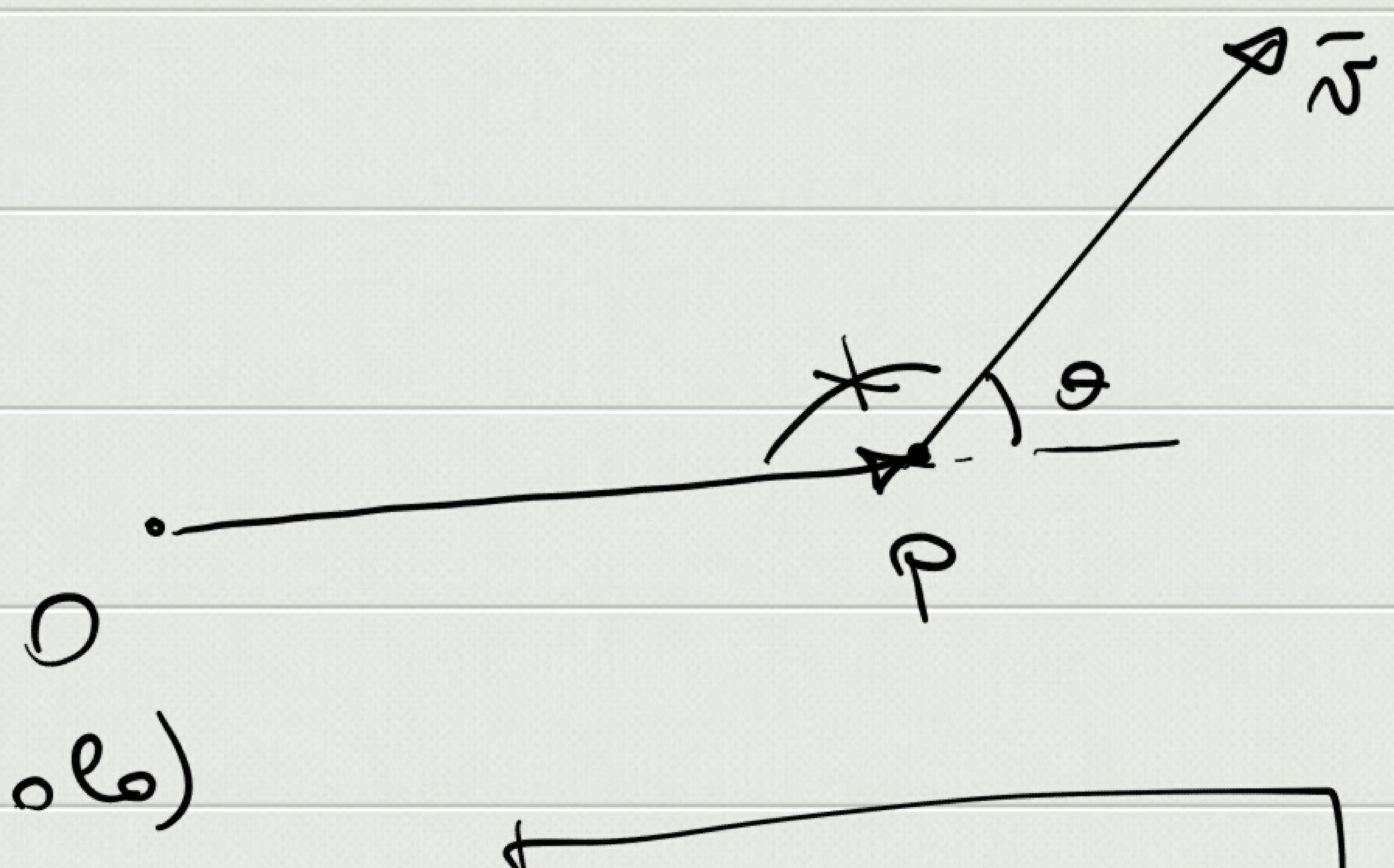
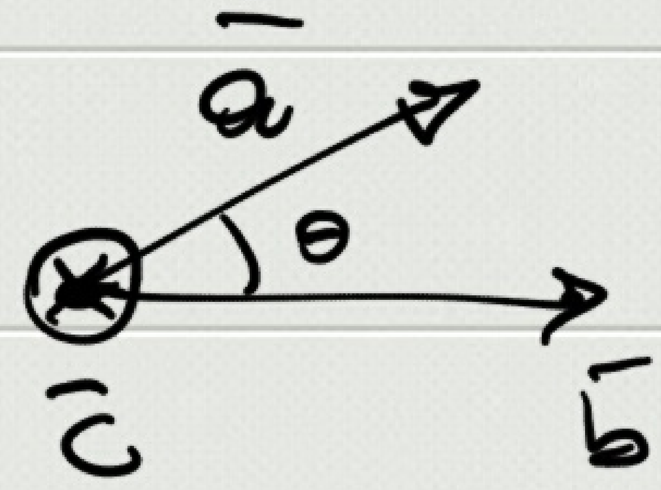
Momento di un vettore

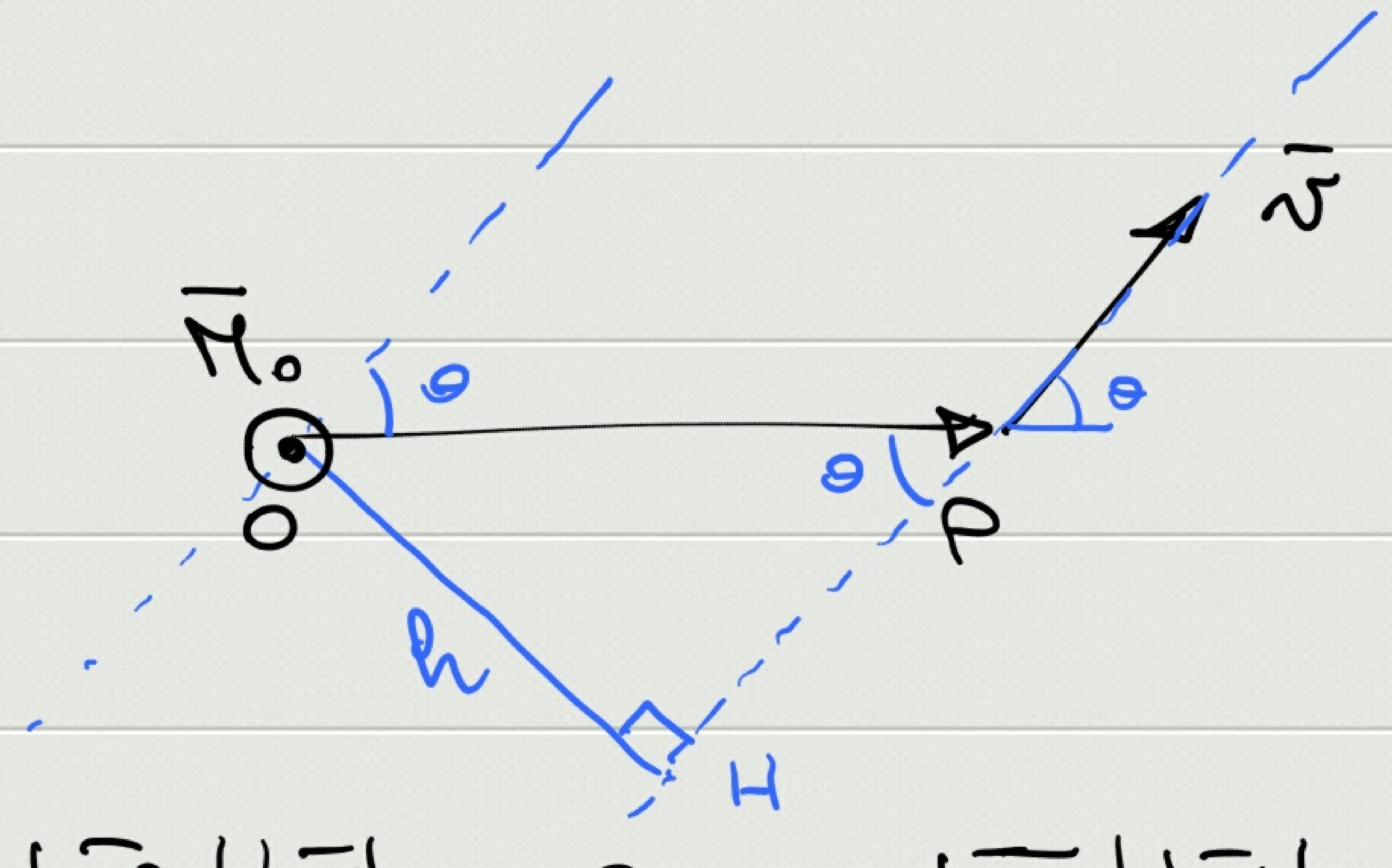


$$\boxed{\bar{M}_O = \overline{OP} \times \bar{v}}$$

$\bar{a} \times \bar{b} = \bar{c}$

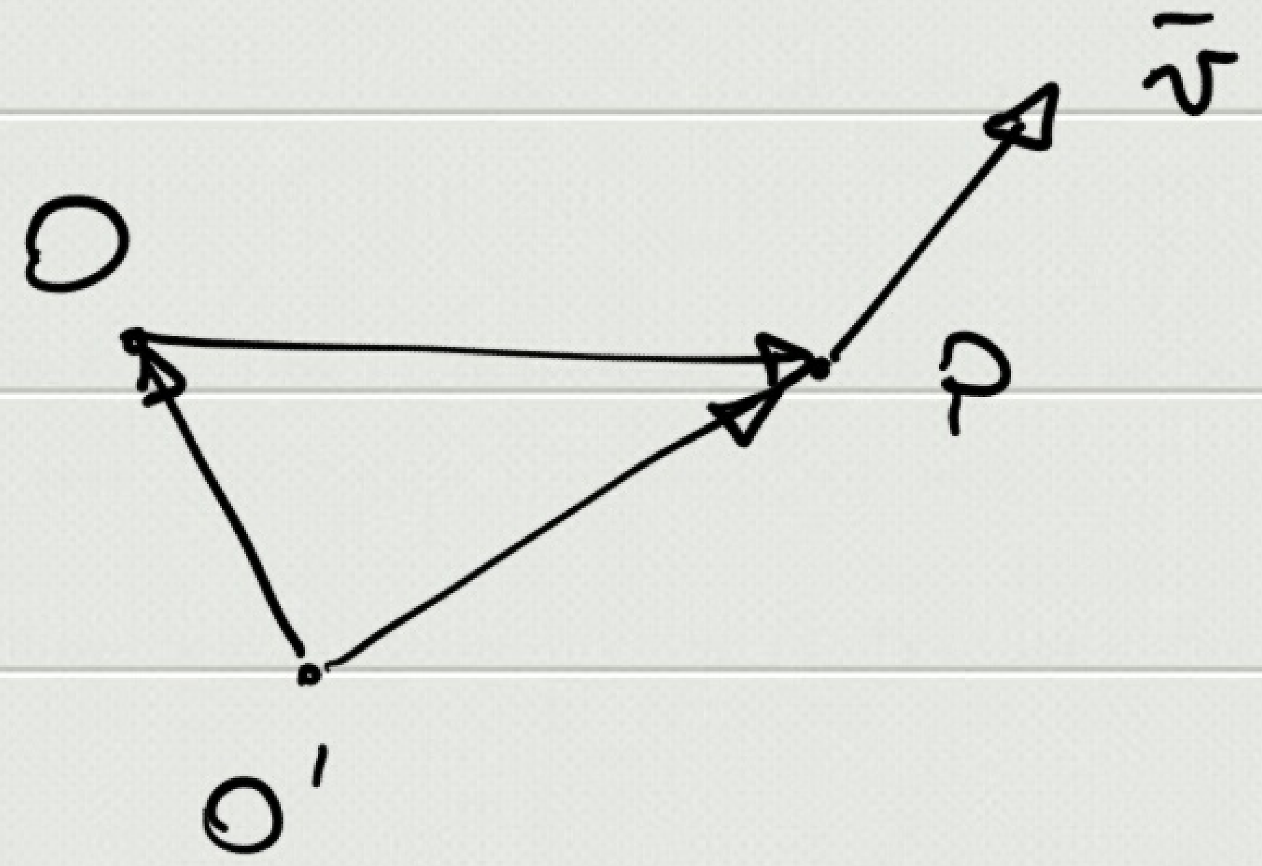


$|\bar{c}| = a\,b \sin\theta$



$$|\bar{M}_O| = |\overline{OP}|\,|\bar{v}| \sin\theta \quad = |\overline{OH}|\,|\bar{v}|$$

h = braccio



$$\bar{M}_O = \overline{OP} \times \bar{v}$$

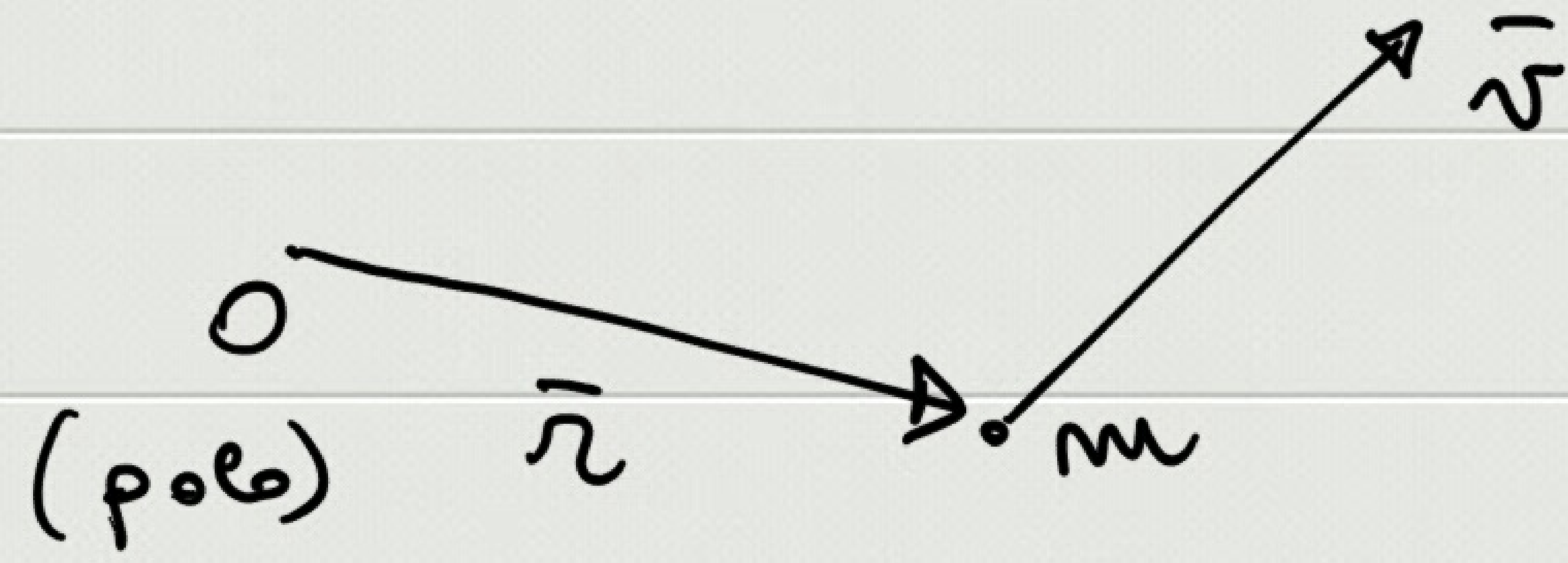
$$\bar{M}_{O'} = \overline{O'P} \times \bar{v} = (\overline{O'O} + \overline{OP}) \times \bar{v} =$$

$$= \overline{O'O} \times \bar{v} + \overline{OP} \times \bar{v} = \overline{O'O} \times \bar{v} + \bar{M}_O$$

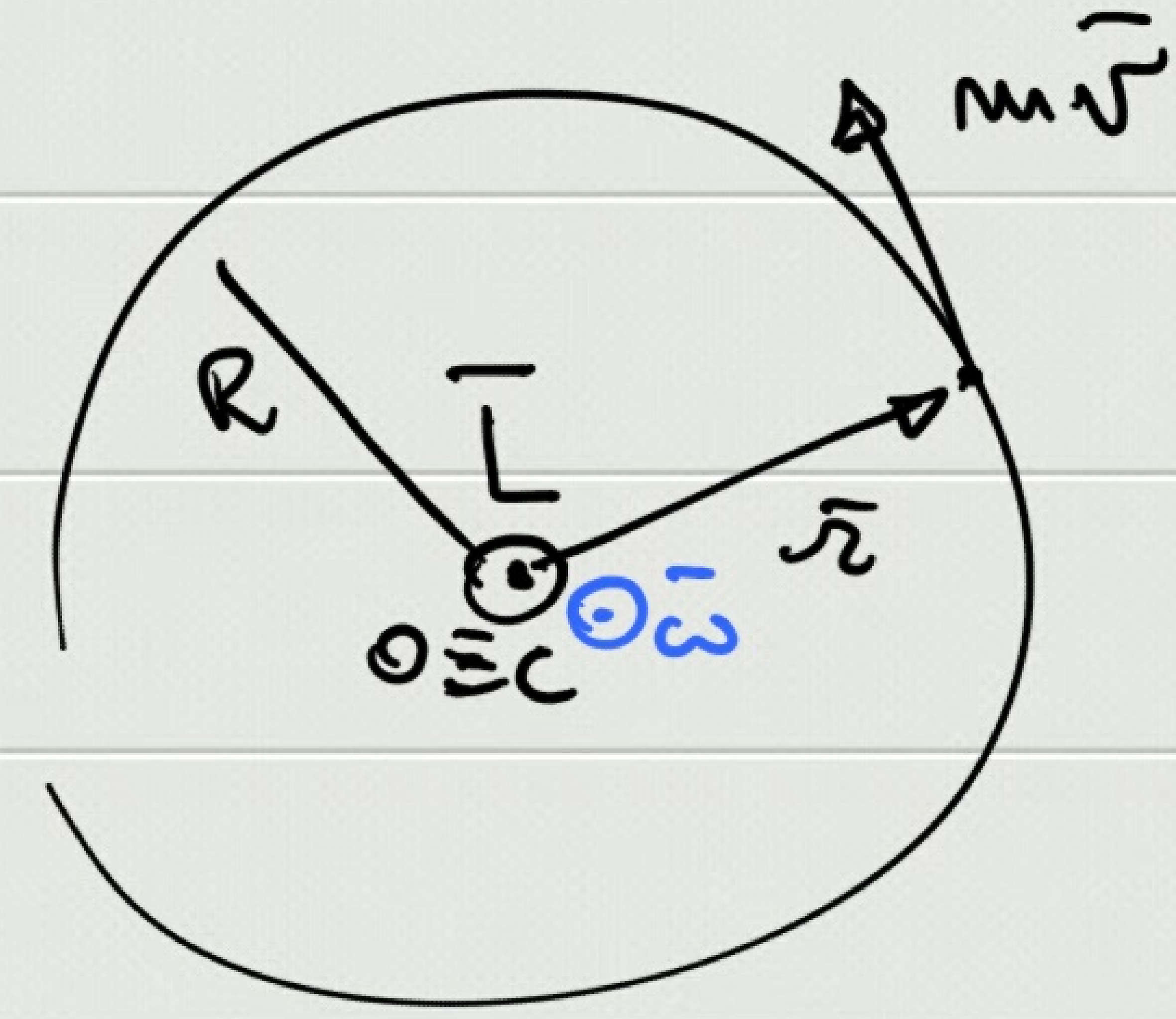
$$\bar{M}_{O'} \neq \bar{M}_O$$

$$\bar{M}_{O'} = \bar{M}_O \quad : \quad \overline{O'O} \times \bar{v} = 0 \qquad \overline{O'O} \parallel \bar{v}$$

Momento angolare : $\boxed{\bar{L} = \bar{r} \times m\bar{v}}$



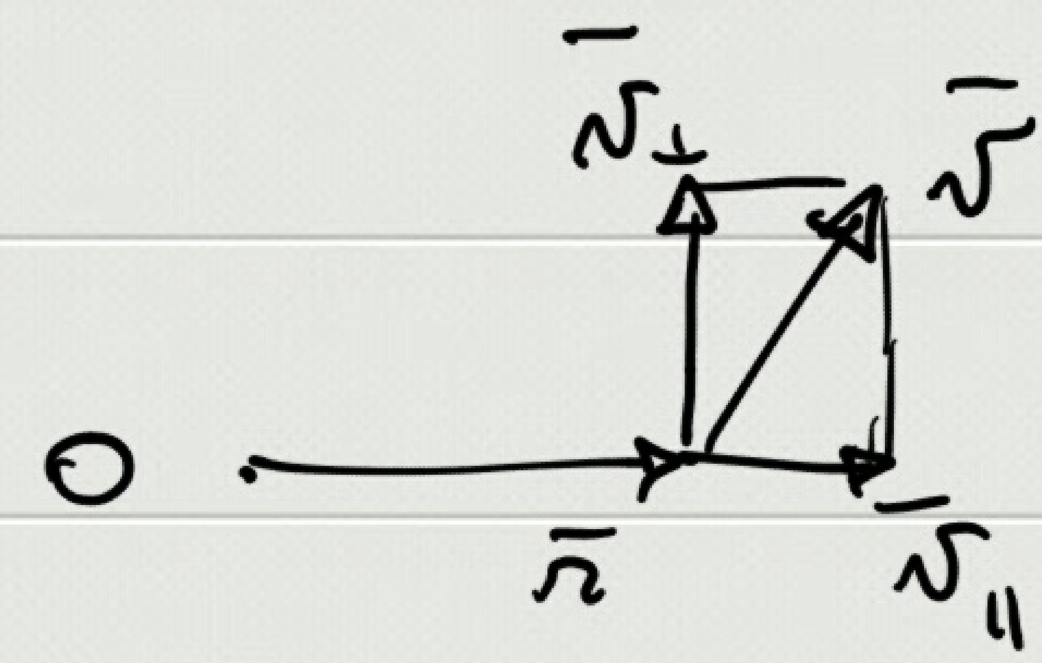
$\bar{L}_{O'} \neq \bar{L}_O \qquad (\overline{O'O} \not\parallel \bar{v})$



$\bar{L}_O = \bar{r} \times m\bar{v} \qquad \bar{v} = \bar{\omega} \times \bar{r}$

$\overset{!}{=} \bar{r} \times m(\bar{\omega} \times \bar{r})$

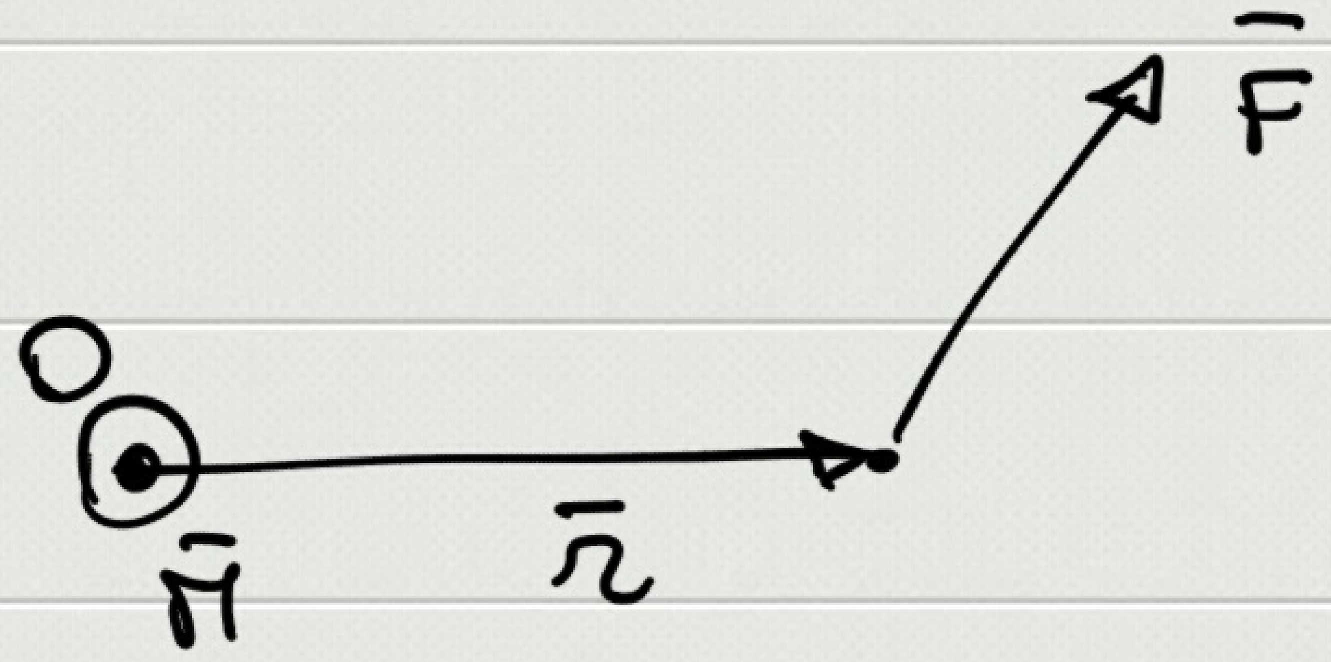
$= mR^2\omega\,\bar{u}_\omega = mR^2\bar{\omega}$



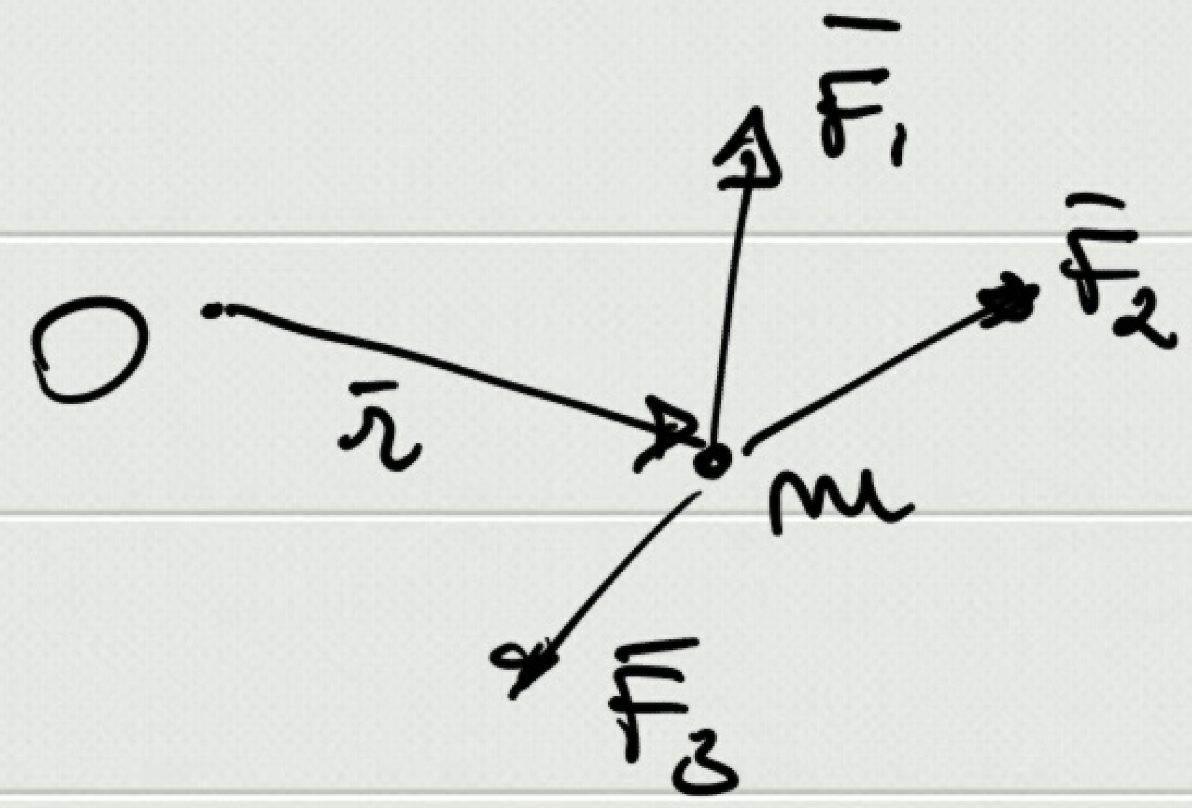
$\bar{L}_O = \bar{r} \times m\bar{v} =$

$= \bar{r} \times m(\bar{v}_\parallel + \bar{v}_\perp) = \bar{r} \times m\bar{v}_\perp$

Momento di una forza : $\bar{M}_O = \bar{r} \times \bar{F}$



$\bar{M}_{O'} \neq \bar{M}_O$



$$\bar{M}_{TOT,O} = \sum_i \bar{M}_i = \bar{r} \times \bar{F}_1 + \bar{r} \times \bar{F}_2 + \dots =$$

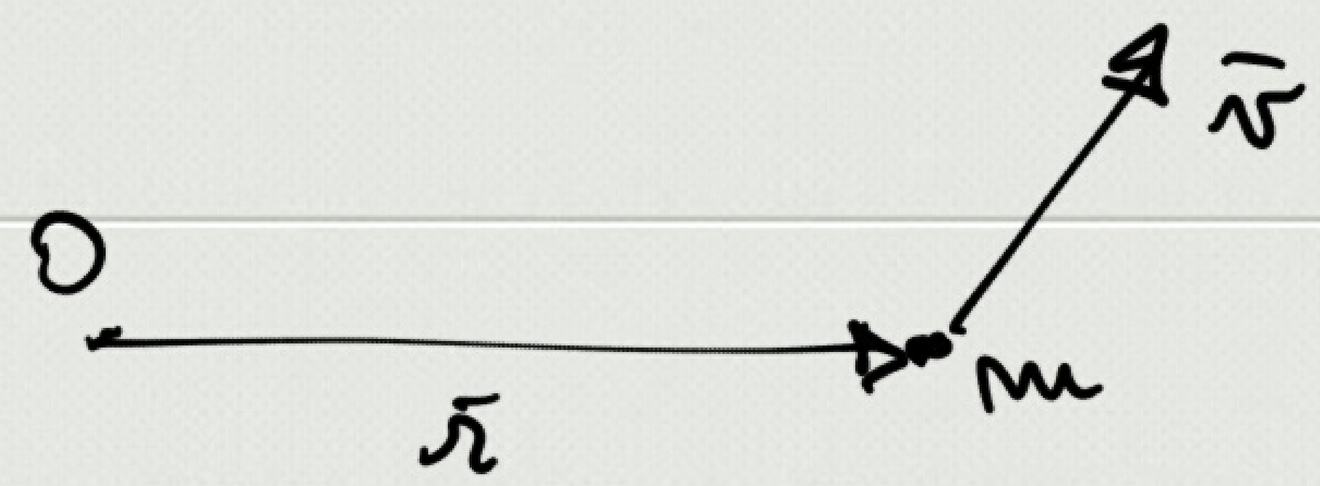
$$= \bar{r} \times (\bar{F}_1 + \bar{F}_2 + \dots) = \bar{r} \times \sum_i \bar{F}_i = \bar{r} \times \bar{R}$$

$[M] = [F \cdot l] = N\,m$

$[L] = [l\,m\,v] = kg\,m^2\,s^{-1} = N\,m\,s$

$N = kg\,m\,s^{-2}$

$$\frac{d\bar{L}_0}{dt} = \frac{d}{dt}(\bar{r} \times m\bar{v}) = \frac{d\bar{r}}{dt} \times m\bar{v} + \bar{r} \times m\frac{d\bar{v}}{dt} =$$



O fermo $(\bar{v}_0 = 0) \Rightarrow \frac{d\bar{r}}{dt} = \bar{v}$

$$= \cancel{\bar{v} \times m\bar{v}} + \bar{r} \times m\bar{a} = \bar{r} \times \bar{F} = \bar{M}_0$$

T. del momento angolare per un punto

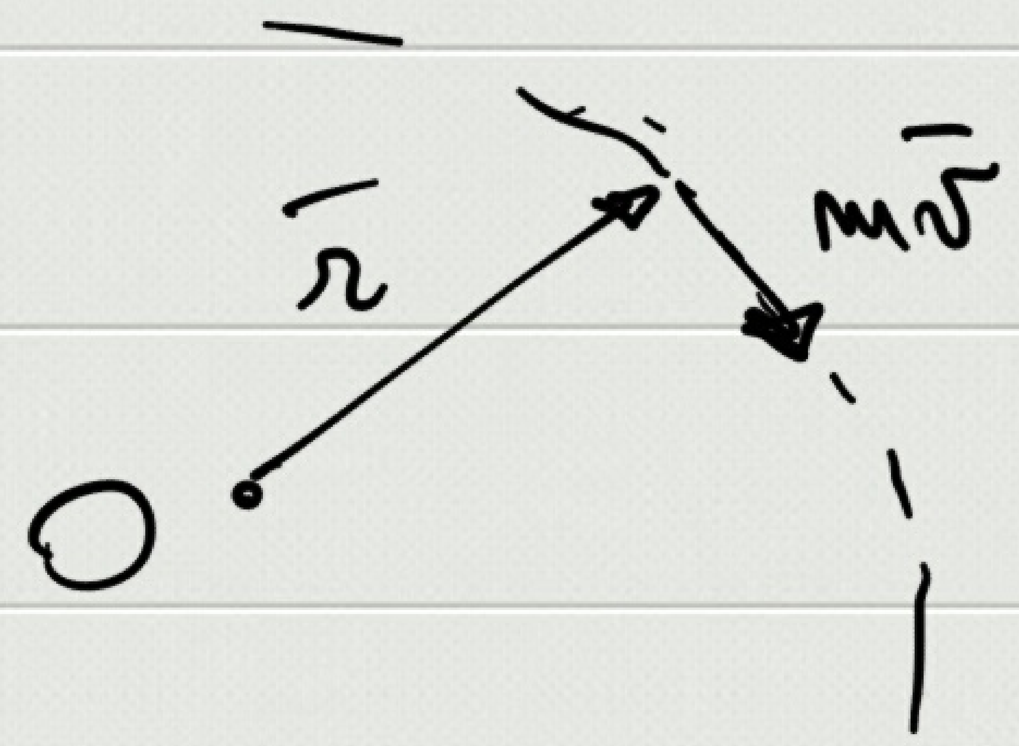
$$\boxed{\frac{d\bar{L}_0}{dt} = \bar{M}_0}$$

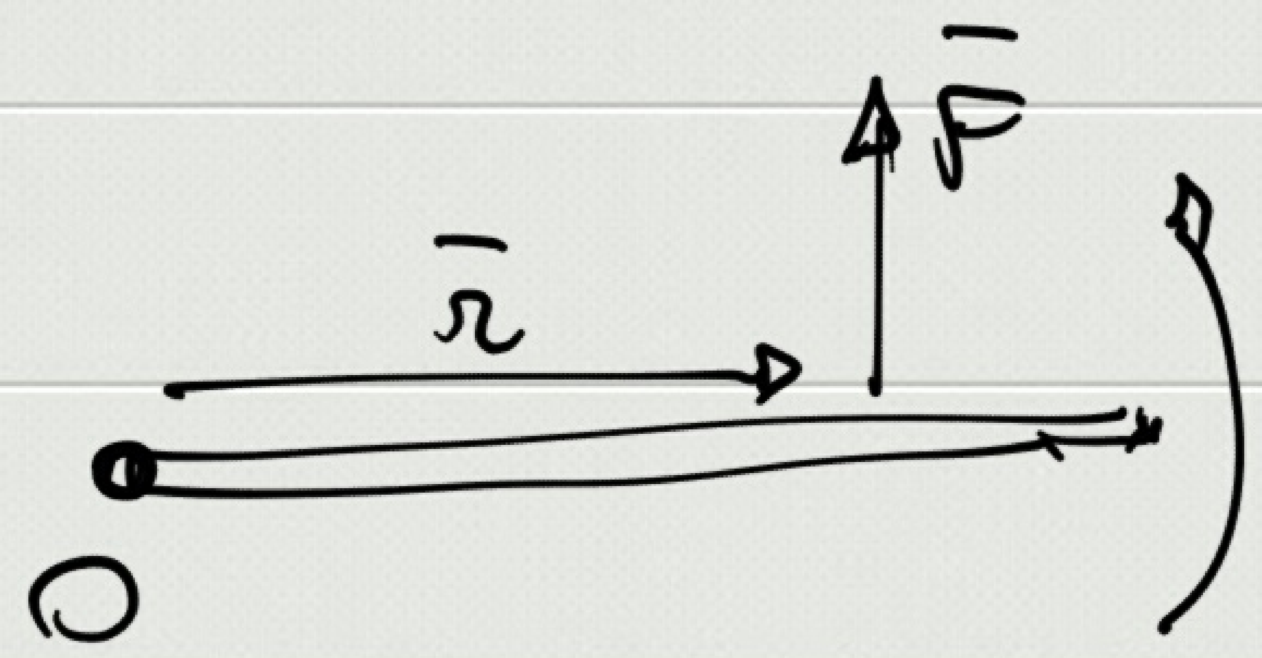
$$\bar{M}_0 = 0 \Rightarrow \frac{d\bar{L}_0}{dt} = 0 \Rightarrow \boxed{\bar{L}_0 = \text{cost}}$$

$$\bar{v} \qquad \frac{d}{dt} \qquad \bar{a}$$

$$\bar{p} = m\bar{v} \qquad " \qquad \bar{F} = m\bar{a}$$

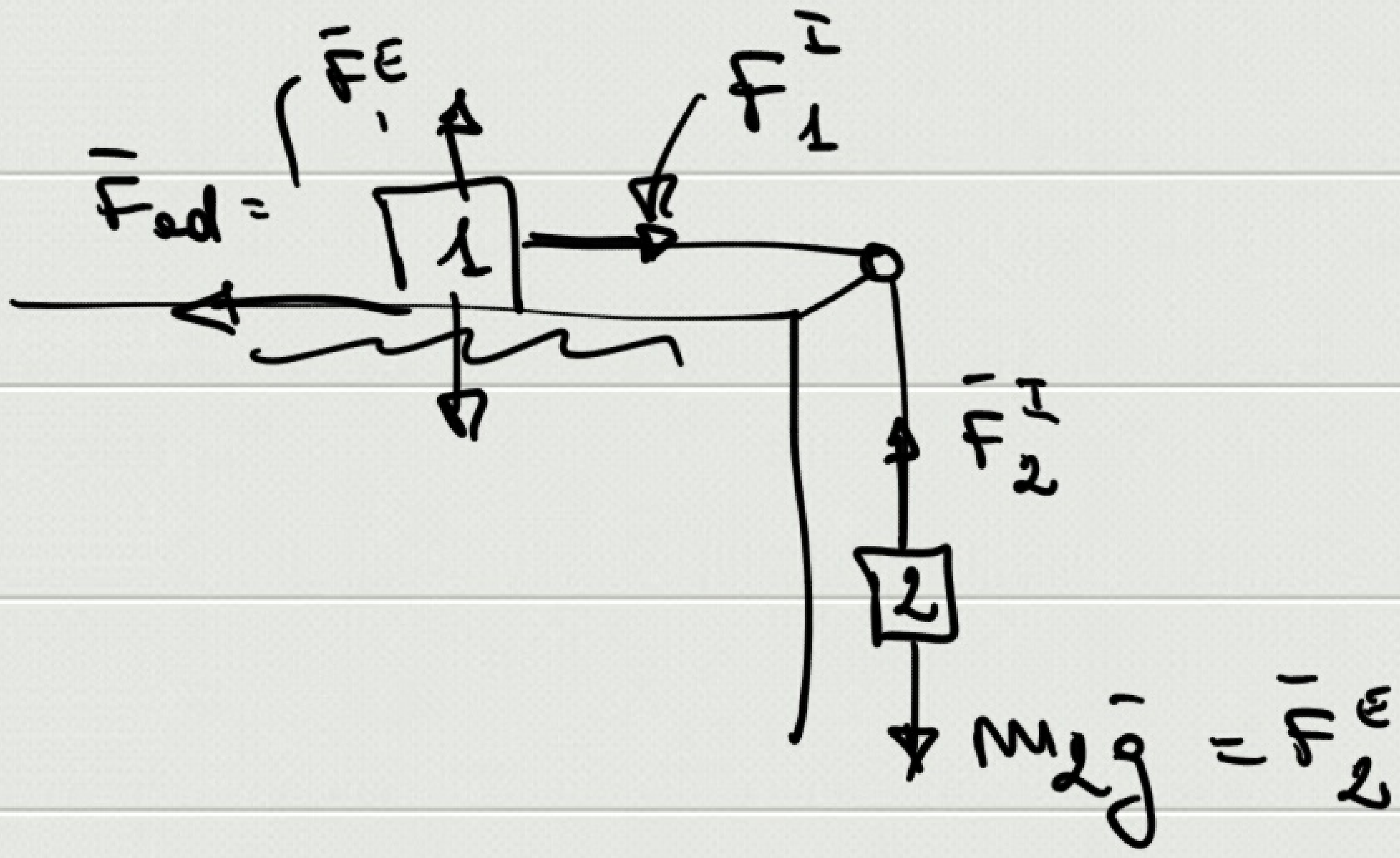
$$\bar{L}_o = \bar{r} \times m\bar{v} \qquad " \qquad \bar{M}_o = \bar{r} \times m\bar{a} \qquad *$$

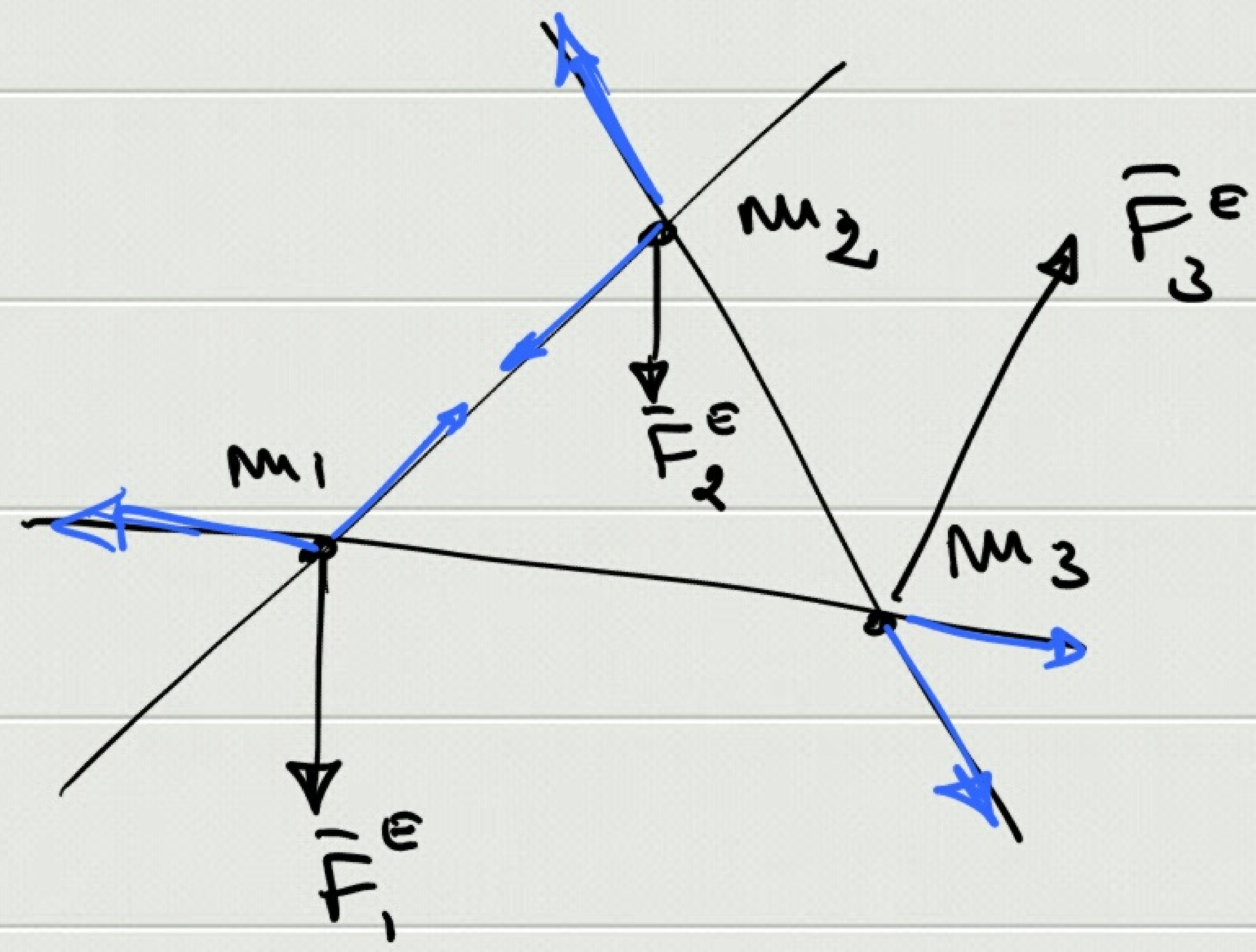




Sistema di punti materiali

$$\bar{F}_i = \bar{F}_i^I + \bar{F}_i^E$$





$$\bar F_{ij} = - \bar F_{ji}$$

$$\bar R^I = \sum_i \bar F_i^I = \sum_i \left(\sum_{j \neq i} \bar F_{ji} \right) = 0$$

$$\bar P = \sum_i \bar p_i = \sum_i m_i \bar v_i$$

$$\bar L_o = \sum_i \bar L_{i,o} = \sum_i \left(\bar r_i \times m_i \bar v_i \right)$$

$$E_k = \sum_i E_{k,i} = \sum_i \frac{1}{2} m_i v_i^2$$