# Tema A

**Es 1.** (a) $P(R) = P(R|A)P(A) + P(R|B)P(B)$

$$= \frac{30}{52} \times \frac{2}{3} + \frac{20}{52} \times \frac{1}{3} = \frac{80}{156} = \frac{20}{39}$$

(b) $P(B|R) = \dfrac{P(R|B)P(B)}{P(R)} = \dfrac{\frac{20}{52} \times \frac{1}{3}}{P(R)} = \dfrac{\frac{20}{156}}{\frac{80}{156}} = \frac{20}{80} = \frac{1}{4}$

(c) $Q(\cdot) = P(\cdot|R_1)$.

$$Q(R_2) = Q(R_2|A)Q(A) + Q(R_2|B)Q(B)$$

Ora $Q(R_2|A) = P(R_2|A \cap R_1) = Q_A(R_2|R_1) \quad (Q_A(\cdot) = P(\cdot|A))$

$\phantom{Ora Q(R_2|A) = P(R_2|A \cap R_1)} = Q_A(R_2) = \frac{30}{52}$

$Q(A) = P(A|R_1) = 1 - P(B|R_1) = \frac{3}{4}$

$Q(R_2|B) = Q_B(R_2|R_1) = Q_B(R_2) = \frac{20}{52}$ ; $Q(B) = P(B|R_1) = \frac{1}{4}$

$$\Rightarrow P(R_2|R_1) = \frac{30}{52} \times \frac{3}{4} + \frac{20}{52} \times \frac{1}{4} = \frac{15}{26} \times \frac{3}{4} + \frac{10}{26} \times \frac{1}{4}$$

$$= \frac{45}{104} + \frac{10}{104} = \frac{55}{104}.$$

(ii) $P(R_2) = P(R_2|A)P(A) + P(R_2|B)P(B) = P(R_1) = \frac{20}{39} \neq P(R_2|R_1)$:

gli eventi non sono indipendenti.

**Es 2.** a) $X \sim B\left(20.000, \frac{1}{10.000}\right)$

$$P(X \leq 2) = \sum_{k=0}^{2} \binom{20000}{k} \left(\frac{1}{10000}\right)^k \left(1 - \frac{1}{10000}\right)^{20000-k}$$

$(\simeq 0.6766)$

b) Sì, $p = \frac{1}{10.000}$ è piccolo e $n = 20.000$ è "grande".

$Y \sim Po\left(20.000 \times \frac{1}{\phantom{10.000}}\right) = Po(2)$ e

$$P(X \le 2) \simeq P(Y \le 2) = \sum_{k=0}^{2} e^{-2} \frac{2^k}{k!} = e^{-2}\left(1+2+\frac{2^2}{2}\right)$$ 0.2706705664732254

$$= \boxed{5e^{-2}}$$

$$\simeq 0.6766$$

c) $P(X \le 2) \simeq P(W \le 2)$ con

$W \sim N\left(20.000 \times \frac{1}{10000}, \sigma^2\right)$,

$$\sigma^2 = 20000 \times \frac{1}{10.000} \times \left(1 - \frac{1}{10.000}\right) = 2 \times \frac{9999}{10.000} = \frac{9.999}{5.000}$$

3. $f_{X,Y}(x,y) = \begin{cases} c(x^2+xy) & x,y \in [0,1] \\ 0 & \text{altrimenti} \end{cases}$

a) $\int f_{X,Y}(x,y)\,dx\,dy = 1 \iff c \int_{[0,1]\times[0,1]} x^2+xy\,dx\,dy = 1$

ed è $\int_{[0,1]\times[0,1]} x^2+xy\,dx\,dy = \int_0^1 \left[\int_0^1 x^2+xy\,dx\right] dy$

$$= \int_0^1 \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 y\right]_{x=0}^{x=1} dy = \int_0^1 \frac{1}{3} + \frac{1}{2}y\,dy$$

$$= \left[\frac{y}{3} + \frac{y^2}{4}\right]_0^1 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$ da cui $\boxed{c = \frac{12}{7}}$

b) $P(X>Y) = \int_{x>y} f_{X,Y}(x,y)\,dx\,dy$

$$= c\int_0^1 \int_0^x (x^2+xy)\,dy\,dx$$

$$= c\int_0^1 \left[x^2 y + x\frac{y^2}{2}\right]_{y=0}^{y=x} dx$$

$$= c\int^1 x^3 + \frac{x^2}{2}\,dx = c\left[\frac{1}{4} + \frac{1}{6}\right] = c \cdot \frac{10}{24} = c\,\frac{5}{12}$$

$$= \frac{12}{7} \cdot \frac{5}{12} = \boxed{\frac{5}{7}}.$$

c) $$f_X(x) = \begin{cases} 0 & \text{se } x \notin [0,1] \\ c\int_0^1 x^2 + xy \, dy & \text{se } x \in [0,1] \end{cases}$$

$$c\left[x^2 y + x\frac{y^2}{2}\right]_{y=0}^1 = c\left(x^2 + \frac{x}{2}\right)$$

$$f_Y(y) = \begin{cases} 0 & \text{se } y \notin [0,1] \\ c\int_0^1 x^2 + xy \, dx = c\left(\frac{1}{3} + \frac{y}{2}\right) \end{cases}$$

Si ha $f_X(x) f_Y(y) = \begin{cases} c^2 x\left(x + \frac{1}{2}\right)\left(\frac{1}{3} + \frac{y}{2}\right) & \text{se } x, y \in [0,1] \\ 0 & \text{altrimenti} \end{cases}$

Dato che $f_X(x) f_Y(y) \neq f_{X,Y}(x,y)$ segue che $X, Y$ non sono indipendenti.