Seminars on Linear Algebra

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1 Matrices and Matrix operations

1.1 Matrix addition, subtraction & scalar multiplication

[1]: p.60 Exercise 1; p. 61 Exercise 3, 4

1.2 Matrix multiplication

[1]: p. 68 Exercise 2; p.69 Exercise 5, 10; p. 70 Exercise 13, 14, 21
[3]: p.46 Exercise 1, 6, 7; p. 47 Exercise 8

1.3 Inverse Matrix

[1]: p. 111 Exercise 1, 2; p.112 Exercise 7, 8, 9, 11

1.4 Determinant

[1]: p. 125 Exercise 1; p. 126 Exercise 2, 3; p.127 Exercise 11, 13, 14

1.5 Matrix rank

[1]: p. 43 Exercise 3, 4, 5

2 Systems of linear equations

2.1 Cramer's rule

[1]: p. 127 Exercise 13, 14

2.2 Gauss-Jordan elimination method

[1]: p. 30 Exercise 4; p. 31 Exercise 5, 6

3 Linear Spaces

3.1 Subspaces

[1]: p. 168 Exercise 1 – 8

3.2 Linear Combinations

[1]: p.183 Exercise 3, 4

3.3 Basis & Dimension

[1]: p. 196 Exercise 1, 2, 4; p. 197 Exercise 7

3.4 Norms & Inner product

1. For the following vectors

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

calculate their normes $\|\cdot\|_1, \|\cdot\|_2, \|\cdot\|_3, \|\cdot\|_\infty$

2. For the following vectors

$$x_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad x_2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad x_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

calculate $(x_1, x_2), (x_1, x_3), (x_2, x_3)$

3.5 Quadratic Forms & Symmetric Matrices

Which of the followeng matrices are

- positive/negative definite,
- positive/negative semidefinite
- indefinite?

$$\begin{pmatrix} 4 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 4 \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 & -4 \\ 1 & 2 & 1 \\ -4 & 1 & 2 \end{pmatrix} \qquad \begin{pmatrix} -3 & 1 & -1 \\ 1 & -4 & 1 \\ -1 & 1 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 3 & 6 \end{pmatrix} \qquad \begin{pmatrix} -2 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

3.6 Eigenvalues & Eigenvectors

[1]: p. 261 Exercise 1, 2

References

[1] Thomas S. Shores, «Applied Linear Algebra and Matrix Analysis», Springer Verlag, 2007

[2] Belkacem Said-Houari, «Linear Algebra», Birkhäuser, 2017

[3] Lorenzo Robbiano, «Linear Algebra for everyone», Springer Verlag, 2011

[4] Jonathan S. Golan, «The Linear Algebra a Beginning Graduate Student Ought to Know», Springer Verlag, 2007

[5] Charles L. Byrne, «Applied and Computational Linear Algebra: A First Course»