Formulas for Determinant and Characteristic Polynomial of Seven-Like Matrices – The Algorithms

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Below are our algorithms for computing determinants and characteristic polynomials of seven-like matrices (their definition can be found in our conference theses at https://probability.knu.ua/shv2024/ShV_2024.pdf).

Algorithm 1 Determinant of an HM-7 Matrix (Structure-Preserving)

function DeterminantHM7StructurePreserving(M):
    $n := M.\text{size}-1$
    $A :=$ **new** array$[1..n]$
    $A[i] := M[i+1][i+1]$, $i = \overline{1,n}$
    for $i := n-1$ **downto** 1 **do**
        $A[i] := A[i] \cdot A[i+1]$
    end for
    $s := 0$
    $b := 1$
    for $i := 1$ **to** n **do**
        $s := s + (-1)^{i+1} \cdot b \cdot M[1][i] \cdot A[i]$
        $b := b \cdot M[i+1][i]$
    end for
    return $s + (-1)^n \cdot b \cdot M[1][n+1]$
end function

Algorithm 2 Determinant of an HM-7 Matrix (Structure-Modifying)

function DeterminantHM7StructureModifying(M):
    $n := M.\text{size}-1$
    for $i := n$ **downto** 2 **do**
        $M[i][i] := M[i][i] \cdot M[i+1][i+1]$
    end for
    $s := 0$
    $b := 1$
    for $i := 1$ **to** n **do**
        $s := s + (-1)^{i+1} \cdot b \cdot M[1][i] \cdot M[i+1][i+1]$
        $b := b \cdot M[i+1][i]$
    end for
    return $s + (-1)^n \cdot b \cdot M[1][n+1]$
end function

Algorithm 3 Characteristic Polynomial of an HM-7 Matrix (Structure-Preserving)

function CharacteristicPolynomialHM7StructurePreserving(M):
    $n \coloneqq M.\text{size}-1$
    $\lambda \coloneqq$ **new** symbolic variable
    if $n = 0$ **then**
        return $\lambda - M[1][1]$
    end if
    $A \coloneqq$ **new** array$[1..n]$
    $A[i] \coloneqq \lambda - M[i+1][i+1]$, $i = \overline{1,n}$
    for $i \coloneqq n$ **downto** 1 **do**
        $A[i] \coloneqq A[i] \cdot A[i+1]$
    end for
    $s \coloneqq (\lambda - M[1][1]) \cdot A[1]$
    $b \coloneqq -M[2][1]$
    for $i \coloneqq 2$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot (-M[1][i]) \cdot A[i]$
        $b \coloneqq b \cdot (-M[i+1][i])$
    end for
    return $s + (-1)^n \cdot b \cdot (-M[1][n+1])$
end function

Algorithm 4 Characteristic Polynomial of an HM-7 Matrix (Structure-Modifying)

function CharacteristicPolynomialHM7StructureModifying(M):
    $n \coloneqq M.\text{size}-1$
    $\lambda \coloneqq$ **new** symbolic variable
    if $n = 0$ **then**
        return $\lambda - M[1][1]$
    end if
    $M[1][1] \coloneqq \lambda - M[1][1]$
    $M[n+1][n+1] \coloneqq \lambda - M[n+1][n+1]$
    for $i \coloneqq n$ **downto** 2 **do**
        $M[i][i] \coloneqq (\lambda - M[i][i]) \cdot M[i+1][i+1]$
    end for
    $s \coloneqq M[1][1] \cdot M[2][2]$
    $b \coloneqq -M[2][1]$
    for $i \coloneqq 2$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot (-M[1][i]) \cdot M[i+1][i+1]$
        $b \coloneqq b \cdot (-M[i+1][i])$
    end for
    return $s + (-1)^n \cdot b \cdot (-M[1][n+1])$
end function

Algorithm 5 Determinant of a D-7 Matrix (Structure-Preserving)

function DeterminantD7StructurePreserving(M):
    $n \coloneqq M.\text{size}-1$
    $A \coloneqq$ **new** array$[1..n]$
    $A[i] \coloneqq M[i+1][n-i+1]$, $i = \overline{1,n}$
    for $i \coloneqq n-1$ **downto** 1 **do**
        $A[i] \coloneqq A[i] \cdot A[i+1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[1][n-i+2] \cdot A[i]$
        $b \coloneqq b \cdot M[i+1][n-i+2]$
    end for
    return $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[1][1])$
end function

Algorithm 6 Determinant of a D-7 Matrix (Structure-Modifying)

function DeterminantD7StructureModifying(M):
    $n \coloneqq M.\text{size}-1$
    for $i \coloneqq n$ **downto** 2 **do**
        $M[i][n-i+2] \coloneqq M[i][n-i+2] \cdot M[i+1][n-i+1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[1][n-i+2] \cdot M[i+1][n-i+1]$
        $b \coloneqq b \cdot M[i+1][n-i+2]$
    end for
    return $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[1][1])$
end function

Algorithm 7 Determinant of a VM-7 Matrix (Structure-Preserving)

function DeterminantVM7StructurePreserving(M):
    $n \coloneqq M.\text{size}-1$
    $A \coloneqq$ **new** array$[1..n]$
    $A[i] \coloneqq M[n-i+1][n-i+1]$, $i = \overline{1,n}$
    for $i \coloneqq n-1$ **downto** 1 **do**
        $A[i] \coloneqq A[i] \cdot A[i+1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[n+1][n-i+2] \cdot A[i]$
        $b \coloneqq b \cdot M[n-i+1][n-i+2]$
    end for
    return $s + (-1)^n \cdot b \cdot M[n+1][1]$
end function

Algorithm 8 Determinant of a VM-7 Matrix (Structure-Modifying)

function DeterminantVM7StructureModifying(M):
    $n \coloneqq M.\text{size}-1$
    for $i \coloneqq 2$ **to** n **do**
        $M[i][i] \coloneqq M[i][i] \cdot M[i-1][i-1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[n+1][n-i+2] \cdot M[n-i+1][n-i+1]$
        $b \coloneqq b \cdot M[n-i+1][n-i+2]$
    end for
    return $s + (-1)^{n+1} \cdot b \cdot M[n+1][1]$
end function

Algorithm 9 Characteristic Polynomial of a VM-7 Matrix (Structure-Preserving)

function CharacteristicPolynomialVM7StructurePreserving(M):
    $n \coloneqq M.\text{size}-1$
    $\lambda \coloneqq$ **new** symbolic variable
    if $n = 0$ **then**
        return $\lambda - M[1][1]$
    end if
    $A \coloneqq$ **new** array$[1..n]$
    $A[i] \coloneqq \lambda - M[n-i+1][n-i+1]$, $i = \overline{1,n}$
    for $i \coloneqq n-1$ **downto** 1 **do**
        $A[i] \coloneqq A[i] \cdot A[i+1]$
    end for
    $s \coloneqq (\lambda - M[n+1][n+1]) \cdot A[1]$
    $b \coloneqq -M[n][n+1]$
    for $i \coloneqq 2$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot (-M[n+1][n-i+2]) \cdot A[i]$
        $b \coloneqq b \cdot (-M[n-i+1][n-i+2])$
    end for
    return $s + (-1)^{n+1} \cdot b \cdot (-M[n+1][1])$
end function

Algorithm 10 Characteristic Polynomial of a VM-7 Matrix (Structure-Modifying)

function CharacteristicPolynomialVM7StructureModifying(M):
    $n \coloneqq M.\text{size}-1$
    $\lambda \coloneqq$ **new** symbolic variable
    if $n = 0$ **then**
        return $\lambda - M[1][1]$
    end if
    $M[1][1] \coloneqq \lambda - M[1][1]$
    $M[n+1][n+1] \coloneqq \lambda - M[n+1][n+1]$
    for $i \coloneqq 2$ **to** n **do**
        $M[i][i] \coloneqq (\lambda - M[i][i]) \cdot M[i-1][i-1]$
    end for
    $s \coloneqq M[n+1][n+1] \cdot M[n][n]$
    $b \coloneqq -M[n][n+1]$
    for $i \coloneqq 2$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot (-M[n+1][n-i+2]) \cdot M[n-i+1][n-i+1]$
        $b \coloneqq b \cdot (-M[n-i+1][n-i+2])$
    end for
    return $s + (-1)^{n+1} \cdot b \cdot (-M[n+1][1])$
end function

Algorithm 11 Determinant of a VHM-7 Matrix (Structure-Preserving)

function DeterminantVHM7StructurePreserving(M):
    $n \coloneqq M.\text{size}$
    $A \coloneqq$ **new** array$[1..n]$
    $A[i] \coloneqq M[n-i+1][i+1],\ i = \overline{1,n}$
    for $i \coloneqq n-1$ **downto** 1 **do**
        $A[i] \coloneqq A[i] \cdot A[i+1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[n+1][i] \cdot A[i]$
        $b \coloneqq b \cdot M[n-i+1][i]$
    end for
    return $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[n+1][n+1])$
end function

Algorithm 12 Determinant of a VHM-7 Matrix (Structure-Modifying)

function DeterminantVHM7StructureModifying(M):
    $n \coloneqq M.\text{size}-1$
    for $i \coloneqq n$ **downto** 2 **do**
        $M[n-i+2][i] \coloneqq M[n-i+2][i] \cdot M[n-i+1][i+1]$
    end for
    $s \coloneqq 0$
    $b \coloneqq 1$
    for $i \coloneqq 1$ **to** n **do**
        $s \coloneqq s + (-1)^{i+1} \cdot b \cdot M[n+1][i] \cdot M[n-i+1][i+1]$
        $b \coloneqq b \cdot M[n-i+1][i]$
    end for
    return $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[n+1][n+1])$
end function