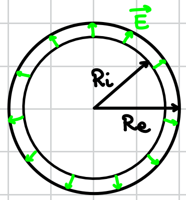
ESERCIZI SCHEDA 4

ESERCIZIO 1

$R_i = 2{,}2\ mm = 2{,}2\cdot 10^{-3}\,m$ $\quad R_e = 3{,}5\ mm = 3{,}5\cdot 10^{-3}\,m$

$l = 2{,}8\ m$ $\quad \Delta V = 350\ mV = 0{,}35\,V$



$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln(r_e/r_i)$ $\quad C = \frac{Q}{\Delta V} = \frac{Q}{\frac{\lambda}{2\pi\varepsilon_0}\ln(r_e/r_i)} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon_0 L}\cdot\ln(r_e/r_i)} = \frac{2\pi\varepsilon_0 L}{\ln(r_e/r_i)}$

$$\Rightarrow \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(r_e/r_i)} = \boxed{1{,}198\cdot 10^{-10}\,F/m}$$

$C = \frac{C}{L}\cdot L = \boxed{3{,}353\cdot 10^{-10}\,F}$

$Q = C\Delta V$ $\quad$ La superficie di un cilindro è pari a: $S = 2\pi r L$ $\Rightarrow \sigma_i = \frac{Q}{S_i} = \frac{C\Delta V}{2\pi r_i L} = \boxed{3{,}032\ C/m^2}$

$$\Rightarrow \sigma_e = \frac{Q}{S_e} = \frac{C\Delta V}{2\pi r_e L} = \boxed{1{,}906\ C/m^2}$$

ESERCIZIO 2

$\sigma = \pm 0{,}5\cdot 10^{-3}\,C/m^2$ $\quad d = 5\,mm = 5\cdot 10^{-9}\,m$ $\quad \varepsilon_r = 5{,}4$ $\quad \bar{V} = 10^{-16}\,m^3$

$E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \boxed{10{,}46\cdot 10^{6}\,N/C}$

La membrana esterna caricata positivamente ha potenziale maggiore: $\Delta V = Ed = \boxed{0{,}052\,V}$

$V_{guscio} = \frac{4}{3}\pi r_e^3 - \frac{4}{3}\pi r_i^3 = \frac{4}{3}\pi(r_e^3 - r_i^3) = \frac{4}{3}\pi(r_e - r_i)(r_e^2 + r_i^2 + r_i r_e)$ $\quad r_i \sim r_e$

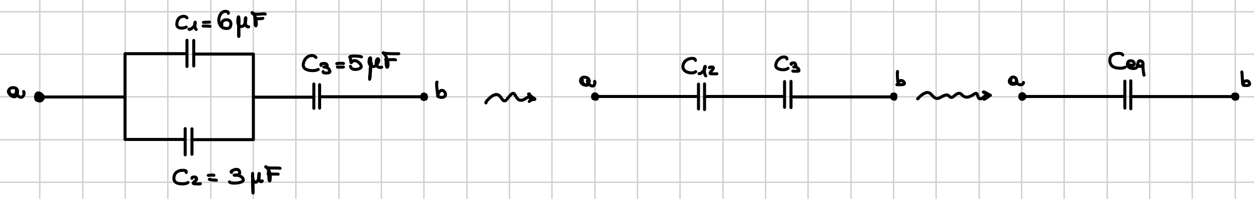
$= \frac{4}{3}\pi\underbrace{(r_e - r_i)}_{=d}\cdot 3 r_e^2 = 4\pi d r_e^2$ $\quad \bar{V} = \frac{4}{3}\pi r_e^3 \Rightarrow r_e = \left(\frac{3\bar{V}}{4\pi}\right)^{\frac{1}{3}}$

$= 4\pi d\left(\frac{3\bar{V}}{4\pi}\right)^{\frac{2}{3}}$

$U = u\,V_{guscio} = \frac{1}{2}\varepsilon E^2 V_{guscio} = \frac{1}{\cancel{2}}\varepsilon_r\varepsilon_0\frac{\sigma^2}{(\varepsilon_r\varepsilon_0)^2}\cdot \cancel{4}^{2}\pi d\left(\frac{3\bar{V}}{4\pi}\right)^{\frac{2}{3}} = \frac{2\pi d\sigma^2}{\varepsilon_r\varepsilon_0}\left(\frac{3\bar{V}}{4\pi}\right)^{\frac{2}{3}} = \boxed{6{,}32\cdot 10^{-15}\,J}$

ESERCIZIO 3

Costruisco lo schema equivalente



$C_{12} = C_1 + C_2 = 9\,\mu F$

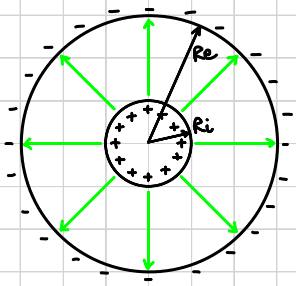
C_1 e C_2 sono in parallelo $\Rightarrow \Delta V_1 = \Delta V_2 \Leftrightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Leftrightarrow Q_1 = Q_2 \frac{C_1}{C_2} = 60\,\mu C$

$Q_3 = Q_1 + Q_2 = 90\,\mu C = Q_{eq}$

$$\frac{1}{C_{eq}} = \frac{1}{C_{12}} + \frac{1}{C_3} \Rightarrow C_{eq} = \frac{C_{12} C_3}{C_{12} + C_3} \Rightarrow Q_{eq} = V_{ab} \cdot C_{eq} \Rightarrow V_{ab} = \frac{Q_{eq}(C_{12} + C_3)}{C_{12} C_3} = 28V$$

Esercizio 4

Per praticità, assumiamo una coordinata radiale attraverso cui calcolare la differenza di potenziale.



$\Delta V = V_e - V_i$

$$\Rightarrow \Delta V = -\int_{Ri}^{Re} E(r)\,dr = -\int_{Ri}^{Re} \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} = \frac{Q}{4\pi\varepsilon_0} \int_{Ri}^{Re} -\frac{1}{r^2}\,dr = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r}\right]_{Ri}^{Re} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{Re} - \frac{1}{Ri}\right) = \frac{Q(Ri - Re)}{4\pi\varepsilon_0 \cdot Ri\,Re}$$

$$= -\frac{Q(Re - Ri)}{4\pi\varepsilon_0\, Ri\, Re}$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{-\frac{\cancel{Q}(Re - Ri)}{4\pi\varepsilon_0\, Ri\, Re}} = -\frac{4\pi\varepsilon_0\, Ri\, Re}{Re - Ri}$$

Analisi dimensionale del membro di sinistra:

$$\text{so che } \Delta U = q\Delta V \Rightarrow 1V = \frac{1J}{1C} \qquad \frac{J}{C} = kg\,\frac{m^2}{s^2} \cdot \frac{1}{C} = \frac{kg\,m^2}{C\,s^2} \Rightarrow \left[\frac{Q}{\Delta V}\right] = \frac{C}{\frac{kg\,m^2}{C\,s^2}} = \frac{C^2 s^2}{kg\,m^2}$$

Analisi dimensionale del membro di destra:

$$\left[-\frac{4\pi\varepsilon_0\, Ri\, Re}{Re - Ri}\right] = \frac{1 \cdot 1 \cdot \left(N\frac{m^2}{C^2}\right)^{-1} \cdot \cancel{m} \cdot m}{\cancel{m}} = \frac{C^2}{\cancel{N m^2}} \cdot m = \frac{C^2}{kg\,\frac{m}{s^2} \cdot m} = \frac{C^2 s^2}{kg\, m^2}$$

VERIFICATO