

Formulas for Determinant and Characteristic Polynomial of Seven-Like Matrices – The Algorithms

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Below are our algorithms for computing determinants and characteristic polynomials of seven-like matrices (their definition can be found at our conference theses at https://probability.knu.ua/shv2024/ShV_2024.pdf).

Algorithm 1 Determinant of an HM-7 Matrix (Structure-Preserving)

```
1: function DETERMINANTHM7STRUCTUREPRESERVING( $M$ ):  
2:    $n := M.size - 1$   
3:    $A := \text{new array}[1..n]$   
4:    $A[i] := M[i + 1][i + 1], i = \overline{1, n}$   
5:   for  $i := n - 1$  downto 1 do  
6:      $A[i] := A[i] \cdot A[i + 1]$   
7:   end for  
8:    $s := 0$   
9:    $b := 1$   
10:  for  $i := 1$  to  $n$  do  
11:     $s := s + (-1)^{i+1} \cdot b \cdot M[1][i] \cdot A[i]$   
12:     $b := b \cdot M[i + 1][i]$   
13:  end for  
14:  return  $s + (-1)^n \cdot b \cdot M[1][n + 1]$   
15: end function
```

Algorithm 2 Determinant of an HM-7 Matrix (Structure-Modifying)

```
1: function DETERMINANTHM7STRUCTUREMODIFYING( $M$ ):  
2:    $n := M.size - 1$   
3:   for  $i := n$  downto 2 do  
4:      $M[i][i] := M[i][i] \cdot M[i + 1][i + 1]$   
5:   end for  
6:    $s := 0$   
7:    $b := 1$   
8:   for  $i := 1$  to  $n$  do  
9:      $s := s + (-1)^{i+1} \cdot b \cdot M[1][i] \cdot M[i + 1][i + 1]$   
10:     $b := b \cdot M[i + 1][i]$   
11:  end for  
12:  return  $s + (-1)^n \cdot b \cdot M[1][n + 1]$   
13: end function
```

Algorithm 3 Characteristic Polynomial of an HM-7 Matrix (Structure-Preserving)

```
1: function CHARACTERISTICPOLYNOMIALHM7STRUCTUREPRESERVING( $M$ ):
2:    $n := M.size - 1$ 
3:    $\lambda := \text{new symbolic variable}$ 
4:   if  $n = 0$  then
5:     return  $\lambda - M[1][1]$ 
6:   end if
7:    $A := \text{new array}[1..n]$ 
8:    $A[i] := \lambda - M[i+1][i+1], i = \overline{1, n}$ 
9:   for  $i := n$  downto 1 do
10:     $A[i] := A[i] \cdot A[i+1]$ 
11:  end for
12:   $s := (\lambda - M[1][1]) \cdot A[1]$ 
13:   $b := -M[2][1]$ 
14:  for  $i := 2$  to  $n$  do
15:     $s := s + (-1)^{i+1} \cdot b \cdot (-M[1][i]) \cdot A[i]$ 
16:     $b := b \cdot (-M[i+1][i])$ 
17:  end for
18:  return  $s + (-1)^n \cdot b \cdot (-M[1][n+1])$ 
19: end function
```

Algorithm 4 Characteristic Polynomial of an HM-7 Matrix (Structure-Modifying)

```
1: function CHARACTERISTICPOLYNOMIALHM7STRUCTUREMODIFYING( $M$ ):
2:    $n := M.size - 1$ 
3:    $\lambda := \text{new symbolic variable}$ 
4:   if  $n = 0$  then
5:     return  $\lambda - M[1][1]$ 
6:   end if
7:    $M[1][1] := \lambda - M[1][1]$ 
8:    $M[n+1][n+1] := \lambda - M[n+1][n+1]$ 
9:   for  $i := n$  downto 2 do
10:     $M[i][i] := (\lambda - M[i][i]) \cdot M[i+1][i+1]$ 
11:  end for
12:   $s := M[1][1] \cdot M[2][2]$ 
13:   $b := -M[2][1]$ 
14:  for  $i := 2$  to  $n$  do
15:     $s := s + (-1)^{i+1} \cdot b \cdot (-M[1][i]) \cdot M[i+1][i+1]$ 
16:     $b := b \cdot (-M[i+1][i])$ 
17:  end for
18:  return  $s + (-1)^n \cdot b \cdot (-M[1][n+1])$ 
19: end function
```

Algorithm 5 Determinant of a D-7 Matrix (Structure-Preserving)

```
1: function DETERMINANTD7STRUCTUREPRESERVING( $M$ ):
2:    $n := M.size - 1$ 
3:    $A := \text{new array}[1..n]$ 
4:    $A[i] := M[i + 1][n - i + 1], i = \overline{1, n}$ 
5:   for  $i := n - 1$  downto 1 do
6:      $A[i] := A[i] \cdot A[i + 1]$ 
7:   end for
8:    $s := 0$ 
9:    $b := 1$ 
10:  for  $i := 1$  to  $n$  do
11:     $s := s + (-1)^{i+1} \cdot b \cdot M[1][n - i + 2] \cdot A[i]$ 
12:     $b := b \cdot M[i + 1][n - i + 2]$ 
13:  end for
14:  return  $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[1][1])$ 
15: end function
```

Algorithm 6 Determinant of a D-7 Matrix (Structure-Modifying)

```
1: function DETERMINANTD7STRUCTUREMODIFYING( $M$ ):
2:    $n := M.size - 1$ 
3:   for  $i := n$  downto 2 do
4:      $M[i][n - i + 2] := M[i][n - i + 2] \cdot M[i + 1][n - i + 1]$ 
5:   end for
6:    $s := 0$ 
7:    $b := 1$ 
8:   for  $i := 1$  to  $n$  do
9:      $s := s + (-1)^{i+1} \cdot b \cdot M[1][n - i + 2] \cdot M[i + 1][n - i + 1]$ 
10:     $b := b \cdot M[i + 1][n - i + 2]$ 
11:  end for
12:  return  $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[1][1])$ 
13: end function
```

Algorithm 7 Determinant of a VM-7 Matrix (Structure-Preserving)

```
1: function DETERMINANTVM7STRUCTUREPRESERVING( $M$ ):
2:    $n := M.size - 1$ 
3:    $A := \text{new array}[1..n]$ 
4:    $A[i] := M[n - i + 1][n - i + 1], i = \overline{1, n}$ 
5:   for  $i := n - 1$  downto 1 do
6:      $A[i] := A[i] \cdot A[i + 1]$ 
7:   end for
8:    $s := 0$ 
9:    $b := 1$ 
10:  for  $i := 1$  to  $n$  do
11:     $s := s + (-1)^{i+1} \cdot b \cdot M[n + 1][n - i + 2] \cdot A[i]$ 
12:     $b := b \cdot M[n - i + 1][n - i + 2]$ 
13:  end for
14:  return  $s + (-1)^n \cdot b \cdot M[n + 1][1]$ 
15: end function
```

Algorithm 8 Determinant of a VM-7 Matrix (Structure-Modifying)

```
1: function DETERMINANTVM7STRUCTUREMODIFYING( $M$ ):
2:    $n := M.size - 1$ 
3:   for  $i := 2$  to  $n$  do
4:      $M[i][i] := M[i][i] \cdot M[i-1][i-1]$ 
5:   end for
6:    $s := 0$ 
7:    $b := 1$ 
8:   for  $i := 1$  to  $n$  do
9:      $s := s + (-1)^{i+1} \cdot b \cdot M[n+1][n-i+2] \cdot M[n-i+1][n-i+1]$ 
10:     $b := b \cdot M[n-i+1][n-i+2]$ 
11:   end for
12:   return  $s + (-1)^{n+1} \cdot b \cdot M[n+1][1]$ 
13: end function
```

Algorithm 9 Characteristic Polynomial of a VM-7 Matrix (Structure-Preserving)

```
1: function CHARACTERISTICPOLYNOMIALVM7STRUCTUREPRESERVING( $M$ ):
2:    $n := M.size - 1$ 
3:    $\lambda := \text{new symbolic variable}$ 
4:   if  $n = 0$  then
5:     return  $\lambda - M[1][1]$ 
6:   end if
7:    $A := \text{new array}[1..n]$ 
8:    $A[i] := \lambda - M[n-i+1][n-i+1], i = \overline{1, n}$ 
9:   for  $i := n-1$  downto  $1$  do
10:     $A[i] := A[i] \cdot A[i+1]$ 
11:   end for
12:    $s := (\lambda - M[n+1][n+1]) \cdot A[1]$ 
13:    $b := -M[n][n+1]$ 
14:   for  $i := 2$  to  $n$  do
15:      $s := s + (-1)^{i+1} \cdot b \cdot (-M[n+1][n-i+2]) \cdot A[i]$ 
16:      $b := b \cdot (-M[n-i+1][n-i+2])$ 
17:   end for
18:   return  $s + (-1)^{n+1} \cdot b \cdot (-M[n+1][1])$ 
19: end function
```

Algorithm 10 Characteristic Polynomial of a VM-7 Matrix (Structure-Modifying)

```
1: function CHARACTERISTICPOLYNOMIALVM7STRUCTUREMODIFYING( $M$ ):
2:    $n := M.size - 1$ 
3:    $\lambda := \text{new symbolic variable}$ 
4:   if  $n = 0$  then
5:     return  $\lambda - M[1][1]$ 
6:   end if
7:    $M[1][1] := \lambda - M[1][1]$ 
8:    $M[n+1][n+1] := \lambda - M[n+1][n+1]$ 
9:   for  $i := 2$  to  $n$  do
10:     $M[i][i] := (\lambda - M[i][i]) \cdot M[i-1][i-1]$ 
11:  end for
12:   $s := M[n+1][n+1] \cdot M[n][n]$ 
13:   $b := -M[n][n+1]$ 
14:  for  $i := 2$  to  $n$  do
15:     $s := s + (-1)^{i+1} \cdot b \cdot (-M[n+1][n-i+2]) \cdot M[n-i+1][n-i+1]$ 
16:     $b := b \cdot (-M[n-i+1][n-i+2])$ 
17:  end for
18:  return  $s + (-1)^{n+1} \cdot b \cdot (-M[n+1][1])$ 
19: end function
```

Algorithm 11 Determinant of a VHM-7 Matrix (Structure-Preserving)

```
1: function DETERMINANTVHM7STRUCTUREPRESERVING( $M$ ):
2:    $n := M.size$ 
3:    $A := \text{new array}[1..n]$ 
4:    $A[i] := M[n-i+1][i+1], i = \overline{1, n}$ 
5:   for  $i := n-1$  downto  $1$  do
6:      $A[i] := A[i] \cdot A[i+1]$ 
7:   end for
8:    $s := 0$ 
9:    $b := 1$ 
10:  for  $i := 1$  to  $n$  do
11:     $s := s + (-1)^{i+1} \cdot b \cdot M[n+1][i] \cdot A[i]$ 
12:     $b := b \cdot M[n-i+1][i]$ 
13:  end for
14:  return  $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[n+1][n+1])$ 
15: end function
```

Algorithm 12 Determinant of a VHM-7 Matrix (Structure-Modifying)

```
1: function DETERMINANTVHM7STRUCTUREMODIFYING( $M$ ):  
2:    $n := M.size - 1$   
3:   for  $i := n$  downto 2 do  
4:      $M[n - i + 2][i] := M[n - i + 2][i] \cdot M[n - i + 1][i + 1]$   
5:   end for  
6:    $s := 0$   
7:    $b := 1$   
8:   for  $i := 1$  to  $n$  do  
9:      $s := s + (-1)^{i+1} \cdot b \cdot M[n + 1][i] \cdot M[n - i + 1][i + 1]$   
10:     $b := b \cdot M[n - i + 1][i]$   
11:  end for  
12:  return  $(-1)^{\lfloor \frac{n+1}{2} \rfloor} \cdot (s + (-1)^n \cdot b \cdot M[n + 1][n + 1])$   
13: end function
```
