ESERCIZI SCHEDA 3

ESERCIZIO 1

ⓐ $f(x) = \log x$ dom f: $x>0 \Rightarrow \text{dom} f = (0,+\infty) \Rightarrow$ pt. acc. $= [0,+\infty) \cup \{+\infty\}$

ⓑ $f(x) = \arcsin\left(\frac{1}{x}\right)$

dom f: $\begin{cases} x \neq 0 \\ -1 \leq \frac{1}{x} \leq 1 \end{cases}$ $\begin{cases} x \neq 0 \\ x \leq -1 \vee x \geq 1 \end{cases}$ $\Rightarrow \text{dom} f = (-\infty,-1] \cup [1,+\infty) \Rightarrow$ pt. acc $= (-\infty,-1] \cup [1,+\infty) \cup \{\pm\infty\}$

ⓒ $f(x) = \sqrt{\frac{3+x}{2-x}}$

dom f: $\frac{3+x}{2-x} \geq 0$

$N(x) \geq 0$: $3+x \geq 0 \Leftrightarrow x \geq -3$

$D(x) > 0$: $2-x > 0 \Leftrightarrow x < 2$

	$x<-3$	$-3<x<2$	$x>2$
N	−	+	+
D	+	+	−
N/D	−	+	−

$\Rightarrow \text{dom} f = [-3,2) \Rightarrow$ pt. acc. $= [-3,2]$

ⓓ $f(x) = \arctan\left(\frac{3x^2+9x+2}{x}\right)$ dom $f = \mathbb{R} \setminus \{0\} \Rightarrow$ pt. acc. $= \overline{\mathbb{R}}$

ESERCIZIO 2

ⓐ $\forall M>0, \exists \delta>0 : f(x) < -M \ \forall x \in \text{dom} f \cap (-1, \delta-1)$

ⓑ $\forall \varepsilon>0, \exists N \in \mathbb{R} : |f(x)-2| < \varepsilon \ \forall x \in \text{dom} f : x < -N$

ⓒ $\forall M>0, \exists N \in \mathbb{R} : f(x) < -M \ \forall x \in \text{dom} f : x < -N$

ⓓ $\forall \varepsilon>0, \exists N \in \mathbb{R} : |f(x)+40| < \varepsilon \ \forall x \in \text{dom} f : x > N$

ⓔ $\forall M>0, \exists N \in \mathbb{R} : f(x) < -M \ \forall x \in \text{dom} f : x > N$

ⓕ $\forall \varepsilon>0, \exists N \in \mathbb{R} : |f(x)-3| \ \forall x \in \text{dom} f \cap (2-\delta, 2)$

ESERCIZIO 3

$f(x) = \begin{cases} x^n & \text{per } x \neq 0 \\ 1 & \text{per } x = 0 \end{cases}$ con $n \in \mathbb{N} \setminus \{0\}$

Esercizio 4

$$\lim_{x\to x_0}(3x+1) = 3x_0+1 \quad \text{con } x_0 \in \mathbb{R}$$

$\forall \varepsilon>0, \exists \delta>0 : \left|f(x)-(3x_0+1)\right| < \varepsilon \ \forall x \in \mathrm{dom} f \cap (x_0-\delta, x_0+\delta) \setminus \{x_0\}$ $\quad \mathrm{dom} f = \mathbb{R}$

$\forall \varepsilon>0, \exists \delta>0 : \left|f(x)-(3x_0+1)\right| < \varepsilon \ \forall x \in \mathbb{R} : \ 0<|x-x_0|<\delta$

$\longrightarrow |3x+1-(3x_0+1)|<\varepsilon \iff |3x+\cancel{1}-3x_0-\cancel{1}|<\varepsilon \iff |3x-3x_0|<\varepsilon \iff 3|x-x_0|<\varepsilon \iff$

$\iff |x-x_0|<\frac{\varepsilon}{3} \implies$ Scelgo $\boxed{\delta = \frac{\varepsilon}{3}}$

Esercizio 5

ⓐ $\lim_{x\to 0} \sin x = 0$

caso x_0, ℓ finiti : $\forall \varepsilon>0, \exists \delta>0 : |f(x)-\ell|<\varepsilon \ \forall x \in \mathrm{dom} f \cap (x_0-\delta, x_0+\delta)$ $\qquad x_0=\ell=0, \ \mathrm{dom} f = \mathbb{R}$

$\forall \varepsilon>0, \exists \delta>0 : |\sin x|<\varepsilon \ \forall x \in \mathbb{R} : \ 0<|x|<\delta$

* se $\varepsilon \in (0,1]$: $|\sin x|<\varepsilon \iff -\varepsilon < \sin x < \varepsilon$

$-\varepsilon, \varepsilon \in [-1,1]\setminus\{0\}$, quindi posso applicare la funzione arcoseno.

$\iff \arcsin(-\varepsilon) < x < \arcsin(\varepsilon)$

$\iff -\arcsin(\varepsilon) < x < \arcsin(\varepsilon)$

$\iff |x| < \arcsin(\varepsilon) \implies$ Scelgo $\boxed{\delta = \arcsin(\varepsilon)}$

* se $\varepsilon > 1$, è sempre verificato il limite.

ⓑ $\lim_{x\to 0} \cos x = 1$

caso x_0, ℓ finiti : $\forall \varepsilon>0, \exists \delta>0 : |f(x)-\ell|<\varepsilon \ \forall x \in \mathrm{dom} f \cap (x_0-\delta, x_0+\delta)$

$\forall \varepsilon>0, \exists \delta>0 : |\cos x - 1|<\varepsilon \ \forall x \in \mathbb{R} : \ 0<|x|<\delta$

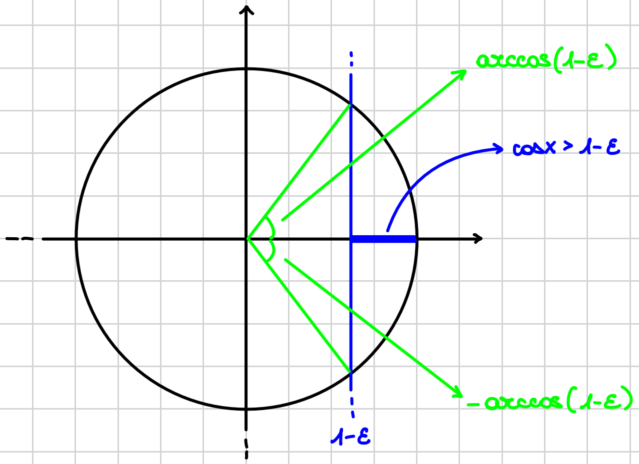
$-\varepsilon < \cos x - 1 < \varepsilon \iff 1-\varepsilon < \cos x < 1+\varepsilon$

$\downarrow$ ovvia, in quanto $1+\varepsilon>1$

$\iff 1-\varepsilon < \cos x$

* se $\varepsilon \in (2,+\infty)$ la disuguaglianza è sempre verificata

* se $\varepsilon \in (0,2]$: $\cos x > 1-\varepsilon$ $\qquad$ Visualizzo il problema sulla circonferenza goniometrica.



$-\arccos(1-\varepsilon) < x < \arccos(1-\varepsilon)$

$|x| < \arccos(1-\varepsilon)$

Inoltre, se si considera $x = \pm\arccos(1-\varepsilon)$ con $\varepsilon \neq 0$, allora $x \neq \arccos(1) \Rightarrow x \neq 0$

$\Rightarrow \quad 0 < |x| < \arccos(1-\varepsilon)$

$\Rightarrow$ Scelgo $\delta = \arccos(1-\varepsilon)$

Esercizio 6

ⓐ $\lim_{x\to+\infty} \frac{1}{x} = 0 \qquad \mathrm{dom} f = \mathbb{R}\setminus\{0\}$

caso $x_0 = +\infty$, $\ell = 0$ finito: $\forall \varepsilon > 0, \exists N \in \mathbb{R} : |f(x) - \ell| < \varepsilon \ \forall x > N \qquad \ell = 0, f(x) = \frac{1}{x}$

$\forall \varepsilon > 0, \exists N \in \mathbb{R} : \left|\frac{1}{x}\right| < \varepsilon \quad \forall x > N$

$\left|\frac{1}{x}\right| < \varepsilon \Leftrightarrow \frac{1}{|x|} < \varepsilon \Leftrightarrow \frac{1}{\varepsilon} < |x| \Leftrightarrow x < -\frac{1}{\varepsilon} \vee x > \frac{1}{\varepsilon}$

$\Rightarrow$ Scelgo $N = \frac{1}{\varepsilon}$

ⓑ $\lim_{x\to+\infty} \frac{\sin x}{x} = 0 \qquad \mathrm{dom} f = \mathbb{R}\setminus\{0\}$

caso $x_0 = +\infty$, $\ell = 0$ finito: $\forall \varepsilon > 0, \exists N \in \mathbb{R} : |f(x) - \ell| < \varepsilon \ \forall x > N \qquad \ell = 0, f(x) = \frac{\sin x}{x}$

$\forall \varepsilon > 0, \exists N \in \mathbb{R} : \left|\frac{\sin x}{x}\right| < \varepsilon \ \forall x > N$

$\left|\frac{\sin x}{x}\right| < \varepsilon \Leftrightarrow \frac{|\sin x|}{|x|} < \varepsilon$

maggiorazione: $\frac{1}{|x|} < \varepsilon \Rightarrow$ anche qui scelgo $N = \frac{1}{\varepsilon}$

Esercizio 7

So che $-1 \leq \sin x \leq 1$

Per $x \to +\infty$, si ha $x > 0 \Rightarrow -\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

Dal punto ⓐ, $\lim_{x\to+\infty} \frac{1}{x} = 0$. È perfettamente analogo dimostrare che $\lim_{x\to+\infty} -\frac{1}{x} = 0$ (dimostrabile anche tramite l'algebra dei limiti).

$$\Rightarrow \lim_{x\to+\infty} \frac{\sin x}{x} = 0$$

Esercizio 8

ⓐ $$\lim_{x\to+\infty} \frac{x^2+3x+4}{x^2-2x+5} = \lim_{x\to+\infty} \frac{x^2\left(\overbrace{1+\frac{3}{x}+\frac{4}{x^2}}^{\to 1}\right)}{x^2\left(\underbrace{1-\frac{2}{x}+\frac{5}{x^2}}_{\to 1}\right)} = 1 = \lim_{x\to-\infty} f(x)$$

f. indet.

$$\lim_{x\to 0} \frac{x^2+3x+4}{x^2-2x+5} = \frac{4}{5}$$

ⓑ $$\lim_{x\to+\infty} \frac{9x^3+3x-1}{7x^2+4x} = \lim_{x\to+\infty} \frac{x^3\left(\overbrace{9+\frac{3}{x^2}-\frac{1}{x^3}}^{\to 9}\right)}{x^2\left(\underbrace{7+\frac{4}{x}}_{\to 7}\right)} = \frac{+\infty\cdot 9}{7} = +\infty$$

f. indet.

analogamente, $\lim_{x\to-\infty} f(x) = -\infty$

$$\lim_{x\to 0^+} \frac{9x^3+3x-1}{7x^2+4x} = -\infty\, ; \quad \lim_{x\to 0^-} f(x) = +\infty \Rightarrow \nexists \lim_{x\to 0} f(x)$$

ⓒ $$\lim_{x\to+\infty} \frac{-2x^2+4x+3}{3x^5-4x^4} = \lim_{x\to+\infty} \frac{x^2\left(\overbrace{-2+\frac{4}{x}+\frac{3}{x^2}}^{\to -2}\right)}{x^5\left(\underbrace{3-\frac{4}{x}}_{\to 3}\right)} = \frac{-2}{+\infty\cdot 3} = 0 = \lim_{x\to-\infty} f(x)$$

$$\lim_{x\to 0} \frac{-2x^2+4x+3}{3x^5-4x^4} = \lim_{x\to 0} \frac{\overbrace{-2x^2+4x+3}^{\to 3}}{\underbrace{x^4}_{\to 0^+}\underbrace{(3x-4)}_{\to -4}} = -\infty$$

Esercizio 9

ⓐ $$\lim_{x\to+\infty} \frac{e^{3x}+9e^{2x}+4}{e^{2x}+7e^x}$$

cambio di variabile: $t = e^x$ $\qquad \lim_{x\to+\infty} t = \lim_{x\to+\infty} e^x = +\infty \Rightarrow t\to+\infty$ per $x\to+\infty$

$$\lim_{t\to+\infty} \frac{t^3+9t^2+4}{t^2+7t} = \lim_{t\to+\infty} \frac{\overbrace{t^3}^{\to+\infty}\left(\overbrace{1+\frac{9}{t}+\frac{4}{t^3}}^{\to 1}\right)}{t^2\left(\underbrace{1+\frac{7}{t}}_{\to 1}\right)} = +\infty$$

ⓑ $$\lim_{x\to+\infty} \frac{\tan^2\left(\frac{\pi}{2}-\frac{1}{x}\right)+2\tan\left(\frac{\pi}{2}-\frac{1}{x}\right)+14}{3\tan^2\left(\frac{\pi}{2}-\frac{1}{x}\right)+7\tan\left(\frac{\pi}{2}-\frac{1}{x}\right)-25}$$

cambio di variabile $t = \tan\left(\frac{\pi}{2}-\frac{1}{x}\right)$

$$\lim_{x\to+\infty} t = \lim_{x\to+\infty} \tan\left(\frac{\pi}{2}-\frac{1}{x}\right) = \tan\left(\frac{\pi}{2}-0^+\right) = \tan\left(\frac{\pi}{2}^-\right) = +\infty$$

$\Rightarrow t\to+\infty$ per $x\to+\infty$

$$\lim_{t\to+\infty}\frac{t^2+2t+14}{3t^2+7t-25}=\lim_{t\to+\infty}\frac{t^2\overbrace{\left(1+\frac{2}{t}+\frac{14}{t^2}\right)}^{\to 1}}{t^2\underbrace{\left(3+\frac{7}{t}-\frac{25}{t^2}\right)}_{\to 3}}=\frac{1}{3}$$

Esercizio 10

ⓐ

$$\lim_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{x}\right)=\lim_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{x}\right)\frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=\lim_{x\to+\infty}\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\lim_{x\to+\infty}\frac{1}{\underbrace{\sqrt{x+1}+\sqrt{x}}_{\to+\infty}}=0$$

ⓑ

$$\lim_{x\to+\infty}\left(\sqrt[3]{x^2+1}-\sqrt[3]{x^2+x\sqrt[3]{x}-2}\right)=$$

Per avere due cubi (quindi, per togliere le radici) bisogna moltiplicare per il falso quadrato di un binomio. Piccola dimostrazione:

$$(A+B)^3=(A+B)(A+B^2)$$
$$(A+B)^3=A^3+3A^2B+3AB^2+B^3$$

$$\Rightarrow A^3+3A^2B+3AB^2+B^3=(A+B)(A+B)^2$$

$$A^3+B^3=(A+B)(A^2+2AB+B^2)-3A^2B-3AB^2$$

$$A^3+B^3=(A+B)(A^2+2AB+B^2)-3AB(A+B)$$

$$A^3+B^3=(A+B)(A^2+2AB+B^2-3AB)=(A+B)\underbrace{(A^2+B^2-AB)}_{\text{FALSO QUADRATO del binomio }(A+B)}$$

$$=\lim_{x\to+\infty}\left(\sqrt[3]{x^2+1}-\sqrt[3]{x^2+x\sqrt[3]{x}-2}\right)\cdot\frac{\left(\sqrt[3]{x^2+1}\right)^2+\left(\sqrt[3]{x^2+x\sqrt[3]{x}-2}\right)^2+\sqrt{x^2+1}\sqrt{x^2+x\sqrt[3]{x}-2}}{\left(\sqrt[3]{x^2+1}\right)^2+\left(\sqrt[3]{x^2+x\sqrt[3]{x}-2}\right)^2+\sqrt{x^2+1}\sqrt{x^2+x\sqrt[3]{x}-2}}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^2}+1-\cancel{x^2}-x\sqrt[3]{x}+2}{\left(\sqrt[3]{x^2+1}\right)^2+\left(\sqrt[3]{x^2+x\sqrt[3]{x}-2}\right)^2+\sqrt{x^2+1}\sqrt{x^2+x\sqrt[3]{x}-2}}=$$

$$=\lim_{x\to+\infty}\frac{3-x^{\frac{4}{3}}}{(x^2+1)^{\frac{2}{3}}+\left(x^2+x^{\frac{4}{3}}-2\right)^{\frac{2}{3}}+(x^2+1)^{\frac{1}{3}}\left(x^2+x^{\frac{4}{3}}-2\right)^{\frac{1}{3}}}=$$

$$=\lim_{x\to+\infty}\frac{3-x^{\frac{4}{3}}}{x^{\frac{4}{3}}(1+x^{-2})^{\frac{2}{3}}+x^{\frac{4}{3}}\left(1+x^{-\frac{2}{3}}-2x^{-2}\right)+x^{\frac{2}{3}}(1+x^{-2})^{\frac{1}{3}}x^{\frac{2}{3}}\left(1+x^{-\frac{2}{3}}-2x^{-2}\right)^{\frac{1}{3}}}$$

$$=\lim_{x\to+\infty}\frac{x^{\frac{4}{3}}\overbrace{\left(-1+3x^{-\frac{4}{3}}\right)}^{\to -1}}{x^{\frac{4}{3}}\underbrace{\left[\underbrace{(1+x^{-2})^{\frac{2}{3}}}_{\to 1}+\underbrace{\left(1+x^{-\frac{2}{3}}-2x^{-2}\right)}_{\to 1}+\underbrace{(1+x^{-2})^{\frac{1}{3}}\left(1+x^{-\frac{2}{3}}-2x^{-2}\right)^{\frac{1}{3}}}_{\to 1}\right]}_{\to 3}}=-\frac{1}{3}$$

ⓒ

$$\lim_{x\to+\infty}x\left(x-\sqrt{x^2-1}\right)=\lim_{x\to+\infty}x\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2+1}}=\lim_{x\to+\infty}x\cdot\frac{\cancel{x^2}-\cancel{x^2}+1}{x+\sqrt{x^2+1}}=\lim_{x\to+\infty}\frac{x}{x+\sqrt{x^2+1}}=$$

$$=\lim_{x\to+\infty}\frac{x}{x+|x|\sqrt{1+\frac{1}{x^2}}}=\left(x\to+\infty\Rightarrow x>0\Rightarrow |x|=x\right)=\lim_{x\to+\infty}\frac{\cancel{x}}{\cancel{x}+\cancel{x}\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{1}{1+\sqrt{1+\frac{1}{x^2}}}=\frac{1}{2}$$

ⓓ

$$\lim_{x\to-\infty}x\left(x+\sqrt{x^2-1}\right)=\lim_{x\to-\infty}x\left(x+\sqrt{x^2-1}\right)\cdot\frac{x-\sqrt{x^2-1}}{x-\sqrt{x^2+1}}=\lim_{x\to-\infty}x\cdot\frac{\cancel{x^2}-\cancel{x^2}+1}{x-\sqrt{x^2+1}}=\lim_{x\to-\infty}\frac{x}{x-\sqrt{x^2+1}}=$$

$$=\lim_{x\to+\infty}\frac{x}{x-|x|\sqrt{1+\frac{1}{x^2}}}=\left(x\to-\infty\Rightarrow x<0\Rightarrow |x|=-x\right)=\lim_{x\to+\infty}\frac{\cancel{x}}{\cancel{x}+\cancel{x}\sqrt{1+\frac{1}{x^2}}}=\lim_{x\to+\infty}\frac{1}{1+\sqrt{1+\frac{1}{x^2}}}=\frac{1}{2}$$

(e) $$\lim_{x\to+\infty}\left[\log_2(x+1)-\log_2\left(\sqrt{x^2-3}\right)\right]=\lim_{x\to+\infty}\log_2\left(\frac{x+1}{\sqrt{x^2-3}}\right)=\lim_{x\to+\infty}\log_2\left(\frac{x+1}{|x|\sqrt{1-\frac{3}{x^2}}}\right)=\quad(x\to+\infty\Rightarrow x>0\Rightarrow|x|=x)$$

$$=\lim_{x\to+\infty}\log_2\left(\frac{x\overbrace{\left(1+\frac{1}{x}\right)}^{\to 1}}{x\underbrace{\sqrt{1-\frac{3}{x^2}}}_{\to\sqrt{1}=1}}\right)=\log_2 1=0$$

(f) $$\lim_{x\to0^+}\frac{\log_2(2x)}{\log_2 x}=\lim_{x\to0^+}\frac{\log_2 2+\log_2 x}{\log_2 x}=\lim_{x\to0^+}\frac{1+\log_2 x}{\log_2 x}=\lim_{x\to0^+}\overbrace{\frac{1}{\log_2 x}}^{\to 0^-}+1=1$$

(g) $$\lim_{x\to+\infty}\left[5\log_9(2x-5)-3\log_9(x+2)+2\log_9(x+1)\right]=\lim_{x\to+\infty}\left[\log_9(2x-5)^5-\log_9(x+2)^3+\log_9(x+1)^2\right]=$$

$$=\lim_{x\to+\infty}\left(\frac{(2x+5)^5(x+1)^2}{(x+2)^3}\right)=\lim_{x\to+\infty}\left(\frac{x^5\overbrace{\left(2+\frac{5}{x}\right)^5}^{\to 32}\cdot x^2\overbrace{\left(1+\frac{1}{x}\right)^2}^{\to 1}}{x^3\underbrace{\left(1+\frac{2}{x}\right)^3}_{\to 1}}\right)^{\to+\infty}=\log(+\infty)=+\infty$$

(h) $$\lim_{x\to+\infty}\frac{e^{2x}+9e^{x}+4}{7e^{2x}-2e^{x}+1}=\lim_{x\to+\infty}\frac{e^{2x}\overbrace{(1+9e^{-x}+4e^{-2x})}^{\to 1}}{e^{2x}\underbrace{(7-2e^{-x}+e^{-2x})}_{\to 7}}=\frac{1}{7}$$

Esercizio 11

(a) $x\in\mathbb{R}$ è punto di accumulazione destro o sinistro di $A\in\mathbb{R}$ $\Rightarrow$ $\forall\delta>0$ si ha $(x-\delta,x)\cap A\neq\emptyset$ oppure $(x,x+\delta)\cap A\neq\emptyset$

$\Rightarrow$ $\forall\delta>0$ si ha $\big((x-\delta,x)\cap A\big)\cup\big((x,x+\delta)\cap A\big)\neq\emptyset$

Facendo un ragionamento logico sugli insiemi: $\big((x-\delta,x)\cap A\big)\cup\big((x,x+\delta)\cap A\big)=$

$=\big((x-\delta,x)\cup(x,x+\delta)\big)\cap A=$

$=\big((x-\delta,x+\delta)\cap A\big)\setminus\{x\}$

$\Rightarrow$ $\forall\delta>0$ si ha $\big((x-\delta,x+\delta)\cap A\big)\setminus\{x\}\neq\emptyset$ $\Rightarrow$ Per definizione, x è punto di accumulazione di A.

(b) $x\in\mathbb{R}$ è punto di accumulazione di $A\subset\mathbb{R}$ $\Rightarrow$ $\forall\delta>0$ si ha che $\big((x-\delta,x+\delta)\cap A\big)\setminus\{x\}\neq\emptyset$

Si procede al contrario rispetto al punto (a).

Facendo un ragionamento logico sugli insiemi: $\big((x-\delta,x+\delta)\cap A\big)\setminus\{x\}=$

$=\big((x-\delta,x)\cup(x,x+\delta)\big)\cap A=$

$=\big((x-\delta,x)\cap A\big)\cup\big((x,x+\delta)\cap A\big)$

$\Rightarrow$ $\forall\delta>0$ si ha $\big((x-\delta,x)\cap A\big)\cup\big((x,x+\delta)\cap A\big)\neq\emptyset$

$\Rightarrow$ $\forall\delta>0$ si ha $(x-\delta,x)\cap A\neq\emptyset$ oppure $(x,x+\delta)\cap A\neq\emptyset$ $\Rightarrow$ Per definizione, x è punto di accumulazione destro o sinistro o entrambi.

ESERCIZIO 12

ⓐ $f(x) = \operatorname{settcosh}(\log(\log x))$

$$\text{dom}f: \begin{cases} \log(\log x) \geq 0 \\ \log x > 0 \\ x > 0 \end{cases} \quad \begin{cases} \log x \geq 1 \\ x > 1 \\ x > 0 \end{cases} \quad \begin{cases} x \geq e \\ x > 1 \\ x > 0 \end{cases} \Rightarrow x \geq e$$

$\text{dom}f = [e, +\infty) \rightarrow$ pt. acc. $= [e, +\infty) \cup \{+\infty\}$

ⓑ $f(x) = \operatorname{settsinh}\left(\sqrt[3]{\frac{3+x^2}{9-3x}}\right)$

$\text{dom}f:\ 9-3x \neq 0 \Leftrightarrow x \neq 3 \Rightarrow \text{dom}f = \mathbb{R} \setminus \{3\} \Rightarrow$ pt. acc. $= \mathbb{R}^*$

ESERCIZIO 13

$\text{dom}f = \mathbb{R}$, $\lim_{x \to +\infty} f(x) = \ell \in \mathbb{R} \Rightarrow \forall \varepsilon > 0, \exists M \in \mathbb{R} : |f(x) - \ell| < \varepsilon \ \forall x \in \mathbb{R} : x > M$

ⓐ $\lim_{x \to -\infty} f(x)$ cambio di variabile: $t = -x$ $\quad \lim_{x \to -\infty} t = \lim_{x \to -\infty} -x = +\infty \Rightarrow t \to +\infty$

$\lim_{t \to +\infty} f(-t) = (f \text{ è pari}) = \lim_{t \to +\infty} f(t)$ Questo limite esiste ed è pari a ℓ.

ⓑ $\lim_{x \to -\infty} f(x)$ cambio di variabile: $t = -x$ $\quad \lim_{x \to -\infty} t = \lim_{x \to -\infty} -x = +\infty \Rightarrow t \to +\infty$

$\lim_{t \to +\infty} f(-t) = (f \text{ è dispari}) = \lim_{t \to +\infty} [-f(t)]$ Questo limite esiste ed è pari a ℓ.

Per il teorema sull'algebra dei limiti: $\lim_{t \to +\infty} [-f(t)] = -\underbrace{\lim_{t \to +\infty} f(t)} = -\ell$

→ Questo limite esiste ed è pari a ℓ.