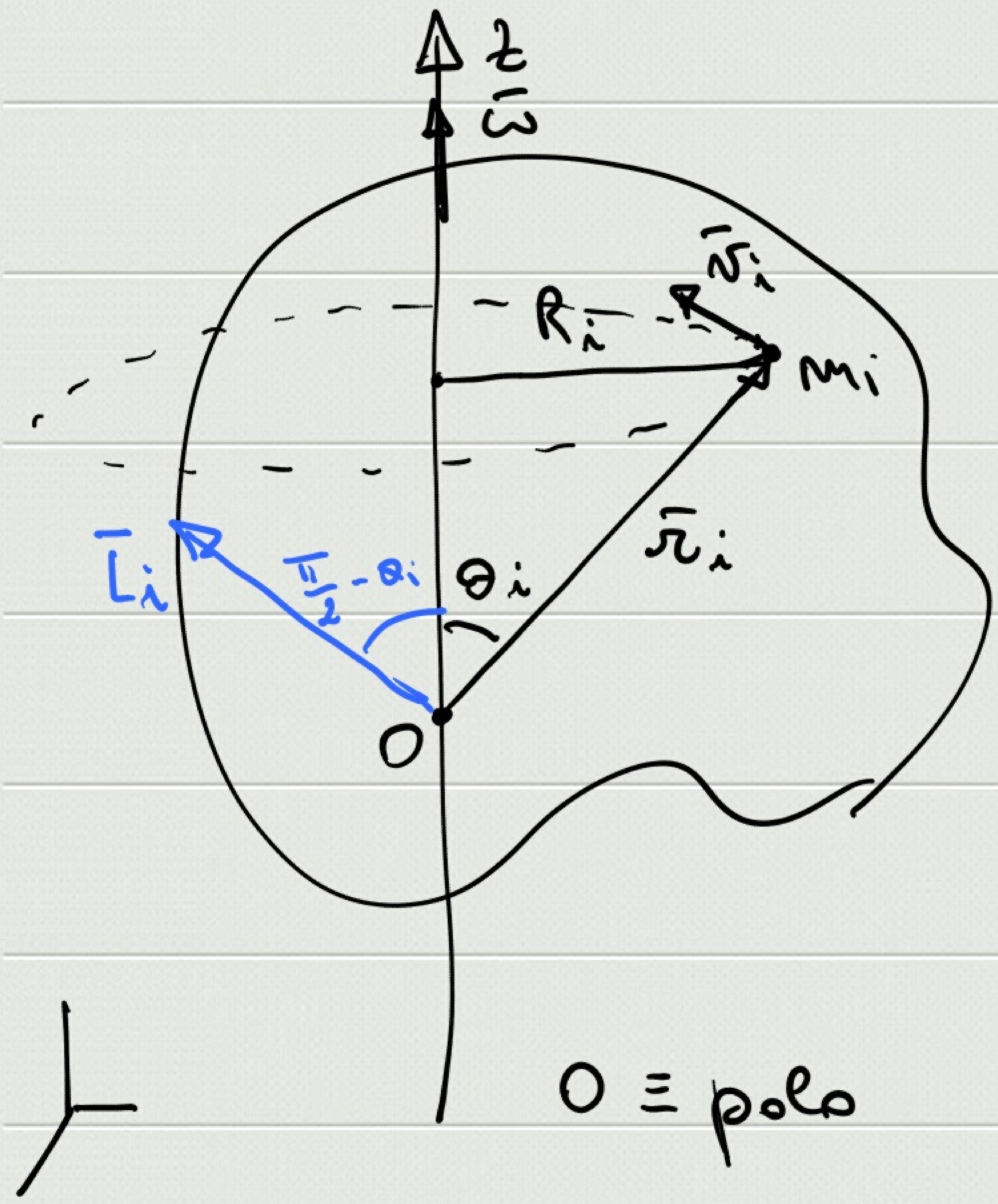


$$v_i = \omega R_i$$

$$\bar{L}_i = \bar{r}_i \times m_i \bar{v}_i$$

$$|\bar{L}_i| = r_i m_i v_i = r_i m_i \omega R_i$$

$O \equiv$ polo

$$L_{iz} = m_i r_i R_i \omega \cos\left(\frac{\pi}{2} - \theta_i\right) = m_i r_i R_i \omega \sin\theta_i =$$

$$= m_i R_i^2 \omega$$

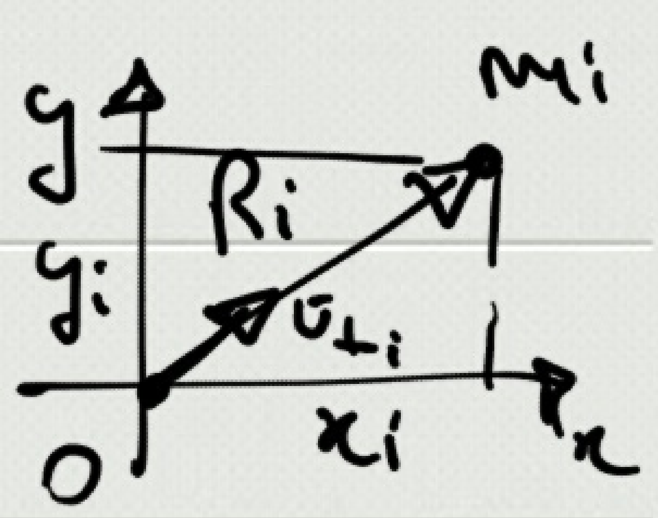
$$L_{i\perp} = m_i r_i R_i \omega \sin\left(\frac{\pi}{2} - \theta_i\right) = m_i r_i R_i \omega \cos\theta_i =$$

$$= m_i z_i R_i \omega$$

$$L_z = \left(\sum_i m_i R_i^2\right)\omega = I_{zz}\,\omega$$: non dipende dal polo

$$\bar{L}_\perp = \left(\sum_i m_i z_i R_i \bar{u}_{\perp i}\right)\omega$$: dipende dal polo

$$= \Big(\underbrace{\sum_i m_i x_i z_i}_{I_{xz}} \bar{u}_x + \underbrace{\sum_i m_i y_i z_i}_{I_{yz}} \bar{u}_y\Big)\omega$$

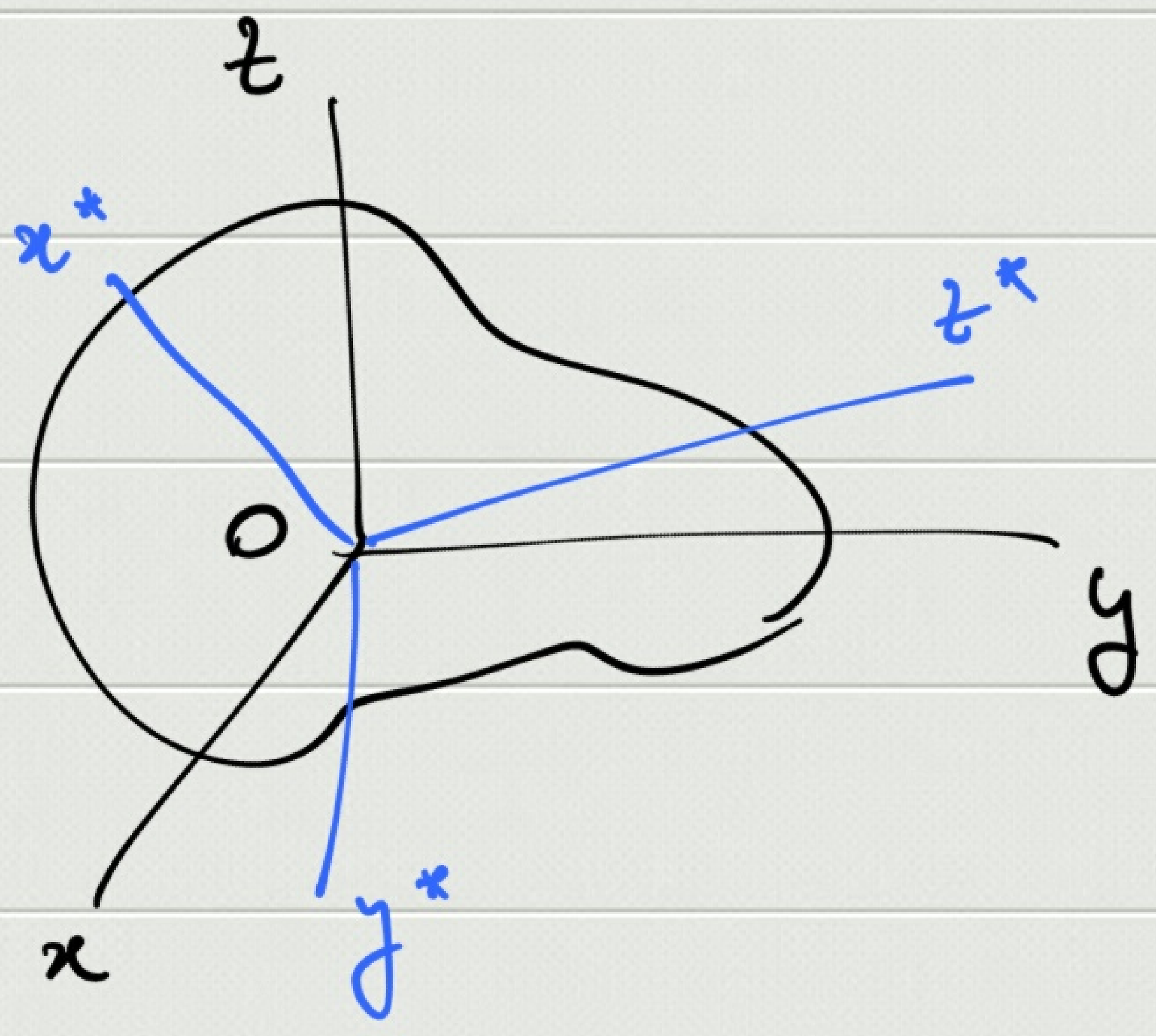


$$\Rightarrow \bar{L}_O = \left(I_{xz}\,\bar{u}_x + I_{yz}\,\bar{u}_y + I_{zz}\,\bar{u}_z \right)\omega \qquad (\bar{\omega} = \omega\,\bar{u}_z)$$

$$\Rightarrow \begin{pmatrix} L_{O,x} \\ L_{O,y} \\ L_{O,z} \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\boxed{\bar{L}_O = \bar{\bar{I}}\,\bar{\omega}}$$

$\bar{\bar{I}} \rightarrow$ tensore di inerzia



$Oxyz \rightarrow \bar{\bar{I}}$

$Ox^*y^*z^* \rightarrow$

$$\bar{\bar{I}}^* = \begin{pmatrix} I_x & & 0 \\ & I_y & \\ 0 & & I_z \end{pmatrix}$$

Assi principali di inerzia

$$\Rightarrow \boxed{\bar{L}_O = \left(I_x\,\omega_x\,\bar{u}_x + I_y\,\omega_y\,\bar{u}_y + I_z\,\omega_z\,\bar{u}_z \right)}$$

z è asse princ. inerzia

Se $\bar{\omega} = \omega \bar{u}_z \Rightarrow \boxed{\bar{L}_0 = I_z \bar{\omega}}$ (*)

$I_z \rightarrow$ momento di inerzia $\quad [I_z] = kg\,m^2$

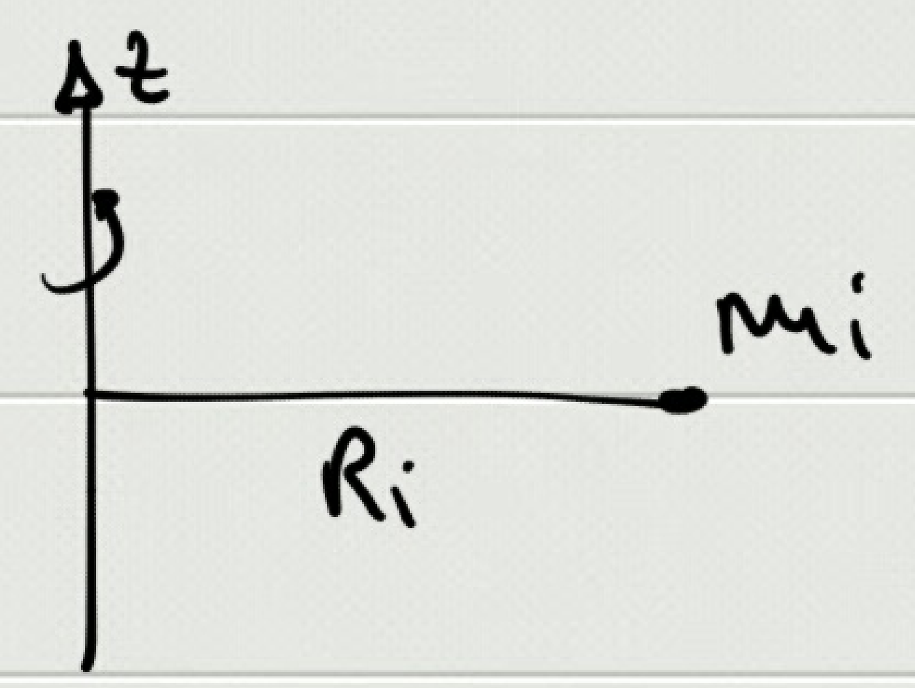
* $\bar{p} = m \bar{v}$ $\qquad m \rightarrow$ costante

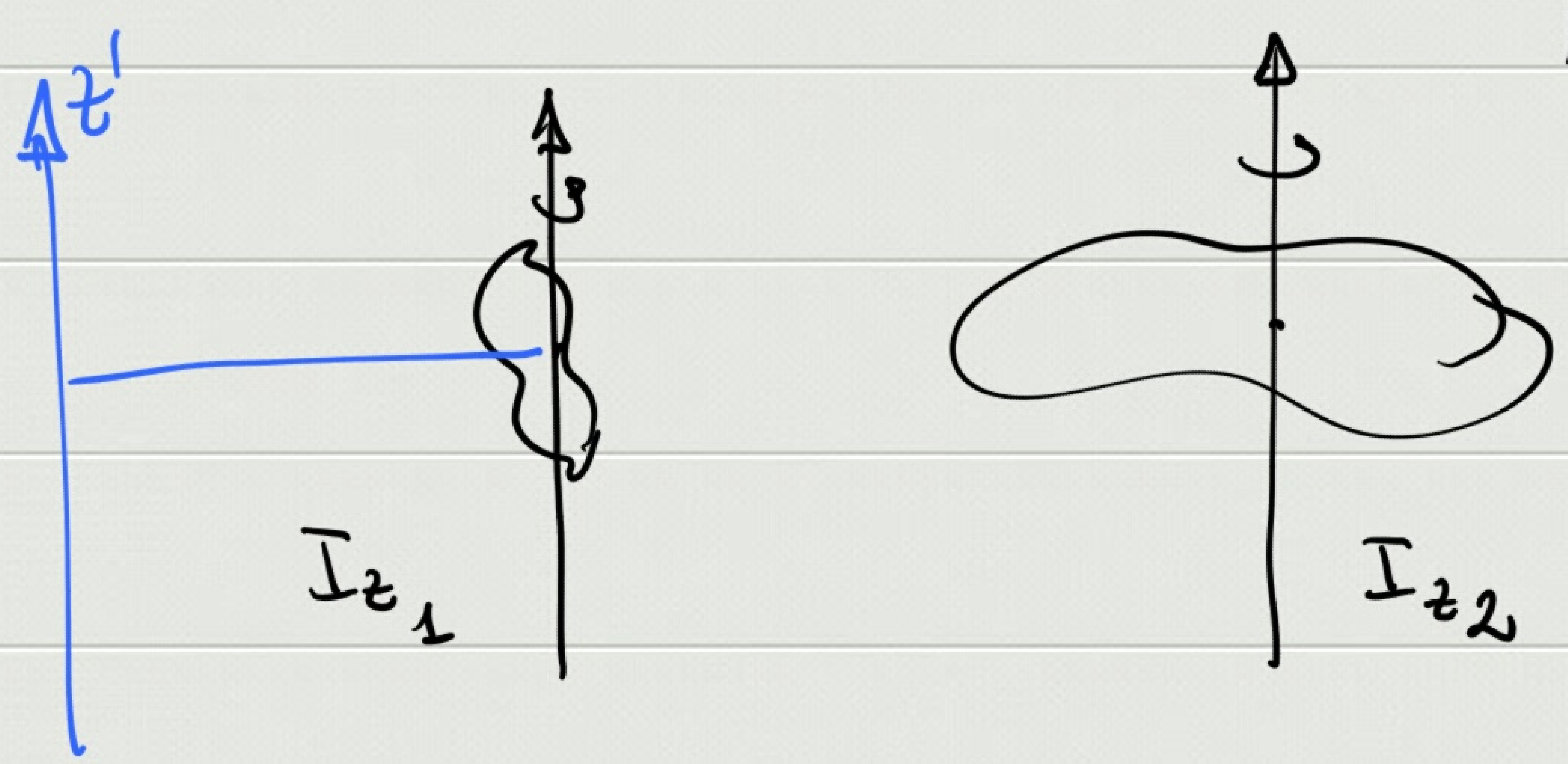
$\bar{L}_0 = \bar{r} \times m\bar{v}$ (punto)

* $\bar{L}_0 = \bar{\bar{I}}\,\bar{\omega}$ (corpo rigido)

↳ dipende dall'asse di rotazione

$$\boxed{I_z = \sum_i m_i R_i^2}$$





$I_{z_1} << I_{z_2}$