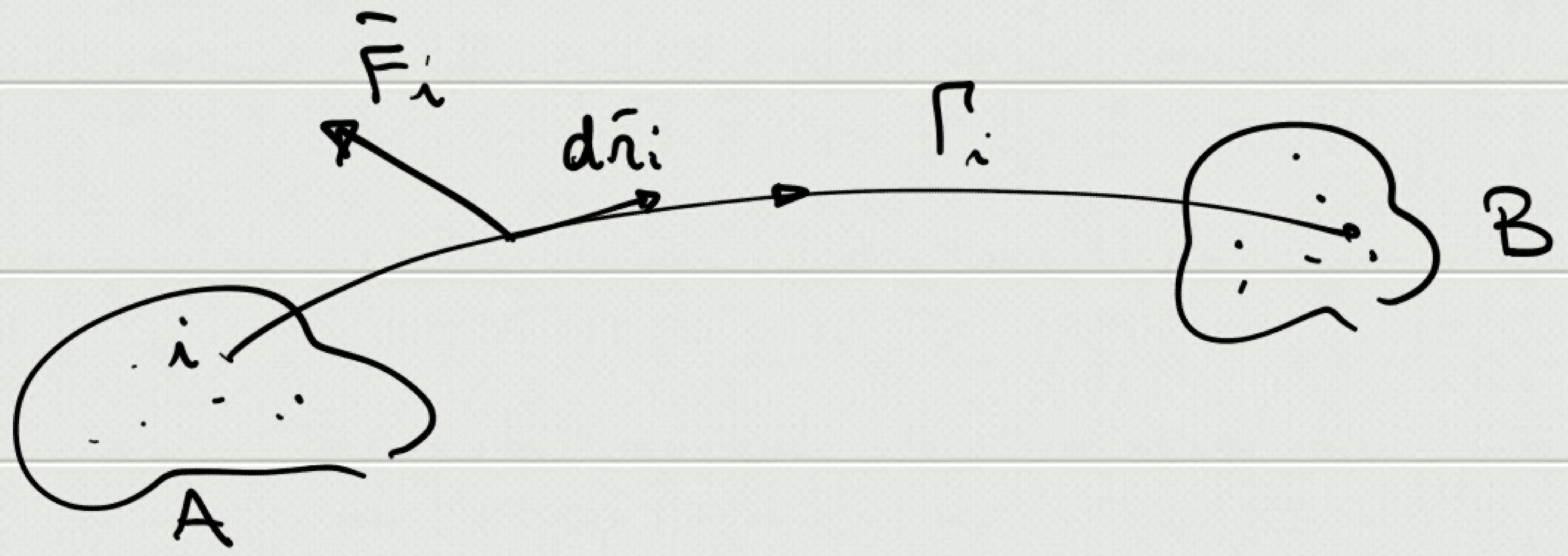
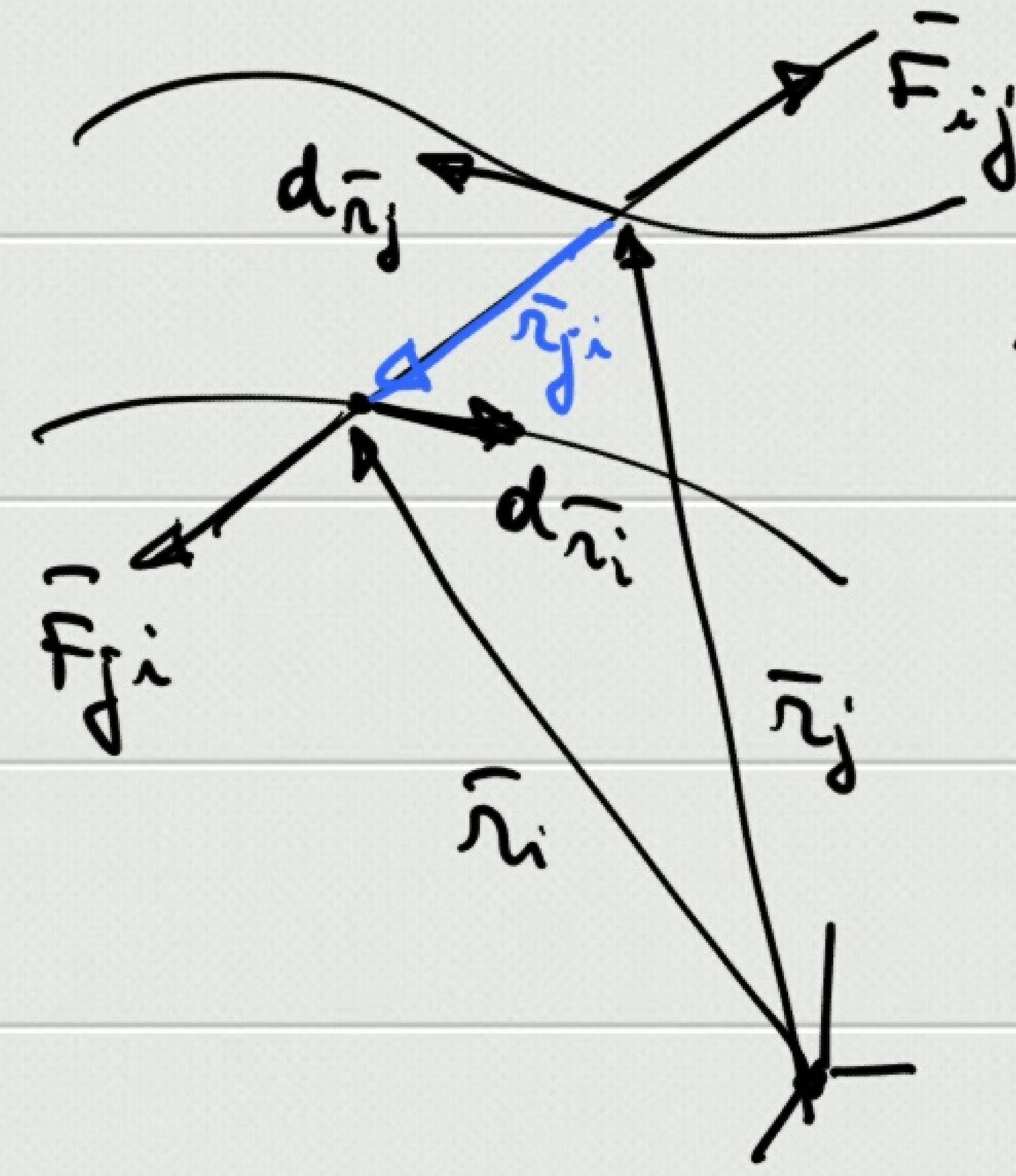



$$dW_i = \bar F_i\, d\bar r_i = \left(\bar F_i^{I} + \bar F_i^{E}\right) d\bar r_i = dW_i^{I} + dW_i^{E}$$

$$\Rightarrow W_{A\to B} = \sum_i W_{i,A\to B} = \sum_i \int_A^B dW_i = W^{I}_{A\to B} + W^{E}_{A\to B}$$




$$\bar F_{ij} = -\bar F_{ji}$$

$$dW^{ij} = \bar F_{ji}\, d\bar r_i + \bar F_{ij} \cdot d\bar r_j =$$

$$= \bar F_{ji}\, d\bar r_i - \bar F_{ji}\, d\bar r_j =$$

$$= \bar F_{ji} \left(d\bar r_i - d\bar r_j\right) =$$

$$= \bar F_{ji}\, d\left(\bar r_i - \bar r_j\right) =$$

$$= \bar F_{ji}\, d\bar r_{ji} \neq 0 \qquad \Rightarrow \boxed{W^{I} \neq 0}$$

T. energia cinetica per un sistema di punti

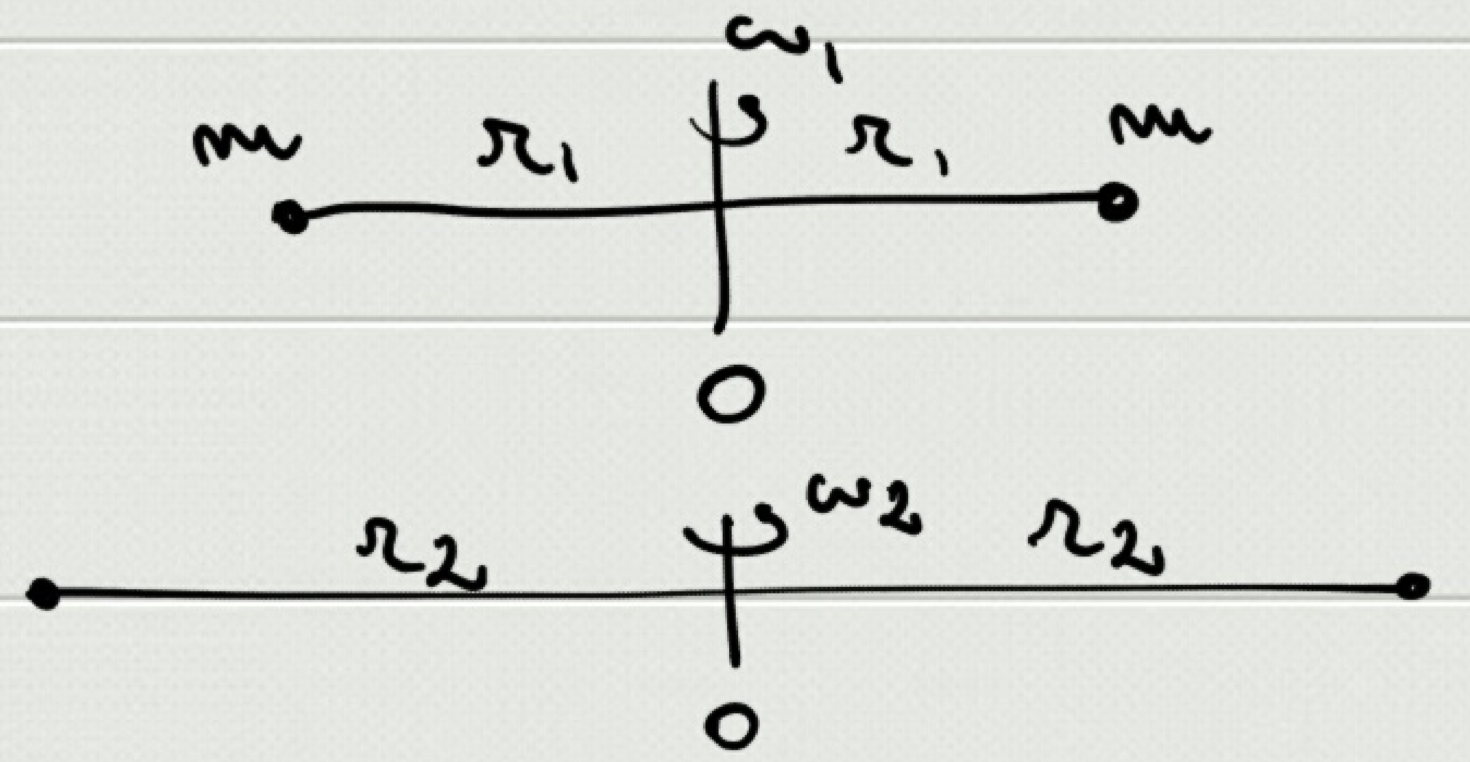
$$\boxed{W_{TOT, A\to B} = E_{k,B} - E_{k,A} = \Delta E_k}$$

Solo forze conservative : $\boxed{W_{A\to B} = -\Delta E_p}$

In presenza di forze non conservative :

$$\left.\begin{array}{l} W_{A\to B} = W_{f.c., A\to B} + W_{nc, A\to B} = -\Delta E_p + W_{nc} \\ \quad\;\; \overset{!}{=} \Delta E_k \end{array}\right\} \Rightarrow$$

$$\underbrace{\Delta E_k + \Delta E_p}_{\Delta E_m} = W_{nc} \qquad \Rightarrow \qquad \boxed{\Delta E_m = W_{nc}}$$




$$\bar{L}_0 = cost = \omega r^2$$

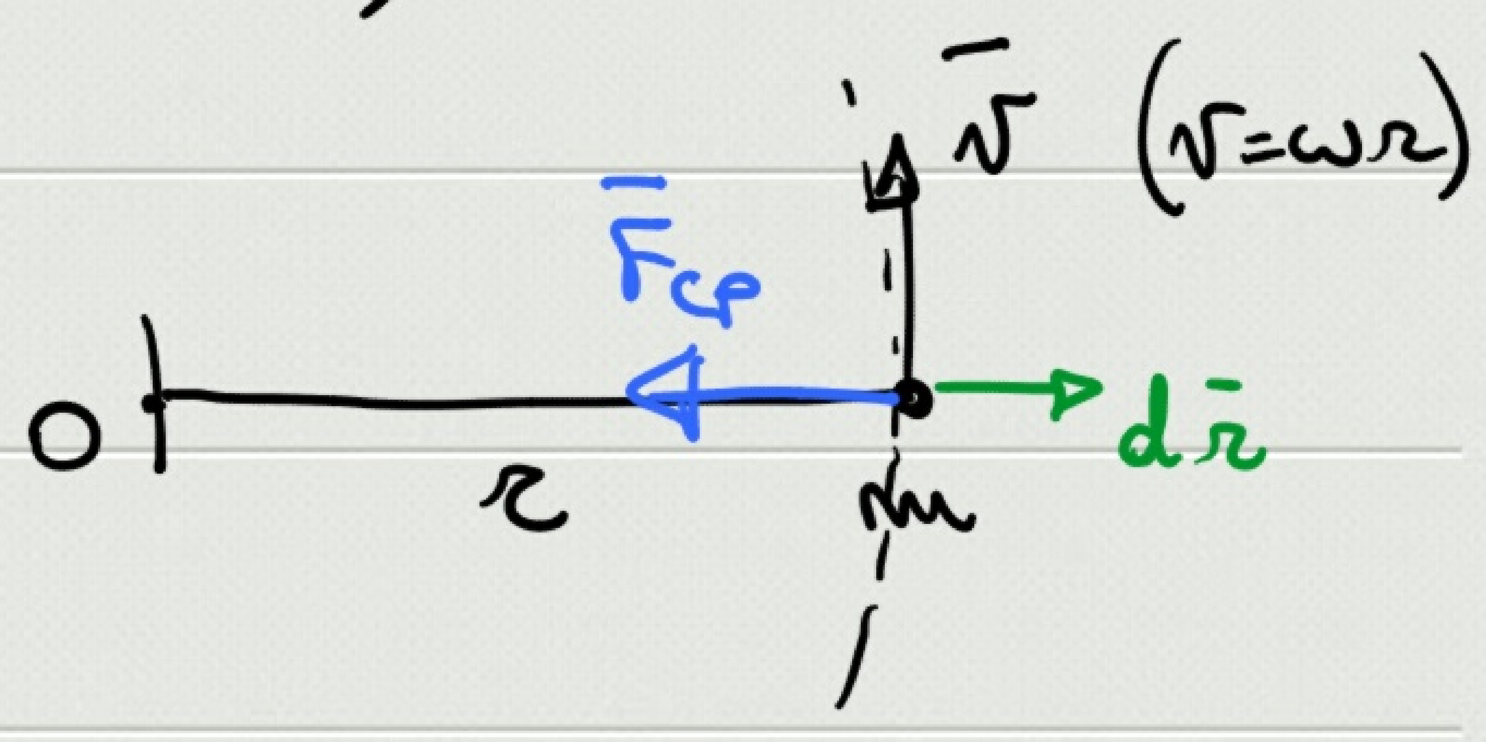
$$\Rightarrow \underbrace{\omega_2 = \omega_1 \left(\frac{r_1}{r_2}\right)^2}$$

$$E_{k,1} = \cancel{2} \cdot \frac{1}{\cancel{2}} m v_1^2 = m(\omega_1 r_1)^2$$

$$E_{k,2} = 2 \cdot \frac{1}{2} m v_2^2 = m(\omega_2 r_2)^2 = m\omega_1^2 \frac{r_1^4}{r_2^4} r_2^2 = m\omega_1^2 \frac{r_1^4}{r_2^2}$$

$$\Delta E_k = E_{k,2} - E_{k,1} = m\omega_1^2 r_1^2 \left(\frac{r_1^2}{r_2^2} - 1\right) < 0$$

$$\Delta E_k = W_{TOT} = W^{I}_{1 \to 2}$$




$$F_{cp} = m\omega^2 r$$

$$W^{I}_{1 \to 2} = \int_1^2 dW^{I} = 2 \int_1^2 \bar{F}_{cp} \, d\bar{r} =$$

$$= -2 \int_{r_1}^{r_2} m \omega^2 r \, dr \qquad \omega = \omega(r)$$

$$L_0 = cost \Rightarrow \omega_1 r_1^2 = \omega(r) r^2$$

$$\Rightarrow \omega(r) = \omega_1 \frac{r_1^2}{r^2}$$

$$W^{I}_{1\to 2} = -2\int_{r_1}^{r_2} m\,\omega_1^2\,\frac{r_1^4}{r^4}\,r\,dr =$$

$$= -2\,m\,\omega_1^2\,r_1^4\int_{r_1}^{r_2}\frac{1}{r^3}\,dr =$$

$$= \cancel{-2}\,m\,\omega_1^2\,r_1^4\,\cancel{\left(-\frac{1}{2}\right)}\,\frac{1}{r^2}\Bigg|_{r_1}^{r_2} =$$

$$= m\,\omega_1^2\,r_1^4\left(\frac{1}{r_2^2}-\frac{1}{r_1^2}\right) =$$

$$= m\,\omega_1^2\,r_1^2\left(\frac{r_1^2}{r_2^2}-1\right)$$

$$\overset{\text{ok}}{\Longrightarrow}\ \boxed{\Delta E_k = W^{I}_{1\to 2}}$$