# Esercizi scheda 2

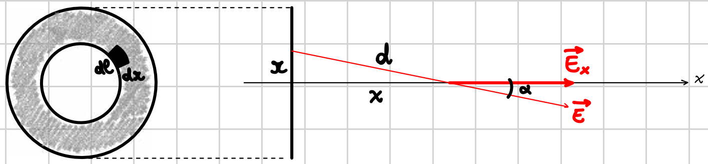
## Esercizio 1

ⓐ Data la superficie della corona circolare $S = \pi R_2^2 - \pi R_1^2 = \pi(R_2^2 - R_1^2)$ : $\sigma = \frac{Q}{S} \Rightarrow \boxed{Q = \sigma\pi(R_2^2 - R_1^2)}$

ⓑ Vista la conformazione simmetrica del problema, $\vec{E}$ sarà diretto nel semiasse positivo dell'asse $x$ nella parte positiva e al contrario nella parte negativa.

Consideriamo la corona circolare come un insieme di anelli infinitesimi, che a loro volta sono costituiti da porzioni infinitesime:

$Q = \sigma S \Rightarrow dQ = \sigma \, dl \, dr$




$$\Rightarrow dE = k\frac{dQ}{d^2}\cos\alpha = k\frac{\sigma \, dr \, dl}{x^2+r^2}\cdot\frac{x}{d} = k\frac{\sigma \, dr \, dl}{x^2+r^2}\cdot\frac{x}{\sqrt{x^2+r^2}} = k\sigma x\cdot\frac{1}{(x^2+r^2)^{3/2}}\,dr\,dl$$

$$E = \int_{R_1}^{R_2}\!\!\int k\sigma x\cdot\frac{1}{(x^2+r^2)^{3/2}}\,dl\,dr = \int_{R_1}^{R_2} k\sigma x\cdot\frac{1}{(x^2+r^2)^{3/2}}\int dl\,dr = \int_{R_1}^{R_2} k\sigma x\cdot\frac{1}{(x^2+r^2)^{3/2}}\cdot 2\pi r\,dr =$$

$$= 2\pi k\sigma x\int_{R_1}^{R_2}\frac{r}{(x^2+r^2)^{3/2}}\,dr = 2\pi k\sigma x\left[-\frac{1}{\sqrt{x^2+r^2}}\right]_{R_1}^{R_2} = \boxed{\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)}$$

## Esercizio 2

$R = 0{,}16\,m$ $\qquad \rho = 7{,}2\cdot10^{-9}\,C/m^3$ $\qquad q = 3{,}4\cdot10^{-6}\,C$

$E(r) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho V}{r^2}$ $\quad$ per $r > R$ (facilmente dimostrabile se non lo si ricorda)

Lungo un asse (radiale in questo caso): $E(r) = -\frac{dV}{dr} \Rightarrow V_\infty - V_R = \int_R^{+\infty} -E(r)\,dr = \int_R^{+\infty} -\frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho V}{r^2}\,dr = \frac{\rho V}{4\pi\varepsilon_0}\int_R^{+\infty} -\frac{1}{r^2}\,dr =$

$$= \frac{\rho V}{4\pi\varepsilon_0}\cdot\lim_{b\to+\infty}\left[\frac{1}{r}\right]_R^b = \frac{\rho V}{4\pi\varepsilon_0}\cdot\lim_{b\to+\infty}\left(\frac{1}{b} - \frac{1}{R}\right) = \frac{\rho}{\cancel{4\pi}\varepsilon_0}\cdot\frac{\cancel{4\pi}R^{\cancel{3}\,2}}{3}\left(-\frac{1}{\cancel{R}}\right) = -\frac{\rho R^2}{3\varepsilon_0}$$

$$\Delta U = q\Delta V \Rightarrow L = -\Delta U = -q\Delta V = -q(V_\infty - V_R) = -q\left(-\frac{\rho R^2}{3\varepsilon_0}\right) = q\rho\frac{R^2}{3\varepsilon_0} = \boxed{23{,}604\,\mu J}$$

## Esercizio 3

$V(x,y,z) = y(x+2)^3\sin\left(\frac{\pi}{2}x\right)$ dato in volt (V)

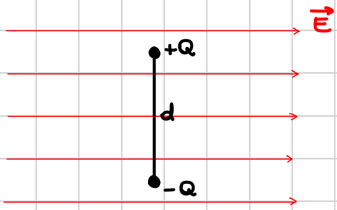
$$\vec{F} = q\vec{E} = -q\nabla V = \begin{pmatrix} -q\dfrac{\partial V}{\partial x} \\ -q\dfrac{\partial V}{\partial y} \\ -q\dfrac{\partial V}{\partial z} \end{pmatrix} = \begin{pmatrix} -q\cdot 3y(x+2)^2 \sin\left(\frac{\pi}{2}z\right) \\ 27 \\ -q(x+2)^3 \sin\left(\frac{\pi}{2}z\right) \\ -q\cdot\frac{\pi}{2}y(x+2)^2\cos\left(\frac{\pi}{2}z\right) \end{pmatrix}$$

$$\Rightarrow \vec{F}(1,2,1) = -q\begin{pmatrix} 54V \\ 27V \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}324\ mN \\ -0{,}162\ mN \\ 0\ mN \end{pmatrix} \Rightarrow |\vec{F}| = 0{,}362\ mN$$

$\theta = 206{,}57°$ sul piano xy

## Esercizio 4

$\sigma = 2\ C/m^2 \quad Q = 5\ \mu C \quad \theta_0 = 90° \quad d = 0{,}1\ \mu m \quad m = 1\ pg$




Tra le due lastre: $E = \frac{\sigma}{\varepsilon_0}$

$\vec{M} = \vec{p} \times \vec{E} \Rightarrow |\vec{M}| = M(\theta) = pE\sin\theta = dQ\frac{\sigma}{\varepsilon_0}\sin\theta$

$M = I\alpha \Leftrightarrow \alpha = \frac{M}{I}$ $\quad$ I (momento d'inerzia rispetto al centro) $= 2m\left(\frac{d}{2}\right)^2 = 2m\frac{d^2}{4} = \frac{1}{2}md^2$

$$\Rightarrow \alpha(\theta) = \frac{dQ\frac{\sigma}{\varepsilon_0}\sin\theta}{\frac{1}{2}md^2} = \frac{2Q\sigma\sin\theta}{\varepsilon_0 md}$$

legge oraria: $\theta(t) = \theta_0 + \cancel{\omega_0 t} - \frac{1}{2}\alpha t^2$

$\cancel{\theta_f} = \theta_0 - \frac{1}{2}\alpha t^2$

$-\theta_0 = -\frac{1}{2}\alpha t^2$

senso orario

Bisogna trovare un'accelerazione media:

$$\alpha = \frac{1}{\cancel{\theta_f} - \theta_0}\int_{\theta_0=\frac{\pi}{2}}^{\theta_f=0}\alpha(\theta)\,d\theta = \frac{1}{-\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{0}\frac{2Q\sigma\sin\theta}{\varepsilon_0 md}\,d\theta = \frac{2}{\pi}\cdot\frac{2Q\sigma}{\varepsilon_0 md}\int_{\frac{\pi}{2}}^{0}-\sin\theta\,d\theta =$$

$$= \frac{4Q\sigma}{\pi\varepsilon_0 md}\left[\cos\theta\right]_{\frac{\pi}{2}}^{0} = \frac{4Q\sigma}{\pi\varepsilon_0 md}$$

$$\cancel{-}\frac{\pi}{2} = \cancel{-}\frac{4Q\sigma}{\pi\varepsilon_0 md}t^2 \Leftrightarrow t = \sqrt{\frac{\pi^2\varepsilon_0 md}{8Q\sigma}} = 1{,}045\cdot 10^{-14}\,s$$