# List 07: Heteroskedasticity

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## Contents



## Tests

### sleep equation

For the dataset `sleep75` consider a regression **sleep ~ totwrk + age + I(age^2) + male + smsa + south**. Its specification is $sleep = \beta_0 + \beta_1 totwrk + \beta_2 age + \beta_3 age^2 + \beta_4 male + \beta_5 smsa + \beta_6 south + u$.

BP-test is based on the following heteroskedasicity model $Var(u|X) = f(x'\gamma)$

We test $H_0 : \gamma_1 = \cdots = \gamma_k = 0$ (homoskedastic error term)

Fit the regression and perform BP-test. Result is

```
	studentized Breusch-Pagan test

data:  mod
BP = 8.31, df = 6, p-value = 0.2163
```

significant level 5%.

Evaluate necessary critical value **Round the answer to 2 decimal places.**

```
[1] 12.59
```

Conclusion

```
[1] "Evidence for homoskedasticity"
```

## wage equation

For the dataset `wage1` consider a regression **log(wage) ~ exper + I(exper^2) + female + married + smsa**.

BP-test is based on the following heteroskedasicity model $Var(u|X) = f(x'\gamma)$

We test $H_0 : \gamma_1 = \dots = \gamma_k = 0$ (homoskedastic error term)

Fit the regression and perform BP-test. Result is

```
    studentized Breusch-Pagan test

data:  mod
BP = 8.9399, df = 5, p-value = 0.1115
```

significant level 5%.

Evaluate necessary critical value **Round the answer to 2 decimal places.**

```
[1] 11.07
```

Conclusion

```
[1] "Evidence for homoskedasticity"
```

## output equation

For the dataset `Labour` consider a regression **log(output) ~ log(capital) + log(labour) + I(log(capital)^2) + , I(log(labour)^2)**.

BP-test is based on the following heteroskedasicity model $Var(u|X) = f(x'\gamma)$

We test $H_0 : \gamma_1 = \dots = \gamma_k = 0$ (homoskedastic error term)

Fit the regression and perform BP-test. Result is

```
    studentized Breusch-Pagan test

data:  mod
BP = 44.534, df = 4, p-value = 4.97e-09
```

significant level 5%.

Evaluate necessary critical value **Round the answer to 2 decimal places.**

```
[1] 9.49
```

Conclusion

```
[1] "Evidence for heteroskedasticity"
```

## cost equation #1

For the dataset `Electricity` consider a regression **log(cost) ~ log(q) + I(log(q)^2) + log(pf) + log(pl) + log(pk)**.

BP-test is based on the following heteroskedasicity model $Var(u|X) = f(x'\gamma)$

We test $H_0 : \gamma_1 = \dots = \gamma_k = 0$ (homoskedastic error term)

Fit the regression and perform BP-test. Result is

```
    studentized Breusch-Pagan test

data:  mod
BP = 45.076, df = 5, p-value = 1.4e-08
```

significant level 5%.

Evaluate necessary critical value **Round the answer to 2 decimal places.**

```
[1] 11.07
```

Conclusion

```
[1] "Evidence for heteroskedasticity"
```

### cost equation #2

For the dataset `Electricity` consider a regression **log(cost) ~ log(q) + I(log(q)^2) + log(pf) + log(pl) + log(pk) + , log(pk) + I(log(pf)^2) + I(log(pl)^2) + I(log(pk)^2)**.

BP-test is based on the following heteroskedasicity model $Var(u|X) = f(x'\gamma)$

We test $H_0 : \gamma_1 = \cdots = \gamma_k = 0$ (homoskedastic error term)

Fit the regression and perform BP-test. Result is

```
    studentized Breusch-Pagan test

data:  mod
BP = 49.299, df = 8, p-value = 5.57e-08
```

significant level 5%.

Evaluate necessary critical value **Round the answer to 2 decimal places.**

```
[1] 15.51
```

Conclusion

```
[1] "Evidence for heteroskedasticity"
```

## Robust inferences: t-test (HC s.e.)

By default we use HC3 s.e.

### output equation

For the dataset `Labour` consider a regression **log(output) ~ log(capital) + log(labour) + I(log(capital)^2) + , I(log(labour)^2)**.

Perform non-robust & robust t-test and compare results

Non-robust t-test (OLS-s.e.)

```
t test of coefficients:

                   Estimate Std. Error t value  Pr(>|t|)
(Intercept)       -1.3039430  0.1885929 -6.9141 1.279e-11 ***
log(capital)       0.1831076  0.0165635 11.0549 < 2.2e-16 ***
```

```
log(labour)        0.5152974  0.0833632  6.1814 1.220e-09 ***
I(log(capital)^2)  0.0227484  0.0050350  4.5181 7.606e-06 ***
I(log(labour)^2)   0.0202628  0.0095958  2.1116   0.03516 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Robust t-test (HC3 s.e.)

```
t test of coefficients:

                    Estimate Std. Error t value  Pr(>|t|)
(Intercept)       -1.3039430  0.4932710 -2.6435  0.008435 **
log(capital)       0.1831076  0.0294634  6.2148 9.999e-10 ***
log(labour)        0.5152974  0.2064002  2.4966  0.012823 *
I(log(capital)^2)  0.0227484  0.0083099  2.7375  0.006386 **
I(log(labour)^2)   0.0202628  0.0209889  0.9654  0.334755
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

significant level 5%.

Which coefficients are significant?

```
         regressors sign.regressors
1       (Intercept)     Significant
2      log(capital)     Significant
3       log(labour)     Significant
4 I(log(capital)^2)     Significant
5  I(log(labour)^2)   Insignificant
```

## cost equation

For the dataset `Electricity` consider a regression **log(cost) ~ log(q) + I(log(q)^2) + log(pf) + log(pl) + log(pk)**.

Perform non-robust & robust t-test and compare results

Non-robust t-test (OLS-s.e.)

```
t test of coefficients:

              Estimate Std. Error t value  Pr(>|t|)
(Intercept) -6.7386606  0.7062673 -9.5412 < 2.2e-16 ***
log(q)       0.4029811  0.0316454 12.7343 < 2.2e-16 ***
I(log(q)^2)  0.0304398  0.0021706 14.0236 < 2.2e-16 ***
log(pf)      0.6847054  0.0426794 16.0430 < 2.2e-16 ***
log(pl)      0.1460853  0.0704738  2.0729  0.039870 *
log(pk)      0.1570790  0.0577194  2.7214  0.007259 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Robust t-test (HC3 s.e.)

```
t test of coefficients:

              Estimate Std. Error t value  Pr(>|t|)
```

```
(Intercept) -6.7386606  0.8472223 -7.9538 3.838e-13 ***
log(q)       0.4029811  0.0662199  6.0855 9.048e-09 ***
I(log(q)^2)  0.0304398  0.0041028  7.4194 7.752e-12 ***
log(pf)      0.6847054  0.0519179 13.1882 < 2.2e-16 ***
log(pl)      0.1460853  0.0853617  1.7114   0.08905 .
log(pk)      0.1570790  0.0622924  2.5216   0.01271 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

significant level 1%.

Which coefficients are significant?

```
   regressors sign.regressors
1 (Intercept)     Significant
2      log(q)     Significant
3 I(log(q)^2)     Significant
4     log(pf)     Significant
5     log(pl)   Insignificant
6     log(pk)   Insignificant
```

# Robust inferences: F-test (HC estimator for the cov matrix)

By default we use HC3 estimator for covariance matrix

## output equation

For the dataset `Labour` consider a regression **log(output)** ~ **log(capital)** + **log(labour)** + **I(log(capital)^2)** + **,** **I(log(labour)^2)**.

Robust t-test

```
t test of coefficients:

                    Estimate Std. Error t value  Pr(>|t|)
(Intercept)       -1.3039430  0.4932710 -2.6435  0.008435 **
log(capital)       0.1831076  0.0294634  6.2148 9.999e-10 ***
log(labour)        0.5152974  0.2064002  2.4966  0.012823 *
I(log(capital)^2)  0.0227484  0.0083099  2.7375  0.006386 **
I(log(labour)^2)   0.0202628  0.0209889  0.9654  0.334755
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

significant level 1%.

Let's test the significance of **labour**, i.e. $H_0 : \beta_{\log(labour)} = \beta_{\log^2(labour)} = 0$

Result of non-robust F-test:

```
==============
F      Pr(> F)
--------------
19.524    0
--------------
```

Result of robust F-test:

```
=============
F     Pr(> F)
-------------
4.534  0.011
-------------
```

Evaluate necessary critical value. **Round the answer to 2 decimal places.**

```
[1] 4.64
```

Is **labour** significant?

```
[1] "Insignificant"
```

## cost equation

For the dataset `Electricity` consider a regression **log(cost) ~ log(q) + I(log(q)^2) + log(pf) + log(pl) + log(pk) + , log(pk) + I(log(pf)^2) + I(log(pl)^2) + I(log(pk)^2)**.

Robust t-test

```
t test of coefficients:

                Estimate  Std. Error t value  Pr(>|t|)    
(Intercept)  -44.4034734  32.7822805 -1.3545    0.1776    
log(q)         0.3963198   0.0664186  5.9670 1.690e-08 ***
I(log(q)^2)    0.0308516   0.0041096  7.5071 5.111e-12 ***
log(pf)        0.8988395   1.8426423  0.4878    0.6264    
log(pl)        8.3334364   7.5068929  1.1101    0.2687    
log(pk)        0.4362352   1.6217309  0.2690    0.7883    
I(log(pf)^2)  -0.0305097   0.2694516 -0.1132    0.9100    
I(log(pl)^2)  -0.4554265   0.4172544 -1.0915    0.2768    
I(log(pk)^2)  -0.0360438   0.2003201 -0.1799    0.8575    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

significant level 5%.

### Hypothesis 1

Let's test the significance of **pl**, i.e. $H_0 : \beta_{\log(pl)} = \beta_{\log^2(pl)} = 0$

Result of non-robust F-test:

```
=============
F     Pr(> F)
-------------
3.480  0.033
-------------
```

Result of robust F-test:

```
=============
F     Pr(> F)
-------------
1.978  0.142
```

-------------

Evaluate necessary critical value. **Round the answer to 2 decimal places.**

```
[1] 3.06
```

Is **pl** significant?

```
[1] "Insignificant"
```

**Hypothesis 2**

Let's test the significance of **pl**, i.e. $H_0 : \beta_{\log(pk)} = \beta_{\log^2(pk)} = 0$

Result of non-robust F-test:

| F | Pr(> F) |
|---|---|
| 2.982 | 0.054 |

Result of robust F-test:

| F | Pr(> F) |
|---|---|
| 2.209 | 0.113 |

Evaluate necessary critical value. **Round the answer to 2 decimal places.**

```
[1] 3.06
```

Is **pl** significant?

```
[1] "Insignificant"
```

**Hypothesis 3**

Let's test the significance of **pl**, i.e. $H_0 : \beta_{\log(pf)} = \beta_{\log^2(pf)} = 0$

Result of non-robust F-test:

| F | Pr(> F) |
|---|---|
| 129.374 | 0 |

Result of robust F-test:

| F | Pr(> F) |
|---|---|
| 79.131 | 0 |

Evaluate necessary critical value. **Round the answer to 2 decimal places.**

```
[1] 3.06
```

Is **pl** significant?

```
[1] "Significant"
```