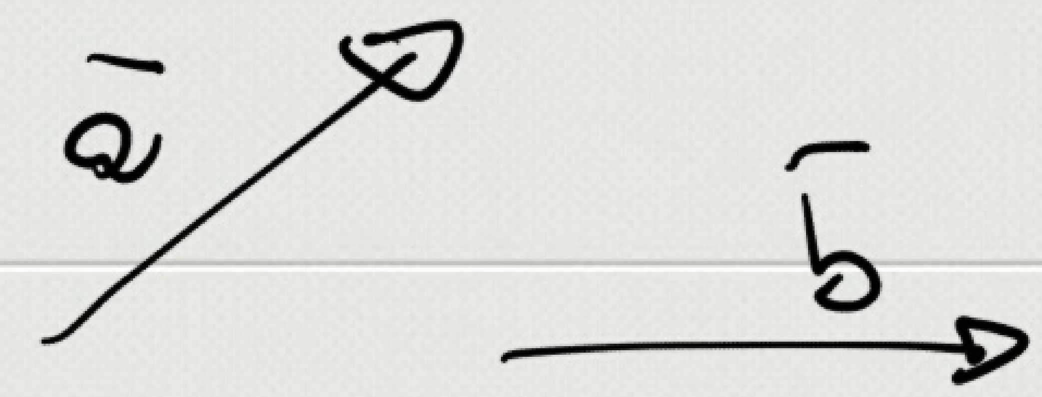
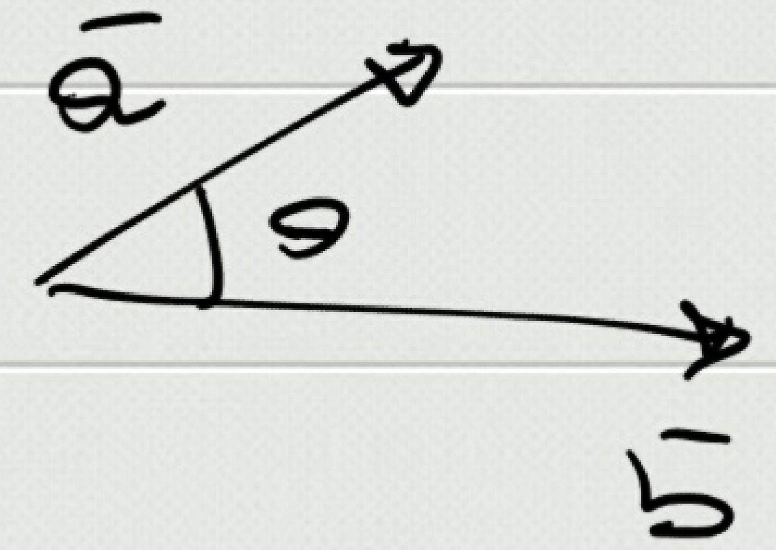
Prodotto scalare

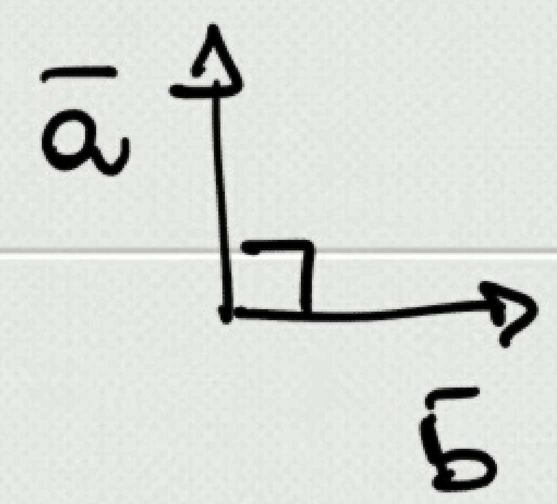


$$\boxed{\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta}$$



$$= a\, b \cos\theta$$

- $\bar{a} \cdot \bar{b} = 0 \quad \begin{cases} a, b = 0 \\ \cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2} \end{cases}$



- $\bar{a}\,\bar{b} = a\, b \cos\theta = b\, a \cos\theta = \bar{b}\,\bar{a}$

- $\bar{b} = a \Rightarrow \bar{a} \cdot \bar{b} = \bar{a} \cdot \bar{a} = a\, a \cos 0° = a^2$

$$\Rightarrow \boxed{\bar{a} \cdot \bar{a} = a^2}$$

- $\cancel{\bar{a} \cdot \bar{b} \cdot \bar{c}}$

- $\bar{a} \cdot (\bar{b} + \bar{c}) = \bar{a} \cdot \bar{b} + \bar{a} \cdot \bar{c}$

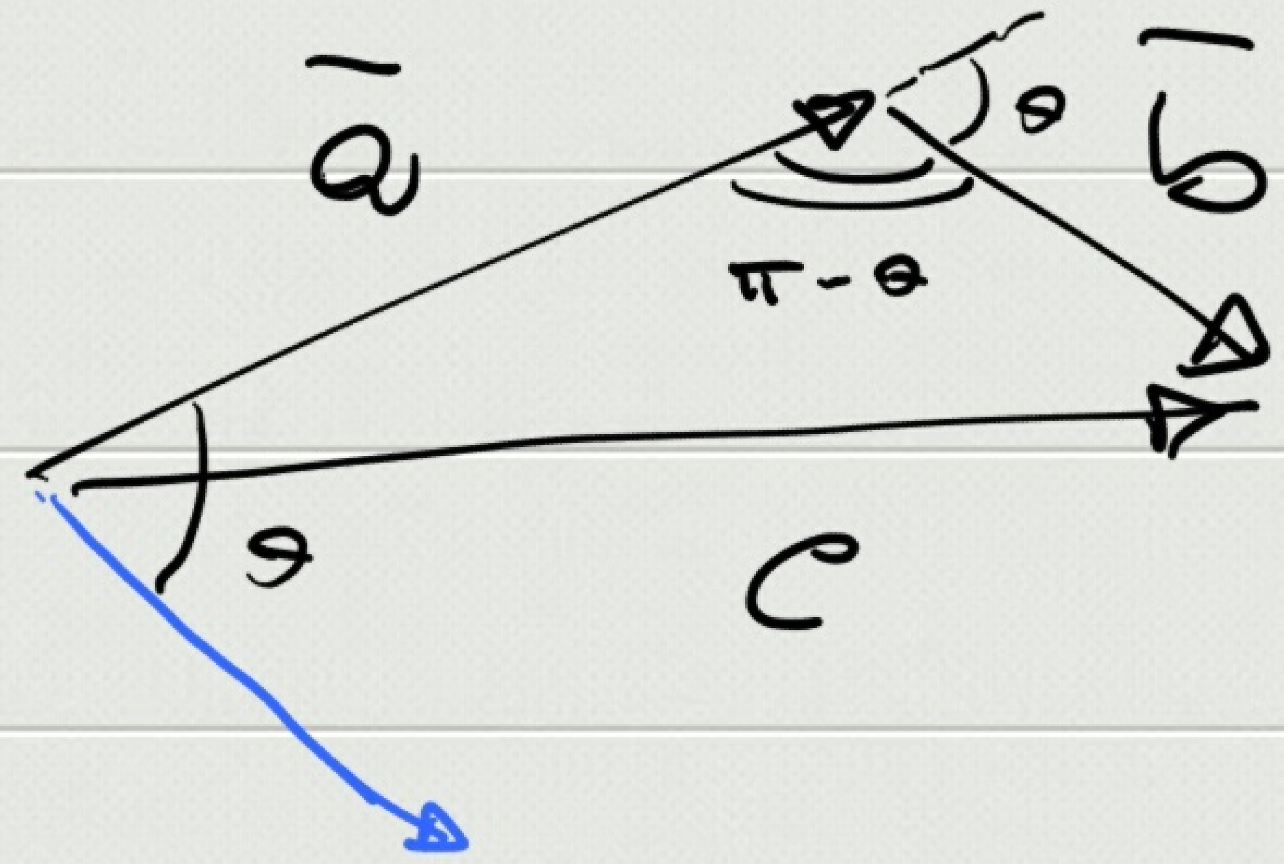
$$\bar{c} = \bar{a} + \bar{b}$$

$$c^2 = \bar{c} \cdot \bar{c} = (\bar{a} + \bar{b}) \cdot (\bar{a} + \bar{b}) =$$

$$= \bar{a} \cdot \bar{a} + \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{a} + \bar{b} \cdot \bar{b} =$$

$$= a^2 + b^2 + 2\, \bar{a} \cdot \bar{b} =$$

$$= a^2 + b^2 + 2ab\cos\theta$$



$$= a^2 + b^2 - 2ab\cos(\pi - \theta)$$

$$\bar{a} = a_x \bar{u}_x + a_y \bar{u}_y + a_z \bar{u}_z$$

$$\bar{b} = b_x \bar{u}_x + b_y \bar{u}_y + b_z \bar{u}_z$$

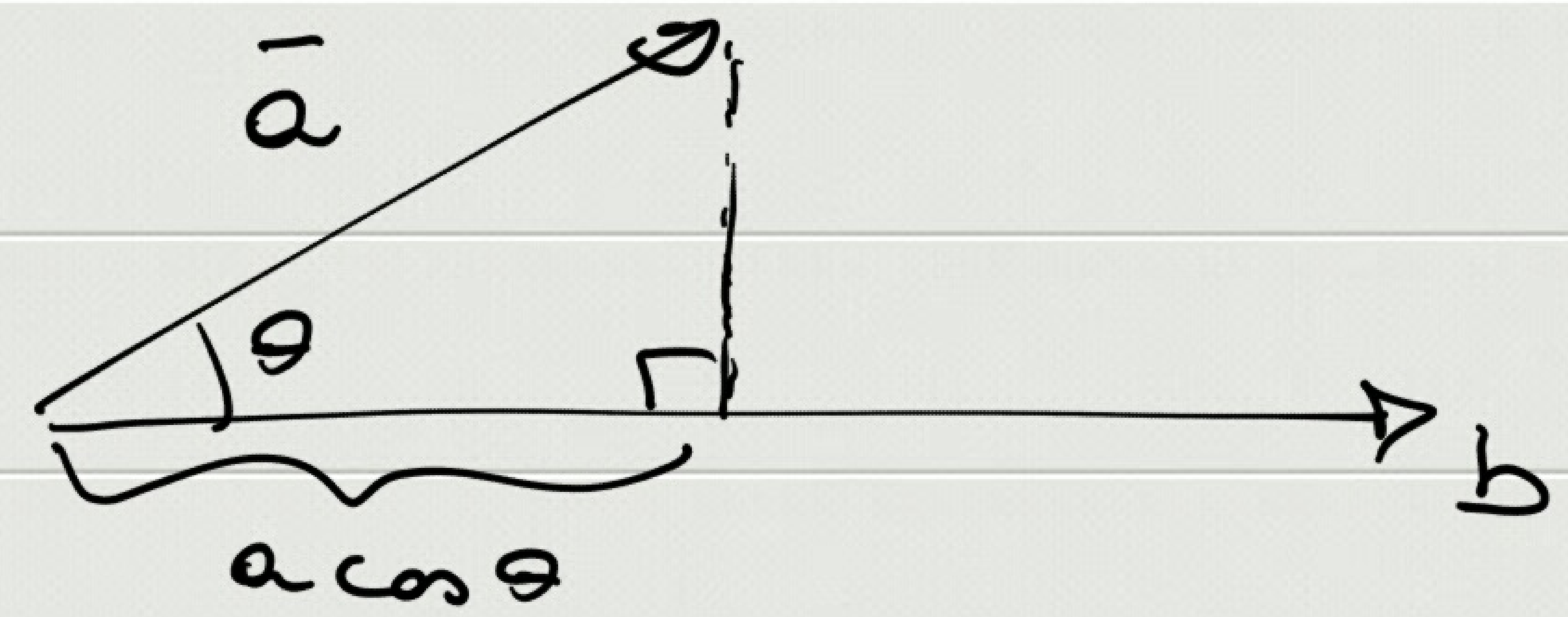
$$\bar{u}_i \cdot \bar{u}_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\bar{a} \cdot \bar{b} = (a_x \bar{u}_x + a_y \bar{u}_y + \dots) \cdot (b_x \bar{u}_x + b_y \bar{u}_y + \dots) =$$

$$= a_x b_x + a_y b_y + a_z b_z$$

$$\bar{a} \cdot \bar{a} = a_x^2 + a_y^2 + a_z^2$$

$$\stackrel{!}{=} a^2$$



$$\bar{a} \cdot \bar{b} = a\,b \cos\theta = (a \cos\theta)\, b$$