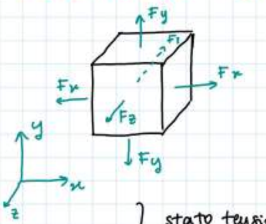
ESERCIZIO: stato di tensione triassiale

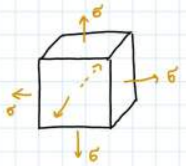


$|F_x| = |F_y| = |F_z| = |F| = 1\,N$

Area $A_0 = 1\,mm^2$

materiale lineare elastico isotropo $\nu = 0.3$, $E = 10\,GPa$

stato tensionale



$$\{\sigma\} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}$$

non si sviluppano tensioni tangenziali τ

$\sigma = F/A_0 = 1\,MPa$

Calcolare stato deformativo

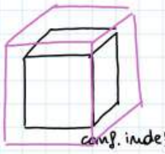
$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = \frac{1\,MPa}{10 \cdot 10^3\,MPa} - \frac{0.3}{10 \cdot 10^3\,MPa}(2\,MPa)$$

$$= 10^{-4} - 0.3 \cdot 10^{-4} = 0.4 \cdot 10^{-4}$$

$\varepsilon_y = \varepsilon_z = \varepsilon_x = 0.4 \cdot 10^{-4}$

$\gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$

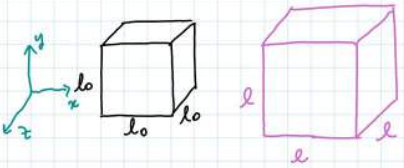
$$\{\varepsilon\} = 0.4 \cdot 10^{-4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



conf. indeformata
conf. deformata

} Espansione nelle 3 direzioni (no cambio di forma)

ESEMPIO: stato di deformazione triassiale



$l_0 = 10\,mm$

$l = 10.001\,mm$

mat. lineare isotropo

$\nu = 0.3$

$E = 10\,GPa$

Calcolare stato deformativo e tensionale

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{10.001 - 10}{10} = 0.0001$$

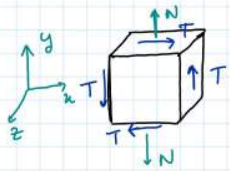
$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$

$$\sigma_x = \sigma_y = \sigma_z = \sigma = \frac{E}{(1+\nu)}\varepsilon + \frac{\nu E}{(1+\nu)(1-2\nu)}(3\varepsilon) = 2.5\,MPa$$

$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

$$\{\varepsilon\} = 10^{-4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \{\sigma\} = 2.5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} MPa$$

ESEMPIO: stato di tensione generico



$E = 10\ GPa$

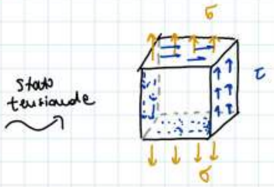
$G = 3.85\ GPa$

$\nu = 0.3$

$l_0 = 10\ mm$

$N = 100\ N$

$T = 200 N$



Calcolare stato deformativo e tensionale

$\sigma_y = \frac{N}{A_0} = \frac{100}{10^2} = 1\ MPa$

$\sigma_x = \sigma_z = 0$

$\tau_{xy} = \tau_{yx} = \frac{T}{A_0} = \frac{200}{10^2} = 2\ MPa$

$\tau_{xz} = \tau_{zy} = 0$

$$\{\sigma\} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa$$

$$\varepsilon_x = \underbrace{\frac{\sigma_x}{E}}_{=0} - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\underbrace{\sigma_z}_{=0} = -\frac{0.3}{10 \cdot 10^3\ MPa} \cdot 1\ MPa = -0.3 \cdot 10^{-4} = \varepsilon_z$$

$$\varepsilon_y = -\frac{\nu}{E}\underbrace{\sigma_x}_{=0} + \frac{\sigma_y}{E} - \frac{\nu}{E}\underbrace{\sigma_z}_{=0} = \frac{1\ MPa}{10 \cdot 10^3\ MPa} = 10^{-4}$$

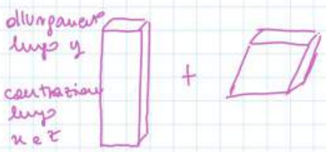
$$\gamma_{xy} = \gamma_{yx} = \frac{\tau_{xy}}{G} = \frac{2\ MPa}{3.85 \cdot 10^3\ MPa} = 0.519 \cdot 10^{-3} = 5.19 \cdot 10^{-4}$$

$\gamma_{xz} = \gamma_{zy} = 0$

$\gamma_{xy}/2$!

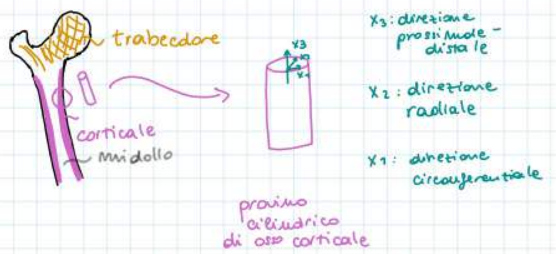
$$\{\varepsilon\} = 10^{-4} \begin{bmatrix} -0.3 & 2.59 & 0 \\ 2.59 & 1 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}$$

la conf DEFORMATA del cubetto è data dalla sovrapposizione degli effetti



ESERCIZIO: mat. trasversalmente isotropo

tessuto osseo corticale del femore



dopo dei test a trazione eseguiti lungo x_1, x_2, x_3
il tessuto mostra un comportamento trasversalmente isotropo
nel piano $\perp$ a x_3

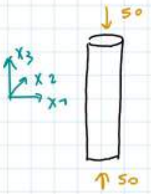
$E_3 = 17$ GPa

$E_1 = E_2 = 11.5$ GPa

$\nu_{31} = 0.3$

$\nu_{12} = 0.38$

- Calcolare gli stati deformativi per uno stato di tensione mono-assiale di compressione in dir. x_3 pari a 50 MPa



$$\varepsilon_{x1} = \varepsilon_{11} = \frac{\overset{=0}{\sigma_{11}}}{E_1} - \nu_{12}\frac{\overset{=0}{\sigma_{22}}}{E_2} - \nu_{31}\frac{\sigma_{33}}{E_3} = -0.3\,\frac{(-50\text{ MPa})}{17\cdot 10^3\text{ MPa}} = 0.0009$$

$$\varepsilon_{x2} = \varepsilon_{22} = -\nu_{12}\frac{\overset{=0}{\sigma_{11}}}{E_1} + \frac{\overset{=0}{\sigma_{22}}}{E_2} - \underbrace{\nu_{32}}_{=\nu_{31}\text{ per mat. trasv. isotropo}}\frac{\sigma_{33}}{E_3} = -0.3\,\frac{(-50\text{ MPa})}{17\cdot 10^3\text{ MPa}} = 0.0009$$

$$\varepsilon_{x3} = \varepsilon_{33} = -\nu_{13}\frac{\sigma_{11}}{E_1} - \nu_{23}\frac{\sigma_{22}}{E_2} + \frac{\sigma_{33}}{E_3} = \frac{-50\text{ MPa}}{17\cdot 10^3\text{ MPa}} = -0.003$$

$$\gamma_{12} = \gamma_{23} = \gamma_{31} = 0$$

$$\{\sigma\} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -50 \end{bmatrix} \text{MPa}$$

$$\{\varepsilon\} = \begin{bmatrix} 0.0009 & 0 & 0 \\ 0 & 0.0009 & 0 \\ 0 & 0 & -0.003 \end{bmatrix}$$