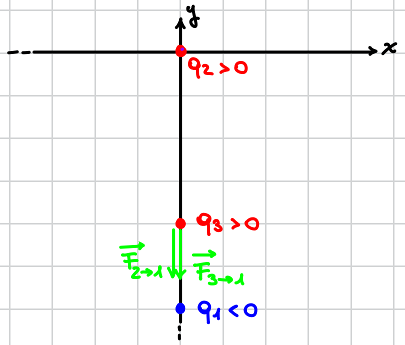
ESERCIZI SCHEDA 1

ESERCIZIO 1

$q_1 = -1{,}50\ nC = -1{,}50 \cdot 10^{-9} C$ $\qquad y_1 = -0{,}600\,m$

$q_2 = +3{,}20\,nC = +3{,}20 \cdot 10^{-9} C$ $\qquad y_2 = 0\,m$

$q_3 = +5{,}00\,nC = +5{,}00 \cdot 10^{-9} C$ $\qquad y = -0{,}400\,m$



$$\vec{F}_{3,1} = -k\frac{|q_1 q_3|}{(y_1-y_3)^2}\hat{y}$$

$$\vec{F}_{3,1} = -k\frac{|q_2 q_3|}{(y_2-y_3)^2}\hat{y}$$

$$\Rightarrow \vec{F}_3 = \vec{F}_{1\to3} + \vec{F}_{2\to3} = \left(-k\frac{|q_1 q_3|}{(y_1-y_3)^2} - k\frac{|q_2 q_3|}{(y_2-y_3)^2}\right)\hat{y}$$

$$= -k|q_3|\left(\frac{|q_1|}{(y_1-y_3)^2} + \frac{|q_2|}{(y_2-y_3)^2}\right)\hat{y}$$

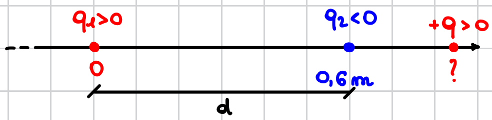
$$= -8{,}988 \cdot 10^{9}\,\frac{Nm^2}{C^2} \cdot 5 \cdot 10^{-9} C\left(\frac{1{,}5 \cdot 10^{-9} C}{(0{,}2\,m)^2} + \frac{3{,}2 \cdot 10^{-9} C}{(0{,}4\,m)^2}\right)\hat{y}$$

$$= (-2{,}584 \cdot 10^{-6} N)\hat{y} = (-2{,}584\,\mu N)\hat{y} = \begin{bmatrix} 0 \\ -2{,}584\,\mu N \\ 0 \end{bmatrix}$$

ESERCIZIO 2

$q_1 = 2{,}50\,\mu C = 2{,}50 \cdot 10^{-6} C$ $\qquad x_1 = 0\,m$

$q_2 = -3{,}50\,\mu C = -3{,}50 \cdot 10^{-6} C$ $\qquad x_2 = 0{,}600\,m$



Devo trovare un punto x_0 tale che $\vec{F}(x_0) = \vec{F}_1(x_0) + \vec{F}_2(x_0) = 0$:

$$\vec{F}(x_0) = k\frac{|qq_1|}{x_0^2}\hat{x} - k\frac{|qq_2|}{(d-x_0)^2}\hat{x} = 0$$

L'eliminazione algebrica del termine $|q|$ giustifica la risposta affermativa alla seconda domanda.

$$\cancel{k|q|\hat{x}}\left(\frac{|q_1|}{x_0^2} - \frac{|q_2|}{(d-x_0)^2}\right) = 0 \Leftrightarrow |q_1|(d-x_0)^2 - |q_2|x_0^2 = 0$$

$$\Leftrightarrow |q_1|(d^2 + x_0^2 - 2dx_0) - |q_2|x_0^2 = 0$$

$$\Leftrightarrow (|q_1| - |q_2|)x_0^2 - 2d|q_1|x_0 + |q_1|d^2 = 0$$

Per chi non se la ricordasse, sto usando la formula del $\Delta/4$, utilizzabile quando in $ax^2+bx+c=0$ b è pari: $x_{1,2} = \dfrac{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a}$

$$x_{0_{1,2}} = \frac{d|q_1| \pm \sqrt{d^2|q_1|^2 - |q_1|d^2(|q_1|-|q_2|)}}{|q_1|-|q_2|}$$

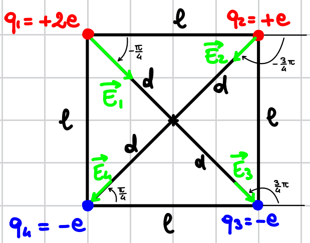
$$= \frac{d|q_1| \pm \sqrt{d^2(|q_1|^2 - |q_1|^2 + |q_1 q_2|}}{|q_1|-|q_2|}$$

$$= d\,\frac{|q_1| \pm \sqrt{|q_1 q_2|}}{|q_1|-|q_2|}$$

$$x_{0_1} = d\,\frac{|q_1| - \overset{2,958}{\sqrt{|q_1 q_2|}}}{|q_1|-|q_2|} = 0,275\ m \qquad \text{impossibile}$$

$$x_{0_2} = d\,\frac{|q_1| + \sqrt{|q_1 q_2|}}{|q_1|-|q_2|} = -3,275\ m$$

Esercizio 3



$e = 1,602 \cdot 10^{-19}\,C$

$\ell = 2\ m$ geometricamente, $d = \frac{\sqrt{2}\,\ell}{2}$

Viene comoda la scrittura vettoriale

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = \begin{pmatrix} k\frac{|q_1|}{d^2}\cos\left(-\frac{\pi}{4}\right) \\ k\frac{|q_1|}{d^2}\sin\left(-\frac{\pi}{4}\right) \end{pmatrix} + \begin{pmatrix} k\frac{|q_1|}{d^2}\cos\left(-\frac{3}{4}\pi\right) \\ k\frac{|q_1|}{d^2}\sin\left(-\frac{3}{4}\pi\right) \end{pmatrix} + \begin{pmatrix} -k\frac{|q_1|}{d^2}\cos\left(\frac{3}{4}\pi\right) \\ -k\frac{|q_1|}{d^2}\sin\left(\frac{3}{4}\pi\right) \end{pmatrix} + \begin{pmatrix} -k\frac{|q_1|}{d^2}\cos\left(\frac{\pi}{4}\right) \\ -k\frac{|q_1|}{d^2}\sin\left(\frac{\pi}{4}\right) \end{pmatrix}$$

$$= \begin{pmatrix} k\frac{2e}{d^2}\frac{\sqrt{2}}{2} \\ -k\frac{2e}{d^2}\frac{\sqrt{2}}{2} \end{pmatrix} + \begin{pmatrix} -k\frac{e}{d^2}\frac{\sqrt{2}}{2} \\ -k\frac{e}{d^2}\frac{\sqrt{2}}{2} \end{pmatrix} + \begin{pmatrix} k\frac{e}{d^2}\frac{\sqrt{2}}{2} \\ -k\frac{e}{d^2}\frac{\sqrt{2}}{2} \end{pmatrix} + \begin{pmatrix} -k\frac{e}{d^2}\frac{\sqrt{2}}{2} \\ -k\frac{e}{d^2}\frac{\sqrt{2}}{2} \end{pmatrix}$$

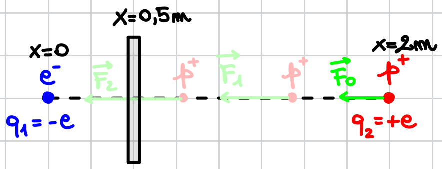
$$\left(d = \frac{\sqrt{2}}{2}\ell \Rightarrow d^2 = \frac{\ell^2}{2}\right) \qquad = \begin{pmatrix} k2\sqrt{2}\frac{e}{\ell^2} \\ -k2\sqrt{2}\frac{e}{\ell^2} \end{pmatrix} + \begin{pmatrix} -k\sqrt{2}\frac{e}{\ell^2} \\ -k\sqrt{2}\frac{e}{\ell^2} \end{pmatrix} + \begin{pmatrix} k\sqrt{2}\frac{e}{\ell^2} \\ -k\sqrt{2}\frac{e}{\ell^2} \end{pmatrix} + \begin{pmatrix} -k\sqrt{2}\frac{e}{\ell^2} \\ -k\sqrt{2}\frac{e}{\ell^2} \end{pmatrix}$$

$$= \begin{bmatrix} k\frac{e}{\ell^2}\left(2\sqrt{2} - \cancel{\sqrt{2}} + \cancel{\sqrt{2}} - \sqrt{2}\right) \\ k\frac{e}{\ell^2}\left(-2\sqrt{2} - \sqrt{2} - \sqrt{2} - \sqrt{2}\right) \end{bmatrix} = \begin{bmatrix} \sqrt{2}\,k\frac{e}{\ell^2} \\ -5\sqrt{2}\,k\frac{e}{\ell^2} \end{bmatrix} = \begin{bmatrix} 5,091 \cdot 10^{-10}\ N/C \\ -25,454 \cdot 10^{-10}\ N/C \end{bmatrix}$$

$$\Rightarrow |\vec{E}| = \sqrt{E_x^2 + E_y^2} = 25,958 \cdot 10^{-10}\ N/C$$

$$\Rightarrow \theta = \operatorname{arctg}\left(\frac{E_y}{E_x}\right) = -78,68^\circ = -1,373\ rad$$

Esercizio 4



$x_i = 2m$

piastra: $x_p = 0{,}5$

Cerco il tempo di collisione t_c: $s(t) = s_0 + \cancel{v_0 t} - \frac{1}{2} a t^2$

$$s(t_c) = x_p = x_i - \frac{1}{2} a t_c^2$$

L'accelerazione è un'accelerazione media, ma in questo caso la forza non è costante, quindi neanche l'accelerazione. Uso il teorema della media integrale:

$$a = \frac{\overline{F}}{m} = \frac{1}{m} \cdot \frac{1}{x_p - x_i} \int_{x_i}^{x_p} F(x)\,dx = \frac{1}{m} \cdot \frac{1}{x_p - x_i} \cdot \int_{x_i}^{x_p} \frac{k|q_1 q_2|}{x^2}\,dx = \frac{1}{m} \cdot \frac{1}{x_p - x_i} \cdot \int_{x_i}^{x_p} \frac{ke^2}{x^2}\,dx = \frac{ke^2}{m(x_p - x_i)} \int_{x_i}^{x_p} \frac{1}{x^2}\,dx =$$

$$= \frac{ke^2}{m(x_p - x_i)} \left[-\frac{1}{x}\right]_{x_i}^{x_p} = \frac{ke^2}{m(x_p - x_i)} \left(-\frac{1}{x_p} + \frac{1}{x_i}\right) = \frac{ke^2}{m\cancel{(x_p - x_i)}} \cdot \frac{\cancel{x_p - x_i}}{x_i x_p} = \frac{ke^2}{m x_i x_p}$$

$$\Rightarrow (x_i - x_p) - \frac{ke^2}{2m x_i x_p} t_c^2 = 0 \iff t_c = \sqrt{\frac{2m x_i x_p (x_i - x_p)}{ke^2}} = \boxed{4{,}66\ s}$$

massa protone $\simeq 1{,}673 \cdot 10^{-27}$ kg