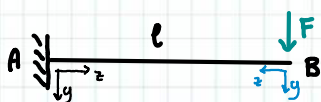
$$M(z) = \frac{ql}{2}z - \frac{qz^2}{2}$$



Massimo?

$$\begin{aligned} \frac{dM(z)}{dz} &= 0 & \frac{ql}{2} - qz &= 0 & z &= \frac{l}{2} \\ M\left(\frac{l}{2}\right) &= \frac{ql^2}{4} - \frac{ql^2}{8} = \frac{ql^2}{8} \end{aligned}$$

ES 2



Metodo 1:

$$\begin{cases} \frac{dN}{dz} = 0 & N = \text{cost} = 0 \\ N(z=0) = 0 \end{cases}$$

$$\begin{cases} \frac{dT}{dz} = 0 & T = \text{cost} = F \\ T(z=0) = F \end{cases}$$

$$\begin{cases} \frac{dM}{dz} = T = F & M = Fz + C \\ M(z=0) = -Fe & C = -Fe \end{cases} \quad M(z) = Fz - Fe$$

Metodo 2 (sist zy)

$$N(z) = 0$$

$$T(z) - F = 0 \quad T(z) = F$$

$$M(z) = M(z) + Fe - Fz = 0$$

$$M(z) = Fz - Fe \quad \rightarrow z=0 \quad M = -Fe$$

$$\rightarrow z=l \quad M = 0$$

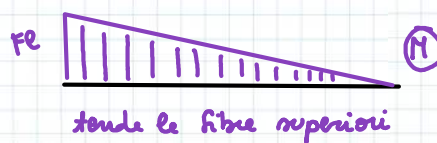
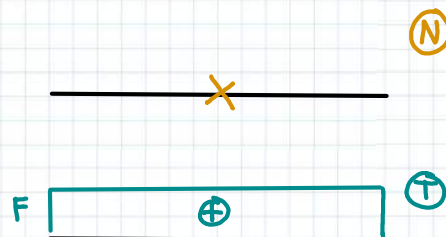
sist. zy

$$N(z) = 0$$

$$T(z) - F = 0 \quad T(z) = F$$

$$M(z) = M(z) + Fz = 0 \quad M(z) = -Fz \quad \rightarrow M(z=0) = 0$$

$$\rightarrow M(z=l) = -Fe$$

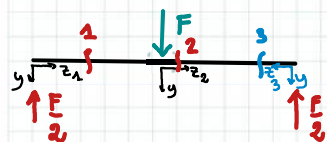


tende le fibre superiori

ES 3



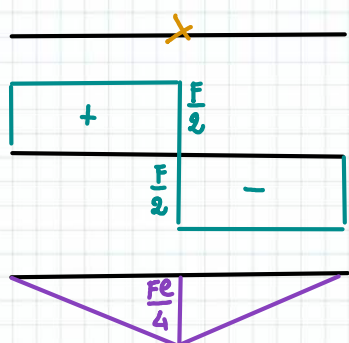
NB Una F concentrata causa un SALTO nel diagramma di TAGLIO pari all'entità del modulo della forza.



$$\begin{aligned} \textcircled{1} \quad N(z_1) &= 0 \\ T(z_1) &= \frac{F}{2} \\ H(z_1) - \frac{F}{2}z_1 &= 0 \quad H(z_1) = \frac{F}{2}z_1 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad N(z_2) &= 0 \\ T(z_2) + F - \frac{F}{2} &= 0 \quad T(z_2) = -\frac{F}{2} \\ H(z_2) + Fz_2 - \frac{F}{2}\left(\frac{l}{2} + z_2\right) &= 0 \\ H(z_2) &= \frac{Fl}{4} - \frac{F}{2}z_2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad N(z_3) &= 0 \\ T(z_3) + \frac{F}{2} &= 0 \quad T(z_3) = -\frac{F}{2} \\ H(z_3) &= \frac{F}{2}z_3 \end{aligned}$$



(N)

(T)

(M)

Salto
pari alla
forza
 F

Integrazione

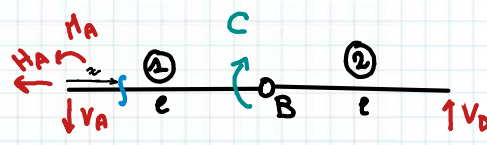
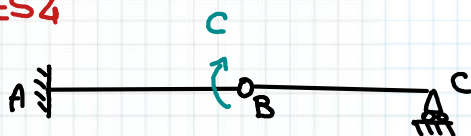
$$\begin{aligned} \textcircled{1} \quad N(z) &= 0 \\ \begin{cases} \frac{dT}{dz_1} = 0 & T = \frac{F}{2} \\ T(z_1) = \frac{F}{2} \end{cases} \\ \begin{cases} \frac{dH}{dz_1} = T = \frac{F}{2} & H(z_1) = \frac{F}{2}z_1 + C \\ H(z_1=0) = 0 & H(z_1) = \frac{F}{2}z_1 \end{cases} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad N(z_2) &= 0 \\ \begin{cases} \frac{dT}{dz_2} = 0 & T(z_2) = -\frac{F}{2} \\ T(z_2=0) = -F + \frac{F}{2} \end{cases} \\ \begin{cases} \frac{dH}{dz_2} = -\frac{F}{2} & H = -\frac{F}{2}z_2 + \frac{Fl}{4} \\ H(z_2=0) = \frac{Fl}{4} \end{cases} \end{aligned}$$

ATTENZIONE

Se integro in z_3 , cambia $\frac{dH}{dz} = -T$ (da eq. del taglio)

ES 4



Eq^{ne} ausiliaria in B del corpo 2

$$\begin{aligned} \hookrightarrow M(B) &= V_D \cdot l = 0 \quad V_D = 0 \Rightarrow V_A = 0 \quad (\text{eq globale } \uparrow) \\ H_A &= 0 \quad (\text{eq globale } \rightarrow) \\ \hookrightarrow H(A) &= H_A - C = 0 \quad H_A = C \end{aligned}$$

Diagrammi:

$$H(z) = -C$$

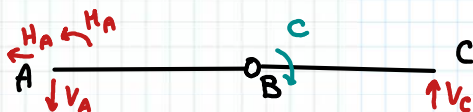
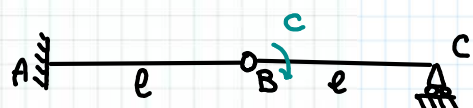
(N)

(T)

(M)



ES 5



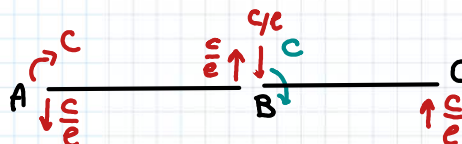
(T)



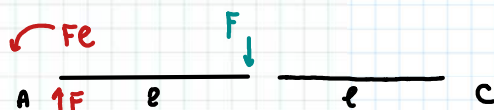
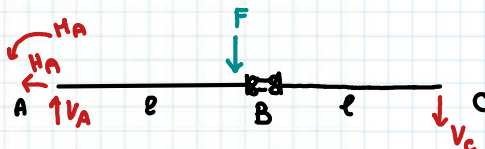
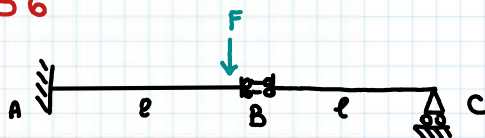
(M)

Eq. ^{na} ausiliaria corpo 2
 $\sum M(B) = V_C \cdot e - C = 0 \quad V_C = \frac{C}{e} = V_A$ eq. \uparrow globale
 $H_A = 0$ eq. \rightarrow globale

$\sum M(A) = H_A - C + V_C \cdot 2e = 0 \quad H_A - C + 2C = 0 \quad H_A = -C$



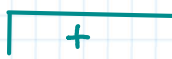
ES 6



Eq. ^{na} ausiliaria corpo 2 $V_C = 0$

Eq. globale \uparrow $V_A = F$
 $H_A = 0$
 $H_A = Fe$

Diagrammi (N scarico)



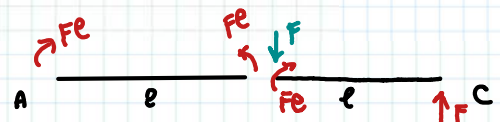
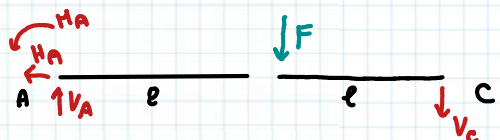
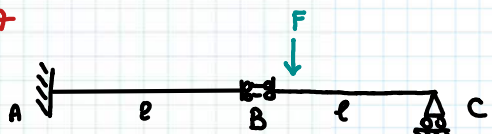
(T)

sotto pari alla forza applicata



(M)

ES 7



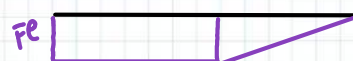
Eq. ^{na} ausiliaria corpo 2 : $V_C = -F$

Eq. globale \uparrow $V_A = 0$
 $\rightarrow H_A = 0$
 $\sum M(A) = H_A - Fe - V_C \cdot 2e = 0$
 $H_A - Fe + 2Fe = 0 \quad H_A = -Fe$

Diagrammi (N scarico)

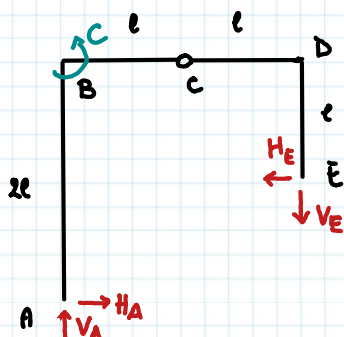
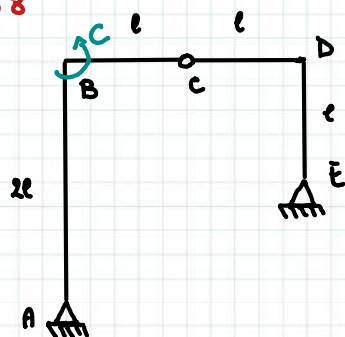


(T)



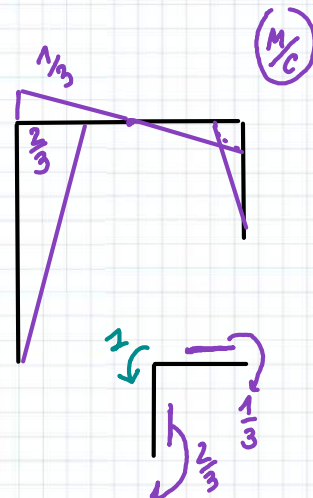
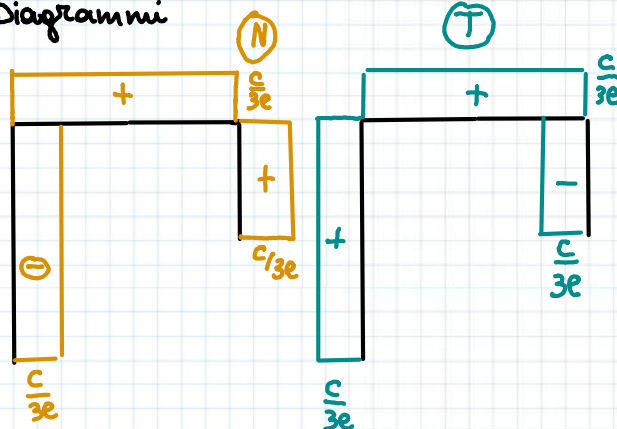
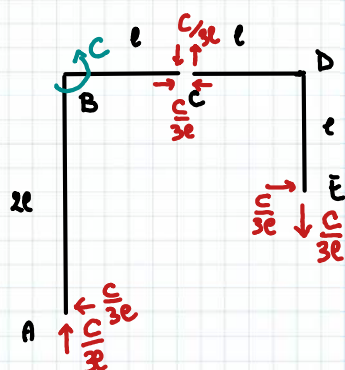
(M)

ES 8



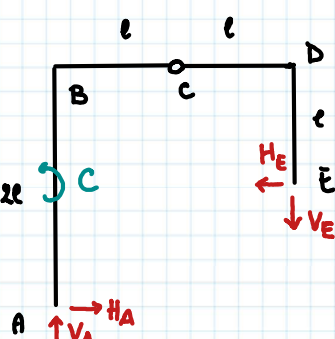
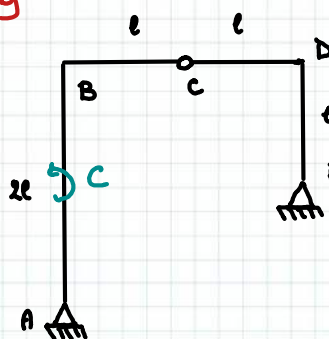
$$\begin{cases} H_A - H_E = 0 & H_A = -\frac{C}{3e} \\ V_A - V_E = 0 & V_A = \frac{C}{3e} \\ \sum M(A) = C + H_E l - V_E 2l = 0 & C - V_E l - 2V_E l = 0 \\ \sum M(C) = -H_E l - V_E l = 0 & H_E = -V_E \end{cases} \quad \begin{matrix} H_E = -\frac{C}{3e} \\ V_E = \frac{C}{3e} \end{matrix}$$

Diagrammi

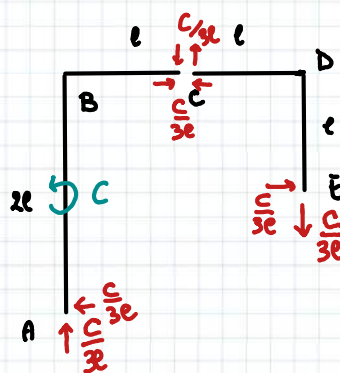


NODO EQUILIBRATO

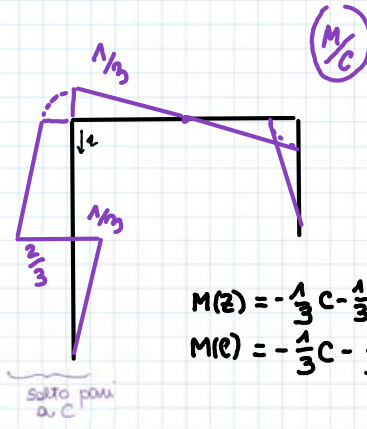
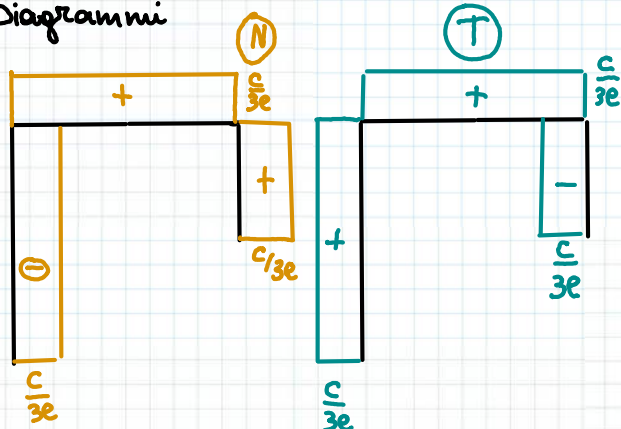
ES 9



Stesse reazioni di prima (es 8)



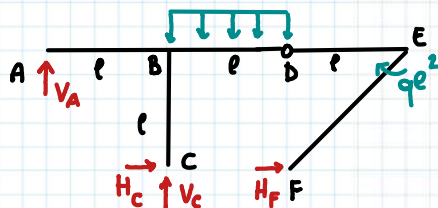
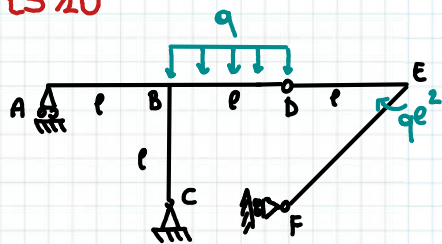
Diagrammi



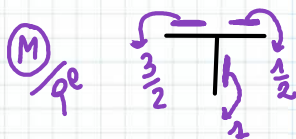
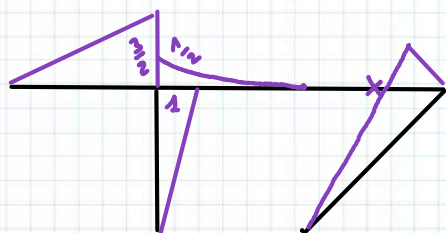
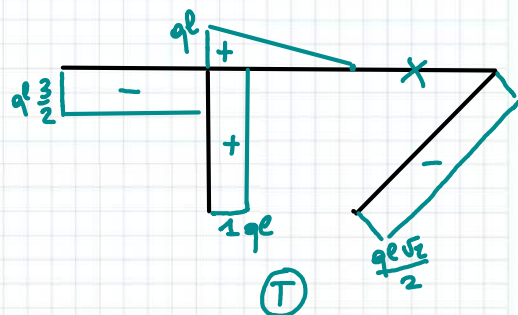
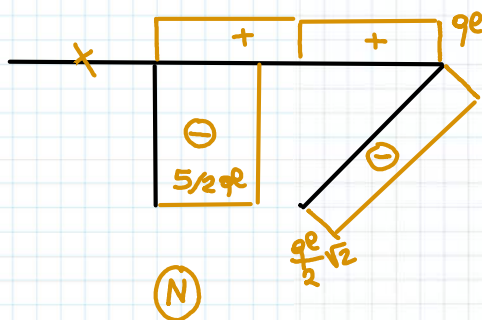
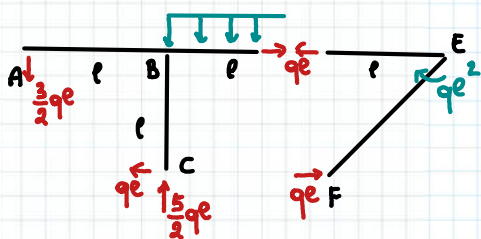
$$\begin{aligned} M(z) &= -\frac{1}{3} C - \frac{1}{3} \frac{C}{e} \cdot z \\ M(e) &= -\frac{1}{3} C - \frac{1}{3} C = -\frac{2}{3} C \end{aligned}$$

salto pari a C

ES 10



$$\begin{aligned}
 H_C + H_F &= 0 & H_C &= -qe \\
 V_A + V_C - qe &= 0 & V_C &= -V_A + qe = \frac{5}{2}qe \\
 \sum M(C) &= -V_A \cdot l - \frac{qe \cdot l^2}{2} - qe^2 = 0 & V_A &= -\frac{3}{2}qe \\
 \sum M(D) &= H_F \cdot l - qe^2 = 0 & H_F &= qe
 \end{aligned}$$



$$\begin{aligned}
 & qe \\
 & \frac{qe \cdot l^2}{2} \rightarrow \frac{qe \cdot l^2}{2}
 \end{aligned}$$

